

Please write ***Your name:*** _____

Show all work. You should either write at a sentence explaining your reasoning, or annotate your math work with brief explanations. There is no need to simplify, and no calculators are needed.

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- (1a) Suppose that X is a random variable with the outcomes $\{1, 2, 3\}$. The corresponding probabilities are given by

$$\mathbb{P}(X = 1) = \frac{2}{5}, \quad \mathbb{P}(X = 2) = \frac{1}{5}, \quad \mathbb{P}(X = 3) = \frac{2}{5}$$

Find its expected value $\mathbb{E}X$ and $\mathbb{E}X^2$

- (1b) Find the variance $\text{Var}(X)$

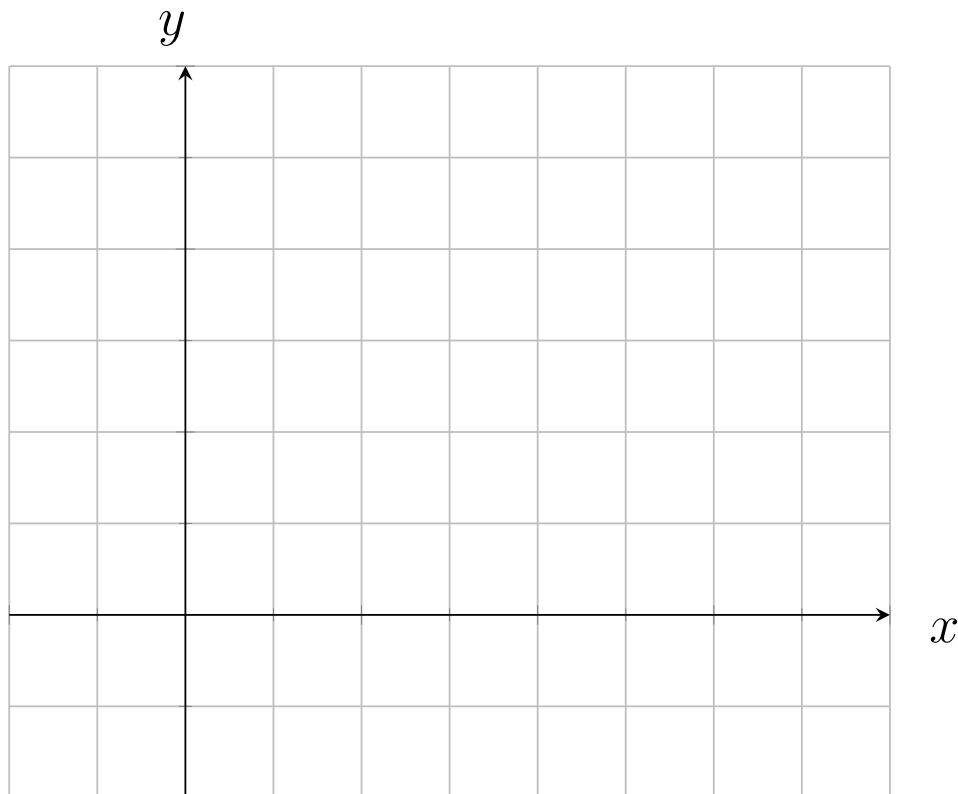
- (2a) Suppose that X is a random variable with the outcomes $\{1, 2, 3\}$. The corresponding probabilities are given as in question (1) by

$$\mathbb{P}(X = 1) = \frac{2}{5}, \quad \mathbb{P}(X = 2) = \frac{1}{5}, \quad \mathbb{P}(X = 3) = \frac{2}{5}$$

Find the cumulative distribution function F_X of X using the cases provided below.

$$F_X(x) = \begin{cases} 0, & \text{for } -\infty < x < \rule{1cm}{0.4pt} \\ \rule{1cm}{0.4pt}, & \text{for } \rule{1cm}{0.4pt} x \rule{1cm}{0.4pt} \\ \rule{1cm}{0.4pt}, & \text{for } \rule{1cm}{0.4pt} x \rule{1cm}{0.4pt} \\ \rule{1cm}{0.4pt}, & \text{for } \rule{1cm}{0.4pt} \leq x < \infty \end{cases}$$

- (2b) Plot the cumulative distribution function F_X of X using the chart provided below. Accurately label values at x and y axes.



- (3a) Toss two fair coins and define the random variable X as the number of heads. Thus, X can have values 0, 1, or 2. Find the probability mass function of X .
- (3b) Find the variance of X from the previous question.