Quiz 10

***Please write* Your name:** _____

Show all work. You should either write at a sentence explaining your reasoning, or annotate your math work with brief explanations. There is no need to simplify, and no calculators are needed.

. . . . . .

(1) Let X be the time that a car can run until a major repair. If $\mathbb{E}X = 2$, and X is exponentially distributed, what is $\mathbb{P}(2 < X < 5)$?

(2) Given that this car has ran 2 years without a repair, what is the conditional probability that it will run 3 more years without a major repair?

(3) What is the probability density function of $Y = X^2$?

(... more questions on the other side of the paper ...)

(4) Find $\mathrm{Var}(X)$ if X is uniformly distributed on the interval $[-1, 5]$. Show all steps.

[(*optional questions for extra credit*)]: Let a, b, c be positive numbers, $b > 1$, and the probability density function $f(x)$ of a random variable X be defined by $f(x) = ax^{-b}$ for $x > c$ and $f(x) = 0$ for $x \leqslant c$.

- What is the relation between a, b, c?

- What is the necessary and sufficient condition for b so that $\mathbb{E}X < +\infty$?

- What is the necessary and sufficient condition for b so that $\mathrm{Var}(X) < +\infty$?