Please write ***Your name:*** ________________________________

Show all work. You should either write at a sentence explaining your reasoning, or annotate your math work with brief explanations. There is no need to simplify, and no calculators are needed.

. . . .

Let X be a discrete random variables uniformly distributed on the numbers $\{1, 2, 3\}$.

(1) Find the moment generating function for X. ***Answer:*** $m(t) = \frac{1}{3}(e^t + e^{2t} + e^{3t})$

(2) Find $m''(0)$. ***Answer:*** $m''(0) = \frac{1}{3}(1 + 4 + 9) = 14/3$

(3) Find $\mathbb{E}X^2$ and $\mathrm{Var}X$. ***Answer:*** $\mathbb{E}X^2 = m''(0) = \frac{1}{3}(1 + 4 + 9) = 14/3$, $\mathrm{Var}X = 14/3 - 2^2 = 2/3$

On this page $X_1, X_2, \ldots, X_{25}$ are independent identically distributed random variables and $S_{25} = \sum_{k=1}^{25} X_k$.

(1) If $X_1, X_2, \ldots, X_{25}$ are discrete random variables uniformly distributed on the numbers $\{1, 2, 3\}$, use the Central Limit Theorem to approximate $\mathbb{P}(S_{25} > 55)$. Your final answer should contain Φ, square roots, and fractions, but should not contain symbols μ, σ.

Answer: $n = 25, \sqrt{n} = 5, \mu = 2, \sigma^2 = 2/3, \sigma = \sqrt{2/3}$, therefore
$\mathbb{P}(S_{25} > 55) \approx \mathbb{P}\left(25 \cdot 2 + 5 \cdot \sqrt{2/3} \cdot Z > 55\right) = \mathbb{P}\left(Z > \sqrt{3/2}\right) = 1 - \Phi\left(\sqrt{3/2}\right)$

(2) If $X_1, X_2, \ldots, X_{25}$ are continuous random variables uniformly distributed on the interval $[1, 3]$, use the Central Limit Theorem to approximate $\mathbb{P}(S_{25} > 55)$. Your final answer should contain Φ, square roots, and fractions, but should not contain symbols μ, σ.

Answer: $n = 25, \sqrt{n} = 5, \mu = 2, \sigma^2 = 1/3, \sigma = \sqrt{1/3}$, therefore
$\mathbb{P}(S_{25} > 55) \approx \mathbb{P}\left(25 \cdot 2 + 5 \cdot \sqrt{1/3} \cdot Z > 55\right) = \mathbb{P}\left(Z > \sqrt{3}\right) = 1 - \Phi\left(\sqrt{3}\right)$