

Please write **Your name:** _____

Show all work. You should either write at a sentence explaining your reasoning, or annotate your math work with brief explanations. There is no need to simplify, and no calculators are needed.

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In this quiz we discuss a random variable X with a probability density function $f(x)$ which is given by $f(x) = a(x+1)$ when $-1 < x < 1$, and $f(x) = 0$ for all other x . Here a is a number.

(1) Find a .

$$a = 1/2 \text{ because } \int_{-1}^1 (1+x)dx = (x + x^2/2)|_{x=-1}^{x=1} = 2$$

(2) Find $\mathbb{P}(X < 0)$.

$$\mathbb{P}(X < 0) = \int_{-1}^0 \frac{1}{2}(1+x)dx = \frac{1}{2}(x + x^2/2)|_{x=-1}^0 = \frac{1}{4}$$

(3) Find $\mathbb{E}X$.

$$\mathbb{E}X = \int_{-1}^1 \frac{1}{2}x(1+x)dx = \frac{1}{2}(x^2/2 + x^3/3)|_{x=-1}^{x=1} = \frac{1}{3}$$

(4) Find $\mathbb{E}X^2$.

$$\mathbb{E}X^2 = \int_{-1}^1 \frac{1}{2}x^2(1+x)dx = \frac{1}{2}(x^3/3 + x^4/4)|_{x=-1}^{x=1} = \frac{1}{3}$$

(5) Find the cumulative distribution function $F(x)$ using the cases provided below.

$$F_X(x) = \begin{cases} 0 & \text{for } -\infty < x < -1 \\ \frac{1}{2}(x + x^2/2 + 1/2) & \text{for } -1 < x < 1 \\ 1 & \text{for } 1 < x < \infty \end{cases}$$

$$\text{Here } \frac{1}{2}(x + x^2/2 + 1/2) = \frac{1}{2}(x + x^2/2)|_{x=-1}^x$$

[(optional question for extra credit)]:

Plot the probability density function $f(x)$ and the cumulative distribution function $F(x)$ using the charts provided below. Accurately label values at x and y axes.

