

Please write ***Your name:*** _____

Show all work. You should either write at a sentence explaining your reasoning, or annotate your math work with brief explanations. There is no need to simplify, and no calculators are needed.

In this quiz use the notation $\Phi(x)$ for the distribution function for $\mathcal{N}(0, 1)$, that is

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-y^2/2} dy = \mathbb{P}(Z < x)$$

where Z is the standard normal random variable. You do not need a table of values of Φ .

- (1) When a die is rolled, you win \$1 if the outcome is divisible by 3, or \$0 otherwise. This means that you win \$1 for each roll which is a 3, and for each roll which is a 6, but win nothing for other results. Let X be the number of dollars you win after 450 rolls. What is the mean and the standard deviation of X ?

- (2) Estimate the probability that $X > 150$ using the normal approximation.

- (3) Estimate the probability that $X > 250$ using the normal approximation.

(4) Find a formula for $\mathbb{P}(0 \leq X \leq 3)$ if X is $\mathcal{N}(-1, 4)$. Your answer should include Φ twice.

(5) If a coin is tossed 16 times, and X is the number of heads, what is the normal approximation for $\mathbb{P}(X > 12)$ using the normal approximation. Your answer should include Φ .

[(*optional question for extra credit*)]:

In the same situation, estimate the probability that $\mathbb{P}(X = 12)$ using the normal approximation.