

Please write ***Your name:*** _____

Show all work. You should either write at a sentence explaining your reasoning, or annotate your math work with brief explanations. There is no need to simplify, and no calculators are needed.

- (1) Suppose we randomly choose 4 pens out of 7 black pens and 5 red pens. What is the probability that all 4 chosen pens are red?

$$\textbf{Answer: } \frac{\binom{5}{4}}{\binom{12}{4}} = \frac{5}{495} = \frac{1}{99}$$

- (2) Suppose we randomly choose 3 pens out of 2 black pens, 2 red pens, and 2 blue pens. What is the probability that all three pens are of different colors?

$$\textbf{Answer: } \frac{\left(\binom{2}{1}\right)^3}{\binom{6}{3}} = \frac{8}{20} = \frac{2}{5}$$

$$\text{Another solution: } 1 - 3 \cdot \frac{\binom{4}{3}}{\binom{6}{3}} = 1 - 3 \cdot \frac{4}{20} = 1 - \frac{3}{5} = \frac{2}{5}$$

- (3) What is the probability that, if we roll 2 dice, the sum is 10?

$$\textbf{Answer: } \frac{3}{36} = \frac{1}{12}$$

- (4) Suppose that you are given a die and asked to roll it twice. What is the probability that the value of the second roll will be less than the value of the first roll? Hint: begin with considering the event that the two rolls will be the same.

$$\textbf{Answer: } \frac{1}{2} \left(1 - \frac{1}{6}\right) = \frac{5}{12}$$

Another solution involves the 6-by-6 table of values, which has 15 small squares above the diagonal. Hence the answer is $\frac{5+4+3+2+1}{36} = \frac{15}{36} = \frac{5}{12}$

(End of the quiz)