Please write Your name: ______________________________

Show all work. You should either write at a sentence explaining your reasoning, or annotate your math work with brief explanations. There is no need to simplify, and no calculators are needed. Hint: use $\Phi(x)$ for the $\mathcal{N}(0,1)$ distribution function: $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2} dy = \mathbb{P}(Z < x)$ where Z is the standard normal.

Solving 5 out of 6 problems will give you 10 points in this quiz.

(1) Let $X_1, X_2, \ldots, X_k$ be independent exponential random variables with parameter $\lambda = 3$. Use the Central Limit Theorem to approximate $\mathbb{P}\left(\sum_{i=1}^{k} X_i > a\right)$. Your answer should contain Φ, k, a, fractions, but should not contain symbols μ, σ.

Answer: $\mathbb{P}\left(\sum_{i=1}^{k} X_i > a\right) \approx 1 - \Phi\left(\frac{3a-k}{\sqrt{k}}\right)$

(2) For which a we have $\mathbb{P}\left(\sum_{i=1}^{25} X_i > a\right) \approx 1 - \Phi(4)$? ***Answer:*** $a = 15$

(3) For which a we have $\mathbb{P}\left(\sum_{i=1}^{9} X_i > a\right) \approx \Phi(1)$? ***Answer:*** $a = 2$

If the joint density function of the random variables X and Y is

$$f(x,y) = \begin{cases} ax^2y^2 & 0 < x < 1,\ 0 < y < 1 \\ 0 & \text{otherwise.} \end{cases}$$

(4) find a ***Answer:*** $a = 9$

(5) find the covariance $\operatorname{Cov}(X,Y)$ ***Answer:*** $\operatorname{Cov}(X,Y) = 0$ because X and Y are independent.
(hint: this is an easy question and you can find the answer without computing any integrals)

(6) find the conditional expectation $\mathbb{E}(X|Y)$

Answer: because X and Y are independent, $\mathbb{E}(X|Y) = \mathbb{E}(X) = \frac{3}{4}$