Please write ***Your name:*** _____

Show all work. You should either write at a sentence explaining your reasoning, or annotate your math work with brief explanations. There is no need to simplify, and no calculators are needed.

. . . . . .

Consider random variables X and Y given by the joint density

$$f(x,y) = \begin{cases} x+y & \text{if } 0 \le x \le 1 \quad \text{and} \quad 0 \le y \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

Find $\mathrm{Cov}(X,Y)$ ***Answer:*** $\frac{1}{3} - \frac{7}{12} \cdot \frac{7}{12} = -\frac{1}{144}$

In the same situation, find $\mathbb{E}(X|Y)$. ***Answer:*** $\mathbb{E}(X|Y) = \dfrac{1/3 + Y/2}{1/2 + Y}$

[(*optional question for extra credit*)]: If Z_1, Z_2 are independent standard normal random variables, and $X = 3Z_1 + 4Z_2$, $Y = 3Z_1 - 4Z_2$, find $\rho(X,Y)$. Do not use any integrals or derivatives.

Answer: Note that Z_1 and Z_2 are independent, but X and Y are not independent.
$\mathbb{E}XY = \mathbb{E}(3Z_1 + 4Z_2)(3Z_1 - 4Z_2) = \mathbb{E}(9Z_1^2 - 16Z_2^2) = 9 - 16 = -7$,
$\mathrm{Var}X = \mathrm{Var}Y = 9 + 16 = 25$,
therefore the correlation coefficient is $\rho(X,Y) = -\dfrac{7}{25}$.