Please write $\mathcal{Y}our\ name:$ _____

Show all work. You should either write at a sentence explaining your reasoning, or annotate your math work with brief explanations. There is no need to simplify, and no calculators are needed.

....

Consider random variables X and Y given by the joint density

$$f(x,y) = \begin{cases} 3(x^2 + y^2)/4 & \text{if } -1 \le x \le 1 \quad \text{and} \quad -1 \le y \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

(A) Find the marginal probability density function (pdf) $f_X(x)$. **Answer:**

$$f_X(x) = \int_{-1}^{1} 3(x^2 + y^2)/4 \, dy = \begin{cases} 3x^2/2 + \frac{1}{2} & \text{if } -1 \le x \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

(B) Find the marginal pdf $f_Y(y)$. *Hint: the correct answer can be obtained with no extra computations based on your answer in (A).* **Answer:**

$$f_Y(y) = \begin{cases} 3y^2/2 + \frac{1}{2} & \text{if } -1 \le y \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

(C) Are X and Y independent random variables? Explain.
Answer: Not independent because $\left(3x^2/2 + \frac{1}{2}\right) \cdot \left(3y^2/2 + \frac{1}{2}\right) \ne 3(x^2 + y^2)/4$

(D) Find $\mathbb{E}X$ and $\mathbb{E}Y$. *Hint: the correct answer is simple, and can be obtained with little or no computations, if you can explain this.* **Answer:** $\mathbb{E}X = \mathbb{E}Y = 0$ because of the symmetry about the origin: integral of an odd function is zero.
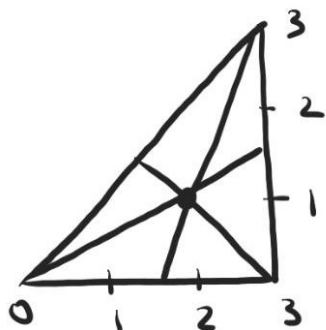
Consider X and Y given by the joint density

$$f(x,y) = \begin{cases} 2/9 & \text{if } 0 \le y \le x \le 3 \\ 0 & \text{otherwise.} \end{cases}$$

(A) Find the marginal pdf $f_X(x)$. **Answer:**

$$f_X(x) = \begin{cases} 2x/9 & \text{if } 0 \le x \le 3 \\ 0 & \text{otherwise.} \end{cases}$$

(B) Find $\mathbb{E}X$. **Answer:** $\mathbb{E}X = \int_0^3 2x^2/9 \, dx = 2$

[(*optional question for extra credit*)]: Can you find from a picture, without making a calculation, what $\mathbb{E}Y$ is?

Answer: $\mathbb{E}Y = 1$