

DMM Assignment 2 Source code

##1. Computing the Loss Function

##1.b

Compute the loss of factorization LR with L2 regularization (parameter lambda)

```
L<-L0.r10
R<-R0.r10
lambda<-2
```

```
loss <- function(L, R, lambda) {
  loss1<-0;
  loss2<-0;
  i<-1;
  while(i<(nrow(summary(D))+1)){
    loss1<-loss1+ (ds[i]-L[is[i],]%*%R[,js[i]])^2;
    i<-i+1;
  }
  loss2<-(lambda/2) * (norm(L, 'F')^2+norm(R, 'F')^2)
  loss<-loss1+loss2;
  return(loss);
}
```

verify loss of starting point with lambda=2

```
loss(L0.r10, R0.r10, 2)
```

##2. Computing the Local Gradients

##2.c

Compute gradient w.r.t. l_i and r_j of the local loss of the p -th revealed
entry for factorization LR with L2 regularization (parameter λ). Returns
a list containing vector elements Li and Rj .

```
dlossp <- function(L, R, lambda, p) {  
  i <- is[p]  
  j <- js[p]  
  d <- ds[p]  
  ## create two length-r vectors dLi and dRj  
  dLi <- rep(0, r)  
  dRj <- rep(0, r)  
  pl <- 1;  
  k <- 1;  
  while(k < (r+1)){  
    item1 <- (-2) * (d - L[i,] %*% R[,j]);  
    dLi[pl] <- item1 * R[k,j] + lambda * L[i,k] / Nis[i];  
    dRj[pl] <- item1 * L[i,k] + lambda * R[k,j] / Njs[j];  
    k <- k+1;  
    pl <- pl+1;  
  }  
  ## * dLi[k] contains the gradient of the local loss with respect to  $l_{ik}$   
  ## * dRi[k] contains the gradient of the local loss with respect to  $r_{kj}$   
  
  ## return the result as a list  
  ## (elements can be accessed with x$Li or x$Ri, where x is the returned list)  
  
  list(Li=dLi, Rj=dRj)  
}  
x <- dlossp(L0.r10, R0.r10, 2, 10)  
x$Li  
x$Ri  
  
## verify the result  
## test local gradient computation for 10th entry  
dlossp(L0.r10, R0.r10, 2, 10)
```

##3. Implementing Gradient Descent

```
## a gradient descent epoch
gdepoch <- function(L, R, lambda, eps) {
  ## create gradient matrices
  dL <- matrix(0, m, r)
  dR <- matrix(0, r, n)

  ## fill the gradient matrices using repeated calls to my dlossp function
  p<-1;
  while(p<(nrow(summary(D))+1)){
    x<-dlossp(L, R, lambda,p);
    dL[is[p],]<- dL[is[p],]+x$Li;
    dR[js[p]]<- dR[js[p]]+x$Rj;
    p<-p+1;
  }
  ## perform a gradient step on L and R with step size eps by using the gradient matrices

  L<-L-eps*dL
  R<-R-eps*dR

  ## return result
  list(L=L, R=R)
}

## test the gd epoch and print 5th row of L; should give
gdepoch(L0.r10, R0.r10, 2, 0.01)$L[5,]

## Visualize the result after 5, 15, and 50 epochs and discuss.

## Run for 5 epochs
result.gd.r10.l2 <- runner(gdepoch, L0.r10, R0.r10, 2, epochs=5, eps=0.01)
## show result
showgray(result.gd.r10.l2$L %*% result.gd.r10.l2$R)

## Run for 15 epochs (continue with the previous result for 10 more epoch)
result.gd.r10.l2 <- runner(gdepoch, result.gd.r10.l2$L, result.gd.r10.l2$R, 2, epochs=10, eps=0.01)
## show result
showgray(result.gd.r10.l2$L %*% result.gd.r10.l2$R)

## Run for 50 epochs (continue with the previous result for 35 more epoch)
result.gd.r10.l2 <- runner(gdepoch, result.gd.r10.l2$L, result.gd.r10.l2$R, 2, epochs=35, eps=0.01)
## show result
showgray(result.gd.r10.l2$L %*% result.gd.r10.l2$R)
```

##4. Implementing Stochastic Gradient Descent

Run a stochastic gradient descent epoch. Uses same paramters and return values as gdepoth.

```
sgdepoth <- function(L, R, lambda, eps) {  
  for (p in sample(length(is))) {  
    i <- is[p]  
    j <- js[p]  
  
    ## perform a stochastic gradient descent step on revealed entry p with step size eps  
    ## Use dlossp to compute the local gradient of entry (i,j), then update  
    ## L[i,] and R[,j] using step size eps. Without scale up this gradient  
    x<-dlossp(L, R, lambda,p);  
    L[i,]<- L[i,]-eps*x$Li;  
    R[,j]<- R[,j]-eps*x$Rj;  
  }  
  ## return result  
  list(L=L, R=R)  
}
```

test the sgd epoch and print 5th row of L

```
set.seed(2, kind="Mersenne-Twister")
```

```
sgdepoth(L0.r10, R0.r10, 2, 0.01)$L[5,]
```

example run (this takes a while)

```
result.sgd.r10.l2 <- runner(sgdepoth, L0.r10, R0.r10, 2)
```

show result

```
showgray(result.sgd.r10.l2$L %*% result.sgd.r10.l2$R)
```

##5. Impact of Parameter Choices

```
##case where (r, λ)=(1,0)
set.seed(1, kind="Mersenne-Twister")
r <- 1
L0.r10 <- matrix(rnorm(m*r), m, r)/sqrt(r)
R0.r10 <- matrix(rnorm(r*n), r, n)/sqrt(r)
result.gd.r10.l2 <- runner(gdepoche, L0.r10, R0.r10, 0,epochs=50)
tr1<-result.gd.r10.l2
showgray(result.gd.r10.l2$L %*% result.gd.r10.l2$R)

##case where (r, λ)=(1,2)
set.seed(1, kind="Mersenne-Twister")
r <- 1
L0.r10 <- matrix(rnorm(m*r), m, r)/sqrt(r)
R0.r10 <- matrix(rnorm(r*n), r, n)/sqrt(r)
result.gd.r10.l2 <- runner(gdepoche, L0.r10, R0.r10, 2,epochs=50)
tr2<-result.gd.r10.l2
showgray(result.gd.r10.l2$L %*% result.gd.r10.l2$R)

##case where (r, λ)=(1,20)
set.seed(1, kind="Mersenne-Twister")
r <- 1
L0.r10 <- matrix(rnorm(m*r), m, r)/sqrt(r)
R0.r10 <- matrix(rnorm(r*n), r, n)/sqrt(r)
result.gd.r10.l2 <- runner(gdepoche, L0.r10, R0.r10, 20,epochs=50)
tr3<-result.gd.r10.l2
showgray(result.gd.r10.l2$L %*% result.gd.r10.l2$R)

##case where (r, λ)=(10,0)
set.seed(1, kind="Mersenne-Twister")
r <- 10
L0.r10 <- matrix(rnorm(m*r), m, r)/sqrt(r)
R0.r10 <- matrix(rnorm(r*n), r, n)/sqrt(r)
result.gd.r10.l2 <- runner(gdepoche, L0.r10, R0.r10, 0,epochs=50)
tr4<-result.gd.r10.l2
showgray(result.gd.r10.l2$L %*% result.gd.r10.l2$R)

##case where (r, λ)=(10,2)
set.seed(1, kind="Mersenne-Twister")
r <- 10
L0.r10 <- matrix(rnorm(m*r), m, r)/sqrt(r)
R0.r10 <- matrix(rnorm(r*n), r, n)/sqrt(r)
result.gd.r10.l2 <- runner(gdepoche, L0.r10, R0.r10, 2,epochs=50)
tr5<-result.gd.r10.l2
showgray(result.gd.r10.l2$L %*% result.gd.r10.l2$R)

##case where (r, λ)=(10,20)
set.seed(1, kind="Mersenne-Twister")
r <- 10
L0.r10 <- matrix(rnorm(m*r), m, r)/sqrt(r)
R0.r10 <- matrix(rnorm(r*n), r, n)/sqrt(r)
result.gd.r10.l2 <- runner(gdepoche, L0.r10, R0.r10, 20,epochs=50)
tr6<-result.gd.r10.l2
showgray(result.gd.r10.l2$L %*% result.gd.r10.l2$R)
```

```
##case where (r, λ)=(20,0)
set.seed(1, kind="Mersenne-Twister")
r <- 20
L0.r10 <- matrix(rnorm(m*r), m, r)/sqrt(r)
R0.r10 <- matrix(rnorm(r*n), r, n)/sqrt(r)
result.gd.r10.l2 <- runner(gdepoche, L0.r10, R0.r10, 0,epochs=50)
tr7<-result.gd.r10.l2
showgray(result.gd.r10.l2$L %*% result.gd.r10.l2$R)
```

```
##case where (r, λ)=(20,2)
set.seed(1, kind="Mersenne-Twister")
r <- 20
L0.r10 <- matrix(rnorm(m*r), m, r)/sqrt(r)
R0.r10 <- matrix(rnorm(r*n), r, n)/sqrt(r)
result.gd.r10.l2 <- runner(gdepoche, L0.r10, R0.r10, 2,epochs=50)
tr8<-result.gd.r10.l2
showgray(result.gd.r10.l2$L %*% result.gd.r10.l2$R)
```

```
##case where (r, λ)=(20,20)
set.seed(1, kind="Mersenne-Twister")
r <- 20
L0.r10 <- matrix(rnorm(m*r), m, r)/sqrt(r)
R0.r10 <- matrix(rnorm(r*n), r, n)/sqrt(r)
result.gd.r10.l2 <- runner(gdepoche, L0.r10, R0.r10, 20,epochs=50)
tr9<-result.gd.r10.l2
showgray(result.gd.r10.l2$L %*% result.gd.r10.l2$R)
```