# Investigation of the MQAM modulation schemes in downlink of space optical communication system

Mi Li\*<sup>a</sup>, Ning Wang, Bowen Li<sup>a</sup>, Xuping Zhang<sup>a</sup>, Yuejiang Song<sup>a</sup>, Yingjie Zhang<sup>b</sup>, Guojie Tu<sup>a</sup>
<sup>a</sup>Institute of Optical Communication Engineering, Nanjing University, Nanjing 210093, People's Republic of China

<sup>b</sup>Shanghai Institute of Satellite engineering, Shanghai 200240, China \*Corresponding author: limi@nju.edu.cn

## **ABSTRACT**

Based on weak fluctuation theory, the expression of bit-error rate (BER) of Multiple Quadrature Amplitude Modulation (MQAM) with the consideration of detector noise in the downlink of space communication system is discussed in this paper. According to the expression, the performance of three typical modulation schemes, which are 4QAM, 16QAM, 64QAM, are specially analyzed. It is known that the higher the order of the modulation scheme is, the more bits of information per symbol can carry. However, when the transmission power is 1 W and the receiver diameter Dr is 1 m, the BER is  $2.12 \times 10^{-13}$  for 4QAM,  $5.98 \times 10^{-8}$  for 16QAM and  $6.22 \times 10^{-5}$  for 64QAM, which means that a higher order modulation scheme shows a higher bit-error rate (BER). Thus considering bandwidth efficiency as well as bit error rate, 16QAM is highly recommended in the real space optical communication system. In addition, the relationships between BER and optimum divergence angle, transmitter beam radius, receiving aperture for downlink are also suggested respectively in this paper, which has important reference significance for the design of the ground-to-satellite laser communication system.

**Keywords:** multiple quadrature amplitude modulation, bit error rate, space optical communication

## 1. INTRODUCTION

Ground-to-satellite laser communication system has made great progress with the advantages of high security, high communication speed, and low power consumption[1,2]. During the process of Ground-to-satellite laser communication, the atmospheric turbulence inevitably deteriorates the communication quality[3,4]. Speaking of downlink space optical communication system, intensity scintillation caused by atmospheric turbulence is the mainly important factor[5]. It affects the bit error rate (BER) performance of communication system when the signal is propagating the atmospheric turbulence.

When it comes to the modulation schemes, traditional on-off keying (OOK) scheme is widely applied because of its great feasibility in communication system. However, with the requirement of communication quality increasing, OOK scheme gradually shows some disadvantages such as higher BER performance and low ability for high data rate transmission[6-8]. Although Erbium-doped Optical fiber amplifier (EDFA) can be used in ground system, it is affected by space radiation[9]. Thus, various modulation schemes are attracting more attention and they are widely utilized in space optical communication system.

Here in this paper, a new and more efficient modulation scheme called M-ary Quadrature Amplitude Modulation (MQAM) is a linear method of digital modulation in which M-ary symbols are transmitted by varying the amplitudes of two carriers in quadrature or equivalently by varying both the amplitude and phase of a single carrier. Because the orthogonal carriers occupy the same frequency band and differ by a 90 degree phase shift, each can be modulated independently, transmitted over the same frequency band, and separated by demodulation at the receiver. For a given available bandwidth, QAM enables data transmission at twice the rate of standard ASK modulation without any degradation in the bit error rate (BER). QAM and its derivatives are used in both mobile radio and satellite communication systems[10].

Many works have been developed to research the BER of MQAM under the ideal situation of narrowband gauss white noise[11-14]. However, in the real ground-to-satellite laser downlink communication system, the expression of the BER

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should be calculated considering combined effects of intensity scintillation and the detector noise while specialized analysis based on these factors can hardly be seen. In this paper, the performance of multiple quadrature amplitude modulation scheme is discussed based on weak fluctuation theory. 4QAM, 16QAM and 64QAM, which are three typical kinds of MQAM schemes, were discussed and the BER of these three schemes are developed. Furthermore, wavelength, optimum divergence angle, transmitter beam radius, receiving aperture based on MQAM schemes are also investigated respectively, which hopefully can be beneficial to the design of ground-to-satellite laser communication system.

#### 2. THEORY

During the process of space optical communication system, the signal is inevitably affected by atmospheric turbulence. For the downlink optical communication system, intensity scintillation is the main factor. If beam is of the Gaussian kind, the probability density function (PDF) of the intensity scintillation under weak fluctuation in the receiving surface can be expressed as[15]

$$P_{r}(I) = \frac{I}{\sqrt{2\pi\sigma_{I}^{2}(r,L)}} \frac{1}{I} \exp \left(-\frac{\left(\ln\frac{I}{\langle I(0,L)\rangle} + \frac{2r^{2}}{W^{2}} + \frac{\sigma_{I}^{2}(r,L)}{2}\right)^{2}}{2\sigma_{I}^{2}(r,L)}\right)$$
(1)

where  $\langle I(0,L)\rangle = \alpha P_T D_r^2/2W^2$  is the mean intensify,  $P_T$  is the transmission power,  $D_r$  is the receiving diameter, W is the radius of beam at the receiving plane,  $\alpha$  is the energy loss of the link, P is the distance between the beam center and receiving point,  $L = (H - h_0)sec(\zeta)$  is the length of the laser link, H and H0 are heights of the receiver and the emitter,  $\sigma_I^2(r,L)$  is the variance [15].

Rectangular QAM signal constellations have the distinct advantage of being easily generated as two PAM signals impressed on the in-phase and quadrature carriers. Provided that the BER of the  $\sqrt{M}$  -ary ASK is  $P_e^{-1}$ , then the probability of correct demodulation in the MQAM system is the product of the probability of correct demodulation of the two  $\sqrt{M}$  -ary ASK modulation schemes, which is[16]

$$P_c = (1 - P_e')^2 (2)$$

So, the BER expression of a MQAM system can be calculated by [16]

$$P_e = 1 - P_c = 1 - (1 - P_e)^2$$
(3)

The total energy must remain unchanged, the amplitudes of the two quadrature signals are only  $1/\sqrt{2}$  of the original signal. So the BER of a MQAM system can be given by [17]

$$BER_{\pm(2i-1)d'} = \int_{-\infty}^{[\pm(2i-1)-1]d} \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left[-\frac{(i\mp\frac{m_i}{\sqrt{2}})^2}{2\sigma_i^2}\right] di + \int_{[\pm(2i-1)+1]d}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left[-\frac{(i\mp\frac{m_i}{\sqrt{2}})^2}{2\sigma_i^2}\right] di, i = 1, 2..., \frac{M}{2} - 1$$

$$(4)$$

$$BER_{+(2i-1)d'} = \int_{-\infty}^{2(i-1)} \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left[-\frac{\left(i - \frac{m_i}{\sqrt{2}}\right)^2}{2\sigma_i^2}\right] di, i = \frac{M}{2}$$
 (5)

$$BER_{-(2i-1)d'} = \int_{-2(i-1)d}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left[-\frac{\left(i + \frac{m_i}{\sqrt{2}}\right)^2}{2\sigma_4^2}\right] di, i = \frac{M}{2}$$
 (6)

$$BER_{MASK'} = \frac{1}{M} \sum_{i=1}^{\frac{M}{2}} BER_{(2i-1)d'}$$
 (7)

$$BER_{MOAM} = 1 - (1 - BER_{\sqrt{MASK'}})^2$$
 (8)

Based on the above expressions, the expressions of BER for 4QAM, 16QAM and 64QAM can be easily deduced as follows. The 4QAM BER performance is [17]

$$BER_{4QAM} = 1 - \left\{1 - \frac{1}{2} \int_{0}^{+\infty} \frac{1}{\sqrt{2\pi\sigma_{1}}} \exp\left[-\frac{\left(i + \frac{m_{1}}{\sqrt{2}}\right)^{2}}{2\sigma_{1}^{2}}\right] di - \frac{1}{2} \int_{-\infty}^{\gamma} \frac{1}{\sqrt{2\pi\sigma_{1}}} \exp\left[-\frac{\left(i - \frac{m_{1}}{\sqrt{2}}\right)^{2}}{2\sigma_{1}^{2}}\right] di\right\}^{2}$$

$$(9)$$

where  $m_1$ ,  $\sigma_1^2$  are the mean and variance of i for '1' bit. For the APD detector,  $m_1$ ,  $\sigma_1^2$  can be expressed as [11]

Furthermore, similarly, in a 4ASK system, the BER expression can be calculated by adding the respective BER of the four possible electrical levels. So the BER of a 16QAM system can be given by [17]

$$BER_{4ASK'} = \frac{1}{4} \cdot \left( \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}\sigma_{1}} \exp\left[ -\frac{\left(i - \frac{m_{1}}{\sqrt{2}}\right)^{2}}{2\sigma_{1}^{2}} \right] di + \int_{0}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma_{1}} \exp\left[ -\frac{\left(i + \frac{m_{1}}{\sqrt{2}}\right)^{2}}{2\sigma_{1}^{2}} \right] di \right) + \int_{-\infty}^{2d} \frac{1}{\sqrt{2\pi}\sigma_{2}} \exp\left[ -\frac{\left(i - \frac{m_{2}}{\sqrt{2}}\right)^{2}}{2\sigma_{2}^{2}} \right] di \right) + \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma_{1}} \exp\left[ -\frac{\left(i - \frac{m_{1}}{\sqrt{2}}\right)^{2}}{2\sigma_{1}^{2}} \right] di \right) + \int_{-\infty}^{2d} \frac{1}{\sqrt{2\pi}\sigma_{1}} \exp\left[ -\frac{\left(i + \frac{m_{1}}{\sqrt{2}}\right)^{2}}{2\sigma_{1}^{2}} \right] di + \int_{-2d}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma_{2}} \exp\left[ -\frac{\left(i + \frac{m_{2}}{\sqrt{2}}\right)^{2}}{2\sigma_{2}^{2}} \right] di \right)$$

$$(10)$$

$$BER_{16QAM} = 1 - (1 - BER_{4ASK'})^2 \tag{11}$$

where  $m_i$  and  $\sigma_i^2$  (i = 1,2) can be given by in the APD detector[18]

$$m_i = G \cdot e \cdot (K_s(I_i) + K_b) + I_{dc}T_s$$

$$\sigma_i^2 = (G \cdot e)^2 \cdot F \cdot (K_s(I_i) + K_b) + \sigma_T^2$$
(12)

where  $K_s$  is the photon count F is the additional noise factor, G is the photomultiplier gain factor,  $\eta$  is quantum efficiency,  $K_b = \eta I_b T_s / h v$  is the photon count of the background light, v is the frequency of the signal light, h is the Planck constant,  $\sigma_T^2 = 2\kappa_c T T_s / R_L$  is the thermal noise,  $T_s$  is bit time, T is the temperature,  $R_L$  is load resistance, and  $I_b$  is the background light in the optical communication system.

In the same way, in a 8ASK system, the BER expression can be calculated by adding the respective BERs of the eight possible electrical levels. Based on it, 64QAM BER performance is[17]

$$BER_{8.45K'} = \frac{1}{8} \cdot \left(\int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}\sigma_{1}} \exp\left[-\frac{(i - \frac{m_{1}}{\sqrt{2}})^{2}}{2\sigma_{1}^{2}}\right] di + \int_{0}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma_{1}} \exp\left[-\frac{(i + \frac{m_{1}}{\sqrt{2}})^{2}}{2\sigma_{1}^{2}}\right] di + \int_{-\infty}^{-2d} \frac{1}{\sqrt{2\pi}\sigma_{1}} \exp\left[-\frac{(i + \frac{m_{1}}{\sqrt{2}})^{2}}{2\sigma_{1}^{2}}\right] di + \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma_{2}} \exp\left[-\frac{(i - \frac{m_{2}}{\sqrt{2}})^{2}}{2\sigma_{2}^{2}}\right] di + \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma_{2}} \exp\left[-\frac{(i - \frac{m_{2}}{\sqrt{2}})^{2}}{2\sigma_{2}^{2}}\right] di + \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma_{2}} \exp\left[-\frac{(i - \frac{m_{2}}{\sqrt{2}})^{2}}{2\sigma_{2}^{2}}\right] di + \int_{-\infty}^{+d} \frac{1}{\sqrt{2\pi}\sigma_{2}} \exp\left[-\frac{(i - \frac{m_{2}}{\sqrt{2}})^{2}}{2\sigma_{2}^{2}}\right] di + \int_{-\infty}^{+d} \frac{1}{\sqrt{2\pi}\sigma_{2}} \exp\left[-\frac{(i - \frac{m_{2}}{\sqrt{2}})^{2}}{2\sigma_{2}^{2}}\right] di + \int_{-\infty}^{+\omega} \frac{1}{\sqrt{2\pi}\sigma_{3}} \exp\left[-\frac{(i - \frac{m_{2}}{\sqrt{2}})^{2}}{2\sigma_{2}^{2}}\right] di + \int_{-d}^{+\omega} \frac{1}{\sqrt{2\pi}\sigma_{3}} \exp\left[-\frac{(i - \frac{m_{2}}{\sqrt{2}})^{2}}{2\sigma_{3}^{2}}\right] di + \int_{-dd}^{+\omega} \frac{1}{\sqrt{2\pi}\sigma_{4}} \exp\left[-\frac{(i - \frac{m_{4}}{\sqrt{2}})^{2}}{2\sigma_{4}^{2}}\right] di + \int_{-dd}^{+\omega} \frac{1}{\sqrt{2\pi}\sigma_{3}} \exp\left[-\frac{(i - \frac{m_{3}}{\sqrt{2}})^{2}}{2\sigma_{3}^{2}}\right] di + \int_{-dd}^{+\omega} \frac{1}{\sqrt{2\pi}\sigma_{4}} \exp\left[-\frac{(i - \frac{m_{4}}{\sqrt{2}})^{2}}{2\sigma_{4}^{2}}\right] di\right)$$

$$+ \int_{-\infty}^{-dd} \frac{1}{\sqrt{2\pi}\sigma_{3}} \exp\left[-\frac{(i + \frac{m_{3}}{\sqrt{2}})^{2}}{2\sigma_{3}^{2}}\right] di + \int_{-dd}^{+\omega} \frac{1}{\sqrt{2\pi}\sigma_{4}} \exp\left[-\frac{(i - \frac{m_{4}}{\sqrt{2}})^{2}}{2\sigma_{4}^{2}}\right] di\right)$$

$$+ \int_{-\infty}^{-dd} \frac{1}{\sqrt{2\pi}\sigma_{3}} \exp\left[-\frac{(i + \frac{m_{3}}{\sqrt{2}})^{2}}{2\sigma_{3}^{2}}\right] di + \int_{-dd}^{+\omega} \frac{1}{\sqrt{2\pi}\sigma_{4}} \exp\left[-\frac{(i - \frac{m_{4}}{\sqrt{2}})^{2}}{2\sigma_{4}^{2}}\right] di\right)$$

$$+ \int_{-\infty}^{-dd} \frac{1}{\sqrt{2\pi}\sigma_{3}} \exp\left[-\frac{(i + \frac{m_{3}}{\sqrt{2}})^{2}}{2\sigma_{3}^{2}}\right] di + \int_{-dd}^{+\omega} \frac{1}{\sqrt{2\pi}\sigma_{4}} \exp\left[-\frac{(i - \frac{m_{4}}{\sqrt{2}})^{2}}{2\sigma_{4}^{2}}\right] di\right)$$

$$BER_{64OAM} = 1 - (1 - BER_{8ASK})^2 \tag{14}$$

Generally speaking, the BER of the system is concerned with the receiving light intensity while the atmospheric turbulence can cause random fluctuation of it. So the BER of the system should be the ensemble average of all the possible value of the light intensity when taking the effect of atmospheric turbulence into consideration. According to the processing model for atmospheric effect from reference [1], the BER of MQAM for ground-to-satellite laser downlink communication system can be expressed as

$$BER_{MQAM} = \int_{0}^{+\infty} BER(I)_{MQAM} P_{r}(I) dI$$
 (15)

## 3. SIMULATIONS

Numerical results are based on the following parameters: wavelength  $\lambda$ =850nm, zenith angle  $\zeta$ =0°, the divergence angle  $\theta$ =30  $\mu$  rad, the altitude of the satellite H=36,000km, the altitude of the ground station  $h_0$ =100m, quantum efficiency of APD  $\eta$ =0.75, photomultiplier gain factor G=100, additional noise factor F= $G^{0.5}$ , temperature T=300K, load resistance  $R_L$ =50  $\Omega$ , the time duration per slot Ts=10ns, spectral density  $I_s$ =10nW/m², the dark current  $I_{dc}$ =1na, the receiving aperture  $D_r$ =0.25m, dissipation coefficient  $\partial$ =1. With the reflected light of the earth, the background is in the range of 0.24-0.38nW [18].

We take 4QAM, 16QAM and 64QAM, which are the most commonly-used MQAM modulation schemes, as examples to analyze in this paper. Considering intensity scintillation only in the downlink communication system, the variation of BER as a function of transmitting power in different M-ary QAM modulation schemes is shown in Fig.1. It is obvious that as the transmitting power increases, the BER of the three modulation schemes declines. What's more, based on the comparison among 4QAM, 16QAM and 64QAM, it is obvious that the BER of 4QAM is consistently lower than that of 16QAM and 64QAM. To be specific, compared with 4QAM, the BER of 16QAM is 37 dB higher. For 64QAM, its BER is 61 dB higher than 4QAM when the transmission power is 4W.

Fig.2 indicates the variation of the BER as a function of the receiver diameter. Here, we set the value of transmission power to 1 W. It is easy to see that with the receiver diameter increasing, the BER of three modulation schemes are decreasing stably. Furthermore, from the figure we can see that the BER of 4QAM is always lower than 16QAM and 64QAM. Specially, compared with 4QAM, BER of 16QAM is 66 dB higher. for 64QAM. And compared with 16QAM, it is 33 dB higher for 64QAM when the receiver diameter is 1.6m.

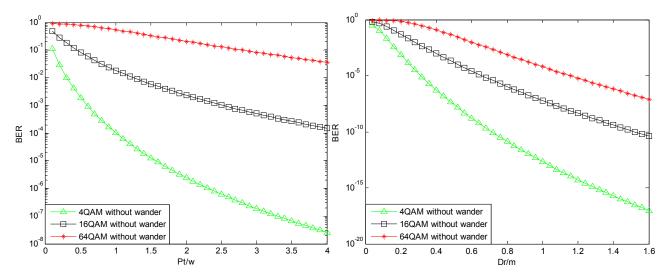


Fig.1. BERs as functions of transmitting power Pt.

Fig.2. BERs as functions of Receiver diameter D<sub>r</sub>

Fig.3 shows the variation of the BER as a function of divergence angle. Here the three modulation schemes show differences. For 4QAM, with the divergence angle increasing, the BER see an increase that is sharp at first and then becoming more and more stable. However, when it comes to 16QAM and 64QAM, it is apparent that there exits optimum divergence angle, which are  $20 \times 10^{-6}$  and  $16.25 \times 10^{-6}$  respectively. Another important parameter is beam radius and the BERs as functions of it are shown in Fig.4. Clearly, the BERs of the three modulation schemes rise as the beam radius increases. Also, the 64QAM modulation scheme showed most stable increase and the 4QAM modulation is in a wider range of BER.

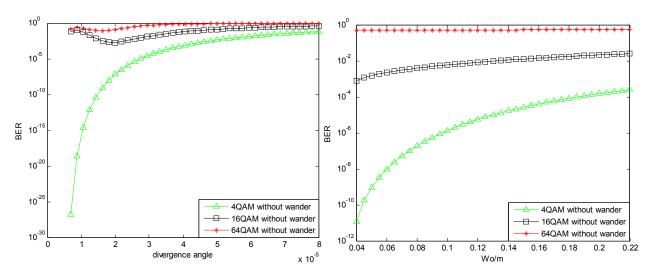


Fig.3. BERs as functions of divergence angle  $\theta$ .

Fig.4. BERs as functions of beam radius.

Furthermore, the advantage of using QAM is that it is a higher order form of modulation and as a result it is able to carry more bits of information per symbol. By selecting a higher order format of QAM, the data rate of a link can be increased. For example, by using 4QAM modulation scheme, the system can transmit 4 bits per symbol while a 64QAM system can carry 6 bits per symbol. However, from all the figures above we can see that while higher order modulation rates are able to offer much faster data rates and higher levels of spectral efficiency for the communications system, this comes at a price. The higher order modulation schemes are considerably less resilient to noise and interference.

For instance, with the consideration of the effects of atmospheric turbulence, when the transmission power is 1 W and the receiver diameter Dr is 1 m, the BER is  $2.11 \times 10^{-13}$  for 4QAM,  $5.98 \times 10^{-8}$  for 16QAM and  $6.22 \times 10^{-5}$  for 64QAM, which means that the BER of 16QAM is 54 dB higher than that of 4QAM and the BER of 64QAM is 30 dB higher than that of 16QAM. So in the real situation of space optical communication system, not only the bandwidth efficiency but also the bit error rate should be considered when designing the communication system. According to the above analysis, 16QAM is highly recommended for its comparatively higher bandwidth efficiency as well as lower BER. Of course, among the three modulation schemes, 4QAM can be the best choice in a BER oriented system while 64QAM is the best in a bandwidth efficiency oriented system. That is, only by choosing a proper M-ary QAM modulation scheme can we develop a better communication system.

Furthermore, the performance of these three modulation schemes show differences when it comes to different parameters. For example, there is an optimal value of the BER as functions of divergence angle for 16QAM and 64QAM while there exists no optimal value for 4QAM. And the designer of the space optical communication system should pay attention to this, too.

## 4. CONCLUSIONS

With the consideration of atmospheric fluctuation and detector noise, performance of 4QAM, 16QAM and 64QAM, which are three typical kinds of MQAM modulation schemes, are discussed in terms of BER in this paper. From the above-mentioned analysis, it is obvious that even though the MQAM modulation scheme with a higher order have more advantages in terms of bandwidth efficiency and data transmitting rate, their BERs are always higher than a lower order

QAM modulation scheme in the real downlink space optical communication system. Thus, the designers of the ground-to-satellite laser communication system should take both of the two factors - bandwidth efficiency and BER - into consideration and then choose the best modulation scheme when designing the system. And 16QAM is highly recommended here. Furthermore, in the real downlink optical communication system, the BERs of different MQAM schemes as functions of different parameters are not all the same and attention should be paid to this.

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