機率與統計 HW5 資訊乙 114 何寬羿 C34104032

Textbook Exercise

(5.14)

probability of win = 90%

(a) by Binomial Dist.

所求 $= b(4;4,0.9) = \sum_{x=0}^{4} b(x;4,0.9) - \sum_{x=0}^{3} b(x;4,0.9) \overset{\text{查表}}{=\!=} 1 - 0.3439$

\# of win

\# of trials

prob. of win

$= 0.6561$ #

(b) The Bulls should win 4 times in 4 or 5 or 6 or 7 games.

by Negative Binomial Dist.

所求 $= b^*(4;4,0.9) + b^*(5;4,0.9) + b^*(6;4,0.9) + b^*(7;4,0.9)$

\# of trials

\# of win

Prob. of win

$= \binom{3}{3} 0.9^4 \times 0.1^0 + \binom{4}{3} 0.9^4 \times 0.1 + \binom{5}{3} 0.9^4 \times 0.1^2 + \binom{6}{3} 0.9^4 \times 0.1^3$

$= 0.6561 + 0.26244 + 0.06561 + 0.013122 = 0.997272$

$\fallingdotseq 0.9973$ #

(c) 每一次 the Bulls 贏的機率都是 90% #

(5.26)

Using Binomial Dist. $b(n;x,p) = \binom{x}{n} \cdot p^n \cdot (1-p)^{x-n}$

\# of violation

\# of trials

prob. of violation

(a) $b(6;8,0.6) = \binom{8}{6} \cdot (0.6)^6 (0.4)^2 = 0.20901888 \fallingdotseq 0.2090$ #

(b) $b(6;8,0.6) = \sum_{x=0}^{6} (x;8,0.6) - \sum_{x=0}^{5} (x;8,0.6) \overset{\text{查表}}{=\!=} 0.8936 - 0.6846$

$= 0.2090$ #

(5.50)

Using Negative Binomial Dist. $b^*(x;k,p) = \binom{x-1}{k-1} p^k (1-p)^{x-k}$

\# of trials

\# of head

Prob. of getting head = 0.5

(a) $b^*(7;3,0.5) = \binom{6}{2} \cdot (0.5)^3 (0.5)^4 = 0.1171875 \fallingdotseq 0.1172$ #

(b) $b^*(4;1,0.5) = \binom{3}{0} \cdot (0.5)^1 (0.5)^3 = 0.0625$ #

(5.56)

Using Poisson Dist. $P(x;\lambda t)$

\# of accidents; t → month = 1; λ → accidents/month = 3

(a) $p(5;3) = \sum_{x=0}^{5} P(x;3) - \sum_{x=0}^{4} P(x;3) \underset{查表}{=} 0.9161 - 0.8153 = 0.1008$ #

(b) $P(X<3) = \sum_{x=0}^{2} p(x;3) \overset{查表}{=\!=} 0.4232$ #

(c) $P(X \geq 2) = 1 - P(X \leq 1) = 1 - \sum_{x=0}^{1} p(x;3) \overset{查表}{=\!=} 1 - 0.1991 = 0.8009$ #

(5.80)

Using Poisson Dist. $P(x;\lambda t) = \frac{e^{-\lambda t}(\lambda t)^x}{x!}$

\# of calls; t → minute; λ → calls/minute = 2.7

(a) $\lambda t = 2.7$

$P(X \leq 4) = \sum_{x=0}^{4} p(x;2.7) = \sum_{x=0}^{4} \frac{e^{-2.7}(2.7)^x}{x!} \fallingdotseq 0.8629$ #

(b) $\lambda t = 2.7$

$P(X < 2) = \sum_{x=0}^{1} \frac{e^{-2.7}(2.7)^x}{x!} \fallingdotseq 0.2487$ #

(c) $\lambda t = 2.7 \times 5 = 13.5$

$P(X > 10) = 1 - P(X \leq 10) = 1 - \sum_{x=0}^{10} \frac{e^{-13.5}(13.5)^x}{x!} \fallingdotseq 1 - 0.211226477$

$\fallingdotseq 0.7888$ #

Matlab Exercise

1. (a),(b) function are in matlab code

(c)

Command Window

Binomial Probability Sums(b(x;n,p), from x = 0 to r)

n	r	p = 0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90
1	0	0.9	0.8	0.75	0.7	0.6	0.5	0.4	0.3	0.2	0.1
1	1	1	1	1	1	1	1	1	1	1	1
2	0	0.81	0.64	0.5625	0.49	0.36	0.25	0.16	0.09	0.04	0.01
2	1	0.99	0.96	0.9375	0.91	0.84	0.75	0.64	0.51	0.36	0.19
2	2	1	1	1	1	1	1	1	1	1	1
3	0	0.729	0.512	0.4219	0.343	0.216	0.125	0.064	0.027	0.008	0.001
3	1	0.972	0.896	0.8438	0.784	0.648	0.5	0.352	0.216	0.104	0.028
3	2	0.999	0.992	0.9844	0.973	0.936	0.875	0.784	0.657	0.488	0.271
3	3	1	1	1	1	1	1	1	1	1	1
4	0	0.6561	0.4096	0.3164	0.2401	0.1296	0.0625	0.0256	0.0081	0.0016	0.0001
4	1	0.9477	0.8192	0.7383	0.6517	0.4752	0.3125	0.1792	0.0837	0.0272	0.0037
4	2	0.9963	0.9728	0.9492	0.9163	0.8208	0.6875	0.5248	0.3483	0.1808	0.0523
4	3	0.9999	0.9984	0.9961	0.9919	0.9744	0.9375	0.8704	0.7599	0.5904	0.3439
4	4	1	1	1	1	1	1	1	1	1	1
5	0	0.5905	0.3277	0.2373	0.1681	0.0778	0.0313	0.0102	0.0024	0.0003	0
5	1	0.9185	0.7373	0.6328	0.5282	0.337	0.1875	0.087	0.0308	0.0067	0.0005
5	2	0.9914	0.9421	0.8965	0.8369	0.6826	0.5	0.3174	0.1631	0.0579	0.0086
5	3	0.9995	0.9933	0.9844	0.9692	0.913	0.8125	0.663	0.4718	0.2627	0.0815
5	4	1	0.9997	0.999	0.9976	0.9898	0.9688	0.9222	0.8319	0.6723	0.4095
5	5	1	1	1	1	1	1	1	1	1	1
6	0	0.5314	0.2621	0.178	0.1176	0.0467	0.0156	0.0041	0.0007	0.0001	0
6	1	0.8857	0.6554	0.5339	0.4202	0.2333	0.1094	0.041	0.0109	0.0016	0.0001
6	2	0.9842	0.9011	0.8306	0.7443	0.5443	0.3438	0.1792	0.0705	0.017	0.0013
6	3	0.9987	0.983	0.9624	0.9295	0.8208	0.6563	0.4557	0.2557	0.0989	0.0158
6	4	0.9999	0.9984	0.9954	0.9891	0.959	0.8906	0.7667	0.5798	0.3446	0.1143
6	5	1	0.9999	0.9998	0.9993	0.9959	0.9844	0.9533	0.8824	0.7379	0.4686
6	6	1	1	1	1	1	1	1	1	1	1

7	0	0.4783	0.2097	0.1335	0.0824	0.028	0.0078	0.0016	0.0002	0	0
7	1	0.8503	0.5767	0.4449	0.3294	0.1586	0.0625	0.0188	0.0038	0.0004	0
7	2	0.9743	0.852	0.7564	0.6471	0.4199	0.2266	0.0963	0.0288	0.0047	0.0002
7	3	0.9973	0.9667	0.9294	0.874	0.7102	0.5	0.2898	0.126	0.0333	0.0027
7	4	0.9998	0.9953	0.9871	0.9712	0.9037	0.7734	0.5801	0.3529	0.148	0.0257
7	5	1	0.9996	0.9987	0.9962	0.9812	0.9375	0.8414	0.6706	0.4233	0.1497
7	6	1	1	0.9999	0.9998	0.9984	0.9922	0.972	0.9176	0.7903	0.5217
7	7	1	1	1	1	1	1	1	1	1	1

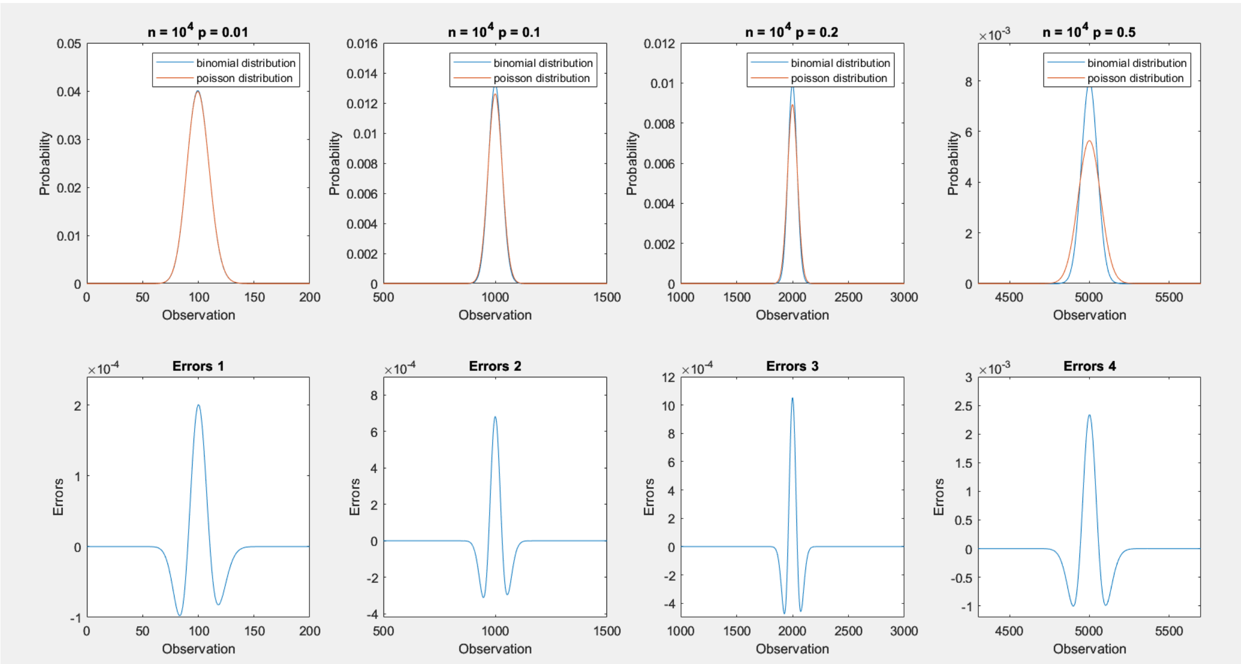
(d)

Command Window

Poisson Probability Sums(p(x;μ), from x = 0 to r)

r	μ = 5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5
0	0.0041	0.0025	0.0015	0.0009	0.0006	0.0003	0.0002	0.0001	0.0001
1	0.0266	0.0174	0.0113	0.0073	0.0047	0.003	0.0019	0.0012	0.0008
2	0.0884	0.062	0.043	0.0296	0.0203	0.0138	0.0093	0.0062	0.0042
3	0.2017	0.1512	0.1118	0.0818	0.0591	0.0424	0.0301	0.0212	0.0149
4	0.3575	0.2851	0.2237	0.173	0.1321	0.0996	0.0744	0.055	0.0403
5	0.5289	0.4457	0.369	0.3007	0.2414	0.1912	0.1496	0.1157	0.0885
6	0.686	0.6063	0.5265	0.4497	0.3782	0.3134	0.2562	0.2068	0.1649
7	0.8095	0.744	0.6728	0.5987	0.5246	0.453	0.3856	0.3239	0.2687
8	0.8944	0.8472	0.7916	0.7291	0.662	0.5925	0.5231	0.4557	0.3918
9	0.9462	0.9161	0.8774	0.8305	0.7764	0.7166	0.653	0.5874	0.5218
10	0.9747	0.9574	0.9332	0.9015	0.8622	0.8159	0.7634	0.706	0.6453
11	0.989	0.9799	0.9661	0.9467	0.9208	0.8881	0.8487	0.803	0.752
12	0.9955	0.9912	0.984	0.973	0.9573	0.9362	0.9091	0.8758	0.8364
13	0.9983	0.9964	0.9929	0.9872	0.9784	0.9658	0.9486	0.9261	0.8981
14	0.9994	0.9986	0.997	0.9943	0.9897	0.9827	0.9726	0.9585	0.94
15	0.9998	0.9995	0.9988	0.9976	0.9954	0.9918	0.9862	0.978	0.9665
16	0.9999	0.9998	0.9996	0.999	0.998	0.9963	0.9934	0.9889	0.9823
17	1	0.9999	0.9998	0.9996	0.9992	0.9984	0.997	0.9947	0.9911
18	1	1	0.9999	0.9999	0.9997	0.9993	0.9987	0.9976	0.9957
19	1	1	1	1	0.9999	0.9997	0.9995	0.9989	0.998
20	1	1	1	1	1	0.9999	0.9998	0.9996	0.9991
21	1	1	1	1	1	1	0.9999	0.9998	0.9996
22	1	1	1	1	1	1	1	0.9999	0.9999
23	1	1	1	1	1	1	1	1	0.9999
24	1	1	1	1	1	1	1	1	1

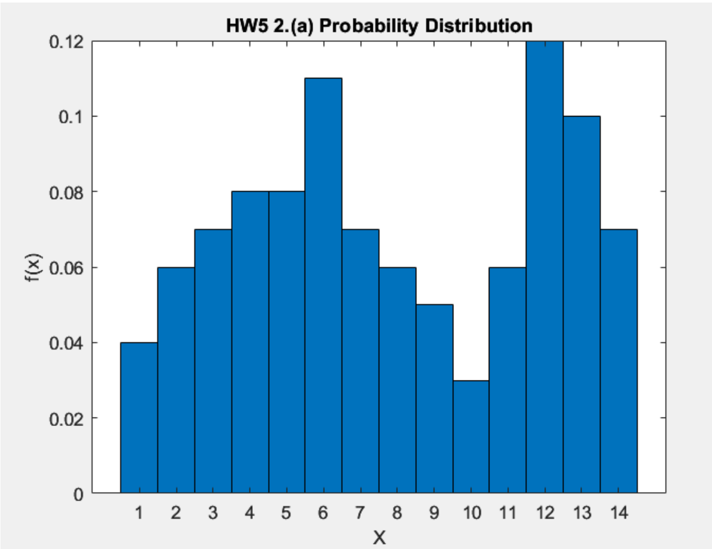
(e)



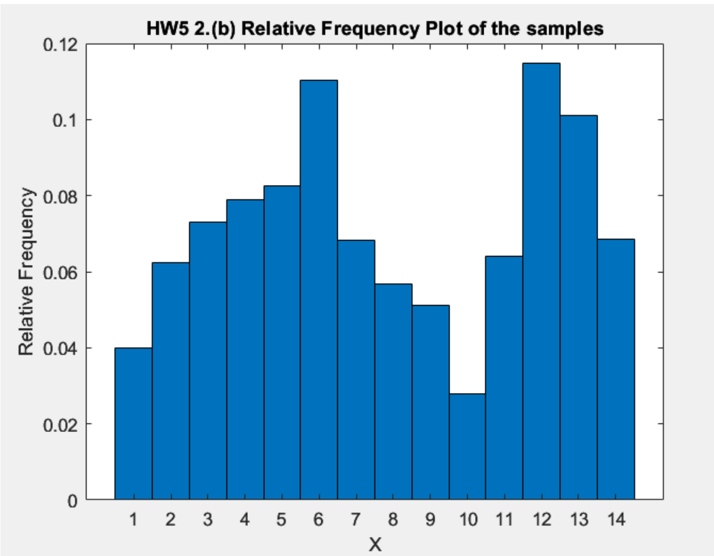
在 n 固定為 10000 的情況下，p 的值越大，the errors from approximating Binomial dist. with Poisson dist. (誤差)會越大。這是因為，當 Binomial Dist. 的 n 趨近無限大、p 趨近於 0 時，較能符合 Poisson Process，使得 Binomial Dist. 會趨近於 Poisson Dist.(在此條件下，n*p 當作 Poisson Dist.的參數 μ 傳入，較接近實際上的 Binomial Dist.)。

2.

(a)



(b)



Yes, the plots from 2.(a) and 2.(b) look alike!

因為當生成的 sample 數量足夠多時，它們的 relative frequency 會很接近用來生成這些 sample 的 Random Variable(X) 所對應到的機率值 f(x)，故兩張圖表各自在 X= x(x = 1,2,…,14)時的 bar，對應到的值會很相似。若 sample 數不足，那可能就會產生較大的誤差。