

機率統 HW 4 資訊 114 何寬羿 C34104032

4.24

from 3.39

$$f(x,y) = \frac{(x)(y)(4-x-y)}{\binom{8}{4} = 70}$$

$$\begin{matrix} \text{orange} & \text{apple} & \text{banana} \\ \binom{3}{x} & \binom{2}{y} & \binom{3}{4-x-y} \end{matrix}$$

$$x=0,1,2,3 \quad y=0,1,2 \quad \text{且 } 1 \leq x+y \leq 4$$

	x				
	0	1	2	3	$h(y)$
0	0	$\frac{3}{70}$	$\frac{9}{70}$	$\frac{27}{70}$	$\frac{15}{70}$
1	$\frac{2}{70}$	$\frac{18}{70}$	$\frac{18}{70}$	$\frac{2}{70}$	$\frac{45}{70}$
2	$\frac{3}{70}$	$\frac{9}{70}$	$\frac{3}{70}$	0	$\frac{15}{70}$
$g(x)$	$\frac{5}{70}$	$\frac{30}{70}$	$\frac{30}{70}$	$\frac{5}{70}$	

	x				
	0	1	2	3	
0	x	0	0	0	
1	0	-1	0	3	
2	0	-2	0	x	

$$(a) E(X^2Y - 2XY) = \sum_{x=0}^3 \sum_{y=0}^2 (x^2y - 2xy) \cdot f(x,y)$$

$$= (-1 \cdot \frac{18}{70}) + (3 \cdot \frac{1}{70}) + (-2 \cdot \frac{9}{70}) = \underline{-\frac{3}{7}} \quad *$$

$$(b) E(X) = \sum_x x \cdot g(x) = 0 + 1 \cdot \frac{30}{70} + 2 \cdot \frac{30}{70} + 3 \cdot \frac{5}{70} = \frac{105}{70} = \frac{3}{2} = \mu_X$$

$$E(Y) = \sum_y y \cdot h(y) = 0 + 1 \cdot \frac{45}{70} + 2 \cdot \frac{15}{70} = \frac{70}{70} = 1 = \mu_Y$$

$$\Rightarrow \mu_X - \mu_Y = \frac{3}{2} - 1 = \underline{0.5} \quad *$$

$$4.44 \quad E(XY) = \sum_x \sum_y (xy) \cdot f(x,y) = (1 \cdot \frac{8}{70}) + (2 \cdot \frac{18}{70}) + (3 \cdot \frac{2}{70})$$

$$+ (2 \cdot \frac{9}{70}) + (4 \cdot \frac{3}{70}) = \underline{\frac{9}{7}}$$

	x				
	0	1	2	3	
0	x	0	0	0	
1	0	1	2	3	
2	0	2	4	x	

$$\text{所求: } G_{XY} = E(XY) - \mu_X \mu_Y = \frac{9}{7} - \frac{3}{2} \cdot 1 = \underline{-\frac{3}{14}} \quad *$$

4.60

y	x		$h(y)$
	2	4	
1	0.15	0.10	0.25
3	0.25	0.25	0.50
5	0.15	0.10	0.25
	$g(x)$	0.55	0.45

$$E(X) = 2 \times 0.55 + 4 \times 0.45 = 2.9$$

$$E(Y) = 1 \times 0.25 + 3 \times 0.5 + 5 \times 0.25 = 3$$

(a)  $E(2X - 3Y) = 2E(X) - 3E(Y) = 2 \times 2.9 - 3 \times 3 = \underline{-3.2}$  #

(b)  $E(XY) \xrightarrow{\text{X, Y indep}} E(X) \cdot E(Y) = 2.9 \times 3 = \underline{8.7}$  #

4.78  $\mu = E(X) = \int_0^1 x \cdot 30x^2(1-x)^2 dx = 30 \cdot \int_0^1 (x^5 - 2x^4 + x^3) dx$

$$= 30 \cdot \left( \frac{1}{6}x^6 - \frac{2}{5}x^5 + \frac{1}{4}x^4 \right) \Big|_0^1 = 5 - 12 + 7.5 = \underline{0.5}$$

$$E(X^2) = \int_0^1 x^2 \cdot 30x^2(1-x)^2 dx = \frac{2}{7}$$

$$\sigma = \sqrt{E(X^2) - \mu^2} = \sqrt{\frac{2}{7} - 0.5^2} \approx \underline{0.18882}$$

\* 找求：

$$P(\mu - 2\sigma < X < \mu + 2\sigma) = P(0.5 - 2 \times 0.18882 < X < 0.5 + 2 \times 0.18882)$$

$$= P(0.122036 < X < 0.877964)$$

$$= \int_{0.122036}^{0.877964} 30 \cdot \frac{x^2(1-x)^2}{x^4 - 2x^3 + x^2} dx \approx \underline{0.97}$$

\* By Chebyshew's Theorem

$$P(\mu - k\sigma < X < \mu + k\sigma) \geq 1 - \frac{1}{k^2}$$

$\xrightarrow{k=2}$  找求： $P(\mu - 2\sigma < X < \mu + 2\sigma) \geq 1 - \frac{1}{4} = \underline{0.75}$

$$\Rightarrow 0.97 \geq 0.75$$

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實際算出的

估算的下界

4.98

	y			$g(x)$
x	0	1	2	
0	0.12	0.04	0.04	0.20
1	0.08	0.19	0.05	0.32
2	0.06	0.12	0.30	0.48
$h(y)$	0.26	0.35	0.39	

(a)

$x$	0	1	2	
$g(x)$	0.20	0.32	0.48	#

y	0	1	2	
$h(y)$	0.26	0.35	0.39	#

$x$	0	1	2	
$f(x Y=2)$	$\frac{4}{39}$	$\frac{5}{39}$	$\frac{30}{39}$	

#

$$(b) E(X) = \sum_x x \cdot g(x) = 0 + 1 \times 0.32 + 2 \times 0.48 = \underline{1.28} \quad \#$$

$$E(X^2) = \sum_x x^2 \cdot g(x) = 0 + 1 \times 0.32 + 4 \times 0.48 = 2.24$$

$$\text{Var}(X) = \sum_x x^2 - E(X)^2 = \underline{0.6016} \quad \#$$

$$(c) E(X|Y=2) = 0 + 1 \times \frac{5}{39} + 2 \times \frac{30}{39}$$

$$= \underline{\frac{5}{3}} \quad \#$$

$$E(X^2|Y=2) = 0 + 1 \times \frac{5}{39} + 4 \times \frac{30}{39} = \underline{\frac{125}{39}}$$

$$\Rightarrow \text{Var}(X|Y=2) = \underline{\frac{125}{39}} - \underline{\left(\frac{5}{3}\right)^2} = \underline{\frac{50}{117}} \quad \#$$

$$E(X^2|Y=2)$$

$$E(X|Y=2)$$