4.1 An intuitive idea behind Kalman filter

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Let us consider a scenario when you are in Mars colony #28 where water is supplied as ice cubes using a cute robot named Boltu instead of through a pipe. And because the average temperature on Mars is -60 degree celsius, storing water as ice is easier. But to drink it you must boil it to a suitable temperature you prefer. Let's say you prefer water at the temperature of 30 degree celsius.

You need to melt one small cube of ice with an electric stove that can heat the ice cube at -60 degree celsius, melt it and raise its temperature to 30 degree celsius in ninety minutes. So, on average the stove can raise the water temperature at a rate of 1 degree per minute. Let us just ignore the fact that you have to actually supply heat to melt a 0 degree celsius ice to a zero degree celsius water and things like atmospheric pressure.

Now let's say that you have an unreliable thermometer that gives measurement which can be up to 5 degree celsius above or below the actual temperature (that you unfortunately have to use until next supply robot truck comes to your colony three weeks from now). For example, if the thermometer measurement shows that it is 30 degree celsius, then the actual temperature can be anywhere between 25 degree and 35 degree celsius.

Now since you know it'll take about 30 minutes for the completely melted water to reach 30 degree celsius, you start to measure it after about 20 minutes of melting. At the time you estimate the temperature should be about 20 degree celsius. But the thermometer says, its 23 degree celsius. We we have the following values:

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mesurement = 23
estimate = 20
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So, in the case the difference between the measured temperature and estimated temperature, which we call residual, is calculated using following simple formula:

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residual = measurement - estimate
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So, in this case residual is 23-20 = 3. We do not need this yet, but will use this in explaining Kalman Filter later.

At this time we know that the temperature can be anywhere between 18 and 28 degree celsius.

Now, one minute later you measured again and the measurement value comes out as 21. So, it can be any temperature between 16 and 26.

But, since temperature is increasing every minute we know that since a minute ago the minimum possible temperature was 18, it can not be less than 18 degree celsius. The temperature is between 18 and 26 degree celsius. This is wonderful, now the possible temperature range is actually 8 degrees instead of 10 degrees.

One minute later you measure the temperature and it comes out as 26 degree celsius. So, the actual temperature can be anything between 21 and 31 degree celsius. One minute later you measure again and you get 18 degree celsius as measurement. Which means the actual temperature is between 13 and 23 degree celsius.

But from previous measurement we know the the water temperature can not be less that 21 because are are continuously adding heat, it has to be more than 21 degree actually. So, the temperature will be between 21 and 23 degree celsius.

As you can see, even with a thermometer that has measurement inaccuracy range of 10 degrees, we have come within 2 degree of actual temperature with just 4 measurements. If we continue to measure the accuracy can improve. We can actually come very close to actual temperature fairly quickly.

This is the basic idea behind how the Kalman Filter works.

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