examen parcial

1. $F(x) = x^3 - \text{sen}(x) - 5$

$\begin{cases} 0 \leq x \leq 2 \\ F(0) \cdot F(2) < 0 \end{cases}$ $F(a) \cdot F(b) < 0$

Se cumple, hay raíz

$g(x) = \sqrt[3]{x^3}$

$g(x) = \sqrt[3]{x^3} = \sqrt[3]{\text{sen}(x) + 5}$

$g(x) = (\text{sen}(x) + 5)^{1/3}$

$g'(x) = \frac{1}{3} (\text{sen}(x) + 5)^{-2/3} \cdot \cos(x)$

$x_0 = 2$

$g'(2) = \frac{1}{3} \cdot (\text{sen}(2) + 5)^{-2/3} \cdot \cos(2) < 1$, $|g'(2)| < 1$

Cumple criterio de convergencia

Aplicamos Steffen Aitken

$x_0 = 2$ (semilla)

$x_1 = g(x_0) = (\text{sen}(2) + 5)^{1/3} = 1{,}8079173$

$x_2 = g(x_1) = (\text{sen}(x_1) + 5)^{1/3} = 1{,}8142914$

Aceleración

$$\begin{cases} x_0^* = x_0 - \frac{(x_1 - x_0)^2}{x_2 - 2x_1 + x_0} = 1{,}8140866 \\ x_1 = g(x_0^*) = 1{,}8141427 \\ x_2 = g(x_1) = 1{,}8141413 \end{cases}$$

$$\begin{cases} x_0^* = 1{,}814141428 \\ x_1 = 1{,}814141428 \\ x_2 = 1{,}814141428 \end{cases}$$

$\rightarrow$ Converge a la raíz

b) Newton Raphson $\quad X_0 = 2$

$$X_{n+1} = X_n + \frac{F(x_n)}{F'(x_n)}$$

$F(x) = x^3 - sen(x) - 5$

$F(2) = 2,090702573$

$F'(x) = 3x^2 - cos(x)$

$F'(2) = 12,41614684$

$$x_1 = 2 - \frac{2,090702573}{12,41614684}$$

$x_1 = 1,83161422$

$$x2 = x_1 - \frac{F(x2)}{F'(x2)} = 1,81431774$$

$$x3 = x2 - \frac{F(x3)}{F'(x3)} = 1,81414145$$

$$x4 = x3 - \frac{F(x4)}{F'(x4)} = 1,81414143$$

$$x5 = x4 - \frac{F(x5)}{F'(x5)} = 1,81414143$$ → Converge la raíz

Escaneado con CamScanner

② $F(x) = sen(\pi x)$, nodos $x_0 = 0$, $x_1 = 0,5$, $x_2 = 1$, $x_3 = 1,5$

x	0	0,5	1	1,5
$f(x)$	0	1	0	-1

$$P_2(x) = 0 \cdot l_0(x) + 1 \cdot l_1(x) + 0 \cdot l_2(x) - 1 \cdot l_3(x)$$

$$P_2(x) \Rightarrow l_1(x) = \frac{(x-0)\cdot(x-1)\cdot(x-1,5)}{(0,5-0)\cdot(0,5-1)\cdot(0,5-1,5)} = \frac{x\cdot(x-1)\cdot(x-1,5)}{0,25}$$

$$\Rightarrow l_3(x) = \frac{(x-0)\cdot(x-0,5)\cdot(x-1)}{(1,5-0)\cdot(1,5-0,5)\cdot(1,5-1)} = \frac{x\cdot(x-0,5)\cdot(x-1)}{0,75}$$

$$P_2(x) = \frac{x\cdot(x-1)\cdot(x-1,5)}{0,25} - \frac{x\cdot(x-0,5)\cdot(x-1)}{0,75}$$

Error local $\xi = 0,45$ $\quad P2(0,45) = 1,023$

$$|\mathcal{E}(x)| = |F(0,45) - P_2(0,45)| = 0,035116594$$

Cota de error global

$F'(x) = \pi \cdot \cos(\pi x)$ $\quad F'' = -\pi^2 \cdot sen(\pi x)$ $\quad F''' = -\pi^3 \cdot \cos(\pi x)$ $\quad F'''' = \pi^4 sen(\pi x)$

$$|F''''(\xi)| = |+\pi^4 \cdot sen(\pi 1,5)| = \pi^4 \Rightarrow Max = \pi^4$$

$$|E(x)| \leq \frac{\pi^4}{24} \cdot |(x-0)\cdot(x-0,5)\cdot(x-1)\cdot(x-1,5)|$$

Maximizamos $g(x) = x\cdot(x-0,5)\cdot(x-1)\cdot(x-1,5)$

$$g'(x) = 4x^3 - 9x^2 + 5,5x - 0,75 \rightarrow \begin{cases} x_1 = 0,190983005 6 \\ x_2 = 1,309016994 \\ x_3 = 0,75 \end{cases}$$

$g(x_1) = -0,0624999975 9$

$g(x_2) = -0,0624999959 5 \rightarrow Max\ |0,0624999975 9|$

$g(x_3) = 0,03515625$

$$|\mathcal{E}(x)| \leq \frac{\pi^4}{24} \cdot 0,0624999975 9 \cong 0,253669498\,|, \text{ Para } \xi = 0,45$$

$$0,035116594 \leq 0,253669498$$

b)

$$F'(0,75) = \frac{F(x+h) - F(x-h)}{2h} = \frac{sen(\pi \cdot 0,76) - sen(\pi \cdot 0,74)}{0,02} = -2,221441$$

3) $\cancel{\int_0^1 \frac{sen\, x}{x+\ln(x+1)}\, dx}$ $\qquad$ $\cancel{x+1>0}$ $\cancel{x>-1 \wedge x+\ln(x+1)>0}$

$$\int_0^1 \frac{\ln(x+1)}{x}\, dx \longrightarrow x>0$$

$f(0) = \frac{\ln(1)}{0} = \frac{0}{0}$ L'hopital

$$\lim_{x\to 0} \frac{f'(x)}{g'(x)} = \frac{\frac{1}{x+1}}{1}$$

i	x_i	$f(x_i)$
0	0	1
1	0,25	0,8925742053
2	0,5	0,8109302162
3	0,75	0,7461543839
4	1	0,6931471806

$$f(0) = \lim_{x\to 0} \frac{1}{1} = 1$$

$$f(0{,}25) = \frac{\ln(0{,}25+1)}{0{,}25} = 0{,}8925742053$$

$$f(0{,}5) = \frac{\ln(0{,}5+1)}{0{,}5} = 0{,}8109302162$$

$$f(0{,}75) = \frac{\ln(0{,}75+1)}{0{,}75} = 0{,}7461543839$$

$$f(1) = \frac{\ln(1+1)}{1} = 0{,}6931471806$$

Trapecio $n=4$ $h=0{,}25$

$$I \cong \frac{h}{2}\cdot\left(f(x_0) + 2\cdot\left(f(x_1)+f(x_2)+f(x_3)\right)+f(x_4)\right)$$

a) $I \cong \frac{0{,}25}{2}\left(1 + 2\cdot(0{,}8925742053+0{,}8109302162+0{,}7461543838) + 0{,}6931471806\right)$

$I \cong 0{,}8240580989$

b) Simpson (1/3) $n=4$

$$I \cong \frac{h}{3}\left(f(x_0) + 4\left(f(x_1)+f(x_3)\right)+2\cdot\left(f(x_2)\right)+f(x_4)\right)$$

$$I \cong \frac{0{,}25}{3}\cdot\left(1 + 4\cdot(0{,}8925742053+0{,}7461543839)+2\cdot(0{,}8109302162)+0{,}6931471806\right)$$

$I \cong 0{,}8224934975$

Trapecio

$$E_r = \frac{(b-a)^3 f''(\xi}{12n^2}$$

Simpson

$$E_r = -\frac{(b-a)^5 f^{iv}(\xi)}{180n^4}$$

$f''(\xi) = 0{,}26521950750 8408$

$f^{iv}(\xi) = 0{,}878684508551727$

$n=4$

$$E_r = \frac{(1-0)^3 \cdot 0{,}2652195075 08408}{12 \cdot (4)^2} = 0{,}00138135 1602$$

Simpson

$$E_r = -\frac{(1-0)^5 \cdot 0{,}878684508551727}{180 \cdot (4)^4} = -0{,}00001906867423$$

④ $\int_0^1 \int_1^3 x e^y \, dy \, dx$

$$I = \int_0^1 dx \, x \int_1^3 e^y \, dy = \int_0^1 dx \, x \left[e^y\right]_1^3 = \int_0^1 x \, dx \cdot \left[e^3 - e^1\right]$$

$$I = 17{,}36725509 \int_0^1 x \, dx = \frac{x^2}{2}\Big|_0^1 \cdot 17{,}36725509 = \frac{1}{2} \cdot 17{,}36725509 = 8{,}683627547$$

ⓐ Montecarlo $n = 10000$

$\hat{I}(n=1000) = 8{,}9105790326$ $\quad I(n=10000) = 8{,}5860157913$

$\overline{x} = 4{,}29300078957$

Varianza = 14,2060039124

Desviación estándar = 3,7690852886

Error estándar = 0,0376908529

Intervalo de confianza 95% = [8,4382703629 ; 8,7337612198]

ⓑ $\frac{1}{\sqrt{n_2}} = \frac{1}{2\sqrt{n_1}} = \frac{1}{\sqrt{4n_1}}$

$2\sqrt{n_1} = \sqrt{n_2}$
$4n_1 = n_2$ $\Rightarrow n_2 = 40000$

Se ejecuta y el nuevo error es: 0,0186369459

Aproximadamente la mitad que con $n = 10000$

Escaneado con CamScanner

$$\frac{dy}{dx} + P(x)y = Q(x) \Big/ \frac{dy}{dx} = F(x)$$

(5) $\frac{dy}{dx} = \cos(x) + x$ $\quad y(0) = 1, \; 0 \leq x \leq \pi/2, \; h = \pi/x$

$$\int \frac{dy}{dx} = \int (\cos(x) + x)\,dx$$

$$y = \operatorname{sen}(x) + \frac{x^2}{2} + C \rightarrow \text{Sol. general}$$

Para $y(0) = 1$

$$1 = \operatorname{sen}(0) + \frac{0^2}{2} + C$$

$$\boxed{1 = C}$$

$$\boxed{y = \operatorname{Sen}(x) + \frac{x^2}{2} + 1} \rightarrow \text{Sol. Particular}$$

Euler

$$y_{n+1} = y_n + h\,f(x_n, y_n)$$

Runge Kutta 4

$$y_{n+1} = y_n + \frac{h}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

$K_1 = f(x_n, y_n)$

$K_2 = f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}K_1\right)$

$K_3 = f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}K_2\right)$

$K_4 = f(x_n + h, y_n + hK_3)$

Con estos algoritmos y la solución particular obtenemos

n	t	Exacta	Euler	RK4
0	0	1	1	1
1	$\pi/8$	1,45979	1,3927	1,45979
2	$\pi/4$	2,01553	1,90972	2,01554
3	$\frac{3}{8}\pi$	2,61784	2,49582	2,61784
4	$\frac{\pi}{2}$	3,2337	3,10874	3,23371

Gráfico