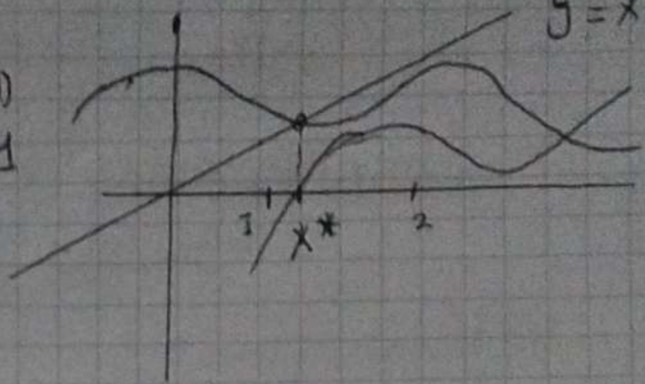
① $f(x) = Ln(x-1) + \cos(x-1)$

$\left\{ \begin{array}{l} 1.3 \leq x \leq 2 \\ \text{Hay raiz} \\ \text{Por Bolzano} \end{array} \right. \quad \left\{ \begin{array}{l} f(1.3) < 0 \\ f(2) > 0 \end{array} \right.$

$g(x) = x \quad , \quad f(x) = 0$

$Ln(x-1) = -\cos(x-1)$

$\Rightarrow x - 1 = e^{-\cos(x-1)} \Rightarrow g(x) = e^{-\cos(x-1)} + 1$



$g'(x) = \operatorname{sen}(x-1) e^{-\cos(x-1)}$

$x_0 = 1,5$

$g'(1,5) = \operatorname{sen}(1,5-1) e^{-\cos(1,5-1)} < 1 \quad , \quad |g'(1,5)| < 1$

Cumple criterio de convergencia con esa semilla.

Aplicando el algoritmo de Steffen Aitken

$x_0 = 1,5$ (semilla)

$x_1 = g(x_0) = e^{-\cos(1,5-1)} + 1 = 1,4157868366739$

$x_2 = g(x_1) = e^{-\cos(x_1 - 1)} + 1 = 1,4005972210981$

Aceleración

$$\left\{ \begin{array}{l} x_0^{*} = x_0 - \dfrac{(x_1 - x_0)^2}{x_2 - 2x_1 + x_0} = 1,3972545296709 \\ x_1 = g(x_0^{*}) = 1,3996724279774 \\ x_2 = g(x_1) = 1,3977367608356 \end{array} \right.$$

$$\left\{ \begin{array}{l} x_0^{*} = 1,3977484664948 \\ x_1 = 1,3977484745007 \\ x_2 = 1,3977484757341 \end{array} \right.$$

$x_0^* = 1{,}39774847 59587$

$x_1 = 1{,}3977484759587$

$x_2 = 1{,}3977484759587$

Converge a la raiz.

b) Newton Raphson $x_0 = 1{,}5$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$f(x) = \ln(x-1) + \cos(x-1)$

$f(1{,}5) = 0{,}184435381330427$

$f'(x) = \frac{1}{x-1} - \operatorname{sen}(x-1)$

$f'(1{,}5) = 1{,}52057446139580$

$$x_1 = 1{,}5 - \frac{0{,}184435381330427}{1{,}52057446139580}$$

$x_1 = 1{,}37870677430612$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1{,}39713581329249$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1{,}39774783699124$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1{,}39774847 59805$$

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} = 1{,}39774847595875$$

$$x_6 = x_5 - \frac{f(x_5)}{f'(x_5)} = 1{,}39774847595875$$

converge a la raiz

$f(x) = \operatorname{Sen} x$, interpolar en $x_0 = 0$, $x_1 = \pi/2$, $x_2 = \pi$

x	0	$\pi/2$	π
$f(x)$	0	1	0

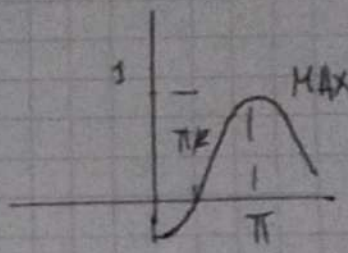
$P_2(x) = \sum_{i=0}^{n} y_i l_i(x)$, $P_2(x) = 0\, l_0(x) + 1 \cdot l_1(x) + 0\, l_2(x)$

$$P_2(x) = \frac{(x-0)}{(\pi/2-0)} \cdot \frac{(x-\pi)}{(\pi/2-\pi)} = -\frac{4}{\pi^2} x(x-\pi)$$

Error local. $\xi = \pi/4$ $\quad P_2(\pi/4) = 0{,}75$

$|E(x)| = |f(\pi/4) - P_2(\pi/4)| = 0{,}0428932188135$

Cota de Error global.



$f'(x) = \cos x$, $f''(x) = -\operatorname{Sen} x$, $f'''(x) = -\cos x$

$|f'''(\xi)| = |-\cos \pi| = 1 \rightarrow \text{MAX}|f'''(x)| = 1$

$$\boxed{|E(x)| \leq \frac{\text{MAX}|f'''(x)|}{(n+1)!} \text{MAX}\left|\prod_{i=0}^{n}(x-x_i)\right|}$$

$x = \xi$

$$|E(x)| \leq \frac{1}{6} \max_{x \in [0,\pi]} |(x-0)(x-\pi/2)(x-\pi)|$$

Maximizamos $g(x) = x(x-\pi/2)(x-\pi)$

$$g(x) = x^3 + x\left(-\pi x - \frac{\pi}{2} x\right) + \frac{\pi^2}{2} x = x^3 - \frac{3}{2}\pi x^2 + \frac{\pi^2}{2} x$$

$$g'(x) = 3x^2 - 3\pi x + \frac{\pi^2}{2} \rightarrow g'(x) = 0 \begin{cases} x_1 = 0{,}663896 \\ x_2 = 2{,}477696 \end{cases}$$

$$\begin{cases} g(x_1) = 1{,}491790 \\ g(x_2) = 1{,}491790 \end{cases} \quad \max_{x \in [0,\pi]} |x(x-\pi/2)(x-\pi)| = 1{,}491790$$

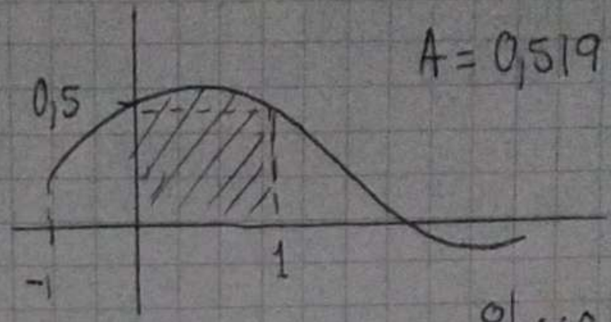
$|E(x)| \leq \frac{1}{6}(1{,}491790) \approx 0{,}248631$, Verificar para $\xi = \pi/4$

$0{,}0428932188135 \leq 0{,}248631$

b) $f'(0{,}785398) = f'(\pi/4) = \dfrac{\operatorname{Sen}(\pi/2) - \operatorname{Sen}(0)}{2(\pi/4)} = 0{,}63661977$

③ $\displaystyle\int_0^1 \frac{\operatorname{sen}(x)}{x+\ln(1+x)}\,dx$

$A = 0{,}5191212086313$



$f(x) = \dfrac{\operatorname{sen} x}{x+\ln(x+1)}$ Dom $x+1>0$ $x>-1$

i	x_i	$f(x_i)$
0	0	0,5
1	0,25	0,522894
2	0,5	0,529479
3	0,75	0,520487
4	1	0,496986

$f(0) = \dfrac{\operatorname{sen} 0}{0+\ln(0+1)} = \dfrac{0}{0} \Rightarrow$ L'Hopital

$\lim\limits_{x\to 0} \dfrac{f(x)}{g(x)} = \lim\limits_{x\to 0} \dfrac{f'(x)}{g'(x)} = \dfrac{\cos x}{1+\frac{1}{x+1}}$

$f(0) = \lim\limits_{x\to 0} \dfrac{\cos(0)}{1+\frac{1}{0+1}} = 0{,}5$

$f(0{,}25) = \dfrac{\operatorname{sen}(0{,}25)}{0{,}25+\ln(1+0{,}25)} = 0{,}522894$

$f(0{,}5) = 0{,}529479$ $f(0{,}75) = 0{,}520487$

$f(1) = 0{,}496986$ trapecio $n=4$, $h=0{,}25$

$I \overset{T}{\approx} \frac{h}{2}\left(f(x_0)+2\left(f(x_1)+f(x_2)+f(x_3)\right)+f(x_4)\right)$

a) $I \overset{T}{\approx} \frac{0{,}2}{2}\left(0{,}5+2(0{,}522894+0{,}529479+0{,}520487)+0{,}496986\right)$

$I \approx 0{,}51983825$

b) Simpson (1/3) $n=4$

$I \overset{S}{\approx} \frac{h}{3}\left(f(x_0)+4\left(f(x_1)+f(x_3)\right)+2\left(f(x_2)\right)+f(x_4)\right)$

$I \overset{S}{\approx} \frac{0{,}25}{3}\left(0{,}5+4(0{,}522894+0{,}520487)+2(0{,}529479)+0{,}496986\right)$

$I \approx 0{,}51912233$

Calculos de los Errores de truncamiento

Trapecio $\xi = 0,5$ $f''(0,5) = \sqrt{2}(3,852076)$

$n=4$ $E_T = \frac{(b-a)^3}{12n^2} f''(\xi)$

$E = \frac{(1-0)^3}{12(4)^2} \cdot \sqrt{2}(3,852076) = 0,028373$

$n=10$

$\Rightarrow$ trapecio $E_T = \frac{(b-a)^3}{12n^2} f''(\xi)$

Simpson $E_n = -\frac{(b-a)^5 f^4(\xi)}{180n^4}$

$f''(\xi) = -0,249730$

$f^4(\xi) = 0,0937049$

$n=4$ $E_T = \frac{-(1-0)^3}{12(4)^2}(-0,249730) = 0,0013006$

Simpson $E_n = \frac{-(1-0)^5 (0,0937049}{180(4)^4} = -0,0000020335265$

Codigo empleado:

```
import sympy as sp
def derivar_evaluar(funcion, variable, orden, punto):
# Definir variable simbólico
  x = sp.Symbol(variable)
  # convertir la funcion en expresión simbólica
    f = sp.symify(funcion)
  # Calcular la enesima derivada
    derivada_n = sp.diff(f, x, orden)
    # Evaluar la derivada en el punto
    valor_evaluado = derivada_n.subs(x, punto)
    return derivada_n, valor_evaluado
  # Ejemplo
  funcion = "sin(x)"
  variable = "x"
  orden = 4 # cuarta derivada
  punto = 0,5   # x=0,5
  derivada, valor = derivar_evaluar(funcion, variable, orden, punto)
  print(f"derivada{orden} es={derivada}")
                              print(f"evaluada en{punto} da={valor}")
```

Dada $I = \int_0^2 \int_0^2 e^{x-y} \, dy\,dx$

$$I = \int_0^2 dx\, e^x \int_0^2 e^{-y} dy = \int_0^2 dx\, e^x \left[-e^{-y}\right]_0^2 = -\int_0^2 e^x dx \left[e^{-2} - e^{-0}\right]$$

$$I = (1-e^{-2}) \int_0^2 e^x dx = (1-e^{-2})(e^2 - e^0) = 5{,}5243913821673$$

a) Ejecutando un codigo (Para montecarlo) $n = 10.000$

$\hat{I} = 5{,}4362356393473l7$ (1000n) $\hat{I} = 5{,}5486616404586 08$

valor exacto (cuadratura/o solución Analítica = 5,524391

$\bar{X} = 1{,}38716541011 4652$

Varianza Muestral = 1,40653426543321

Desviación Estandar Muestral = 1,18597397338 71935

Error Estandar = 0,0118597373337 1935

Intervalo de Confianza 95% : [5,52541655058 1199, 5,71190673033 6617]

b) $\frac{1}{\sqrt{n_1}} = \frac{1}{2\sqrt{n_2}} = \frac{1}{\sqrt{4n_2}}$ $\quad n_2 = 4n_1$

$n_2 = 40.000$

Se ejecuta el código y la simolacción muestra

un nuevo error = 0,0058759178076 1654

Aproximadamente la mitad que con $n = 10.000$

5) $\frac{dy}{dx} = x e^{-\operatorname{sen}(x)} - y\cos x \qquad y(0)=1,\ 0 \leq x \leq \pi,\ h=\pi/4$

Solucion Analitica

$$\mu(x) = e^{\int \cos x\, dx}$$

$$\frac{dy}{dx} + y\cos x = x e^{-\operatorname{sen} x} \qquad \mu(x) = e^{\operatorname{sen} x}$$

$$\frac{dy}{dx} e^{\operatorname{sen} x} + y e^{\operatorname{sen} x} \cos x = x e^{-\operatorname{sen} x} e^{\operatorname{sen} x}$$

$$\int \frac{d}{dx}\left(y e^{\operatorname{sen} x}\right) dx = \int x\, dx$$

$$y e^{\operatorname{sen} x} = \frac{x^2}{2} + c$$

$$y = \frac{1}{2}x^2 e^{-\operatorname{sen} x} + c e^{-\operatorname{sen} x} \qquad \text{Para } y(0)=1$$

$$1 = \frac{1}{2}(0)^2 e^{-\operatorname{sen}(0)} + c e^{-\operatorname{sen}(0)} \Rightarrow c = 1$$

$$\boxed{y = \frac{1}{2}x^2 e^{-\operatorname{sen} x} + e^{-\operatorname{sen} x}} \quad \text{Solucion Particular}$$

Euler $\quad y_{n+1} = y_n + h f(x_n, y_n)$

Runge-kutta 4. $\quad y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$k_1 = f(x_n, y_n)$

$k_2 = f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}k_1\right)$

$k_3 = f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}k_2\right)$

$k_4 = f(x_n + h, y_n + hk_3)$

Con estos algoritmos y la solución particular se obtiene.

n	t	Exacta	Euler	RK4
0	0	1	1	1
1	$\pi/4$	0,645143	0,214602	0,646520
2	$\pi/2$	0,821733	0,399570	0,822013
3	$3\pi/4$	1,861742	0,853423	1,860275
4	π	5,934802	2,239829	5,919802