VYSOKÉ UČENÍ TECHNICKÉ V BRNĚ
FAKULTA INFORMAČNÍCH TECHNOLOGIÍ

Elektronika pro informační technologie

Semestrální projekt

12. decembra 2015

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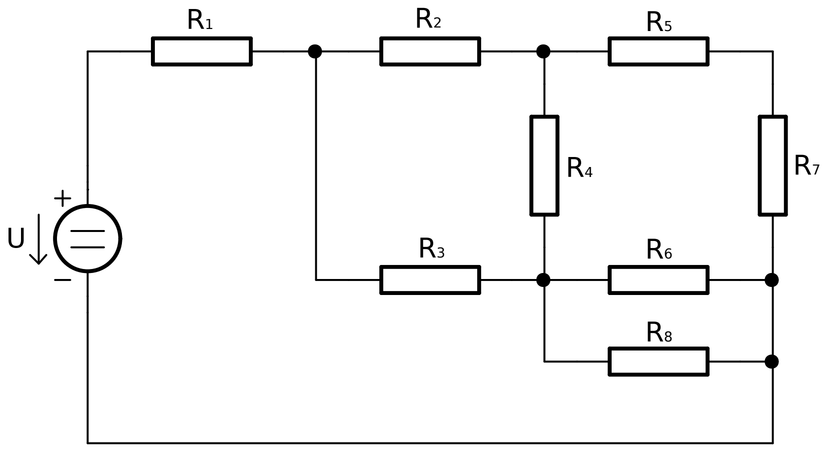
1 Úloha č. 1

1.1 Zadanie

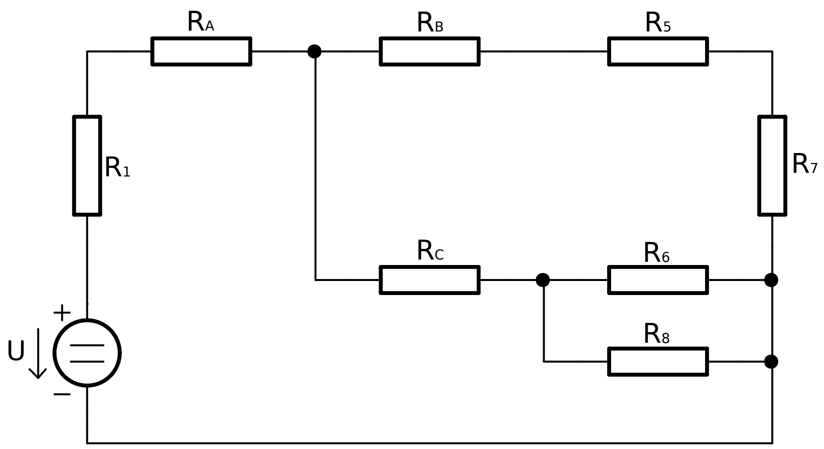
Stanovte napětí U_{R3} a proud I_{R3}. Použijte metodu postupného zjednodušování obvodu.

sk.	$U[V]$	$R_1[\Omega]$	$R_2[\Omega]$	$R_3[\Omega]$	$R_4[\Omega]$	$R_5[\Omega]$	$R_6[\Omega]$	$R_7[\Omega]$	$R_8[\Omega]$
F	125	510	500	550	250	300	800	330	250



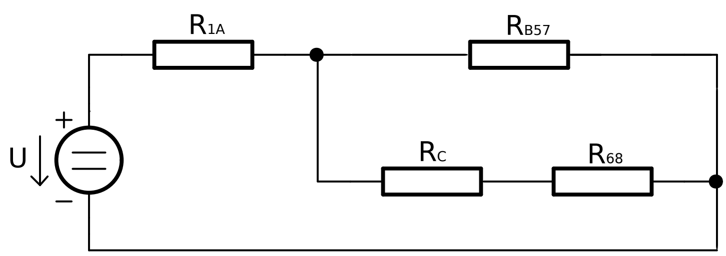
1.2 Riešenie

Prevedieme rezistory R_2, R_3, R_4 z trojuholníka na hviezdu:



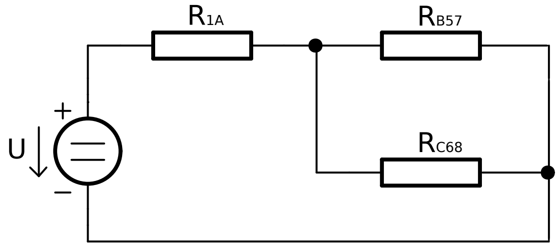
$$R_A = \frac{R_2.R_3}{R_2 + R_3 + R_4} = \frac{500.550}{500 + 550 + 250} = 211,53846\ \Omega$$
$$R_B = \frac{R_2.R_4}{R_2 + R_3 + R_4} = \frac{500.250}{500 + 550 + 250} = 96,153846\ \Omega$$
$$R_C = \frac{R_3.R_4}{R_2 + R_3 + R_4} = \frac{550.250}{500 + 550 + 250} = 105,76923\ \Omega$$

Vypočítame rezistory R_1 a R_A v sérií, ďalej R_B, R_5 a R_7 tiež v sérií, a R_6 a R_8 paralelne:



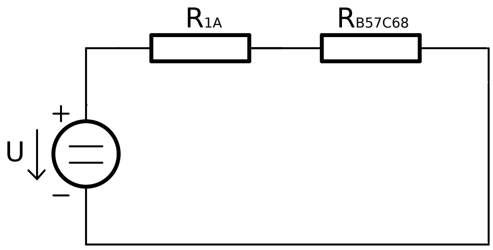
$$R_{1A} = R_1 + R_A = 510 + 211,53846 = 721,53846\ \Omega$$
$$R_{B57} = R_B + R_5 + R_7 = 96,153846 + 300 + 330 = 726,153846\ \Omega$$
$$R_{68} = \frac{R_6.R_8}{R_6 + R_8} = \frac{800.250}{800 + 250} = 190,47619\ \Omega$$

Vypočítame rezistory R_C a R_{68} v sérií:



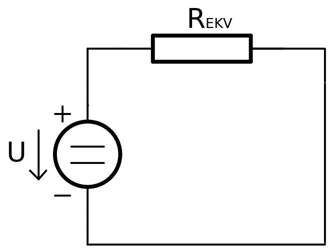
$$R_{C68} = R_C + R_{68} = 105,76923 + 190,47619 = 296,24542\ \Omega$$

Vypočítame rezistory R_{B57} a R_{C68} paralelne:



$$R_{B57C68} = \frac{R_{B57}.R_{C68}}{R_{B57} + R_{C68}} = \frac{726,153846.296,24542}{726,153846 + 296,24542} = 210,40679\ \Omega$$

Vypočítame si výsledný odpor obvodu R_{EKV}:

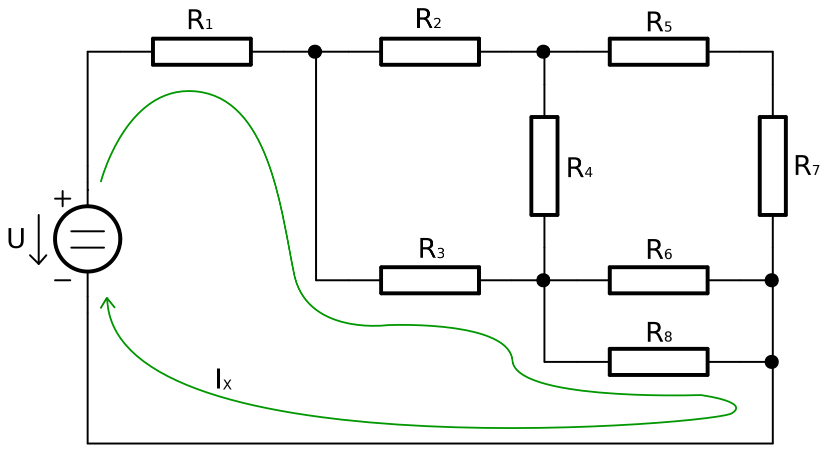


$$R_{EKV} = R_{1A} + R_{B57C68} = 721,53846 + 210,40679 = 931,94525\ \Omega$$

Z odporu R_{EKV} vypočítame prúd I tečúci obvodom:

$$I = \frac{U}{R_{EKV}} = \frac{125}{931,94525} = 0,13413\ A$$

V obvode si vytvoríme slučku, aby sme mohli vypočítať napätie U_{R3}:



Napíšeme si pre slučku I_X rovnicu:

$$I_X : U_{R1} + U_{R3} + U_{R8} - U = 0$$

Aby sme mohli vypočítať U_{R3}, potrebujeme poznať aj U_{R1} a U_{R8}. Prvé vypočítame U_{R1}:

$$U_{R1} = R_1 I = 510.0,13413 = 68,4063\ V$$

Druhé vypočítame U_{R8}:

$$\begin{aligned}
&U_{RB57C68} = R_{B57C68}.I = 210,40679.0,13413 = 28,221862743\ V\\
&I_{RC68} = \frac{U_{RB57C68}}{R_{C68}} = \frac{28,221862743}{296,24542} = 0,095265144\ A\\
&U_{R68} = R_{68}.I_{RC68} = 190,47619.0,095265144 = 18,145741669\ V\\
&U_{R8} = U_{R68} = 190,47619.0,095265144 = 18,145741669\ V
\end{aligned}$$

Dosadíme hodnoty do rovnice a vypočítame U_{R3}:

$$\begin{aligned}
&I_X : U_{R1} + U_{R3} + U_{R8} - U = 0\\
&I_X : U_{R3} = U - U_{R1} - U_{R8}\\
&I_X : U_{R3} = 125 - 68,4063 - 18,145741669 = 38,4480\ V
\end{aligned}$$

Posledné vypočítame I_{R3}:

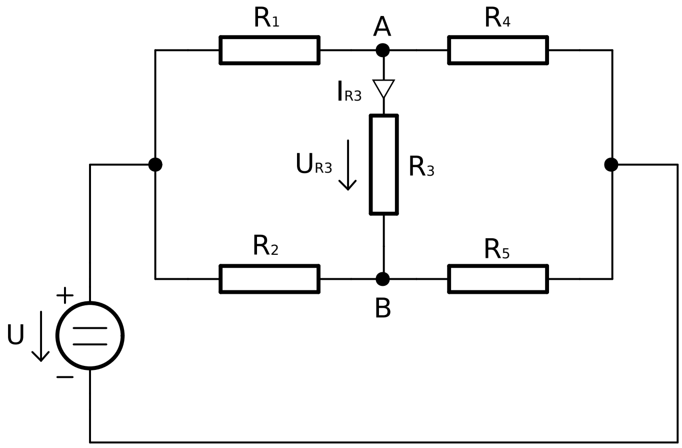
$$I_{R3} = \frac{U_{R3}}{R_3} = \frac{38,447958331}{550} = 0,0699\ A$$

2 Úloha č. 2

2.1 Zadanie

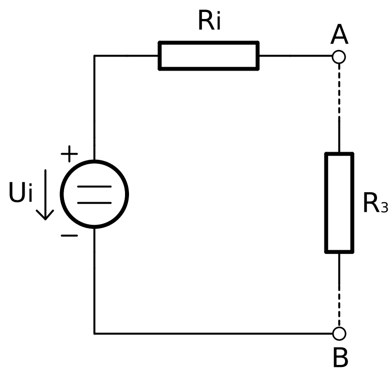
Stanovte napětí U_{R3} a proud I_{R3}. Použijte metodu Théveninovy věty.

sk.	$U[V]$	$R_1[\Omega]$	$R_2[\Omega]$	$R_3[\Omega]$	$R_4[\Omega]$	$R_5[\Omega]$
A	50	525	620	210	530	130



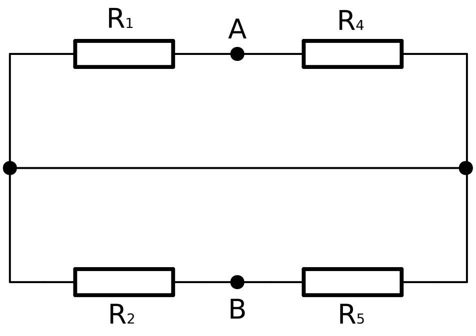
2.2 Riešenie

Vytvoríme si náhradný obvod:



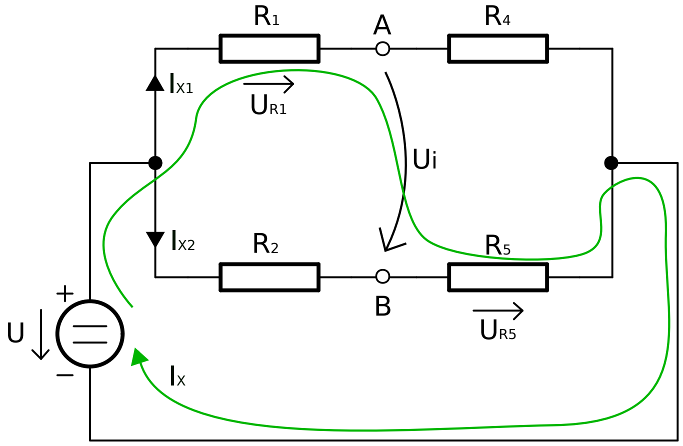
$$I_{R3} = \frac{U_i}{R_3 + R_i}$$

Výpočet odporu R_i v náhradnom v obvode:



$$R_i = R_{AB} = \frac{R_1.R_4}{R_1 + R_4} + \frac{R_2.R_5}{R_2 + R_5}$$

Výpočet napätia U_i v náhradnom v obvode:



Vytvoríme si v obvode slučku aby sme mohli vypočítať napätie U_i a zostavíme pre ňu rovnicu:

$$U_{R1} + U_i + U_{R5} - U = 0$$
$$I_{X1}.R_1 + U_i + I_{X2}.R_5 - U = 0$$
$$U_i = U - I_{X1}.R_1 - I_{X2}.R_5$$

Potrebujeme vypočítať I_{X1} a I_{X2}, to urobíme pomocou metody slučkových prúdov:

$$U_{R1} + U_{R4} - U = 0$$
$$R_2 I_{X1} + R_5 I_{X1} = U$$
$$I_{X1}(R_2 + R_5) = U$$
$$I_{X1} = \frac{U}{R_2 + R_5}$$

$$U_{R2} + U_{R5} - U = 0$$
$$R_1 I_{X2} + R_4 I_{X2} = U$$
$$I_{X2}(R_1 + R_4) = U$$
$$I_{X2} = \frac{U}{R_1 + R_4}$$

Dosadíme I_{X1} a I_{X2} do pôvodnej rovnice a vyjadríme si U_i:

$$U_i = U - R_1.\frac{U}{R_1 + R_4} - R_5.\frac{U}{R_2 + R_5}$$
$$U_i = U.\left(1 - \frac{R_1}{R_1 + R_4} - \frac{R_5}{R_2 + R_5}\right)$$

Máme všetky potrebné hodnoty pre výpočet I_{R3}. Dosadíme a vypočítame:

$$I_{R3} = \frac{U_i}{R_3 + R_i} = \frac{U.\left(1 - \frac{R_1}{R_1+R_4} - \frac{R_5}{R_2+R_5}\right)}{R_{R3} + \frac{R_1.R_4}{R_1+R_4} + \frac{R_2.R_5}{R_2+R_5}} = \frac{50.\left(1 - \frac{525}{525+530} - \frac{130}{620+130}\right)}{210 + \frac{525.530}{525+530} + \frac{620.130}{620+130}} = 0,0283\ A$$

Z prúdu I_{R3} vypočítame aj napätie U_{R3}:

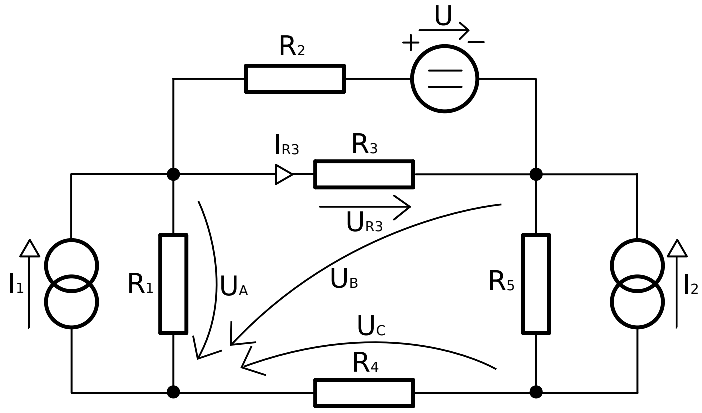
$$U_{R3} = R_3.I_{R3} = 210.0,028306 = 5,9463\ V$$

3 Úloha č. 3

3.1 Zadanie

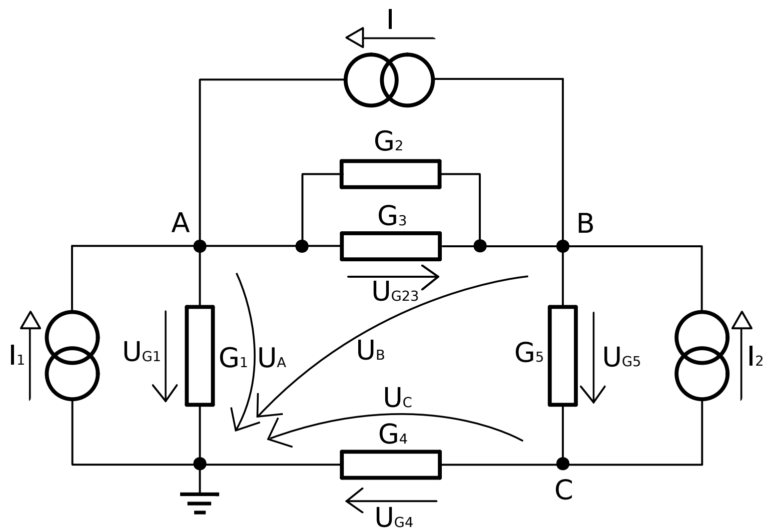
Stanovte napětí U_{R3} a proud I_{R3}. Použijte metodu uzlových napětí (U_A, U_B, U_C).

sk.	$U[V]$	$I_1[A]$	$I_2[A]$	$R_1[\Omega]$	$R_2[\Omega]$	$R_3[\Omega]$	$R_4[\Omega]$	$R_5[\Omega]$
A	50	0,55	0.65	525	620	210	530	130



3.2 Riešenie

V obvode si premeníme napäťový zdroj U na prúdový zdroj I. Ďalej si premeníme rezistory na vodivosti a v obvode si určíme si kde sa nachádza referenčný uzol a zvyšné uzly si označíme písmenami A, B a C. Potom bude obvod vyzerať nasledovne:



Ďalej si zostavíme rovnice podľa I. Kirchohoffoveho zákona:

$A : I_1 + I - I_{G23} - I_{G1} = 0$
$B : I_2 + I_{G23} - I - I_{G5} = 0$
$C : I_{G5} - I_2 - I_{G4} = 0$

$A : I_1 + I - (G_2 + G_3).(U_A - U_B) - U_A.G_1 = 0$
$A : I_1 + I - U_A.G_2 - U_A.G_3 + U_B.G_2 + U_B.G_3 - U_A.G_1 = 0$
$A : U_A.(G_1 + G_2 + G_3) - U_B.(G_2 + G_3) = I_1 + I$

$B: I_2 + (G_2 + G_3).(U_A - U_B) - I - G_5.(U_B - U_C) = 0$
$B: I_2 + U_A.G_2 + U_A.G_3 - U_B.G_2 - U_B.G_3 - I - G_5.U_B + G_5.U_C = 0$
$B: U_A.(G_2 + G_3) - U_B.(G_2 + G_3 + G_5) + U_C.G_5 = I - I_2$

$C: G_5.(U_B - U_C) - G_4.U_C - I_2 = 0$
$C: G_5.U_B - G_5.U_C - G_4.U_C - I_2 = 0$
$C: U_B.G_5 - U_C.(G_4 + G_5) = I_2$

Ďalej riešime sústavu rovníc Cramerovým pravidlom:

$$\begin{pmatrix} G_1+G_2+G_3 & -G_2-G_3 & 0 \\ G_2+G_3 & -G_2-G_3-G_5 & G_5 \\ 0 & G_5 & -G_4-G_5 \end{pmatrix} \begin{pmatrix} U_A \\ U_B \\ U_C \end{pmatrix} = \begin{pmatrix} I_1+I \\ I-I_2 \\ I_2 \end{pmatrix}$$

$$\begin{aligned} D_S = &\begin{vmatrix} G_1+G_2+G_3 & -G_2-G_3 & 0 \\ G_2+G_3 & -G_2-G_3-G_5 & G_5 \\ 0 & G_5 & -G_4-G_5 \end{vmatrix} = \\ &= (G_1+G_2+G_3)\,.\,(-G_2-G_3-G_5)\,.\,(-G_4-G_5) - (G_5)\,.\,(G_5)\,.\,(G_1+G_2+G_3) \\ &\quad - (-G_4-G_5)\,.\,(G_2+G_3)\,.\,(-G_2-G_3) = \\ &= \left(\frac{1}{520}+\frac{1}{420}+\frac{1}{520}\right)\cdot\left(-\frac{1}{420}-\frac{1}{520}-\frac{1}{215}\right)\cdot\left(-\frac{1}{420}-\frac{1}{215}\right) \\ &\quad - \left(\frac{1}{215}\right)\cdot\left(\frac{1}{215}\right)\cdot\left(\frac{1}{520}+\frac{1}{420}+\frac{1}{520}\right) - \left(-\frac{1}{420}-\frac{1}{215}\right)\cdot\left(\frac{1}{420}+\frac{1}{520}\right)\cdot\left(-\frac{1}{420}-\frac{1}{520}\right) = \\ &= 1,27165 \times 10^{-7} \end{aligned}$$

$$\begin{aligned} D_A = &\begin{vmatrix} I_1+I & -G_2-G_3 & 0 \\ I-I_2 & -G_2-G_3-G_5 & G_5 \\ I_2 & G_5 & -G_4-G_5 \end{vmatrix} = \\ &= (I_1+I)\,.\,(-G_2-G_3-G_5)\,.\,(-G_4-G_5) + (-G_2-G_3)\,.\,(G_5)\,.\,(I_2) - (G_5)\,.\,(G_5)\,.\,(I_1+I) \\ &\quad - (-G_4-G_5)\,.\,(I-I_2)\,.\,(-G_2-G_3) = \\ &= \left(\frac{11}{20}+\frac{9}{28}\right)\cdot\left(-\frac{1}{420}-\frac{1}{520}-\frac{1}{215}\right)\cdot\left(-\frac{1}{420}-\frac{1}{215}\right) + \left(-\frac{1}{420}-\frac{1}{520}\right)\cdot\left(\frac{1}{215}\right)\cdot\left(\frac{13}{20}\right) \\ &\quad - \left(\frac{1}{215}\right)\cdot\left(\frac{1}{215}\right)\cdot\left(\frac{11}{20}+\frac{9}{28}\right) - \left(-\frac{1}{420}-\frac{1}{215}\right)\cdot\left(\frac{9}{28}-\frac{13}{20}\right)\cdot\left(-\frac{1}{420}-\frac{1}{520}\right) = \\ &= 3,29579 \times 10^{-5} \end{aligned}$$

$$\begin{aligned} D_B = &\begin{vmatrix} G_1+G_2+G_3 & I_1+I & 0 \\ G_2+G_3 & I-I_2 & G_5 \\ 0 & I_2 & -G_4-G_5 \end{vmatrix} = \\ &= (G_1+G_2+G_3)\,.\,(I-I_2)\,.\,(-G_4-G_5) - (I_2)\,.\,(G_5)\,.\,(G_1+G_2+G_3) \\ &\quad - (-G_4-G_5)\,.\,(G_2+G_3)\,.\,(I_1+I) = \\ &= \left(\frac{1}{520}+\frac{1}{420}+\frac{1}{520}\right)\cdot\left(\frac{9}{28}-\frac{13}{20}\right)\cdot\left(-\frac{1}{420}-\frac{1}{215}\right) - \left(\frac{13}{20}\right)\cdot\left(\frac{1}{215}\right)\cdot\left(\frac{1}{520}+\frac{1}{420}+\frac{1}{520}\right) \\ &\quad - \left(-\frac{1}{420}-\frac{1}{215}\right)\cdot\left(\frac{1}{420}+\frac{1}{520}\right)\cdot\left(\frac{11}{20}+\frac{9}{28}\right) = \\ &= 2,193695 \times 10^{-5} \end{aligned}$$

$$D_C = \begin{vmatrix} G_1+G_2+G_3 & -G_2-G_3 & I_1+I \\ G_2+G_3 & -G_2-G_3-G_5 & I-I_2 \\ 0 & G_5 & I_2 \end{vmatrix} =$$

$$= (G_1+G_2+G_3)\,.\,(-G_2-G_3-G_5)\,.\,(I_2) + (I_1+I)\,.\,(G_2+G_3)\,.\,(G_5)$$
$$-\,(G_5)\,.\,(I-I_2)\,.\,(G_1+G_2+G_3) - (I_2)\,.\,(G_2+G_3)\,.\,(-G_2-G_3) =$$
$$= \left(\frac{1}{520}+\frac{1}{420}+\frac{1}{520}\right).\left(-\frac{1}{420}-\frac{1}{520}-\frac{1}{215}\right).\left(\frac{13}{20}\right)+\left(\frac{11}{20}+\frac{9}{28}\right).\left(\frac{1}{420}+\frac{1}{520}\right).\left(\frac{1}{215}\right)$$
$$-\left(\frac{1}{215}\right).\left(\frac{9}{28}-\frac{13}{20}\right).\left(\frac{1}{520}+\frac{1}{420}+\frac{1}{520}\right)-\left(\frac{13}{20}\right).\left(\frac{1}{420}+\frac{1}{520}\right).\left(-\frac{1}{420}-\frac{1}{520}\right)=$$
$$=2.75524\times 10^{-6}$$

Z determinantov vypočítame napätia U_A, U_B a U_C:

$$U_A = \frac{D_A}{D_S} = \frac{3,29579\times 10^{-5}}{1,27165\times 10^{-7}} = 259,1743\ V$$
$$U_B = \frac{D_B}{D_S} = \frac{2,193695\times 10^{-5}}{1,27165\times 10^{-7}} = 172,50777\ V$$
$$U_C = \frac{D_C}{D_S} = \frac{2.75524\times 10^{-6}}{1,27165\times 10^{-7}} = 21,66665\ V$$

Nakoniec si vyjadríme U_{R3} a vypočítame:

$$U_{R3} + U_B - U_A = 0$$
$$U_{R3} = U_A - U_B$$
$$U_{R3} = 259,1743 - 172,50777 = 86,6667\ V$$

Vypočítame I_{R3}:

$$I_{R3} = \frac{U_{R3}}{R_3} = \frac{86,666}{520} = 0,1667\ A$$

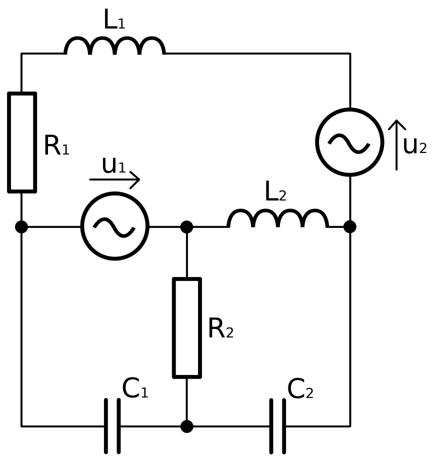
4 Úloha č. 4

4.1 Zadanie

Pro napájecí napětí platí: $u_1 = U_1.sin(2\pi ft)$, $u_2 = U_2.sin(2\pi ft)$.
Ve vztahu pro napětí $u_{C2} = U_{C2}.sin(2\pi ft + \varphi_{C_2})$ určete $|U_{C2}|$ a φ_{C_2}. Použijte metodu smyčkových proudů.

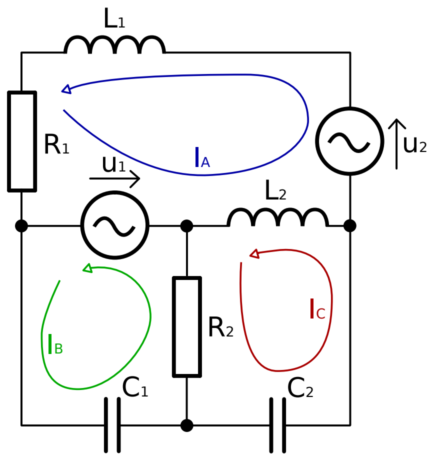
Pozn: Pomocné směry šipek napájecích zdrojů platí pro speciální časový okamžik $\left(t = \frac{\pi}{2\omega}\right)$.

sk.	$U_1[V]$	$U_2[V]$	$R_1[\Omega]$	$R_2[\Omega]$	$L_1[mH]$	$L_2[mH]$	$C_1[\mu F]$	$C_2[\mu F]$	$f[Hz]$
F	20	35	120	100	170	80	150	90	65



4.2 Riešenie

V obvode si nakreslíme tri slučky a zostavíme pre ne rovnice:



Pre zostavenie rovnice potrebujeme poznať reaktanciu kondenzátora a reaktanciu cievky:

$$X_C = \frac{1}{j\omega C} \qquad\qquad X_L = j\omega L$$

Zostavíme rovnice:

$$I_A : I_A\left(j\omega L_1 + R_1 + j\omega L_2\right) - I_C(j\omega L_2) = -U_1 - U_2$$

$$I_B : I_B\left(\frac{1}{j\omega C_1} + R_2\right) - I_C R_2 = U_1$$

$$I_C : I_C\left(R_2 + \frac{1}{j\omega C_2} + j\omega L_2\right) - I_A(j\omega L_2) - I_B R_2 = 0$$

$$\begin{pmatrix} j\omega L_1 + R_1 + j\omega L_2 & 0 & -j\omega L_2 \\ 0 & R_2 + \frac{1}{j\omega C_1} & -R_2 \\ -j\omega L_2 & -R_2 & R_2 + \frac{1}{j\omega C_2} + j\omega L_2 \end{pmatrix} \begin{pmatrix} I_A \\ I_B \\ I_C \end{pmatrix} = \begin{pmatrix} -U_1 - U_2 \\ U_1 \\ 0 \end{pmatrix}$$

Vypočítame sústavu rovníc Cramerovým pravidlom:

$$\omega = 2\pi f = 2\pi 65 = 130\pi$$

$$\begin{pmatrix} 120 + 102,1j & 0 & -32,67j \\ 0 & 100 - 16,32j & -100 \\ -32,76i & -100 & 100 + 5,47j \end{pmatrix} \begin{pmatrix} I_A \\ I_B \\ I_C \end{pmatrix} = \begin{pmatrix} -55 \\ 20 \\ 0 \end{pmatrix}$$

$$\begin{aligned}
D_S =& \begin{vmatrix} 120 + 102,1j & 0 & -32,67j \\ 0 & 100 - 16,32j & -100 \\ -32,67j & -100 & 100 + 5,47j \end{vmatrix} = (120 + 102,1j).(100 - 16,32j).(100 + 5,47j) \\
&- (-32,67j + 5,47j).(100 - 16,32j).(-32,67j) - (-100).(-100).(120 + 102,1j) = \\
=& 2,2831 \times 10^5 - 1.386j \times 10^5 \\
D_A =& \begin{vmatrix} -55 & 0 & -32,67j \\ 20 & 100 - 16,32j & -100 \\ 0 & -100 & 100 + 5,47j \end{vmatrix} = (-55).(100 - 16,32j).(100 + 5,47j) \\
&+ (-32,67j).(20).(-100) - (-100).(-100).(-55) = -4.9079 \times 10^3 + 1.2506j \times 10^5 \\
D_B =& \begin{vmatrix} 120 + 102,1j & -55 & -32,67j \\ 0 & 20 & -100 \\ -32,67j & 0 & 100 + 5,47j \end{vmatrix} = (120 + 102,1j).(20).(100 + 5,47j) \\
&+ (-55).(-100).(-32,67j + 5,47j) - (-32,67j + 5,47j).(20).(-32,67j) = \\
=& 2.5019 \times 10^5 + 3.7624j \times 10^4 \\
D_C =& \begin{vmatrix} 120 + 102,1j & 0 & -55 \\ 0 & 100 - 16,32j & 20 \\ -32,67j & -100 & 0 \end{vmatrix} = -(-32,67j).(100 - 16,32j).(-55) \\
&- (-100).(20).(120 + 102,1j) = 2.1067 \times 10^5 + 2.4504j \times 10^4
\end{aligned}$$

Z determinantov vypočítame jednotlivé prúdy:

$$\begin{aligned}
I_A =& \frac{D_A}{D_S} = \frac{-4.9079 \times 10^3 + 1.2506j \times 10^5}{2,2831 \times 10^5 - 1.386j \times 10^5} = -0.25869 + 0.39072i\ A \\
I_B =& \frac{D_B}{D_S} = \frac{2.5019 \times 10^5 + 3.7624j \times 10^4}{2,2831 \times 10^5 - 1.386j \times 10^5} = 0.72763 + 0.60651i\ A \\
I_C =& \frac{D_C}{D_S} = \frac{2.1067 \times 10^5 + 2.4504j \times 10^4}{2,2831 \times 10^5 - 1.386j \times 10^5} = 0.62664 + 0.48774i\ A
\end{aligned}$$

Vypočítame napätie U_{C2} a $|U_{C2}|$:

$$\begin{aligned}
&I_{C2} = I_C \\
&U_{C2} = X_{C2}.I_{C2} = \frac{1}{j\omega C_2} I_{C2} = \frac{0.627 + 0.488j}{130\pi j 9 \times 10^{-5}} = 13.269 - 17.048j\ V \\
&|U_{C2}| = \sqrt{13.269 + (-17.048j)^2} = 21.6037\ V
\end{aligned}$$

Vypočítame fázový posuv φ_{C2} z napätia U_{C2}:

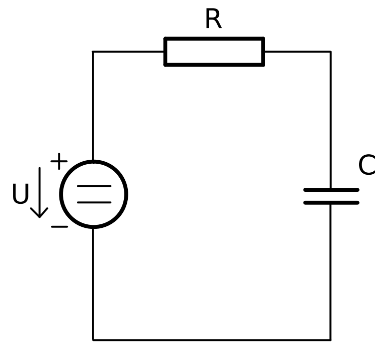
$$\varphi_{C2} = arctan\left(\frac{Im(U_{C2})}{Re(U_{C2})}\right) = arctan\left(\frac{-17.048}{13.269}\right) = -0,9094\ rad = 127^\circ\ 53'\ 42,38''$$

5 Úloha č. 5

5.1 Zadanie

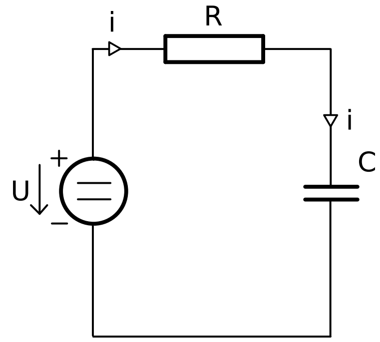
Sestavte diferenciální rovnici popisující chování obvodu na obrázku, dále ji upravte dosazením hodnot parametrů. Vypočítejte analytické řešení $u_C = f(t)$. Proveďte kontrolu výpočtu dosazením do sestavené diferenciální rovnice.

sk.	$U[V]$	$C[F]$	$R[\Omega]$	$u_C(0)[V]$
A	20	40	10	9



5.2 Riešenie

Axiom: $u'_C = \frac{1}{C} i_C$
Obvodom tečie prúd i:



$$i_C = i \qquad\qquad u'_C = \frac{1}{C} i$$

$$\begin{aligned} U_R + u_C - U &= 0 \\ R.i &= U - u_C \\ i &= \frac{U - u_C}{R} \end{aligned}$$

Dosadíme do u'_C:

$$u'_C = \frac{1}{C} i = \frac{1}{C} \frac{U - u_C}{R} = \frac{U - u_C}{CR}$$

Diferenciálna rovnica je:

$$u'_C = \frac{U - u_C}{CR} \qquad\qquad u_C(0) = 9\ V$$

Dosadíme zadané hodnoty:

$$\begin{aligned} u'_C &= \frac{20 - u_C}{40.10} \\ 400u'_C &= 20 - u_C \\ 400u'_C + u_C &= 20 \\ 20u'_C + \frac{u_C}{20} &= 1 \end{aligned}$$

Vytvoríme charakteristrickú rovnicu:

$$\begin{aligned} 20\lambda + \frac{1}{20} &= 0 \\ 20\lambda &= -\frac{1}{20} \\ \lambda &= -\frac{1}{400} \end{aligned}$$

Očakávané riešenie rovnice:

$$\begin{aligned} u_C(t) &= C(t).e^{\lambda.t} \\ u_C(t) &= C(t).e^{-\frac{t}{400}} \end{aligned}$$

Vypočítame u'_C deriváciou u_C:

$$\begin{aligned} u'_C(t) &= C'(t).e^{-\frac{t}{400}} + C(t).e^{-\frac{t}{400}}.\left(-\frac{1}{400}\right) \\ u'_C(t) &= C'(t).e^{-\frac{t}{400}} - \frac{1}{400}C(t).e^{-\frac{t}{400}} \end{aligned}$$

Dosadíme u_C a u'_C do diferenciálnej rovnice:

$$\begin{aligned} 400u'_C + u_C &= 20 \\ 400.\left(C'(t).e^{-\frac{t}{400}} - \frac{1}{400}C(t).e^{-\frac{t}{400}}\right) + C(t).e^{-\frac{t}{400}} &= 20 \\ 400.C'(t).e^{-\frac{t}{400}} - C(t).e^{-\frac{t}{400}} + C(t).e^{-\frac{t}{400}} &= 20 \\ 400.C'(t).e^{-\frac{t}{400}} &= 20 \\ C'(t).e^{-\frac{t}{400}} &= \frac{1}{20} \\ C'(t) &= \frac{1}{20e^{-\frac{t}{400}}} \\ C'(t) &= \frac{e^{\frac{t}{400}}}{20} \end{aligned}$$

Zintegrujeme $C'(t)$:

$$\begin{aligned} \int C'(t)dt &= \int \frac{e^{\frac{t}{400}}}{20}dt \\ C(t) &= 20e^{\frac{t}{400}} + k \end{aligned}$$

Dosadíme $C(t)$ do očakávaného riešenia:

$$\begin{aligned} u_C(t) &= (20e^{\frac{t}{400}} + k)e^{\frac{-t}{400}} \\ u_C(t) &= 20 + ke^{\frac{-t}{400}} \end{aligned}$$

Pre vypočítanie konštanty k využijeme počiatočnú podmienku:

$$\begin{aligned} u_C(t) &= 20 + ke^{\frac{-t}{400}} \\ 9 &= 20 + ke^{\frac{0}{400}} \\ 9 &= 20 + k \\ k &= -11 \end{aligned} \qquad\qquad u_C(0) = 9\ V$$

Úplné riešenie je:

$$u_C(t) = 20 - 11e^{\frac{-t}{400}} \qquad\qquad u_C(0) = 9\ V$$

Spravíme skúšku dosadením do pôvodnej diferenciálnej rovnice. Predtým si však ešte vypočítame u_C':

$$u_C'(t) = \left(21 - 11e^{\frac{-t}{400}}\right)' = \frac{11}{400}e^{\frac{-t}{400}}$$

$$\begin{aligned} 20u_C' + \frac{u_C}{20} &= 1 \\ 20\left(\frac{11}{400}e^{\frac{-t}{400}}\right) + \frac{20 - 11e^{\frac{-t}{400}}}{20} &= 1 \\ 400\left(\frac{11}{400}e^{\frac{-t}{400}}\right) + 20 - 11e^{\frac{-t}{400}} &= 20 \\ 11e^{\frac{-t}{400}} + 20 - 11e^{\frac{-t}{400}} &= 20 \\ 0 &= 0 \end{aligned}$$

Skúška vyšla.

6 Tabuľka výsledkov

Úloha č.	Varianta	Výsledky
1	F	$U_{R3} = 38,4480\ V,\ I_{R3} = 0,0699\ A$
2	A	$U_{R3} = 5,9463\ V,\ I_{R3} = 0,0283\ A$
3	E	$U_{R3} = 86,6667\ V,\ I_{R3} = 0,1667\ A$
4	F	$\lvert U_{C2}\rvert = 21,6037\ V,\ \varphi_{C2} = -0,9094\ rad = 127^\circ\ 53'\ 42,38''$
5	A	$u_C(t) = 20 - 11e^{\frac{-t}{400}}$