Financial Instruments and Pricing
Winter semester 2024/25

Set 2
(due date: December 8th 2024)

Bonds

1. Consider a 10-year bond with maturity on 28th February 2025. The bond has nominal value of 10 000 EUR and coupon rate of 10%. Coupons are paid annually in the end of February (EOM).
   a) Compute the value of coupons paid in 2023, 2024 and 2025.
   b) For a transaction done on the 25th October 2024 the clean price was 99,80. Compute accrued interest and dirty price (cash flow) paid on the spot date (D+2). Remember about weekends !

   Use various day count conventions: 30/360 US, 30E/360, ACT/ACT (ICMA), ACT/365 (Fixed).*

   *Use description of day count conventions from:
   http://en.wikipedia.org/wiki/Day_count_convention

2. Polish State Treasury (ST) issues long-term (> 1 year maturity) bonds. Among the fixed interest rate bonds aimed at institutional investors the most popular ones are 2-year zero-coupon bonds (OK_) and 5-year (PS_), 10-year (DS_), 20-year & 30-year (WS_) bonds with coupon. The nominal value of all these bonds is 1000 PLN. The interest is paid annually using ACT/ACT (ICMA) day count convention. The table below presents prices of some ST bonds from 24th October 2024.*

Name	Maturity	Nominal interest %	Clean Price	Accrued interest	Dirty Price	YTM
PS0425	25/04/2025	0.75	98.61			
OK0426	25/04/2026	-	93.52			
DS0726	25/07/2026	2.50	96.10			
DS0727	25/07/2027	2.50	93.57			
PS0728	25/07/2028	7.50	107.41			

Fill in blank spaces in the table above. Transactions are settled on the spot date (D+2) - remember about weekends !

*Source: BondSpot S.A. (Fixing)

3. For all the bonds from exercise 2 compute:

   a. (Macaulay) duration: $$D = \frac{\sum_t t \times \frac{CF(t)}{(1+y)^t}}{\sum_t \frac{CF(t)}{(1+y)^t}} = \frac{\sum_t t \times \frac{CF(t)}{(1+y)^t}}{P}$$

   b. modified duration: $MD = D/(1+y)$

   Make calculations for the spot date (D+2). Symbols used in the formulae: $CF(t)$ – cash flow in time t , y = YTM (computed in exercise 2), P – present value of the bond (dirty price !).

4. An investor has bought a 2.5 year bond with nominal value 1000 PLN paying annual interest of 5% (last coupon is paid on maturity, i.e in 2.5 years). Current YTM is 6%. Compute the change of the (dirty) price if:
   a. YTM increases by 0.1% (i.e. ΔYTM = +0.1%)
   b. YTM decreases by 0.1%
   c. YTM increases by 0.5%
   d. YTM decreases by 0.5%

   Make calculations both exactly and using the modified duration approximation (see the Lecture): $\Delta P \approx - MD\ P\ \Delta y$.

5. Consider a simplified model of a bank. Bank has granted loans for 1 billion PLN with (weighted) average modified duration (MD): 1 year. The bank also has a portfolio of bonds worth 1 bln PLN with average MD: 3 years. Bank finances his activity by offering 2 bln PLN deposits with average MD: 1.5 years. Compute the value of 10-year bonds (MD = 10 years) that the bank should sell and buy instead 1-year bonds (MD = 1 year) in order to avoid the risk of small parallel shifts of the yield curve.

6. In the Lecture we showed that Convexity: C ≥ 0. As a result the profit caused by the decrease in YTM by Δy ≥ loss caused by the increase in YTM by the same Δy (if all other conditions are equal investors should choose bonds with highest convexity C). Suppose an investor can choose between 2 bonds: bond A has maturity in 4 years and it pays annual coupon of 4%, bond B is zero-coupon and it has 3.75 years to maturity. YTMs of both bonds are equal 9.29%.
   a. compute modified duration (MD) and Convexity (C) for both bonds
   b. check the effect of decrease/increase of YTM by ΔYTM = 5% on returns from the bonds (make exact computations of the % price change, i.e. compute $\Delta P / P$ due to the yield change and check which bond is better)

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Useful Wolfram Mathematica functions:

- FinancialBond[]