Financial Instruments and Pricing
Winter semester 2024/25

Set 1
(due date: November 10^{th} 2024)

Time Value of Money

1. In the beginning of October (October 1^{st} in the morning) the bank account XYZ showed a balance of 100 000 PLN
   a) Below is the list of inflows to / outflows from the account in October:
      - 5 X : inflow 17 000 PLN
      - 12 X: inflow 35 000 PLN
      - 21 X: outflow -55 000 PLN

      Compute interest added to the account for October, assuming the nominal interest rate is 5% p.a. (per annum).
      In November there were some additional inflows to / outflows from the account:
      - 9 XI : outflow -25 000 PLN
      - 30 XI: inflow 100 000 PLN

      Compute interest added to the account for November.

      Assume that interest is due for each day of a month when a given balance stays overnight (including the night from Sept. 30^{th} to Oct 1^{st} for the opening balance) and the total interest for the month is added to the final balance in the last day of that month (e.g. inflow of interest for October is on October 31^{st} in the evening).

2. What time t in years (for simplicity t can be any R^+ number, also not integer) is needed to double the initial value of a bank deposit with nominal interest rate r % p.a. if:
   a) no interest is added (compounded) before the end of the deposit (simple interest)
   b) interest is added once a month, quarter, year, and in general n times a year (compound interest)
   c) interest is added continuously (continuous compounding of interest)

   Derive general formula. Compute t for r = 5% p.a.

3. What should be the nominal interest rate r offered by a bank to ensure that savings of its clients rise at least as fast as the inflation rate i = 10% p.a. Assume yearly, half-yearly, quarterly, monthly, and continuous compounding of interest.

4. Assume the annual effective interest rate (yield) is y = … % / year.
   a) What is the equivalent quarterly, monthly, weekly, daily interest rate (yield).
      E.g. the equivalent quarterly rate should be understood as an effective quarterly rate (i.e. % per quarter NOT per annum) with quarterly compounding frequency. Derive general formula and make computations for y = 5 % p.a.

   b) What is the equivalent effective continuous rate (yield), i.e. the yearly rate but with continuous compounding frequency ?

5. What amount should be put aside on the bank account each quarter in order to accumulate 10 000 PLN after 2 years of saving. Assume interest rate of r =5% p.a. and
   a) no interest is added before the end of the deposit (simple interest)
   b) interest is added quarterly (compound interest).

6. Bank A offers one-year deposit with monthly compounded interest and the interest rate grows each month: from 1% p.a. in the first month, 2% p.a. in the second month, …, to 12% p.a. in the last month. The bank advertises the deposit as "the best deposit with an average interest rate of 6.5%" (arithmetic mean). Bank B offers a standard deposit with monthly compounded interest and constant interest rate 6,5% p.a. Bank C offers a deposit with monthly compounded interest and the interest rate is falling each month: from 12% p.a. in the first month, …, to 1% p.a. in the last month.
   a) Which bank should we choose? (based on the effective interest rate)
   b) Would it matter if the banks were paying out interest cash-flows on a monthly basis instead of adding it to the principal amount of the deposit ?

7. A Bank offers a loan of 10 000 PLN for one year with interest rate 10% p.a. and quarterly amortization. A client can choose between two amortization schemes (systems of CF payments): (1) equal principal payments (each quarter 1/4 of the initial principal value of the loan + accrued interest is paid) or (2) equal total payments (each quarter the client pays the same constant amount, called the "annuity").
   a) Compute payments done by the client (CFs) in both amortization schemes
   b) Prepare Amortization Schedule (a table showing current principal value of the loan in the beginning / end of each quarter and a split of the payment (CF) into Principal payment and Interest payment)
   c) Compute total amount of interest paid to the Bank in both systems
   d) Which system is more favourable for the Bank / Client (check what is the effective interest rate in each case) ?

8. A Bank granted 20-year mortgage loan of 200 000 PLN. The loan is amortized monthly as an annuity, i.e. in the equal total payments scheme (principal + interest = const.). The interest rate is floating (variable) and can change every 3 months. It is set as the WIBOR3M rate + 2% p.a. (the interest rate is recalculated every 3 months based on the current WIBOR3M rate). At the initial moment (day when the loan was granted) the WIBOR3M rate was 6% p.a.
   e) Compute (equal) monthly payments fixed for the first three months
   f) Compute the outstanding amount of the loan (i.e. the remaining balance of the principal amount) after 3 months
   g) After 3 months the WIBOR3M rate increased to 7% p.a. Compute new (equal) monthly payments fixed for next three months.

Jakub Gizbert-Studnicki

Useful Wolfram Mathematica functions:

- DayCount[]
- TimeValue[]
- EffectiveInterest[]
- Annuity[]
- AnnuityDue[]
- CashFlow[]