Ex. 1

Covariance matrix : $C_{ij}^{(x)} \equiv Cov(X_i, X_j) = \langle x_i x_j \rangle - \langle x_i \rangle \langle x_j \rangle$

$Y_\alpha = \sum_i v_{\alpha i} X_i$

$Y_\beta = \sum_j v_{\beta j} X_j$

$Cov(Y_\alpha, Y_\beta) = Cov\left(\sum_i v_{\alpha i} X_i, \sum_j v_{\beta j} X_j\right)$

$Cov(A, B) = E[(A - E[A])(B - E[B])]$

$A = Y_\alpha \quad , \quad B = Y_\beta$

$\cancel{Cov(Y_\alpha, Y_\beta) =}$

$E[A] = E[Y_\alpha] = \sum_i v_{\alpha i} E[X_i]$

$E[B] = E[Y_\beta] = \sum_j v_{\beta j} E[X_j]$

$\cancel{Cov(Y_\alpha, Y_\beta) =}$

$Y_\alpha - E[Y_\alpha] = \sum_i v_{\alpha i} (X_i - E[X_i])$

$Y_\beta - E[Y_\beta] = \sum_j v_{\beta j} (X_j - E[X_j])$

$Cov(Y_\alpha, Y_\beta) = E\left[\left(\sum_i v_{\alpha i} (X_i - E[X_i])\right)\left(\sum_j v_{\beta j} (X_j - E[X_j])\right)\right] =$

$= \sum_i \sum_j v_{\alpha i} v_{\beta j} \cdot E[(X_i - E[X_i])(X_j - E[X_j])] =$

$= \sum_{i,j} v_{\alpha i} Cov(X_i, X_j) v_{\beta j} = \sum_{i,j} v_{\alpha i} C_{ij}^{(x)} v_{\beta j}$