You are currently looking at **version 1.0** of this notebook. To download notebooks and datafiles, as well as get help on Jupyter notebooks in the Coursera platform, visit the <u>Jupyter Notebook FAQ</u> (https://www.coursera.org/learn/python-machine-learning/resources/bANLa) course resource.

Applied Machine Learning: Module 3 (Evaluation)

Evaluation for Classification

Preamble

In [1]:

```
%matplotlib notebook
import numpy as np
import pandas as pd
import seaborn as sns
import matplotlib.pyplot as plt
from sklearn.model_selection import train_test_split
from sklearn.datasets import load_digits

dataset = load_digits()
X, y = dataset.data, dataset.target

for class_name, class_count in zip(dataset.target_names, np.bincount(dataset.target)):
    print(class_name,class_count)
```

```
0 178
```

^{1 182}

^{2 177}

^{3 183}

^{4 181}

^{5 182}

^{6 181}

^{7 179}

^{8 174}

^{9 180}

In [2]:

```
# Creating a dataset with imbalanced binary classes:
# Negative class (0) is 'not digit 1'
# Positive class (1) is 'digit 1'
y_binary_imbalanced = y.copy()
y_binary_imbalanced[y_binary_imbalanced != 1] = 0
print('Original labels:\t', y[1:30])
print('New binary labels:\t', y_binary_imbalanced[1:30])
Original labels:
                       [1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5
6 7 8 9]
New binary labels:
                       0 0 0 01
In [3]:
                                 # Negative class (0) is the most frequent class
np.bincount(y binary imbalanced)
Out[3]:
array([1615, 182])
In [4]:
X_train, X_test, y_train, y_test = train_test_split(X, y_binary_imbalanced, random_state=0)
# Accuracy of Support Vector Machine classifier
from sklearn.svm import SVC
svm = SVC(kernel='rbf', C=1).fit(X_train, y_train)
svm.score(X_test, y_test)
Out[4]:
```

0.9088888888888886

Dummy Classifiers

DummyClassifier is a classifier that makes predictions using simple rules, which can be useful as a baseline for comparison against actual classifiers, especially with imbalanced classes.

```
In [5]:
from sklearn.dummy import DummyClassifier
# Negative class (0) is most frequent
dummy majority = DummyClassifier(strategy = 'most frequent').fit(X train, y train)
# Therefore the dummy 'most_frequent' classifier always predicts class 0
y_dummy_predictions = dummy_majority.predict(X_test)
y_dummy_predictions
Out[5]:
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
In [6]:
```

```
dummy_majority.score(X_test, y_test)
```

Out[6]:

0.90444444444445

In [7]:

```
svm = SVC(kernel='linear', C=1).fit(X_train, y_train)
svm.score(X_test, y_test)
```

Out[7]:

0.977777777777775

Confusion matrices

Binary (two-class) confusion matrix

```
In [8]:
```

```
from sklearn.metrics import confusion matrix
# Negative class (0) is most frequent
dummy_majority = DummyClassifier(strategy = 'most_frequent').fit(X_train, y_train)
y_majority_predicted = dummy_majority.predict(X_test)
confusion = confusion_matrix(y_test, y_majority_predicted)
print('Most frequent class (dummy classifier)\n', confusion)
Most frequent class (dummy classifier)
 [[407
         0]
 [ 43
        011
In [9]:
# produces random predictions w/ same class proportion as training set
dummy classprop = DummyClassifier(strategy='stratified').fit(X train, y train)
y_classprop_predicted = dummy_classprop.predict(X_test)
confusion = confusion matrix(y test, y classprop predicted)
print('Random class-proportional prediction (dummy classifier)\n', confusion)
Random class-proportional prediction (dummy classifier)
 [[365 42]
 Γ 39
        4]]
In [10]:
svm = SVC(kernel='linear', C=1).fit(X_train, y_train)
svm_predicted = svm.predict(X_test)
confusion = confusion matrix(y test, svm predicted)
print('Support vector machine classifier (linear kernel, C=1)\n', confusion)
Support vector machine classifier (linear kernel, C=1)
 [[402
        5]
   5 38]]
In [11]:
from sklearn.linear_model import LogisticRegression
lr = LogisticRegression().fit(X train, y train)
lr predicted = lr.predict(X test)
confusion = confusion matrix(y test, lr predicted)
print('Logistic regression classifier (default settings)\n', confusion)
Logistic regression classifier (default settings)
 [[401
   6 3711
```

In [12]:

```
from sklearn.tree import DecisionTreeClassifier

dt = DecisionTreeClassifier(max_depth=2).fit(X_train, y_train)
tree_predicted = dt.predict(X_test)
confusion = confusion_matrix(y_test, tree_predicted)

print('Decision tree classifier (max_depth = 2)\n', confusion)

Decision tree classifier (max_depth = 2)
[[400 7]
[ 17 26]]
```

Evaluation metrics for binary classification

In [13]:

```
from sklearn.metrics import accuracy_score, precision_score, recall_score, f1_score
# Accuracy = TP + TN / (TP + TN + FP + FN)
# Precision = TP / (TP + FP)
# Recall = TP / (TP + FN) Also known as sensitivity, or True Positive Rate
# F1 = 2 * Precision * Recall / (Precision + Recall)
print('Accuracy: {:.2f}'.format(accuracy_score(y_test, tree_predicted)))
print('Precision: {:.2f}'.format(precision_score(y_test, tree_predicted)))
print('Recall: {:.2f}'.format(recall_score(y_test, tree_predicted)))
print('F1: {:.2f}'.format(f1_score(y_test, tree_predicted)))
```

Accuracy: 0.95 Precision: 0.79 Recall: 0.60 F1: 0.68

In [14]:

```
# Combined report with all above metrics
from sklearn.metrics import classification_report

print(classification_report(y_test, tree_predicted, target_names=['not 1', '1']))
```

support	f1-score	recall	precision	
407	0.97	0.98	0.96	not 1
43	0.68	0.60	0.79	1
450	0.94	0.95	0.94	avg / total

In [15]:

```
print('Random class-proportional (dummy)\n',
        classification_report(y_test, y_classprop_predicted, target_names=['not 1', '1']))
print('SVM\n',
        classification_report(y_test, svm_predicted, target_names = ['not 1', '1']))
print('Logistic regression\n',
        classification_report(y_test, lr_predicted, target_names = ['not 1', '1']))
print('Decision tree\n',
        classification_report(y_test, tree_predicted, target_names = ['not 1', '1']))
```

		12 —	- -	-
Random class	-proportional	(dummy)		
	precision	recall	f1-score	support
not 1	0.90	0.90	0.90	407
1	0.09			43
avg / total	0.83	0.82	0.82	450
SVM				
אויו	precision	recall	f1-score	support
	p. 552525			
not 1	0.99	0.99	0.99	407
1	0.88	0.88	0.88	43
/ +-+-1	0.00	0.00	0.00	450
avg / total	0.98	0.98	0.98	450
Logistic reg	ression			
-88	precision	recall	f1-score	support
	0.99			
1	0.86	0.86	0.86	43
avø / total	0.97	0 97	a 97	450
avg / cocai	0.57	0.57	0.57	430
Decision tree	2			
	precision	recall	f1-score	support
	0.00	0.00	0.07	407
	0.96			
1	0.79	0.00	0.68	43
avg / total	0.94	0.95	0.94	450
-				

Decision functions

In [16]:

```
X_train, X_test, y_train, y_test = train_test_split(X, y_binary_imbalanced, random_state=0)
y_scores_lr = lr.fit(X_train, y_train).decision_function(X_test)
y_score_list = list(zip(y_test[0:20], y_scores_lr[0:20]))
# show the decision_function scores for first 20 instances
y_score_list
```

Out[16]:

```
[(0, -23.172292973469549),
 (0, -13.542576515500066),
 (0, -21.717588760007864),
 (0, -18.903065133316442),
 (0, -19.733169947138638),
 (0, -9.7463217496747667),
 (1, 5.2327155658831117),
 (0, -19.308012306288916),
 (0, -25.099330209728528),
 (0, -21.824312362996),
 (0, -24.143782750720494),
 (0, -19.578811099762504),
 (0, -22.568371393280199),
 (0, -10.822590225240777),
 (0, -11.907918741521936),
 (0, -10.977026853802803),
 (1, 11.206811164226373),
 (0, -27.644157619807473),
 (0, -12.857692102545419),
 (0, -25.848149140240199)]
```

```
In [17]:
```

```
X_train, X_test, y_train, y_test = train_test_split(X, y_binary_imbalanced, random_state=0)
y_proba_lr = lr.fit(X_train, y_train).predict_proba(X_test)
y_proba_list = list(zip(y_test[0:20], y_proba_lr[0:20,1]))
# show the probability of positive class for first 20 instances
y_proba_list
```

Out[17]:

```
[(0, 8.6377579220606466e-11),
 (0, 1.3138118599563736e-06),
 (0, 3.6997386039099659e-10),
 (0, 6.1730972504865241e-09),
 (0, 2.6914925394345074e-09),
 (0, 5.8506057771143608e-05),
 (1, 0.99468934644404694),
 (0, 4.1175302368500096e-09),
 (0, 1.2574750894253029e-11),
 (0, 3.3252290754668869e-10),
 (0, 3.269552979937297e-11),
 (0, 3.1407283576084996e-09),
 (0, 1.5800864117150149e-10),
 (0, 1.9943442430612578e-05),
 (0, 6.7368003023859777e-06),
 (0, 1.7089540581641637e-05),
 (1, 0.9999864188091131),
 (0, 9.8694940340196163e-13),
 (0, 2.6059983600823614e-06),
 (0, 5.9469113009063784e-12)]
```

Precision-recall curves

In [31]:

```
from sklearn.metrics import precision_recall_curve

precision, recall, thresholds = precision_recall_curve(y_test, y_scores_lr)

closest_zero = np.argmin(np.abs(thresholds))

closest_zero_p = precision[closest_zero]

closest_zero_r = recall[closest_zero]

plt.figure()

plt.xlim([0.0, 1.01])

plt.ylim([0.0, 1.01])

plt.plot(precision, recall, label='Precision-Recall Curve')

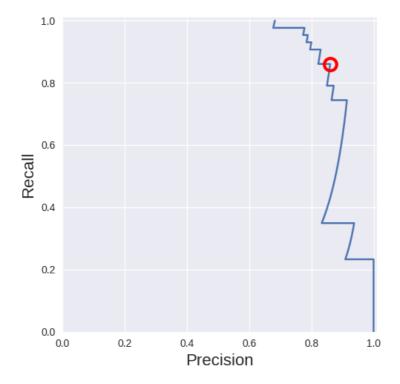
plt.plot(closest_zero_p, closest_zero_r, 'o', markersize = 12, fillstyle = 'none', c='r', m

plt.xlabel('Precision', fontsize=16)

plt.ylabel('Recall', fontsize=16)

plt.axes().set_aspect('equal')

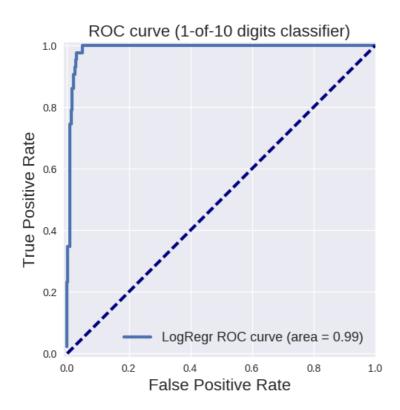
plt.show()
```



ROC curves, Area-Under-Curve (AUC)

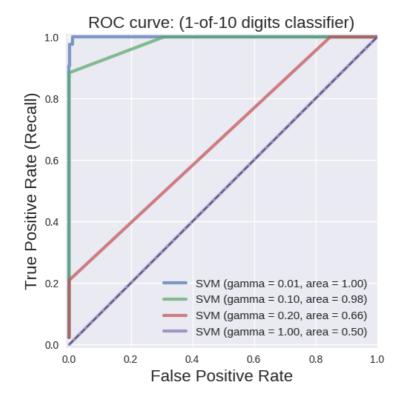
In [32]:

```
from sklearn.metrics import roc_curve, auc
X_train, X_test, y_train, y_test = train_test_split(X, y_binary_imbalanced, random_state=0)
y_score_lr = lr.fit(X_train, y_train).decision_function(X_test)
fpr_lr, tpr_lr, _ = roc_curve(y_test, y_score_lr)
roc_auc_lr = auc(fpr_lr, tpr_lr)
plt.figure()
plt.xlim([-0.01, 1.00])
plt.ylim([-0.01, 1.01])
plt.plot(fpr_lr, tpr_lr, lw=3, label='LogRegr ROC curve (area = {:0.2f})'.format(roc_auc_lr
plt.xlabel('False Positive Rate', fontsize=16)
plt.ylabel('True Positive Rate', fontsize=16)
plt.title('ROC curve (1-of-10 digits classifier)', fontsize=16)
plt.legend(loc='lower right', fontsize=13)
plt.plot([0, 1], [0, 1], color='navy', lw=3, linestyle='--')
plt.axes().set_aspect('equal')
plt.show()
```



In [33]:

```
from matplotlib import cm
X_train, X_test, y_train, y_test = train_test_split(X, y_binary_imbalanced, random_state=0)
plt.figure()
plt.xlim([-0.01, 1.00])
plt.ylim([-0.01, 1.01])
for g in [0.01, 0.1, 0.20, 1]:
    svm = SVC(gamma=g).fit(X_train, y_train)
    y score svm = svm.decision function(X test)
    fpr_svm, tpr_svm, _ = roc_curve(y_test, y_score_svm)
    roc_auc_svm = auc(fpr_svm, tpr_svm)
    accuracy_svm = svm.score(X_test, y_test)
    print("gamma = {:.2f} accuracy = {:.2f}
                                              AUC = {:.2f}".format(g, accuracy_svm,
                                                                     roc_auc_svm))
    plt.plot(fpr_svm, tpr_svm, lw=3, alpha=0.7,
             label='SVM (gamma = {:0.2f}, area = {:0.2f})'.format(g, roc_auc_svm))
plt.xlabel('False Positive Rate', fontsize=16)
plt.ylabel('True Positive Rate (Recall)', fontsize=16)
plt.plot([0, 1], [0, 1], color='k', lw=0.5, linestyle='--')
plt.legend(loc="lower right", fontsize=11)
plt.title('ROC curve: (1-of-10 digits classifier)', fontsize=16)
plt.axes().set_aspect('equal')
plt.show()
```



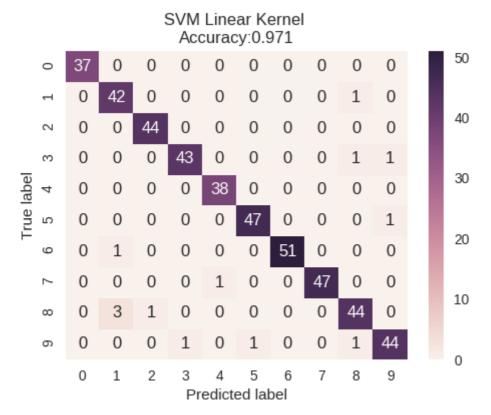
```
gamma = 0.01 accuracy = 0.91 AUC = 1.00
gamma = 0.10 accuracy = 0.90 AUC = 0.98
gamma = 0.20 accuracy = 0.90 AUC = 0.66
gamma = 1.00 accuracy = 0.90 AUC = 0.50
```

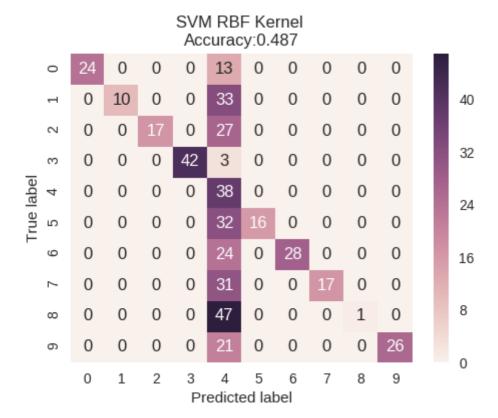
Evaluation measures for multi-class classification

Multi-class confusion matrix

In [34]:

```
dataset = load digits()
X, y = dataset.data, dataset.target
X_train_mc, X_test_mc, y_train_mc, y_test_mc = train_test_split(X, y, random_state=0)
svm = SVC(kernel = 'linear').fit(X_train_mc, y_train_mc)
svm_predicted_mc = svm.predict(X_test_mc)
confusion_mc = confusion_matrix(y_test_mc, svm_predicted_mc)
df_cm = pd.DataFrame(confusion_mc,
                     index = [i for i in range(0,10)], columns = [i for i in range(0,10)])
plt.figure(figsize=(5.5,4))
sns.heatmap(df_cm, annot=True)
plt.title('SVM Linear Kernel \nAccuracy:{0:.3f}'.format(accuracy_score(y_test_mc,
                                                                        svm_predicted_mc)))
plt.ylabel('True label')
plt.xlabel('Predicted label')
svm = SVC(kernel = 'rbf').fit(X_train_mc, y_train_mc)
svm_predicted_mc = svm.predict(X_test_mc)
confusion_mc = confusion_matrix(y_test_mc, svm_predicted_mc)
df_cm = pd.DataFrame(confusion_mc, index = [i for i in range(0,10)],
                  columns = [i for i in range(0,10)])
plt.figure(figsize = (5.5,4))
sns.heatmap(df_cm, annot=True)
plt.title('SVM RBF Kernel \nAccuracy:{0:.3f}'.format(accuracy_score(y_test_mc,
                                                                     svm_predicted_mc)))
plt.ylabel('True label')
plt.xlabel('Predicted label');
```





Multi-class classification report

In [22]:

In [22]:					
<pre>print(classification_report(y_test_mc, svm_predicted</pre>					
	precision	recall	f1-score	support	
0	1.00	0.65	0.79	37	
1	1.00	0.23	0.38	43	
2	1.00	0.39	0.56	44	
3	1.00	0.93	0.97	45	
4	0.14	1.00	0.25	38	
5	1.00	0.33	0.50	48	
6	1.00	0.54	0.70	52	
7	1.00	0.35	0.52	48	
8	1.00	0.02	0.04	48	
9	1.00	0.55	0.71	47	
avg / total	0.93	0.49	0.54	450	

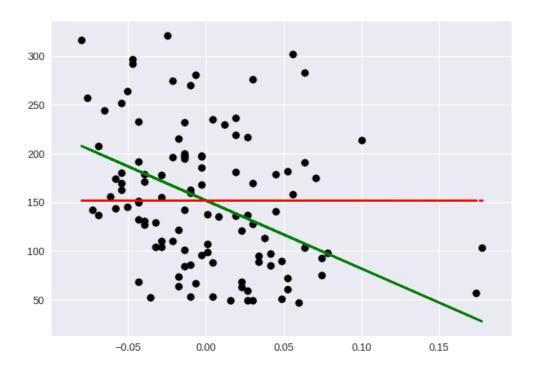
Micro- vs. macro-averaged metrics

```
In [23]:
```

Regression evaluation metrics

In [25]:

```
%matplotlib notebook
import matplotlib.pyplot as plt
import numpy as np
from sklearn.model selection import train test split
from sklearn import datasets
from sklearn.linear_model import LinearRegression
from sklearn.metrics import mean_squared_error, r2_score
from sklearn.dummy import DummyRegressor
diabetes = datasets.load diabetes()
X = diabetes.data[:, None, 6]
y = diabetes.target
X_train, X_test, y_train, y_test = train_test_split(X, y, random_state=0)
lm = LinearRegression().fit(X_train, y_train)
lm_dummy_mean = DummyRegressor(strategy = 'mean').fit(X_train, y_train)
y_predict = lm.predict(X_test)
y_predict_dummy_mean = lm_dummy_mean.predict(X_test)
print('Linear model, coefficients: ', lm.coef_)
print("Mean squared error (dummy): {:.2f}".format(mean_squared_error(y_test,))
                                                                     y predict dummy mean))
print("Mean squared error (linear model): {:.2f}".format(mean_squared_error(y_test, y_predi
print("r2_score (dummy): {:.2f}".format(r2_score(y_test, y_predict_dummy_mean)))
print("r2_score (linear model): {:.2f}".format(r2_score(y_test, y_predict)))
# Plot outputs
plt.scatter(X_test, y_test, color='black')
plt.plot(X_test, y_predict, color='green', linewidth=2)
plt.plot(X_test, y_predict_dummy_mean, color='red', linestyle = 'dashed',
         linewidth=2, label = 'dummy')
plt.show()
Linear model, coefficients: [-698.80206267]
Mean squared error (dummy): 4965.13
Mean squared error (linear model): 4646.74
r2_score (dummy): -0.00
r2 score (linear model): 0.06
```



Model selection using evaluation metrics

Cross-validation example

In [26]:

```
from sklearn.model_selection import cross_val_score
from sklearn.svm import SVC
dataset = load_digits()
# again, making this a binary problem with 'digit 1' as positive class
# and 'not 1' as negative class
X, y = dataset.data, dataset.target == 1
clf = SVC(kernel='linear', C=1)
# accuracy is the default scoring metric
print('Cross-validation (accuracy)', cross_val_score(clf, X, y, cv=5))
# use AUC as scoring metric
print('Cross-validation (AUC)', cross_val_score(clf, X, y, cv=5, scoring = 'roc_auc'))
# use recall as scoring metric
print('Cross-validation (recall)', cross_val_score(clf, X, y, cv=5, scoring = 'recall'))
Cross-validation (accuracy) [ 0.91944444 0.98611111 0.97214485 0.97493036
  0.96935933]
Cross-validation (AUC) [ 0.9641871
                                    0.9976571
                                                0.99372205 0.99699002 0.9
8675611]
Cross-validation (recall) [ 0.81081081 0.89189189 0.83333333 0.83333333
0.83333333]
```

Grid search example

In [27]:

```
from sklearn.svm import SVC
from sklearn.model_selection import GridSearchCV
from sklearn.metrics import roc auc score
dataset = load_digits()
X, y = dataset.data, dataset.target == 1
X_train, X_test, y_train, y_test = train_test_split(X, y, random_state=0)
clf = SVC(kernel='rbf')
grid values = {'gamma': [0.001, 0.01, 0.05, 0.1, 1, 10, 100]}
# default metric to optimize over grid parameters: accuracy
grid_clf_acc = GridSearchCV(clf, param_grid = grid_values)
grid_clf_acc.fit(X_train, y_train)
y_decision_fn_scores_acc = grid_clf_acc.decision_function(X_test)
print('Grid best parameter (max. accuracy): ', grid_clf_acc.best_params_)
print('Grid best score (accuracy): ', grid_clf_acc.best_score_)
# alternative metric to optimize over grid parameters: AUC
grid_clf_auc = GridSearchCV(clf, param_grid = grid_values, scoring = 'roc_auc')
grid_clf_auc.fit(X_train, y_train)
y decision fn scores auc = grid clf auc.decision function(X test)
print('Test set AUC: ', roc_auc_score(y_test, y_decision_fn_scores_auc))
print('Grid best parameter (max. AUC): ', grid_clf_auc.best_params_)
print('Grid best score (AUC): ', grid_clf_auc.best_score_)
Grid best parameter (max. accuracy): {'gamma': 0.001}
Grid best score (accuracy): 0.996288047513
Test set AUC: 0.999828581224
Grid best parameter (max. AUC): {'gamma': 0.001}
Grid best score (AUC): 0.99987412783
```

Evaluation metrics supported for model selection

In [28]:

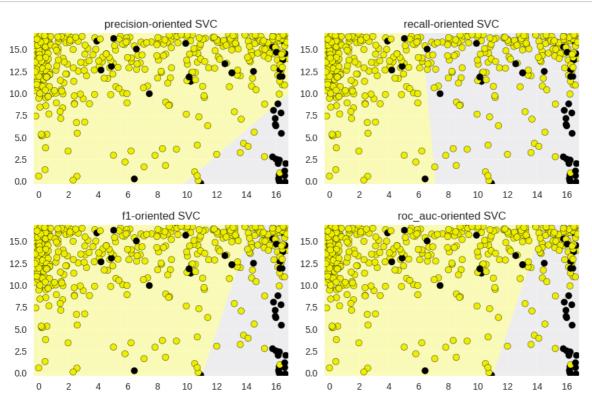
```
from sklearn.metrics.scorer import SCORERS
sorted(list(SCORERS.keys()))
# print(sorted(list(SCORERS.keys())))
Out[28]:
['accuracy',
 'adjusted_rand_score',
 'average_precision',
 'f1',
 'f1_macro',
 'f1_micro',
 'f1_samples',
 'f1_weighted',
 'log_loss',
 'mean_absolute_error',
 'mean_squared_error',
 'median_absolute_error',
 'neg_log_loss',
 'neg_mean_absolute_error',
 'neg_mean_squared_error',
 'neg_median_absolute_error',
 'precision',
 'precision_macro',
 'precision_micro',
 'precision_samples',
 'precision_weighted',
 'r2',
 'recall',
 'recall_macro',
 'recall_micro',
 'recall_samples',
 'recall_weighted',
 'roc_auc']
```

Two-feature classification example using the digits dataset

Optimizing a classifier using different evaluation metrics

In [29]:

```
from sklearn.datasets import load digits
from sklearn.model_selection import train_test_split
from adspy_shared_utilities import plot_class_regions_for_classifier_subplot
from sklearn.svm import SVC
from sklearn.model selection import GridSearchCV
dataset = load_digits()
X, y = dataset.data, dataset.target == 1
X_train, X_test, y_train, y_test = train_test_split(X, y, random_state=0)
# Create a two-feature input vector matching the example plot above
# We jitter the points (add a small amount of random noise) in case there are areas
# in feature space where many instances have the same features.
jitter_delta = 0.25
X_twovar_train = X_train[:,[20,59]]+ np.random.rand(X_train.shape[0], 2) - jitter_delta
X_twovar_test = X_test[:,[20,59]] + np.random.rand(X_test.shape[0], 2) - jitter_delta
clf = SVC(kernel = 'linear').fit(X_twovar_train, y_train)
grid_values = {'class_weight':['balanced', {1:2},{1:3},{1:4},{1:5},{1:10},{1:20},{1:50}]}
plt.figure(figsize=(9,6))
for i, eval_metric in enumerate(('precision', 'recall', 'f1', 'roc_auc')):
    grid_clf_custom = GridSearchCV(clf, param_grid=grid_values, scoring=eval_metric)
    grid_clf_custom.fit(X_twovar_train, y_train)
    print('Grid best parameter (max. {0}): {1}'
          .format(eval_metric, grid_clf_custom.best_params_))
    print('Grid best score ({0}): {1}'
          .format(eval_metric, grid_clf_custom.best_score ))
    plt.subplots_adjust(wspace=0.3, hspace=0.3)
    plot_class_regions_for_classifier_subplot(grid_clf_custom, X_twovar_test, y_test, None,
                                             None, None, plt.subplot(2, 2, i+1))
    plt.title(eval_metric+'-oriented SVC')
plt.tight layout()
plt.show()
```



```
Grid best parameter (max. precision): {'class_weight': {1: 2}}

Grid best score (precision): 0.5296977802916927

Grid best parameter (max. recall): {'class_weight': {1: 50}}

Grid best score (recall): 0.935661321592438

Grid best parameter (max. f1): {'class_weight': {1: 4}}

Grid best score (f1): 0.49965286867474573

Grid best parameter (max. roc_auc): {'class_weight': {1: 4}}

Grid best score (roc_auc): 0.8864081749542332
```

Precision-recall curve for the default SVC classifier (with balanced class weights)

In [30]:

```
from sklearn.model selection import train test split
from sklearn.metrics import precision_recall_curve
from adspy shared utilities import plot class regions for classifier
from sklearn.svm import SVC
dataset = load_digits()
X, y = dataset.data, dataset.target == 1
X_train, X_test, y_train, y_test = train_test_split(X, y, random_state=0)
# create a two-feature input vector matching the example plot above
jitter_delta = 0.25
X_twovar_train = X_train[:,[20,59]]+ np.random.rand(X_train.shape[0], 2) - jitter delta
X_twovar_test = X_test[:,[20,59]] + np.random.rand(X_test.shape[0], 2) - jitter_delta
clf = SVC(kernel='linear', class_weight='balanced').fit(X_twovar_train, y_train)
y_scores = clf.decision_function(X_twovar_test)
precision, recall, thresholds = precision_recall_curve(y_test, y_scores)
closest_zero = np.argmin(np.abs(thresholds))
closest_zero_p = precision[closest_zero]
closest_zero_r = recall[closest_zero]
plot_class_regions_for_classifier(clf, X_twovar_test, y_test)
plt.title("SVC, class_weight = 'balanced', optimized for accuracy")
plt.show()
plt.figure()
plt.xlim([0.0, 1.01])
plt.ylim([0.0, 1.01])
plt.title ("Precision-recall curve: SVC, class_weight = 'balanced'")
plt.plot(precision, recall, label = 'Precision-Recall Curve')
plt.plot(closest_zero_p, closest_zero_r, 'o', markersize=12, fillstyle='none', c='r', mew=3
plt.xlabel('Precision', fontsize=16)
plt.ylabel('Recall', fontsize=16)
# plt.legend(loc="top right")
plt.axes().set_aspect('equal')
plt.show()
print('At zero threshold, precision: {:.2f}, recall: {:.2f}'
      .format(closest zero p, closest zero r))
```