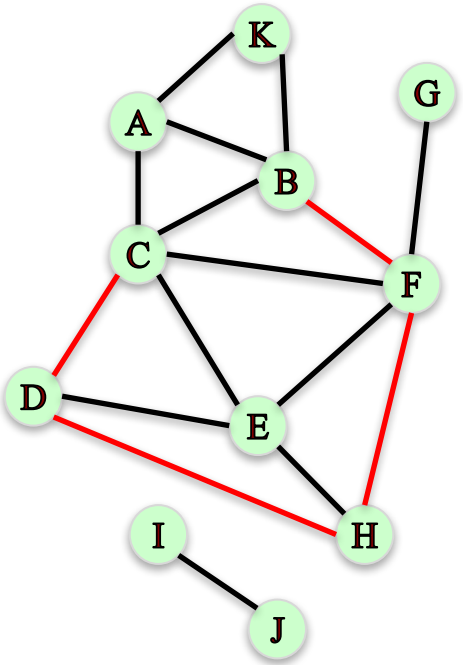
Triadic Closure

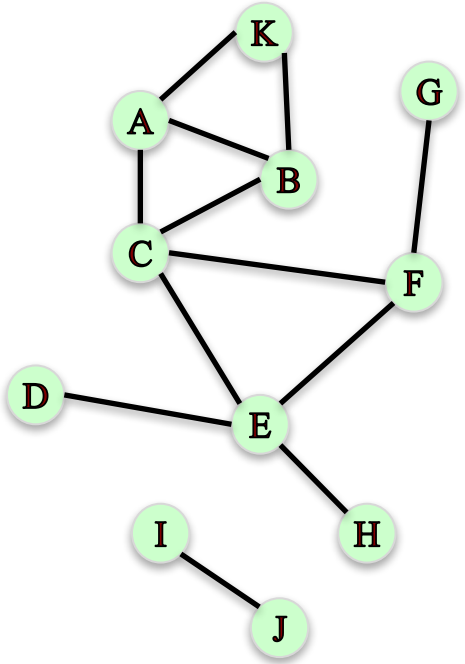
Triadic closure: The tendency for people who share connections in a social network to become connected.

How can we measure the prevalence of triadic closure in a network?



Local Clustering Coefficient

Local clustering coefficient of a node:
Fraction of pairs of the node's friends that are friends with each other.



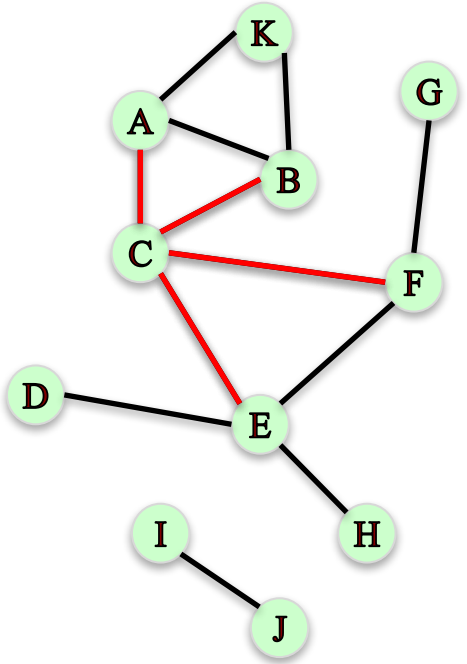
Local Clustering Coefficient

Compute the local clustering coefficient of node C:

$$\frac{\#\text{ of pairs of C's friends who are friends}}{\#\text{ of pairs of C's friends}}$$

# of C's friends = $d_c = 4$ (the "degree" of C)

# of pairs of C's friends =



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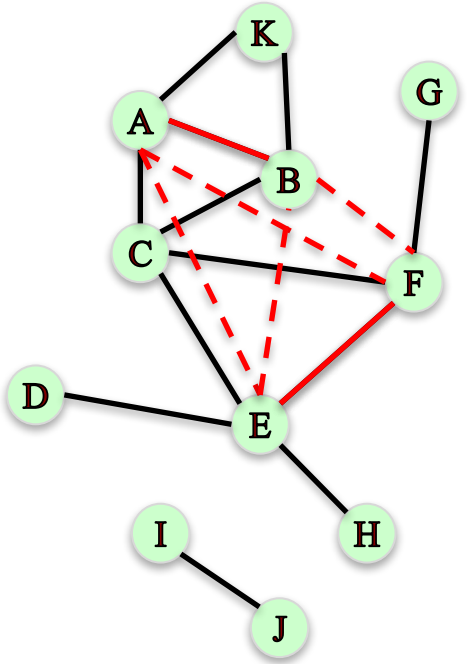

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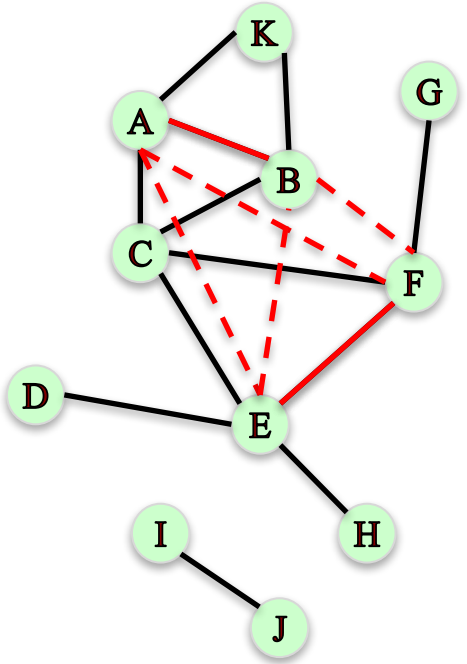
$$\#\text{ of pairs of C's friends} = \frac{d_c(d_c - 1)}{2} = \frac{12}{2} = 6$$



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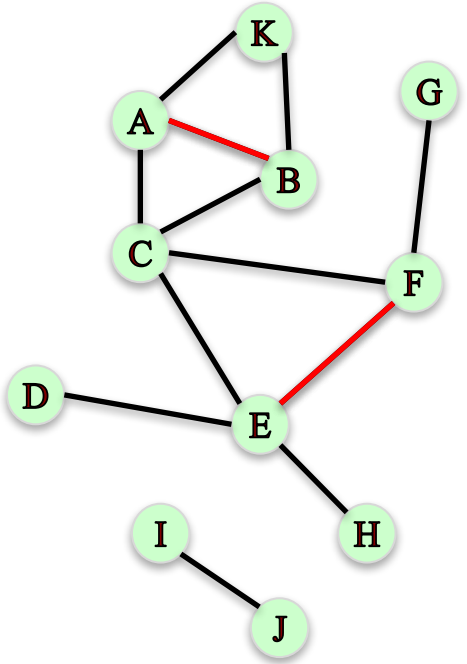
\# of pairs of C's friends who are friends =



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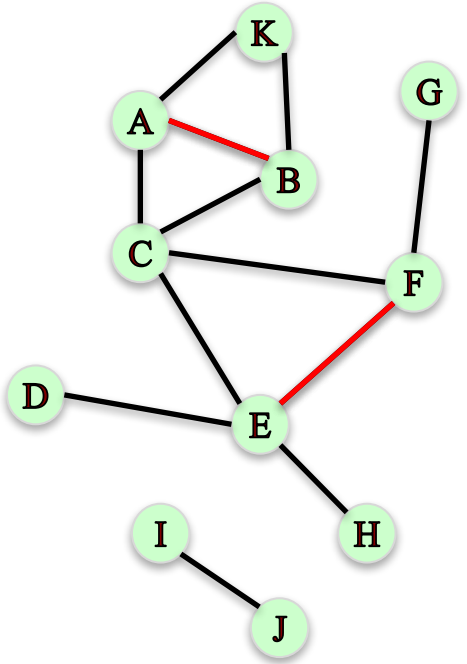
\# of pairs of C's friends who are friends = 2



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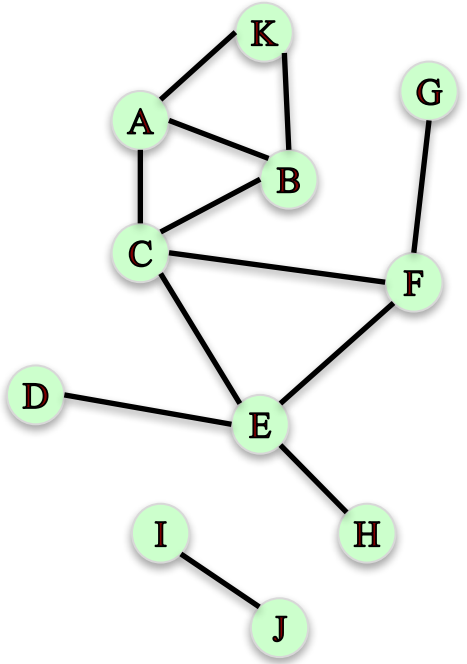
# of pairs of C's friends who are friends = 2

$$\text{Local clustering coefficent of C} = \frac{2}{6} = \frac{1}{3}$$



Local Clustering Coefficient

Compute the local clustering coefficient of node F:

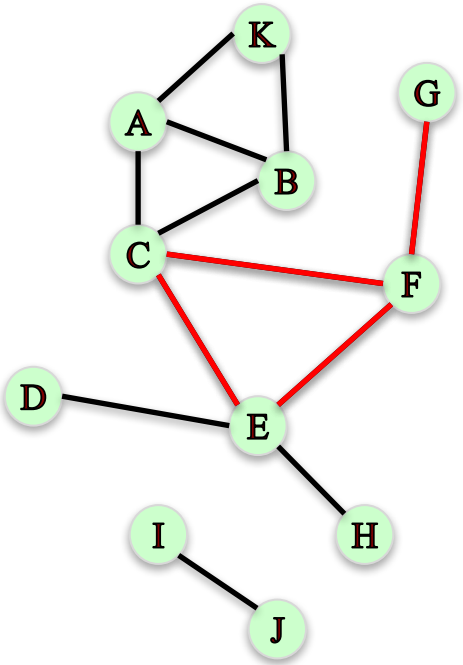
$$\frac{\text{\# of pairs of F's friends who are friends}}{\text{\# of pairs of F's friends}}$$

$d_F = 3$

$$\text{\# of pairs of F's friends} = \frac{d_F(d_F - 1)}{2} = \frac{6}{2} = 3$$

\# of pairs of F's friends who are friends = 1

Local clustering coefficent of F = $\frac{1}{3}$



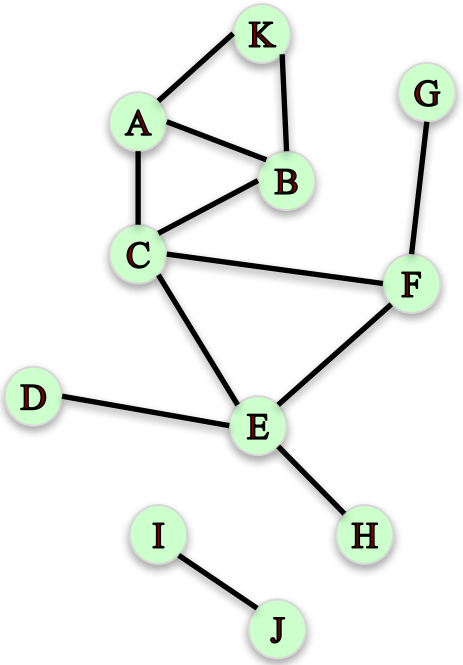
Local Clustering Coefficient

Compute the local clustering coefficient of node J:

$$\frac{\#\text{ of pairs of J's friends who are friends}}{\#\text{ of pairs of J's friends}}$$

# of pairs of J's friends = 0 (Can not divide by 0)

We will assume that the local clustering coefficient of a node of degree less than 2 to be 0.



Local Clustering Coefficient

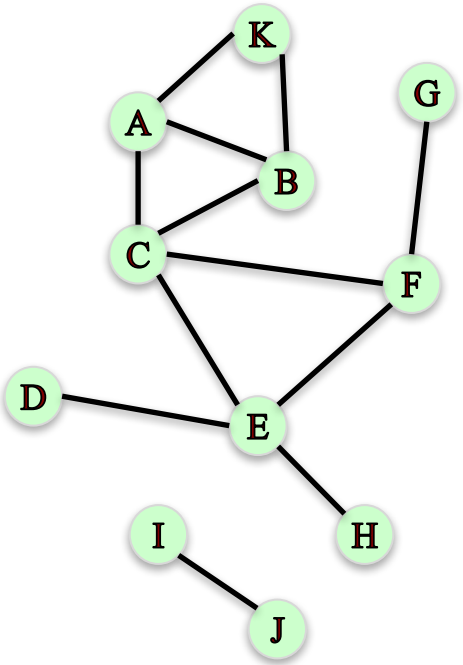
Local clustering coefficient in NetworkX:

```
G = nx.Graph()
G.add_edges_from([('A', 'K'), ('A', 'B'), ('A', 'C'), ('B', 'C'), ('B', 'K'),
('C', 'E'), ('C', 'F'), ('D', 'E'), ('E', 'F'), ('E', 'H'), ('F', 'G'), ('I', 'J')])
```

```
In: nx.clustering(G, 'F')
Out: 0.3333333333333333
```

```
In: nx.clustering(G, 'A')
Out: 0.6666666666666666
```

```
In: nx.clustering(G, 'J')
Out: 0.0
```

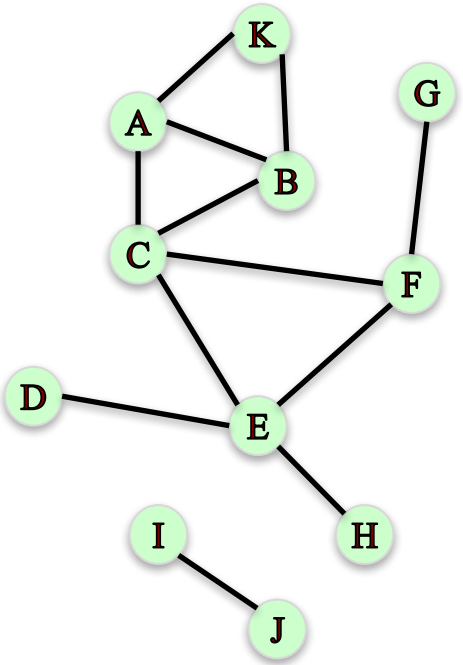


Global Clustering Coefficient

Measuring clustering on the whole network:

Approach 1: Average local clustering coefficient over all nodes in the graph.

In: nx.average_clustering(G)
Out: 0.2878787878787878

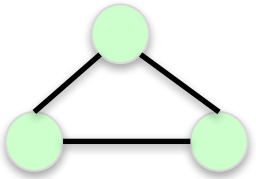


Global Clustering Coefficient

Measuring clustering on the whole network (Approach 2):

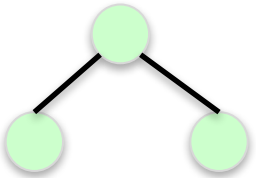
Percentage of "open triads" that are triangles in a network.

Triangles:



Transitivity = $\frac{3*\text{Number of closed triads}}{\text{Number of open triads}}$

Open triads:



Global Clustering Coefficient

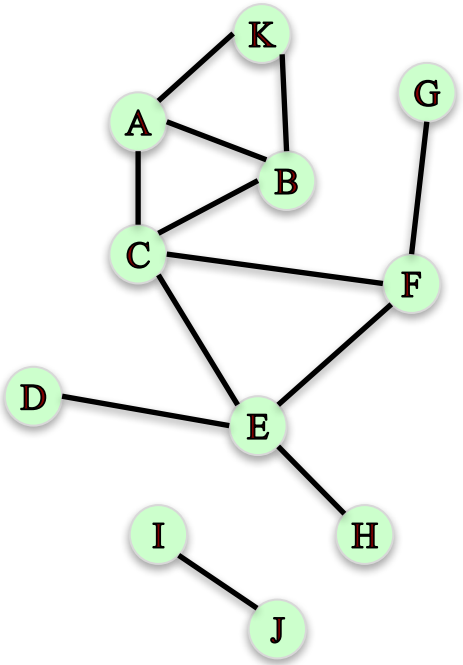
Measuring clustering on the whole network:

Transitivity: Ratio of number of triangles and number of "open triads" in a network.

In: nx.transitivity(G)
Out: 0.409090909091
= 9/22

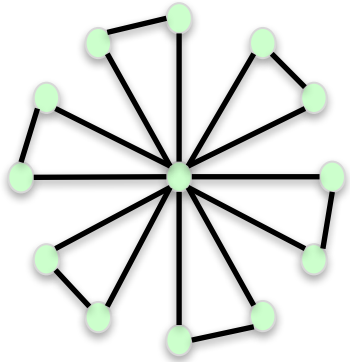
AKB
KAB
ABK
ACB
ABC
CAB
CEF
CFE
ECF

KAC
KBC
ACF
BCF
CFG
ACE
BCE
CED
CEH
DEH
DEF
HEF
EFG



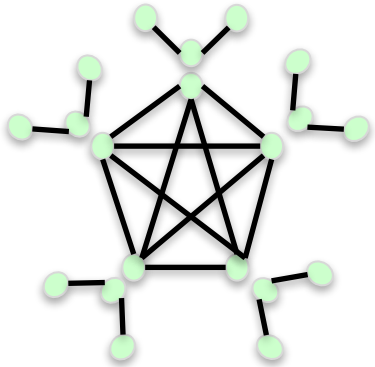
Transitivity vs. Average Clustering Coefficient

Both measure the tendency for edges to form triangles.
Transitivity weights nodes with large degree higher.



- Most nodes have high LCC
- The high degree node has low LCC

Ave. clustering coeff. = 0.93
Transitivity = 0.23



- Most nodes have low LCC
- High degree node have high LCC

Ave. clustering coeff. = 0.25
Transitivity = 0.86

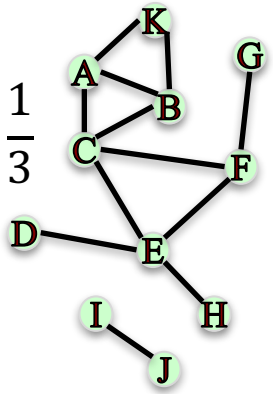
Summary

Clustering coefficient measures the degree to which nodes in a network tend to "cluster" or form triangles.

Local Clustering Coefficient

Fraction of pairs of the node's friends that are friends with each other.

LCC of C = $\frac{2}{6} = \frac{1}{3}$



Global Clustering Coefficient

Average Local Clustering Coefficient

nx.average_clustering(G)

Transitivity

Ratio of number of triangles and number of "open triads".

Puts larger weight on high degree nodes.

nx.transitivity(G)