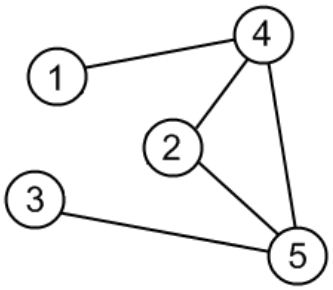
Exercises: Introduction to Graphs - Solutions

Exercise 1

Draw the undirected graph represented by the adjacency matrix below

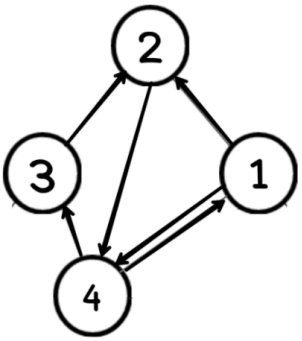
$$\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \\ \left[\begin{array}{ccccc} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{array}\right] & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \end{array} \tag{1}$$



Exercise 2

Draw the directed graph represented by the adjacency matrix below

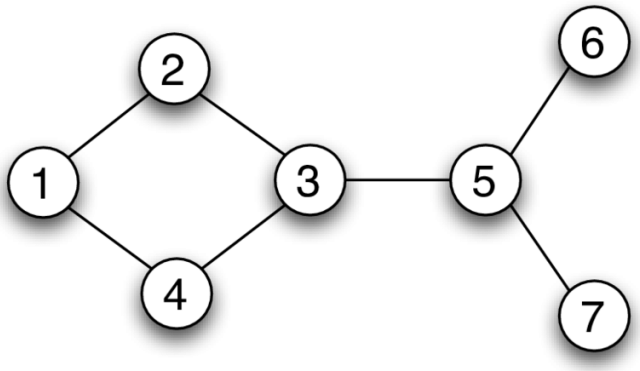
$$\begin{array}{cccc} 1 & 2 & 3 & 4 \\ \left[\begin{array}{cccc} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{array}\right] & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \end{array} \tag{2}$$



Exercise 3

Draw the undirected graph represented by the adjacency list below

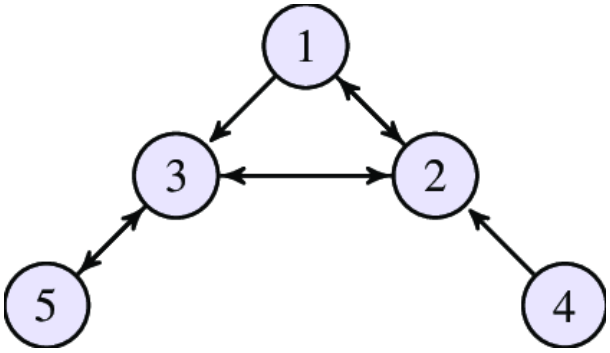
$1 \rightarrow 2 \rightarrow 4$
$2 \rightarrow 1 \rightarrow 3$
$3 \rightarrow 2 \rightarrow 4 \rightarrow 5$
$4 \rightarrow 1 \rightarrow 3$
$5 \rightarrow 3 \rightarrow 6 \rightarrow 7$
$6 \rightarrow 5$
$7 \rightarrow 5$



Exercise 4

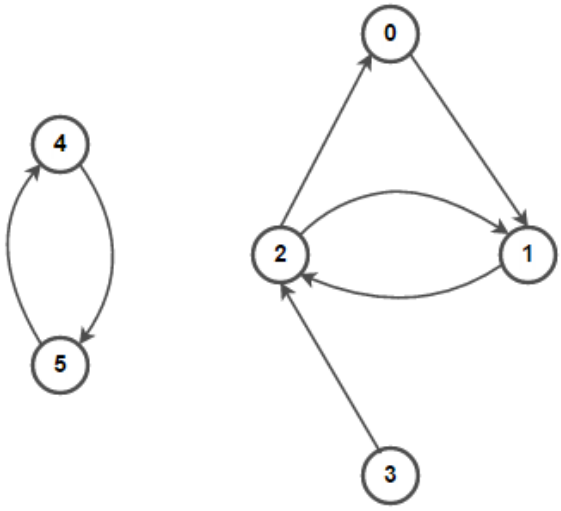
Draw the directed graph represented by the adjacency list below

$1 \to 2 \to 3$
$2 \to 1 \to 3$
$3 \to 2 \to 5$
$4 \to 2$
$5 \to 3$



Exercise 5

Construct both the adjacency matrix and adjacency list corresponding to the graph below.

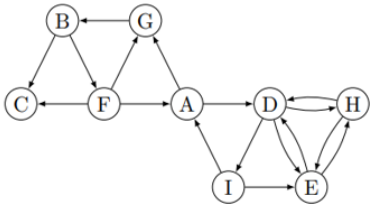


	0	1	2	3	4	5	
0	0	1	0	0	0	0	0
1	0	0	1	0	0	0	1
2	1	1	0	0	0	0	2
3	0	0	1	0	0	0	3
4	0	0	0	0	0	1	4
5	0	0	0	0	1	0	5

$0 \to 1$
$1 \to 2$
$2 \to 0 \to 1$
$3 \to 2$
$4 \to 5$
$5 \to 4$

Exercise 6

For a breadth-first search (BFS) of the graph below starting in vertex A, state the order the vertices are removed from the queue Q in the BFS-algorithm. We assume that the graph is given by adjacency lists, where the adjacency lists are sorted alphabetically.



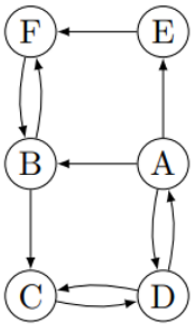
A D G E H I B F C A G D B E H I C F A D E H I G B C F A D G E H I B C F

[A] [B] [C] [D]

Answer: D

Exercise 7

For each of the below set of edges, state whether they make up a legal BFS tree for a breadth-first traversal of the graph below starting in vertex A and for an arbitrary order of the graph's adjacency lists.

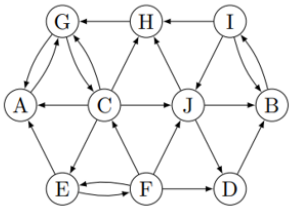


	Yes	No
(A,B) (A,D) (A,E) (B,C) (E,F)	[A]	[B]
(A,B) (A,D) (A,E) (B,C) (B,F)	[A]	[B]
(A,D) (A,E) (D,C) (E,F) (F,B)	[A]	[B]
(A,B) (A,E) (B,C) (B,F) (C,D)	[A]	[B]
(A,B) (A,D) (A,E) (D,C) (E,F)	[A]	[B]

Answer: A A B B A

Exercise 8

Consider a depth-first search (DFS) of the graph below starting in vertex A, where the outgoing edges to a vertex is visited in alphabetical order. State in which order each vertex is assigned finishing time.



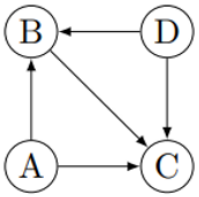
HJIBDFECGA JHIBDFECGA IDBFJHECGA HIBDJFECGA

[A] [B] [C] [D]

Answer: A

Exercise 9

For each of the below sortings of the vertices of the graph below, state whether or not it is a topological sorting.



	Yes	No
ADBC	[A]	[B]
DABC	[A]	[B]
CABD	[A]	[B]
CDBA	[A]	[B]
ABDC	[A]	[B]

Answer: A A B B B