Gør tanke til handling

VIA University College

Recap: DMA

Domain/Range

Consider the following two functions

$h(x) = \frac{1}{x+5}$ and $g(x) = \frac{x}{x-\frac{1}{2}}$

What is the domain of the composite $(h \circ g)(x)$? State your answers as integers between 0 and 99 such that all fractions are irreducible.

$\mathbb{R} \setminus \left\{ \frac{\boxed{5}}{\boxed{12}}, \frac{1}{\boxed{2}} \right\}$

$$\frac{1}{\frac{x}{x-\frac{1}{2}}+5}$$

$$, \quad \frac{x}{x-\frac{1}{2}} + 5 = 0$$

$$\frac{x}{x-\frac{1}{2}} = -5 \iff x = -5\left(x-\frac{1}{2}\right) = -5x + \frac{5}{2}$$

$$6x = \frac{5}{2}, \quad x = \frac{5}{12}$$

Domain/Range

Consider the function

$$f(x) = \frac{102}{3x-24} + 76.$$

What is the range of the **inverse** function f^{-1}? Write your answer as an integer between 0 and 99.

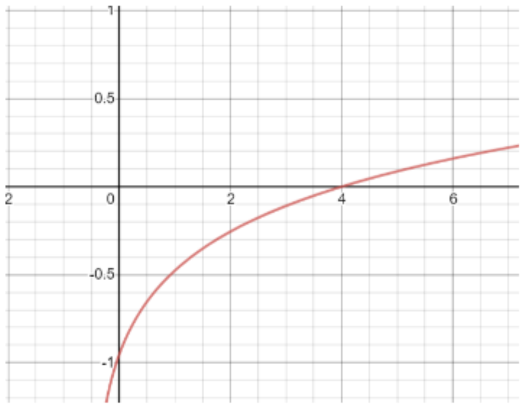
Range of f^{-1}: $\mathbb{R}\backslash\{\boxed{8}\}$

f: Domain = Range of f^{-1}

f range = domain of f^{-1}

Functions

Consider the function f shown in the graph below.



Use the graph to determine the missing value in the expression for f. State your answer as an integer between 0 and 99.

$$f(x) = \log\left(\frac{2x+1}{\boxed{9}}\right)$$

$$\frac{2 \cdot 4 + 1}{a} = 1$$

$$\log(x) = 0 \implies x = 1$$

$$a = 9$$

Sequence

Find the fifth term of the sequence $\{a_n\}_{n=0} = \frac{(-1)^n}{(n-1)!}$, beginning with $n = 0$. State your answer as an integer between 0 and 99.

$$\frac{1}{\boxed{6}}$$

$$a_4 = \frac{(-1)^4}{(4-1)!} = \frac{1}{3 \cdot 2 \cdot 1} = \frac{1}{6}$$

Series

Find the value of the series $\sum_{n=0}^{5} n + 2^n$. Write your answer as an integer between 0 and 99:

$\boxed{78}$

$(0+2^0)+(1+2^1)+(2+2^2)+(3+2^3)+(4+2^4)+(5+2^5)$

$1 + 3 + 6 + 11 + 20 + 37$

$= 78$

Sequence

Consider the recurrence relation and initial condition: $a_n = a_{n-1} + 2n + 4, \quad a_0 = 4$. Fill in the missing values in the expression below such that the correct solution to the recurrence relation and initial condition is given. State your answer as two integers between 0 and 99.

$a_n = n^2 + \boxed{5}n + \boxed{4}$

Method 1:

$a_0 = 4$

$a_1 = 4 + 2\cdot 1 + 4$

$= 2\cdot 4 + 2$

$a_2 = 2\cdot 4 + 2 + 2\cdot 2 + 4$

$= 3\cdot 4 + 6$

$a_3 = 3\cdot 4 + 6 + 2\cdot 3 + 4$

$= 4\cdot 4 + 12$

$a_4 = 4\cdot 4 + 12 + 2\cdot 4 + 4$

$= 5\cdot 4 + 20$

$a_n = (n+1)4 + n^2 + n$

$= 4n + 4 + n^2 + n = n^2 + 5n + 4$

Method 2:

$a_0 = 4$

$a_1 = 4 + 2\cdot 1 + 4$

$= 1^2 + \underbrace{3 + 2}_{5} + 4 = 10$

$a_2 = 10 + 2\cdot 2 + 4$

$= 2^2 + \underbrace{6 + 4}_{=10} + 4$

$a_3 = 18 + 2\cdot 3 + 4$

$= 3^2 + \underbrace{9 + 6}_{=15} + 4$

$\vdots$

$a_n = n^2 + 5n + 4$

Method 3:

$a_0 = 4$

$a_1 = 4 + 2\cdot 1 + 4$

$a_2 = 2\cdot 4 + 2\cdot 1 + 2\cdot 2 + 4 = 3\cdot 4 + 2(1+2)$

$a_3 = 4\cdot 4 + 2(1+2+3)$

$=$

$a_n = (n+1)\cdot 4 + 2\left(n\cdot \frac{n+1}{2}\right)$

Sum

Compute the sum of the series: $\sum_{n=0}^{\infty} \frac{1^n-2^n}{3^n}$. State your answer as an irredeble fraction using two integer between 0 and 99.

$-\frac{\boxed{3}}{\boxed{2}}$

$$\boxed{\text{We use:} \quad \sum |x|^n = \frac{1}{1-x}, \ |x|<1}$$

$$\sum \frac{1^n-2^n}{3^n} = \sum \frac{1^n}{3^n} - \frac{2^n}{3^n} = \sum \left(\frac{1}{3}\right)^n - \left(\frac{2}{3}\right)^n$$

$$= \sum \left(\frac{1}{3}\right)^n - \sum \left(\frac{2}{3}\right)^n = \frac{1}{1-1/3} - \frac{1}{1-2/3} = \frac{3}{2} - 3 = -\frac{3}{2}$$

Logarithms

Fill in the blank in the logarithmic expression below. Write your answers as an integer between 0 and 99:

$\log(14^{71}) = \boxed{71} \times \log(14)$

We use $\boxed{\log x^n = n \cdot \log x}$

Logarithms

Note error in problem
$10^5 = 100000$

Write the exponential equation $10^5 = 10000$ in logarithmic form. State your answer as four integers between 0 and 99.

$\boxed{5} = \log_{\boxed{10}}\left(\boxed{10}^{\boxed{4}}\right)$

Binary/Hex conversion

8 7 6 5 4 3 2 1 0

Fill in the missing values in the below expression such that it converts the binary number $\mathbf{101010101}$ to the decimal number $\mathbf{341_{10}}$. Write your answer as four integers between 0 and 99:

$$1 \times 2^8 + \boxed{0} \times 2^7 + 1 \times 2^6 + \boxed{1} \times 2^4 + 1 \times 2^2 + \boxed{0} \times 2^1 + \boxed{1} \times 2^0$$

Binary/Hex conversion

3 2 1 0

Fill in the missing values in the below expression such that it converts the hexidecimal number 4726_{16} to the decimal number 18214_{10}. Write your answer as three integers between 0 and 99:

$$6 \times 16^{\boxed{0}} + \boxed{4} \times 16^3 + 2 \times 16^1 + \boxed{7} \times 16^2$$

Binary/Hex conversion

What is $110_2 \times B_{16}$ in decimal? Write your answer as an integer between 0 and 99:

6×11

$\boxed{66}$

Number Theory

$a^{\phi(n)} \equiv 1 \mod n$

With reference to Euler's theorem, fill in the missing value in the expression below. State your answer as an integer between 0 and 99.

$17^{\boxed{44}} \equiv 1 \mod 69$

$\phi(69) = (3-1)(23-1) = 2 \cdot 22 = \underline{\underline{44}}$

$69 = 3 \cdot 23$

$\phi(p \cdot q) = (p-1) \cdot (q-1)$ if p, q are prime

Number Theory

$a^x \cdot a^y = a^{x+y}$

Euler's Theorem can be used to calculate exponents in an efficient way. Complete the calculations below with reference to Euler's Theorem. State your answer as integers between 0 and 999.

$$3^{307} \bmod 103 \equiv 3^{3\cdot\boxed{102}+\boxed{1}} \equiv 3$$

Since n is Prime $\Big\{$ $\Phi(103) = 103 - 1 = 102$

$a^{k\cdot\Phi(n)} \equiv 1 \mod n$

$$3^{307} = 3^{3\cdot 102+1} = \underbrace{3^{3\cdot 102}}\cdot 3^1$$

$$= \underbrace{\left(3^{102}\right)}^3 \cdot 3^1 =$$

$a^{\Phi(n)} \equiv 1 \mod n$

Number Theory

Complete the calculations below so that they follow Euler's Phi Function. State your answer as integers betweeen 0 and 99.

$$\phi(300) = \left(2^{\boxed{2}} - \boxed{2}^{\boxed{1}}\right)\left(\boxed{3}^{1} - \boxed{3}^{\boxed{0}}\right)\left(\boxed{5}^{\boxed{2}} - \boxed{5}^{\boxed{1}}\right) = 80$$

$$\phi(n) = \prod \left(p_i^{a_i} - p_i^{a_i - 1}\right)$$

$$300 = 2 \cdot 150 = 2 \cdot 2 \cdot 75 = 2 \cdot 2 \cdot 3 \cdot 25 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5$$

$$= 2^2 \cdot 3^1 \cdot 5^2$$

$$\phi(n) = (2^2 - 2^1)(3^1 - 3^0)(5^2 - 5^1) = 2 \cdot 2 \cdot 20 = 80$$

Number Theory

Let a be positive integer . Find the smallest possible remainder in the expression below. State your answer as an integer between 0 and 99.

$$88 = a \times 7 + \boxed{4}$$

Number Theory

Consider the congruence $ax \equiv 5 \bmod 8$. It is known that the inverse of $a \bmod 8$ is 3. Use this to determine the smallest positive value of x. State your answer as an integer between 0 and 99.

$x = \boxed{7}$

$\underbrace{3 \cdot a}_{b} \cdot x \equiv 3 \cdot 5 \mod 8$

$= 1$ since $a^{-1} = 3 \Rightarrow a^{-1} \cdot a = 1$

$x \equiv 15 \mod 8$, $x = 7$

Number Theory

We want to find 3^{27} $\mathbf{mod\ 11}$ using the square and multiply algorithm. Fill in the missing values below in order to demonstrate correct use of the algorithm. If you only need to square, leave the non-squared box empty. State your answer as integers between 0 and 99.

$$\begin{bmatrix} 1: & 3 & \equiv 3 & \mod 11 \\ 1: & \boxed{3}^2 \cdot \boxed{3} & \equiv \boxed{5} & \mod 11 \\ 0: & \boxed{5}^2 \cdot \boxed{} & \equiv 3 & \mod 11 \\ 1: & 3^2 \cdot \boxed{3} & \equiv \boxed{5} & \mod 11 \\ 1: & \boxed{5}^2 \cdot \boxed{3} & \equiv \boxed{9} & \mod 11 \end{bmatrix}$$

Number Theory

Find the smallest, positive multiplicative inverse of 5 modulo 104. Write your answer as an integer between 0 and 99.

Number Theory

Calculate $3^{207} \pmod{103}$ (e.g. using Euler's theorem or the square and multiply algorithm). Write your answer as an integer between 0 and 99.

27

$$3^{207} = 3^{2\cdot 102+3} = 3^{2\cdot 102}\cdot 3^3 = 1\cdot 3^3 = 27$$

$$\phi(p) = p-1$$

$$\phi(103) = 103-1 = 102$$

$$a^{\phi(n)} \equiv 1 \mod n$$

$$a^{\phi(p)} = a^{p-1} \equiv 1 \mod p$$

Note $a^{k\cdot\phi(n)} \equiv 1 \mod n$,

since $\Big(\underbrace{a^{\phi(n)}}_{=1}\Big)^k \equiv 1 \mod n$

Number Theory

Find $\gcd(508, 1020)$. Write your answer as an integer between 0 and 99.

4

$1020 = 2 \cdot 508 + 4$

$508 = 127 \cdot 4 + 0$

:Asymptotic Notation

In the following, $\log\ n$ refers to log base 2, i.e. the binary logarithm. Be aware that the final three sub-problems involve *Big-Theta* and the previous sub-problems involve *Big-O*. Note, in this assignment you must have more than half correct in order to obtain points.

$$\frac{\text{left}}{\text{right}} \to \circ \quad \text{if left} = O(\text{right})$$

$$n^1 = n^{2/3+1/3} = \cancel{n^{2/3}} \cdot n^{1/3}$$

$$\cancel{n^{2/3}}$$

$$\left(2^{\log n}\right)^2 = n^2$$

$$n^1 = n^{0.01+0.99}$$

notic Θ

	True	False
$\overset{n \cdot \log n}{\log(n!)} = O\left(\log n^2\right)$	○	X
$\cancel{n^{2/3}} + n^3 = O\left(n^2\right)$	○	X
$\cancel{[illegible]} + \overset{n^1}{\underline{n}} \cdot \log n = O\left(n^{2/3}\right)$	○	X
$\cancel{[illegible]} \cdot \overset{n^2}{2^{2\log n}} = O\left((\log n)^3\right)$	○	X
$\log n = O(n \cdot \log n)$	X	○
$\underline{\cancel{(\log n)^2}} + \overset{n}{2^{\log n}} = O\left(n^{0.01}\right)$	○	X
$n! = O(\sqrt{n})$	○	X
$n^{0.001} = O(1)$	○	X
$3^n = O\left(\underline{\overset{n^3}{2^{3\log n}}}\right)$	○	X
$n^2 = \Theta\left(2^{2\log n}\right)$	X	○
$\sqrt{n} = \Theta(n \cdot \log n)$	○	X
$n^2 = \Theta\left(n^{0.1}\right)$	○	X

Loop analysis

Algorithm loop1(n)
$s = 1$
for $i = 1$ **to** n
  for $j = 1$ **to** n
    $s = s + 1$

Algorithm loop2(n)
$s = 1$
while $s \leq n$
  $s = s + 1$

Algorithm loop3(n)
$s = 0$
for $i = 1$ **to** n
  for $j = i$ **to** n
    for $k = i$ **to** j
      $s = s + 1$

For each of the above algorithms, state its execution time as a function of n in Θ-notation.

	$\Theta(n^3)$	$\Theta((\log n)^2)$	$\Theta(\sqrt{n})$	$\Theta(n^2)$	$\Theta(n \log n)$	$\Theta(n\sqrt{n})$	$\Theta(\log n)$	$\Theta(n)$
loop2	○	○	○	○	○	○	○	X
loop1	○	○	○	X	○	○	○	○
loop3	X	○	○	○	○	○	○	○

Recursion

$$a \cdot T\left(\frac{n}{b}\right) + \Theta(n^k)$$

$n^{\log_b a}$ if $a > b^k$

$n^k \cdot \log n$ if $a = b^k$

n^k if $a < b^k$

For each of the recursions below, state its solution where $T(n) = 1$ for $n \leq 1$.

		$\Theta(\log n)$	$\Theta(\sqrt{n})$	$\Theta(n)$	$\Theta(n \log n)$	$\Theta(n^2)$	$\Theta(n^2 \log n)$	$\Theta(n^3)$
1	$T(n) = T(n/3) + 2$	X	○	○	○	○	○	○
2	$T(n) = 2 \cdot T(n/4) + 1$	○	X	○	○	○	○	○
3	$T(n) = 3 \cdot T(n/4) + n$	○	○	X	○	○	○	○
4	$T(n) = 4 \cdot T(n/2) + n^2$	○	○	○	○	○	X	○

(1) $a = 1$, $b = 3$, $k = 0$ $\quad \underline{a = b^k}$

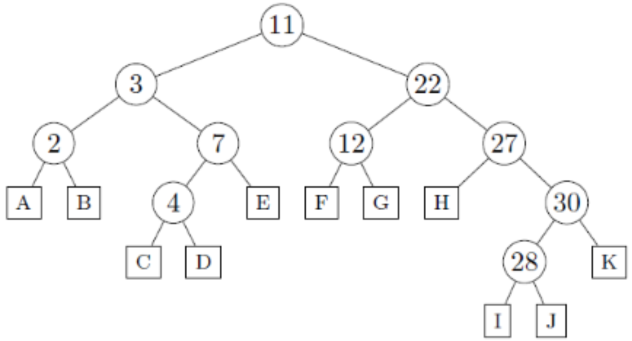
(2) $a = 2$, $b = 4$, $k = 0$ $\quad \underline{a > b^k}$

$n^{\log_4 2}$

(3) $a = 3$, $b = 4$, $k = 1$ $\quad \underline{a < b^k}$

(4) $a = 4$, $b = 2$, $k = 2$ $\quad \underline{a = b^k}$

Preview:Tree Insertion (3%)



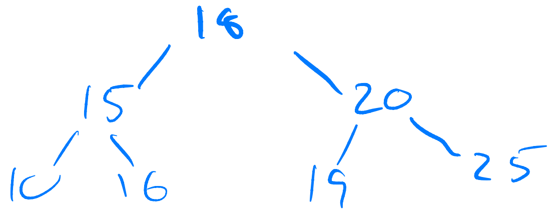
State in which leaf A-K the elements 23, 13, 31, 14 and 6 must be inserted in the above unbalanced binary search tree. Assume that the tree only contains the ten elements above before each insertion (i.e. the elements to be inserted are not present in the tree at a new insertion).

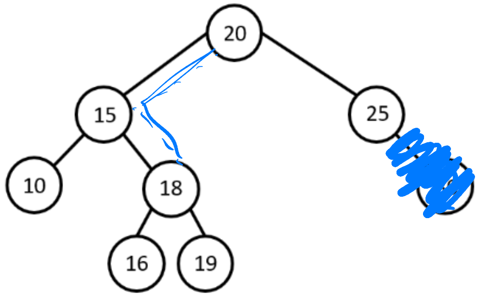
	A	B	C	D	E	F	G	H	I	J	K
INSERT(23)	○	○	○	○	○	○	○	X	○	○	○
INSERT(13)	○	○	○	○	○	○	X	○	○	○	○
INSERT(31)	○	○	○	○	○	○	○	○	○	○	X
INSERT(14)	○	○	○	○	○	○	X	○	○	○	○
INSERT(6)	○	○	○	X	○	○	○	○	○	○	○

Check Answer

AVL Tree

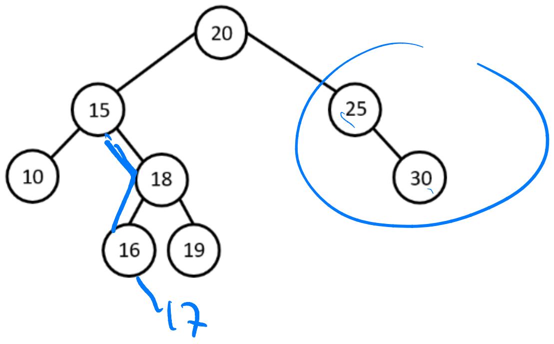
a. If you delete 30 from the following binary search tree using the algorithm that keeps the tree height-balanced by doing rotations, what tree do you get? Drag-and-drop the correct values to the appropriate nodes. Leave the nodes empty that are NIL after rotation.

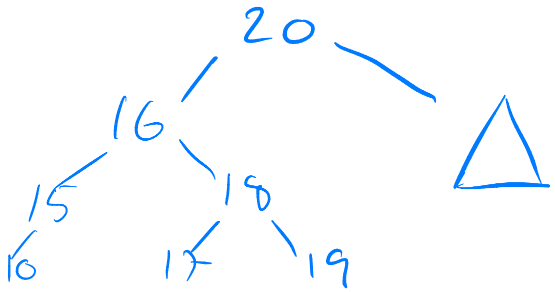




AVL Tree

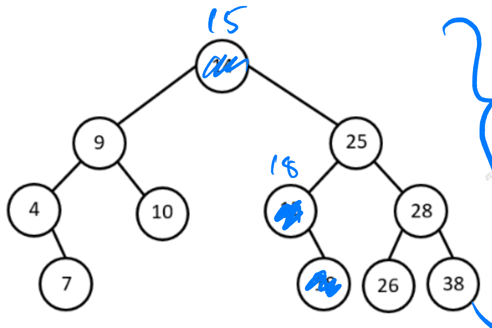
b. If you insert 17 into the following binary search tree using the algorithm that keeps the tree height-balanced by doing rotations, what tree do you get? Drag-and-drop the correct values to the appropriate nodes. Leave the nodes empty that are NIL after rotation.





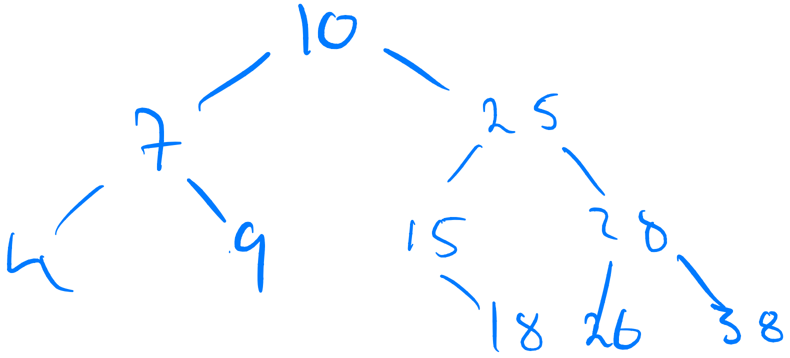
AVL Tree

c. If you delete 14 from the following binary search tree using the algorithm that keeps the tree height-balanced by doing rotations, what tree do you get? Drag-and-drop the correct values to the appropriate nodes. Leave the nodes empty that are NIL after rotation. Note, two answers are accepted for this question.



Solution when looking at right sub

Solution if looking at left:



Linear Probing

0	1	2	3	4	5	6	7	8	9	10
11							17	6	10	21

a. In the hash table above of size 11, *linear probing* with the hash function $h(k) = 3k \bmod 11$ has been used. Find the positions that the five elements 0, 3, 7, 8 and 9 will be inserted into. For each insertion, assume that the hash table only contains the elements shown above, i.e. 6, 10, 11, 17 and 21).

	0	1	2	3	4	5	6	7	8	9	10
INSERT(0)	○	X	○	○	○	○	○	○	○	○	○
INSERT(3)	○	X	○	○	○	○	○	○	○	○	○
INSERT(7)	○	X	○	○	○	○	○	○	○	○	○
INSERT(8)	○	○	X	○	○	○	○	○	○	○	○
INSERT(9)	○	○	○	○	○	X	○	○	○	○	○

Quadratic Probing

b. In the hash table below of size 11, *quadratic probing* with the hash functions $h'(k) = 2k \bmod 11$ and $h = (h'(k) + 3i + 5i^2) \bmod 11$ has been used.

0	1	2	3	4	5	6	7	8	9	10
	4	12			19			15	8	

Find the positions that the five elements 0, 5, 6, 7 and 10 will be inserted into. For each insertion, assume that the hash table only contains the elements shown above, i.e. 4, 8, 12, 15 and19.

	0	1	2	3	4	5	6	7	8	9	10
INSERT(0)	X	○	○	○	○	○	○	○	○	○	○
INSERT(5)	○	○	○	○	○	○	○	○	○	○	X
INSERT(6)	X	○	○	○	○	○	○	○	○	○	○
INSERT(7)	○	○	○	X	○	○	○	○	○	○	○
INSERT(10)	○	○	○	○	○	○	X	○	○	○	○

$1 + 3\cdot 1 + 5\cdot 1^2 = 9$

$1 + 3\cdot 2 + 5\cdot 2^2 = 5$

$1 + 3\cdot 3 + 5\cdot 3^2 =$

$9 + 3\cdot 1 + 5\cdot 1$

Partition: Quicksort

				16	6	12	9	13	14	20	29	25	23	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	5	3	19	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	24
									i			r		

State the result of using $\text{Partition}(A, 4, 14)$ on the array above. Check the correct answer from the choices below.

○

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	3	5	6	9	12	13	14	16	19	20	23	24	25	29

○ X

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	5	3	19	16	6	12	9	13	14	20	29	25	23	24

○

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	5	3	19	16	6	12	9	13	14	20	29	23	25	24

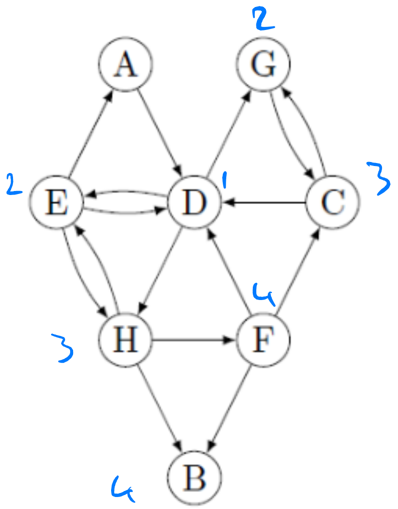
○

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	5	3	6	9	12	13	14	16	19	20	23	25	29	24

PARTITION(A, p, r)

```
x = A[r]
i = p - 1
for j = p to r - 1
    if A[j] ≤ x
        i = i + 1
        exchange A[i] with A[j]
exchange A[i + 1] with A[r]
return i + 1
```

For a breadth-first search (BFS) of the graph below starting in vertex A, state the order the vertices are removed from the queue Q in the BFS-algorithm. We assume that the graph is given by adjacency lists, where the adjacency lists are sorted alphabetically.

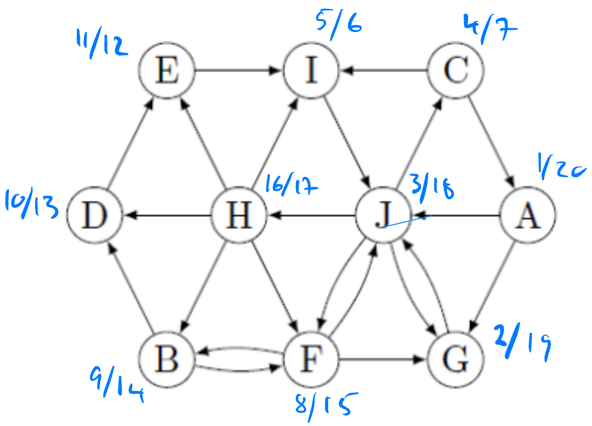


A D E G H C B F

- [ ] ADEGHCBF
- [ ] ADGHECFB
- [ ] ADEGHCFB
- [ ] ADEHBFCG

DepthFirstSearch

a. Consider a depth-first search (DFS) of the graph below starting in **vertex A**, where the outgoing edges to a vertex are visited in **alphabetical** order. State in which order each vertex is assigned **finishing** time.



ICEDBFHJGA

- ICEDBFHJGA ✗
- IEDBGFHCJA ○
- HEDBF ICJGA ○
- EDBIHFCJGA ○

b. Determine the **discovery** times of the following vertices from the graph above. State you answers as integers between 1 and 20.

$D =$ 10

$H =$ 16

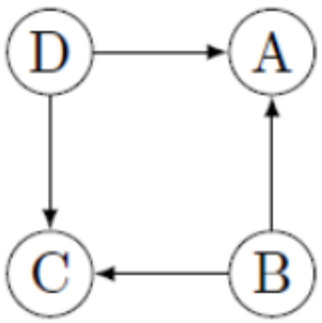
c. Determine the **finishing** times of the following vertices from the graph above. State you answers as integers between 1 and 20.

$B =$ 14

$G =$ 19

Topological Sorting

For each of the below sortings of the vertices of the graph above, state whether or not it is a topological sorting.



	Yes	No
CBAD		
BDAC		
DBAC		
BCAD		
DBCA		

Merge Sort

a. Run MERGE-SORT on the array below and drag-and-drop the values to their correct position as the algorithm is executed.

42	7	10	34	65	61	68	57

42 7 10 34 65 61 68 57

7 42 10 34 6165 57 68

7 10 34 42 57 61 65 68

7 10 34 42 57 61 65 68

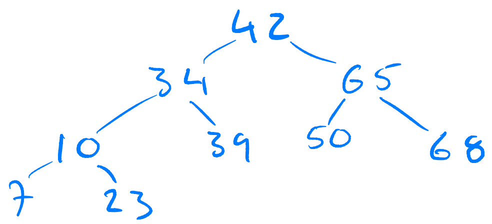
b. For each of the below pair of elements, determine whether MERGE-SORT applied to the above array will compare the two elements as part of its execution. Note, all pairs must be correctly identified in order to obtain points.

		Yes	No
42	10	X	
65	57		X
10	61		X
42	57	X	

Binary Trees

Assume you insert 42, 34, 10, 65, 68, 39, 23, 50, 7 into an initially empty binary tree in the stated order. Identify the correct list which is obtained by subsequently performing each traversal.

	Preorder	Inorder	Postorder	Level order traversal (breadth-first)	Neither DFS nor BFS
42 34 65 10 39 50 68 23 7					X
7 23 10 39 34 42 50 68 65					X
7 10 23 34 39 42 50 65 68		X			
42 34 10 7 23 39 65 50 68	X				
7 23 10 39 34 50 68 65 42			X		



Preorder:

42 34 10 7 23 39 65 50 68

Inorder:

7 10 23 34 39 42 50 65 68

Post order:

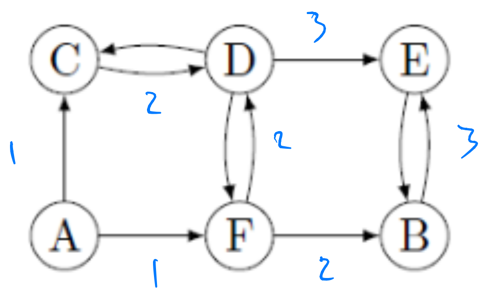
7 23 10 39 34 50 68 65 42

BFS:

42 34 65 10 39 50 68 7 23

Legal BFS

For each of the below set of edges, state whether they make up a legal BFS tree for a breadth-first traversal of the graph below starting in vertex A and for an arbitrary order of the graph's adjacency lists



	Yes	No
(A,C) (A,F) (B,E) (F,B) (F,D)	X	○
(A,C) (A,F) (C,D) (D,E) (F,B)	X	○
(A,C) (A,F) (D,E) (E,B) (F,D)	○	X
(A,C) (C,D) (D,E) (D,F) (F,B)	○	X
(A,F) (D,C) (D,E) (E,B) (F,D)	○	X