

Exercises: Arithmetic and Functions

Exercise 1

Solve the following equations:

a. $\frac{1}{x+1} = \frac{1}{2x}$

b. $2 - \frac{4x+3}{x+x^2} = \frac{2x}{x+1} - \frac{5}{x}$

c. $\frac{y+5}{y-7} - 2 = -\frac{y+7}{y-5}$

Exercise 2

Solve the following equations:

a. $-2 + 2 \ln 3x = 17$

d. $7 \log_4(0.6x) = 12$

g. $\log_{10} x^2 = 6$

b. $\log_5(3x+2) = \log_5(6-x)$

e. $\ln \sqrt{x+2} = 1$

h. $4 \log_{10}(x-6) = 11$

c. $\log_{10}(z-3) = 2$

f. $\ln(x+1)^2 = 2$ (!)

i. $\ln(x^2+1) = 8$

Exercise 3

Solve the following equations:

a. $3^{2x} = 9$

c. $2^{3-x} = 4^{2-x}$

e. $\left(\frac{1}{4}\right)^{x-1} = 4^{2-3x}$

b. $5^{3x+2} = 25^{x-1}$

d. $8^{-x} = 2^{x-8}$

f. $5^{3x+2} = 25^{x-1}$

Exercise 4

According to Einstein's theory of relativity, the mass of a particle is given by

$$m = \frac{m_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

where

m_0 is the mass of the particle at rest

v is the speed of the particle, and

c is the speed of light in a vacuum.

- Make v the subject of the formula given $v > 0$
- Find the speed necessary to increase the mass of a particle to three times its rest mass. Give the value for v as a fraction of c

Exercise 5

Determine the domain and range of each of the real functions below. It is a good idea to plot the functions using some software (e.g. Geogebra):

a. $f(x) = x - 1$

c. $f(x) = \frac{x}{2}$

e. $f(x) = \sqrt{x+3}$

b. $f(x) = x^2 + 1$

d. $f(x) = \frac{1}{x-7}$

Exercise 6

Find each of the following composite functions:

a. $g \circ f$ when $f(x) = 3x + 1$ and $g(x) = x^2$.

b. $f \circ g$ when $f(x) = x^2 + 1$ and $g(x) = \frac{1}{x}$.

c. $g \circ f$ when f and g are defined as in exercise (b).

Exercise 7

Find the inverse of each of the functions below. Verify your result by checking that $f(f^{-1}(x)) = x$ (or, equivalently, that $f^{-1}(f(x)) = x$)

a. $f(x) = 3x - 5$

d. $f(x) = -2 \lg(x - 1) + 2$

g. $f(x) = 2 \cdot 10^{3x} - 1$

b. $f(x) = \frac{6}{5-x}$

e. $f(x) = -\ln(1 - 2x) + 1$

h. $f(x) = -5 \cdot e^{-x} + 2$

c. $f(x) = 3 \cdot \log(x + 3) + 1$

f. $f(x) = 2^x - 3$

Exercise 8

15000 \$ is invested in an account that yields 5% interest per year.

a. Find a functional expression for the amount of money in the account after x years, if the interest is compounded yearly.

b. After how many years will the account be worth 91 221.04 \$?

Exercise 9

Starting at the age of 40, an average man loses 5% of his hair every year.

a. Find an expression for the fraction of hair the average man has got left as a function of his age (after age 40).

b. At what age should an average man expect to have half his hair left?

Exercise 10

A bacteria culture starts with 1000 bacteria at time $t = 0$ and the number doubles every 40 minutes.

- a. Find a functional expression for the number of bacteria at time t (measured in minutes).
- b. Find the number of bacteria after one hour.
- c. After how many minutes will there be 50000 bacteria?