Ex.26 gcd(508, 1020)

1020 = 508 . 2 + 4

508 = 4 . 127 + 0

Ex.27

42

$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$
$2^6 = 64$
$2^7 = 128$
$2^8 = 256$
$2^9 =$ [illegible]

| | True | False |
|---|---|---|
| $2^{\log n} + 5n^n = O(\sqrt{n} \cdot \log n)$ | ○ | ◉ ✔ |
| $n^2 \log n = O(\sqrt{n})$ | ○ | ◉ ✔ |
| $\sqrt{n} \cdot \log n = O(n \cdot \log n)$ | ◉ ✔ | ○ |
| $4n^2 = O\left(n^{0.1}\right)$ | ○ | ◉ ✔ |
| $n^2 = O\left(\log n^2\right)$ | ○ | ◉ ✔ |
| $2^n = O(n \cdot \log n)$ | ○ | ◉ ✔ |
| $n = O(\log n)$ | ○ | ◉ ✔ |
| $2^{3 \log n} = O(n \cdot \log n)$ | ○ | ◉ ✔ |
| $5 = O(n \cdot \log n)$ | ◉ ✔ | ○ |
| $n^{0.01} = \Theta(\sqrt{n})$ | ○ | ◉ ✔ |
| $\log n^2 = O\left(\sqrt{n}\right)$ | ◉ ✔ | ○ |
| $n^2 = O\left(2^{\log n}\right)$ | ○ | ◉ ✔ |

| | | |
|---|---|---|
| $6\sqrt{n} = O(n\sqrt{n})$ | ◉ ✔ | ○ |
| $\sqrt{n} + \sqrt{n} = O(n \cdot \log n)$ | ◉ ✔ | ○ |
| $n \cdot \log n = O\left((\log n)^3\right)$ | ○ | ◉ ✔ |
| $2^n = O(\sqrt{n} \cdot \log n)$ | ○ | ◉ ✔ |
| $n\sqrt{n} = O\left(n^{3/2}\right)$ | ◉ ✔ | ○ |
| $8^{\log n} = O\left(n^{2/3}\right)$ | ○ | ◉ ✔ |
| $n \cdot \log n = O\left((\log n)^2\right)$ | ○ | ◉ ✔ |
| $n^n = O\left(3^n\right)$ | ○ | ◉ ✔ |
| $7n \cdot \log n = \Theta(\log(n!))$ | ◉ ✔ | ○ |
| $\sum_{i=1}^{n} i = O(\sqrt{n} \cdot \log n)$ | ○ | ◉ ✔ |

**Algorithm** loop1($n$)
$s = 0$
**for** $i = 1$ **to** $n$
  **for** $j = 1$ **to** $i * i$
    $s = s + 1$

**Algorithm** loop2($n$)
$s = 1$
**for** $i = 1$ **to** $n$
  **for** $j = 1$ **to** $n$
    $s = s + 1$

**Algorithm** loop3($n$)
$i = 0$
$s = 0$
**while** $s \leq n$
  $i = i + 1$
  $s = s + i$

**Algorithm** loop4($n$)
**for** $i = 0$ **to** $n$
  $j = 0$
  $s = 0$
  **while** $s \leq i$
    $j = j + 1$
    $s = s + j$

For each of the above algorithms, state its execution time as a function of $n$ in $\Theta$-notation.

| | $\Theta\left(n^3\right)$ | $\Theta\left((\log n)^2\right)$ | $\Theta(\sqrt{n})$ | $\Theta\left(n^2\right)$ | $\Theta(n \log n)$ | $\Theta(n\sqrt{n})$ | $\Theta(\log n)$ | $\Theta(n)$ |
|---|---|---|---|---|---|---|---|---|
| loop4 | ○ | ○ | ○ | ○ | ○ | ◉ ✔ | ○ | ○ |
| loop3 | ○ | ○ | ◉ ✔ | ○ | ○ | ○ | ○ | ○ |
| loop2 | ○ | ○ | ○ | ◉ ✔ | ○ | ○ | ○ | ○ |
| loop1 | ◉ ✔ | ○ | ○ | ○ | ○ | ○ | ○ | ○ |

**Algorithm** loop1($n$)
  $s = 1$
  **for** $i = n$ **to** 1 **step** $-1$
    $s = s + 1$

**Algorithm** loop2($n$)
  **for** $i = 1$ **to** $n$
    $j = i$
    **while** $j > 0$
      $j = j - 1$

**Algorithm** loop3($n$)
  $s = 0$
  $i = n$
  **while** $i > 0$
    **for** $j = 1$ **to** $i$
      $s = s + 1$
    $i = i - 1$

**Algorithm** loop4($n$)
  $i = 0$
  $j = 0$
  **while** $i \leq n$
    **if** $i < j$ **then**
      $i = i + 1$
    **else**
      $j = j + 1$
      $i = 0$

For each of the above algorithms, state its execution time as a function of $n$ in $\Theta$-notation.

| | $\Theta\left(n^3\right)$ | $\Theta\left((\log n)^2\right)$ | $\Theta(\sqrt{n})$ | $\Theta\left(n^2\right)$ | $\Theta(n \log n)$ | $\Theta(n\sqrt{n})$ | $\Theta(\log n)$ | $\Theta(n)$ |
|---|---|---|---|---|---|---|---|---|
| loop2 | ○ | ○ | ○ | ◉ ✔ | ○ | ○ | ○ | ○ |
| loop1 | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ◉ ✔ |
| loop3 | ○ | ○ | ○ | ◉ ✔ | ○ | ○ | ○ | ○ |
| loop4 | ○ | ○ | ○ | ◉ ✔ | ○ | ○ | ○ | ○ |

**Algorithm** loop1($n$)
  $s = 1$
  **for** $i = 1$ **to** $n$
    $s = s + 1$

**Algorithm** loop2($n$)
  $i = 1$
  **while** $i \leq n$
    $i = 2 * i$

**Algorithm** loop3($n$)
  $i = 1$
  **while** $i * i \leq n$
    $i = i + i$

**Algorithm** loop4($n$)
  $i = 1$
  **while** $i \leq n$
    $j = 0$
    **while** $j \leq i$
      $j = j + 1$
    $i = 2 * i$

For each of the above algorithms, state its execution time as a function of $n$ in $\Theta$-notation.

| | $\Theta\left(n^3\right)$ | $\Theta\left((\log n)^2\right)$ | $\Theta(\sqrt{n})$ | $\Theta\left(n^2\right)$ | $\Theta(n \log n)$ | $\Theta(n\sqrt{n})$ | $\Theta(\log n)$ | $\Theta(n)$ |
|---|---|---|---|---|---|---|---|---|
| loop4 | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ◉ ✔ |
| loop3 | ○ | ○ | ○ | ○ | ○ | ○ | ◉ ✔ | ○ |
| loop1 | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ◉ ✔ |
| loop2 | ○ | ○ | ○ | ○ | ○ | ○ | ◉ ✔ | ○ |