Exercises: Sequences, Series and Recursion

Exercise 1

Given the sequence $\{a_n\}_{n=1}^{5} = \{1, 3, 5, 7, 9\}$

a. What is the value of a_3?

b. Find the value of $\sum_{n=1}^{5} a_n$.

Exercise 2

Expand the following series and find the sum

$$\sum_{n=0}^{4} 2n$$

Exercise 3

List the first four terms of the following sequence, beginning with $n = 0$

$$\{a_n\}_{n=0} = \frac{(-1)^n}{(n+1)!}$$

Exercise 4

Find the sum of the first six terms of $\{a_n\}_{n=1}$ where $a_n = 2a_{n-1} + a_{n-2}$, $a_1 = 1$ and $a_2 = 1$.

Exercise 5

Write the following series using summation notation, beginning with $n = 1$:

$$2 \ -4 \ +6 \ -8 \ +10$$

Exercise 6

Write the following using summation notation

$$\frac{5}{6+3} + \frac{5}{7+3} + \frac{5}{8+3} + \cdots + \frac{5}{31+3}$$

Exercise 7

Use summation formulae to determine the values of the sums below:

a. $\sum_{k=1}^{10} k$

b. $\sum_{k=1}^{10} k^2$

c. $1+2+3+4+\cdots+99+100$

d. $\sum_{k=0}^{\infty} 0.6^k$

e. $\sum_{k=0}^{42} 3 \cdot 2^k$

Exercise 8

A person deposits \$1000 in an account that yields 9% interest compounded annually.

a. Set up a recurrence relation for the amount in the account at the end of n years.

b. Find an explicit formula for the amount in the account at the end of n years.

c. How much money will the account contain after 100 years?

Exercise 9

Suppose that the number of bacteria in a colony triples every hour.

a. Set up a recurrence relation for the number of bacteria after n hours have elapsed.

b. If 100 bacteria are used to begin a new colony, how many bacteria will be in the colony after 10 hours?

Exercise 10

A factory makes custom sports cars at an increasing rate. In the first month only one car is made, in the second month two cars are made, and so on, with n cars made in the n'th month.

a. Set up a recurrence relation for the number of cars produced in the first n months by this factory.

b. How many cars are produced in the first year?

c. Find an explicit formula for the number of cars produced in the first n months by this factory.