Gør tanke til handling

VIA University College

Binary numbers

Decimal numbers

The most commonly used number system today is the decimal number system. In this system, each digit represent a value that is 10 times larger than the value to the right. For example:

$$23{,}948 = 2 \cdot 10000 + 3 \cdot 1000 + 9 \cdot 100 + 4 \cdot 10 + 8 \cdot 1$$

or, equivalently

$$23{,}948 = 2 \cdot 10^4 + 3 \cdot 10^3 + 9 \cdot 10^2 + 4 \cdot 10^1 + 8 \cdot 10^0$$

To write decimal numbers, we use the 10 symbols $\{0,1,2,3,4,5,6,7,8,9\}$.

Binary numbers

Modern computers only have two "symbols": 0 and 1. Therefore, it seems natural to use a number system which only need two symbols – this is exactly what the binary number system does.

In binary, the first digit from the right represent the value $1 = 2^0$ (just as in decimal), the second digit represent the value $2 = 2^1$, the third digit represent the value $4 = 2^2$ and so on. So e.g.

$$10110_2 = 1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 21_{10}$$

we use a subscript number to denote which number representation we mean. If we omit this, we usually mean decimal.

or, equivalently,

$$10110_2 = 1 \cdot 16 + 0 \cdot 8 + 1 \cdot 4 + 1 \cdot 2 + 0 \cdot 1 = 21_{10}$$

Converting from binary to decimal

Example:

Convert 1101_2 to decimal.

Solution:

$$1101_2 = 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0$$
$$= 1 \cdot 8 \ + 1 \cdot 4 \ + 0 \cdot 2 \ + 1 \cdot 1$$
$$= 13$$

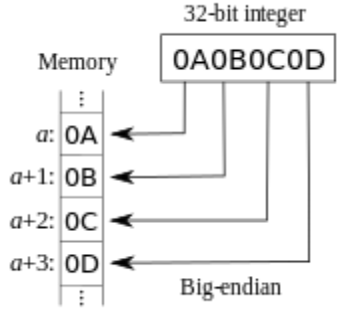
Hexadecimal numbers – why?

The computer uses binary numbers to represent addresses in the computer register. Unfortunately, it takes up a lot of space on the screen to print these addresses, which can for example be

00001010000010110000110000001101.

Hexadecimal (or simply "hex") numbers are often used in computers, because it is really easy to convert between binary and hexadecimal numbers, and the hex representation of any given number is much shorter than the binary representation. For example, the address above in hexadecimal is

0A0B0C0D



Hexadecimal numbers

In hex, the first digit from the right represent the value $1 = 16^0$, the second digit represent the value $16 = 16^1$, the third digit represent the value $16^2 = 256$ and so on. So e.g.

$$9201_{16} = 9 \cdot 16^3 + 2 \cdot 16^2 + 0 \cdot 16^1 + 1 \cdot 16^0 = 37{,}377_{10}$$

or, equivalently,

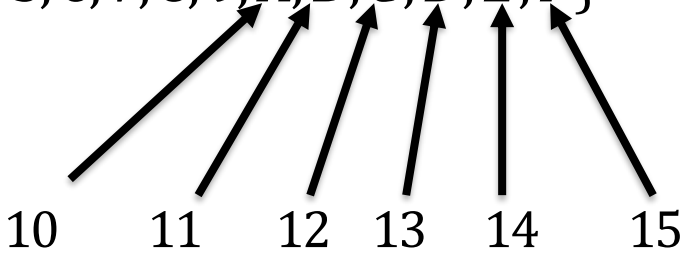
$$9201_{16} = 9 \cdot 4{,}096 + 2 \cdot 256 + 0 \cdot 16 + 1 \cdot 16 = 37{,}377_{10}$$

Extra symbols in hex

In binary, we have 2 symbols: $\{0,1\}$, in decimal we have 10 symbols: $\{0,1,2,3,4,5,6,7,8,9\}$. In hex, we have 16 symbols. It is natural to use $0-9$ for the first 10. For the remaining 6, we use the first 6 (uppercase) letters of the alphabet – these then represent the values from 10 to 15:

<u>Hexadecimal:</u>

Sixteen symbols: $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F\}$



Converting from hexadecimal to decimal

Convert $D802_{16}$ to decimal.

Solution:

$$\begin{aligned} D802_{16} &= D \cdot 16^3 + 8 \cdot 16^2 + 0 \cdot 16^1 + 2 \cdot 16^0 \\ &= 13 \cdot 4096 + 8 \cdot 256 + 0 \cdot 16 + 2 \cdot 1 \\ &= 55298 \end{aligned}$$

Converting from decimal to binary

In the following slides, we will see how to convert a number from decimal to binary, and how to convert between binary and hex. If you know and prefer another method, feel free to use that instead ☺

Converting from decimal to binary

Example:

Convert the decimal number 88 to binary:

1) Identify the largest power of 2 which is less than or equal to 88 – this is 64.
2) We now have $88 - 64 = 24$ left. Identify the largest power of 2 which is less than or equal to 24 – this is 16.
3) We now have $24 - 16 = 8$ left. Identify the largest power of 2 which is less than or equal to 8 – this is 8.

These considerations tells us that $88 = 64 + 16 + 8$. We could write this as

$$88 = 1 \cdot 64 + 0 \cdot 32 + 1 \cdot 16 + 1 \cdot 8 + 0 \cdot 4 + 0 \cdot 2 + 0 \cdot 1$$

or $88 = 1 \cdot 2^6 + 0 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0$, so

$$88 = 1011000_2$$

Converting a binary number to hex

Starting from the right, divide the number into blocks of 4 digits (add zeros in front if necessary) and convert each block individually:

$$10111101011010010$$

$$0101 \quad 1110 \quad 1011 \quad 0101 \quad 0010$$

$$=$$

$$5 \quad E \quad B \quad 5 \quad 2_{16}$$

Converting a hex number to binary

To convert a hexadecimal number to binary, we simply convert each digit into a block of 4 binary digits – e.g.:

$$A \rightarrow 1010$$
$$F \rightarrow 1111$$
$$5 \rightarrow 0101$$
$$0 \rightarrow 0000$$

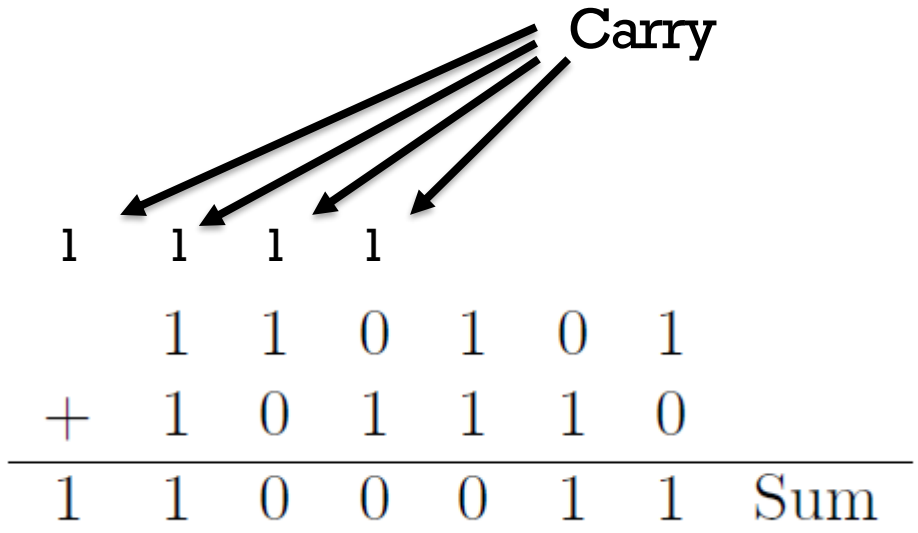
Note that it is important that each digit is converted to **four** – no less! – digits (however, you are of course allowed to remove zeros in front of the hex result).

Example:

$$4BC03_{16} = 0100\ 1011\ 1100\ 0000\ 0011_2 = 100\ 1011\ 1100\ 0000\ 0011_2$$

Adding and multiplying in binary

Adding in binary



Multiplying in binary

			1	0	0	1	1	1	0	Multiplicand
$\times$							1	0	1	Multiplier
			1	0	0	1	1	1	0	times 1
		0	0	0	0	0	0	0		Shift left one and times 0
	1	0	0	1	1	1	0			Shift left two and times 1
	1	1	0	0	0	0	1	1	0	Add to get the product