Exercises: Arithmetic and Functions - Solutions

Exercise 1

Solve the following equations:

    a. $x = 1$

    b. $x = -\dfrac{2}{3}$

    c. $y = 6$

Exercise 2

Solve the following equations:

    a. $x = \dfrac{e^{\frac{19}{2}}}{3}$

    b. $x = 1$

    c. $z = 103$

    d. $x \approx 17.945$

    e. $x = -2 + e^2$

    f. $x = \pm e - 1$

    g. $x = \pm 1000$

    h. $x = 6 + 100 \times 10^{3/4}$

    i. $x = \pm\sqrt{-1 + e^8}$

Exercise 3

Solve the following equations:

    a. $x = 1$

    b. $x = -4$

    c. $x = 1$

    d. $x = 2$

    e. $x = \dfrac{1}{2}$

    f. $x = -4$

Exercise 4

According to Einstein's theory of relativity, the mass of a particle is given by

$$m = \frac{m_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

where

    m_0 is the mass of the particle at rest

    v is the speed of the particle, and

    c is the speed of light in a vacuum.

    a. Make v the subject of the formula given $v > 0$: $v = c \cdot \sqrt{1 - \left(\dfrac{m_0}{m}\right)^2}$

    b. Find the speed necessary to increase the mass of a particle to three times its rest mass. Give the value for v as a fraction of c: $v = 0.943c$

Exercise 5

Determine the domain and range of each of the real functions below. It is a good idea to plot the functions using some software (e.g. Geogebra):

a. Domain: $\mathbb{R}$; Range: $\mathbb{R}$

b. Domain: $\mathbb{R}$; Range: $\mathbb{R}_{\geq 1}$

c. Domain: $\mathbb{R}$; Range: $\mathbb{R}$

d. Domain: $\mathbb{R}\backslash\{7\}$;
Range: $\mathbb{R}\backslash\{0\}$

e. Domain: $\mathbb{R}_{\geq -3}$;
Range: $\mathbb{R}_{\geq 0}$

Exercise 6

Find each of the following composite functions:

a. $(g \circ f)(x) = 9x^2 + 1 + 6x$

b. $(f \circ g)(x) = \dfrac{1}{x^2} + 1$

c. $(g \circ f)(x) = \dfrac{1}{x^2 + 1}$

Exercise 7

Find the inverse of each of the functions below. Verify your result by checking that $f\left(f^{-1}(x)\right) = x$ (or, equivalently, that $f^{-1}(f(x)) = x$)

a. $f^{-1}(x) = \dfrac{x+5}{3}$

b. $f^{-1}(x) = 5 - \dfrac{6}{x}$

c. $f^{-1}(x) = 10^{(x-1)/3} - 3$

d. $f^{-1}(x) = 2^{(2-x)/2} + 1$

e. $f^{-1}(x) = \left(1 - e^{1-y}\right)/2$

f. $f^{-1}(x) = \lg(x+3)$

g. $f^{-1}(x) = \dfrac{\log\left(\frac{x+1}{2}\right)}{3}$

h. $f^{-1}(x) = -\ln\left(\dfrac{2-x}{5}\right)$

Exercise 8

$15000\,\$$ is invested in an account that yeilds 5% interest per year.

a. $f(x) = 15000 \cdot 1.05^x$

b. After 37 years

Exercise 9

Starting at the age of 40 , an average man looses 5% of his hair every year.

a. $f(x) = 0.95^{x-40}$

b. At age 53.5

Exercise 10

A bacteria culture starts with 1000 bacteria at time $t = 0$ and the number doubles every 40 minutes.

    a. $f(t) = 1000 \cdot 2^{t/40}$

    b. $f(60) \approx 2828$

    c. 225.75 minutes