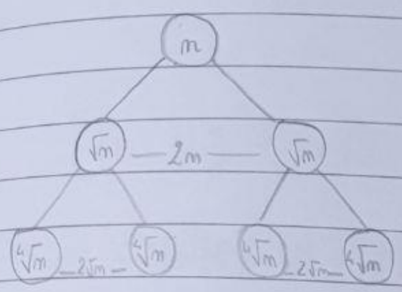
EQUAZIONE 1

$$T(m) = \begin{cases} 1 & \text{se } m \leq 2 \\ 2m\, T(\sqrt{m}) + m^2 & \text{se } m > 2 \end{cases}$$



LIV.	ISTANZA	N° NODI	CONTR. NODO	CONTR. LIV.
0	m	1	m^2	m^2
1	$\sqrt{m}$	$2m$	m	$2m^2$
2	$\sqrt[4]{m}$	$2m \cdot 2\sqrt{m}$	$\sqrt{m}$	$2m \cdot 2\sqrt{m} \cdot \sqrt{m} = 4m^2$
i	$\sqrt[2^i]{m}$	$2m \cdot 2\sqrt{m} \cdot 2^i\sqrt{m}$	$\left(\sqrt[2^i]{m}\right)^2$	$2^i m^2$

Calcolo $h := \sqrt[2^i]{m} = 2 \iff m^{\frac{1}{2^i}} = 2 \Rightarrow \frac{1}{2^i} = \log_m 2 \iff 2^i = \frac{1}{\log_m 2} \Rightarrow i = \log_2(\log_2 m)$

Calcoliamo la sommatoria

$$T(m) = \sum_{i=0}^{h} 2^i \cdot m^2 = \sum_{i=0}^{\log_2(\log_2 m)} 2^i \cdot m^2 \Rightarrow m^2 \cdot \sum_{i=0}^{\log_2(\log_2 m)} 2^i$$

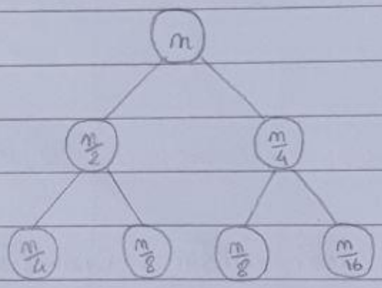
serie geometrica con ragione > 1

$$= m^2 \cdot \left(\frac{2^{\log_2 \log_2 m + 1} - 1}{2 - 1} \right) = m^2 \left(2^{\log_2 \log_2 m} \cdot 2 - 1 \right) = m^2 \cdot (2 \log_2 m - 1) = 2m^2 \cdot \log_2 m - m^2 =$$

$$= \Theta(m^2 \log_2 m)$$

EQUAZIONE 2

$$T(n) = \begin{cases} 1 & \text{se } n \leq 1 \\ T\left(\frac{n}{2}\right) + T\left(\frac{n}{4}\right) + n^2 & \text{se } n > 1 \end{cases}$$



LIVELLO	CONTRIBUTO
0	n^2
1	$\left(\frac{n}{2}\right)^2 + \left(\frac{n}{4}\right)^2 = \frac{5}{16} n^2$
2	$\left(\frac{n}{4}\right)^2 + \left(\frac{n}{8}\right)^2 + \left(\frac{n}{8}\right)^2 + \left(\frac{n}{16}\right)^2 = \frac{25}{256} n^2$
i	$\left(\frac{5}{16}\right)^i n^2$

Calcolo h_1 := $\frac{n}{2^i} = 1 \iff 2^i = n \Rightarrow i = \log_2 n$

Calcolo h_2 := $\frac{n}{4^i} = 1 \iff 4^i = n \Rightarrow i = \log_4 n$

Calcolo la sommatoria:

$$T_{h_1}(n) = \sum_{i=0}^{h_1} \left(\frac{5}{16}\right)^i \cdot n^2 = \sum_{i=0}^{\log_2 n} \left(\frac{5}{16}\right)^i \cdot n^2 = n^2 \sum_{i=0}^{\log_2 n} \left(\frac{5}{16}\right)^i$$ serie geometrica con ragione < 1

$$= n^2 \cdot \frac{1}{1 - \frac{5}{16}} = n^2 \cdot \frac{1}{\frac{11}{16}} = 4n^2 = \Theta(n^2)$$

Calcolo la sommatoria:

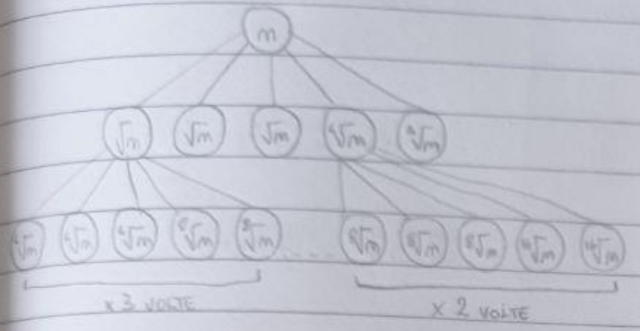
$$T_{h_2}(n) = \sum_{i=0}^{h_2} \left(\frac{5}{16}\right)^i \cdot n^2 = \sum_{i=0}^{\log_4 n} \left(\frac{5}{16}\right)^i \cdot n^2 = n^2 \cdot \sum_{i=0}^{\log_4 n} \left(\frac{5}{16}\right)^i$$ serie geometrica con ragione < 1

$$= n^2 \cdot \frac{1}{1 - \frac{5}{16}} = n^2 \cdot \frac{1}{\frac{11}{16}} = 4n^2 = \Theta(n^2)$$

Sapendo che $T_{h_1}(n) \leq T(n) \leq T_{h_2}(n) \Rightarrow \Theta(n^2) \leq T(n) \leq \Theta(n^2) \Rightarrow T(n) = \Theta(n^2)$

EQUAZIONE 3

$$T(n) = \begin{cases} 1 & \text{se } n \le 2 \\ 3T(\sqrt{n}) + 2T(\sqrt[4]{n}) + \log n & \text{se } n > 2 \end{cases}$$



LIVELLO	CONTRIBUTO
0	$\log n$
1	$3\log n^{\frac{1}{2}} + 2\log n^{\frac{1}{4}} = \left(\frac{3}{2} + \frac{1}{2}\right)\log n = 2\log n$
2	$3(3\log n^{\frac{1}{4}} + 2\log n^{\frac{1}{8}}) + 2(3\log n^{\frac{1}{8}} + 2\log n^{\frac{1}{16}}) = \left(\frac{9}{4} + \frac{3}{4}\right)\log n + \left(\frac{3}{4} + \frac{1}{4}\right)\log n = 3\log n + \log n = 4\log n$
i	$2^i \log n$

Calcolo $h_1 := \sqrt[2^i]{n} = 2 \iff n^{\frac{1}{2^i}} = 2 \iff \frac{1}{2^i} = \log_n 2 \iff 2^i = \log_2 n \Rightarrow i = \log_2(\log_2 n)$

Calcolo $h_2 := \sqrt[4^i]{n} = 2 \iff n^{\frac{1}{4^i}} = 2 \iff \frac{1}{4^i} = \log_n 2 \iff i = \log_4(\log_2 n) = \frac{\log_2(\log_2 n)}{2}$

Calcolo la sommatoria

$$T_{h_1}(n) = \sum_{i=0}^{h_1} 2^i \cdot \log n = \sum_{i=0}^{\log_2(\log_2 n)} 2^i \cdot \log n = \log n \cdot \sum_{i=0}^{\log_2 \log_2 n} 2^i$$ serie geometrica con ragione > 1

$$= \log n \cdot \left(\frac{2^{\log_2(\log_2 n)+1} - 1}{2 - 1} \right) = \log n \left(2^{\log_2(\log_2 n)} \cdot 2 - 1 \right) = (2\log_2 n - 1) \cdot \log n = 2\log^2 n - \log n = \Theta(\log^2 n)$$

Calcolo la sommatoria:

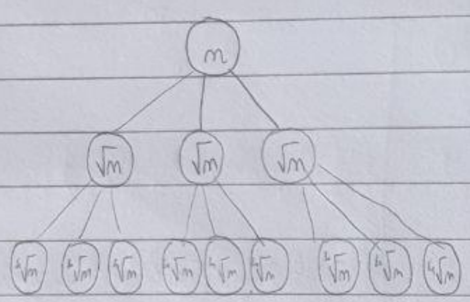
$$T_{h_2}(n) = \sum_{i=0}^{h_2} 2^i \cdot \log n = \sum_{i=0}^{\log_4(\log_2 n)} 2^i \cdot \log n = \log n \cdot \sum_{i=0}^{\log_4(\log_2 n)} 2^i$$ serie geometrica con ragione > 1

$$= \log n \cdot \left(\frac{2^{\log_4(\log_2 n)+1} - 1}{2 - 1} \right) = \log n \cdot \left(2^{\frac{\log_2(\log_2 n)}{2}} \cdot 2 - 1 \right) =$$

$$= \log n \left[2(\log_2 n)^{\frac{1}{2}} - 1 \right] = 2(\log_2 n)^{\frac{3}{2}} - \log_2 n = \Theta\left(\log^{\frac{3}{2}}(n)\right) \Rightarrow \Theta(\log^2 n) \le T(n) \le \Theta\left(\log^{\frac{3}{2}}(n)\right)$$

EQUAZIONE 4

$$T(n) = \begin{cases} 1 & \text{se } n \le 2 \\ 3T(\sqrt{n}) + \log n & \text{se } n > 2 \end{cases}$$



LIVELLO	CONTRIBUTO
0	$\log n$
1	$3 \log n^{\frac{1}{2}} = \frac{3}{2} \log n$
2	$9 \log n^{\frac{1}{4}} = \frac{9}{4} \log n$
i	$\left(\frac{3}{2}\right)^i \cdot \log n$

Calcolo h: $\sqrt[2^i]{n} = 2 \iff n^{\frac{1}{2^i}} = 2 \iff \frac{1}{2^i} = \log_n 2 \Rightarrow i = \log_2(\log_2 n)$

Calcolo la sommatoria:

$$T(n) = \sum_{i=0}^{h} \left(\frac{3}{2}\right)^i \cdot \log n = \sum_{i=0}^{\log_2 \log_2 n} \left(\frac{3}{2}\right)^i \cdot \log n = \log n \cdot \sum_{i=0}^{\log_2 \log_2 n} \left(\frac{3}{2}\right)^i$$ serie geometrica con ragione > 1

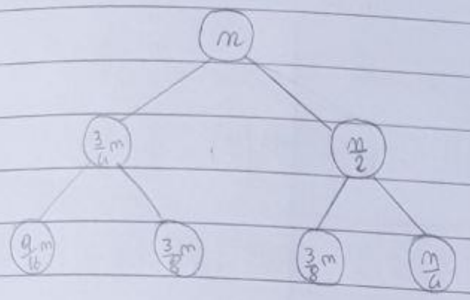
$$= \log n \cdot \frac{\left(\frac{3}{2}\right)^{\log_2(\log_2 n)+1} - 1}{\frac{3}{2} - 1} = \log n \cdot 2\left(\left(\frac{3}{2}\right)^{\log_2 \log_2 n} \cdot \frac{3}{2} - 1\right) = 3 \log n \cdot \left(\frac{3}{2}\right)^{\log_2 \log_2 n} - 2 \log n$$

$$= 3 \log n \cdot (\log_2 n)^{\log_2\left(\frac{3}{2}\right)} - 2 \log n = 3 \log n \cdot (\log_2 n)^{\log_2(3) - 1} - 2 \log n$$

$$= 3 \cancel{\log n} \cdot \frac{(\log_2 n)^{\log_2(3)}}{\cancel{\log_2 n}} - 2 \log n = 3(\log_2 n)^{\log_2(3)} - 2 \log n \Rightarrow \Theta\left((\log_2 n)^{\log_2(3)}\right)$$

EQUAZIONE 5

$$T(n) = \begin{cases} 1 & \text{se } n \le 1 \\ T\left(\frac{3}{4}n\right) + T\left(\frac{n}{2}\right) + n & \text{se } n > 1 \end{cases}$$



LIVELLO	CONTRIBUTO
0	n
1	$\frac{3}{4}n + \frac{1}{2}n = \frac{5}{4}n$
2	$\frac{9}{16}n + \frac{3}{8}n + \frac{3}{8}n + \frac{1}{4}n = \frac{25}{16}n = \left(\frac{5}{4}\right)^2 n$
i	$\left(\frac{5}{4}\right)^i \cdot n$

Calcolo $h_1 := \left(\frac{3}{4}\right)^i n = 1 \Rightarrow \left(\frac{4}{3}\right)^i = n \Rightarrow i = \log_{\frac{4}{3}} n$

Calcolo $h_2 := \frac{n}{2^i} = 1 \Rightarrow 2^i = n \Rightarrow i = \log_2 n$

Calcolo la sommatoria:

$T_{h_1}(n) = \sum_{i=0}^{h_1} \left(\frac{5}{4}\right)^i \cdot n = \sum_{i=0}^{\log_{\frac{4}{3}} n} \left(\frac{5}{4}\right)^i \cdot n = n \cdot \sum_{i=0}^{\log_{\frac{4}{3}} n} \left(\frac{5}{4}\right)^i$ serie geometrica con ragione > 1

$= n \frac{\left(\frac{5}{4}\right)^{h+1} - 1}{\frac{5}{4} - 1} = n \cdot 4\left(\left(\frac{5}{4}\right)^h \cdot \frac{5}{4} - 1\right) = 5n \cdot \left(\frac{5}{4}\right)^{\log_{\frac{4}{3}} n} - 4n = 5n \cdot n^{\log_{\frac{4}{3}} \frac{5}{4}} - 4n$

$= 5 \cdot n^{\log_{\frac{4}{3}}\left(\frac{5}{4}\right) + 1} - 4n \Rightarrow \Theta\left(n^{\log_{\frac{4}{3}}\left(\frac{5}{4}\right) + 1}\right)$

Calcolo la sommatoria:

$T_{h_2}(n) = \sum_{i=0}^{h_2} \left(\frac{5}{4}\right)^i \cdot n = \sum_{i=0}^{\log_2 n} \left(\frac{5}{4}\right)^i \cdot n = n \cdot \sum_{i=0}^{\log_2 n} \left(\frac{5}{4}\right)^i$ serie geometrica con ragione > 1

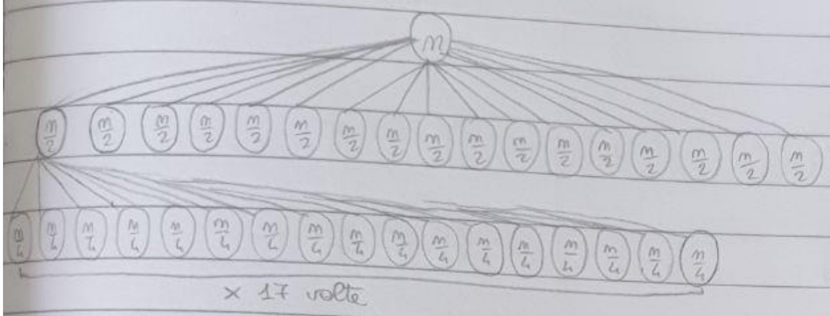
$= 5n \left(\frac{5}{4}\right)^{\log_2 n} - 4n = 5n \cdot n^{\log_2\left(\frac{5}{4}\right)} - 4n = 5n^{\log_2\left(\frac{5}{4}\right) + 1} - 4n \Rightarrow \Theta\left(n^{\log_2\left(\frac{5}{4}\right) + 1}\right)$

Essendo $T_{h_2}(n) \le T(n) \le T_{h_1}(n)$ avremo $\Theta\left(n^{\log_2\left(\frac{5}{4}\right) + 1}\right) \le T(n) \le \Theta\left(n^{\log_{\frac{4}{3}}\left(\frac{5}{4}\right) + 1}\right)$

N.B. Abbiamo scritto che $T_{h_2}(n) \le T(n) \le T_{h_1}(n)$ poiché $T_{h_2}(n)$ decresce più velocemente e quindi termina prima di $T_{h_1}(n)$

EQUAZIONE 7

$$T(m) = \begin{cases} 1 & \text{se } m \leq 1 \\ 17T\left(\frac{m}{2}\right) + m^4 & \text{se } m > 1 \end{cases}$$



LIVELLO	CONTRIBUTO
0	m^4
1	$17 \cdot \left(\frac{m}{2}\right)^4 = 17 \cdot \frac{m^4}{16}$
2	$17^2 \left(\frac{m}{4}\right)^4 = 17^2 \cdot \frac{m^4}{256} = 17^2 \cdot \frac{m^4}{16^2}$
i	$17^i \cdot \frac{m^4}{16^i} = \left(\frac{17}{16}\right)^i \cdot m^4$

Calcolo h: $\frac{m}{2^i} = 1 \Rightarrow 2^i = m \Rightarrow i = \log_2 m$

Calcolo la sommatoria:

$$T(m) = \sum_{i=0}^{h} \left(\frac{17}{16}\right)^i \cdot m^4 = \sum_{i=0}^{\log_2 m} \left(\frac{17}{16}\right)^i \cdot m^4 = m^4 \cdot \sum_{i=0}^{\log_2 m} \left(\frac{17}{16}\right)^i$$ serie geometrica con ragione > 1

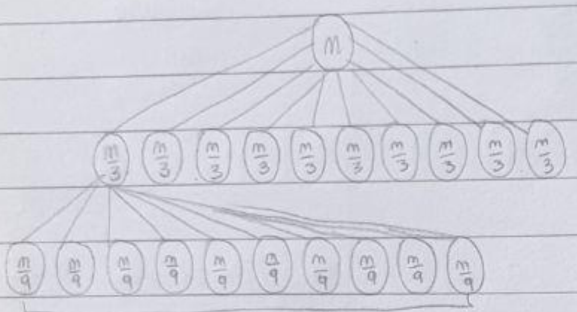
$$= m^4 \cdot \frac{\left(\frac{17}{16}\right)^{h+1} - 1}{\frac{17}{16} - 1} = m^4 \cdot \frac{\left(\frac{17}{16}\right)^{\log_2 m + 1} - 1}{\frac{1}{16}} = m^4 \left[16\left(\left(\frac{17}{16}\right)^{\log_2 m} \cdot \frac{17}{16} - 1\right)\right]$$

$$= 17\, m^4 \cdot \left(\frac{17}{16}\right)^{\log_2 m} - 16\, m^4 = 17\, m^4 \cdot m^{\log_2\left(\frac{17}{16}\right)} - 16\, m^4 = 17\, m^{\log_2\left(\frac{17}{16}\right) + 4} - 16\, m^4$$

$$\Rightarrow T(m) = \Theta\left(m^{\log_2\left(\frac{17}{16}\right) + 4}\right)$$

EQUAZIONE 8

$$T(m) = \begin{cases} 1 & \text{se } m \leq 2 \\ 10T\left(\frac{m}{3}\right) + m^2 & \text{se } m > 2 \end{cases}$$



x 10 VOLTE

LIVELLO	CONTRIBUTO
0	m^2
1	$10\left(\frac{m}{3}\right)^2 = 10 \cdot \frac{m^2}{9}$
2	$10^2\left(\frac{m}{9}\right)^2 = 10^2 \cdot \frac{m^2}{81}$
i	$10^i \cdot \left(\frac{m}{3^i}\right)^2 = 10^i \cdot \frac{m^2}{9^i} = \left(\frac{10}{9}\right)^i m^2$

Calcolo $h := \frac{m}{3^i} = 2 \Rightarrow 3^i = \frac{m}{2} \Rightarrow i = \log_3\left(\frac{m}{2}\right)$

Calcolo la sommatoria:

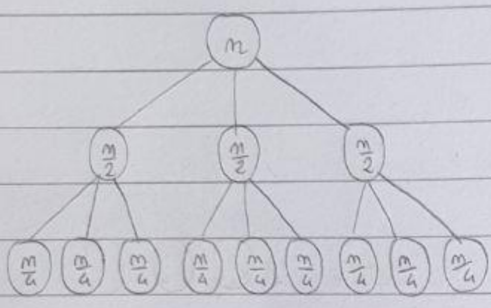
$$T(m) = \sum_{i=0}^{h}\left(\frac{10}{9}\right)^i \cdot m^2 = \sum_{i=0}^{\log_3\left(\frac{m}{2}\right)}\left(\frac{10}{9}\right)^i \cdot m^2 = m^2 \cdot \sum_{i=0}^{\log_3\left(\frac{m}{2}\right)}\left(\frac{10}{9}\right)^i$$ serie geometrica con ragione > 1

$$= m^2 \cdot \frac{\left(\frac{10}{9}\right)^{h+1} - 1}{\frac{10}{9} - 1} = m^2 \cdot \frac{\left(\frac{10}{9}\right)^{\log_3\left(\frac{m}{2}\right)+1} - 1}{\frac{1}{9}} = m^2\left[9\left(\left(\frac{10}{9}\right)^{\log_3\left(\frac{m}{2}\right)} \cdot \frac{10}{9} - 1\right)\right]$$

$$= m^2\left[\left(\frac{10}{9}\right)^{\log_3\left(\frac{m}{2}\right)} \cdot 10 - 9\right] = 10m^2 \cdot \left(\frac{10}{9}\right)^{\log_3\left(\frac{m}{2}\right)} - 9m^2 = 10m^2 \cdot \left(\frac{m}{2}\right)^{\log_3\left(\frac{10}{9}\right)} - 9m^2$$

EQUAZIONE 10

$$T(m) = \begin{cases} 1 & \text{se } m \leq 1 \\ 3T\left(\frac{m}{2}\right) + m & \text{se } m > 1 \end{cases}$$



LIVELLO	CONTRIBUTO
0	m
1	$\frac{m}{2} + \frac{m}{2} + \frac{m}{2} = \frac{3}{2} m$
2	$9\left(\frac{m}{4}\right) = \frac{9}{4} m = \left(\frac{3}{2}\right)^2 m$
i	$\left(\frac{3}{2}\right)^i \cdot m$

Calcolo h: $\frac{n}{2^i} = 1 \Rightarrow 2^i = m \Rightarrow i = \log_2 m$

Calcolo la sommatoria:

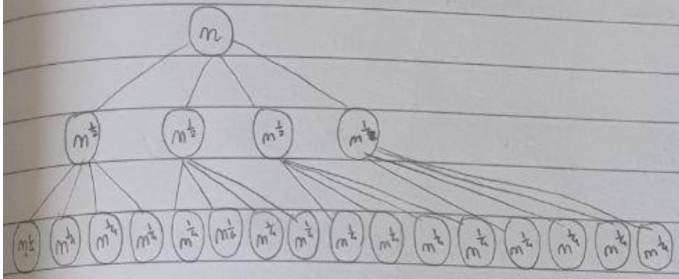
$$T(m) = \sum_{i=0}^{h} \left(\frac{3}{2}\right)^i \cdot m = \sum_{i=0}^{\log_2 m} \left(\frac{3}{2}\right)^i \cdot m = m \cdot \sum_{i=0}^{\log_2 m} \left(\frac{3}{2}\right)^i$$ serie geometrica con ragione > 1

$$= m \cdot \frac{\left(\frac{3}{2}\right)^{h+1} - 1}{\frac{3}{2} - 1} = m\left[2\left(\frac{3}{2} \cdot \left(\frac{3}{2}\right)^{\log_2 m} - 1\right)\right] = 3m \left(\frac{3}{2}\right)^{\log_2 m} - 2m$$

$$= 3m \cdot m^{\log_2\left(\frac{3}{2}\right)} - 2m = 3m^{\log_2\left(\frac{3}{2}\right)+1} - 2m \Rightarrow T(m) = \Theta\left(m^{\log_2\left(\frac{3}{2}\right)+1}\right)$$

EQUAZIONE 11

$$T(m) = \begin{cases} 1 & \text{se } m \le 2 \\ 4T(\sqrt{m}) + \log m & \text{se } m > 2 \end{cases}$$



LIVELLO	CONTRIBUTO
0	$\log m$
1	$4 \log \sqrt{m} = 4 \log m^{\frac{1}{2}}$
2	$16 \log \sqrt[4]{m} = 4^2 \log m^{\frac{1}{4}}$
i	$4^i \log \sqrt[2^i]{m} = 4^i \log m^{\frac{1}{2^i}} = 2^i \cdot \log m$

Calcolo h:: $m^{\frac{1}{2^i}} = 2 \Rightarrow \frac{1}{2^i} = \log_m 2 \Rightarrow 2^i = \log_2 m \Rightarrow i = \log_2 \log_2 m$

Calcolo la sommatoria:

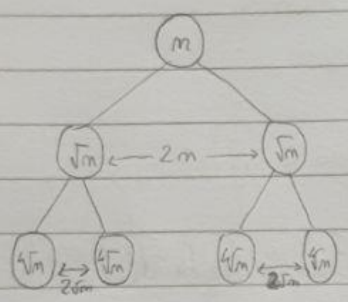
$T(m) = \sum_{i=0}^{h} 2^i \cdot \log m = \sum_{i=0}^{\log_2 \log_2 m} 2^i \cdot \log m = \log m \cdot \sum_{i=0}^{\log_2 \log_2 m} 2^i$ serie geometrica con ragione > 1

$= \log m \cdot \frac{2^{h+1} - 1}{2 - 1} = \log m \cdot \left(2^{\log_2 \log_2 m + 1} - 1\right) = \log m \left(2^{\log_2 \log_2 m} \cdot 2 - 1\right)$

$= 2 \cdot \log m \cdot \log_2 m - \log m \Rightarrow T(m) = \Theta(\log^2 m)$

EQUAZIONE 12

$$T(m) = \begin{cases} 1 & \text{se } m \leq 2 \\ 2m\,T(\sqrt{m}) + m^2 & \text{se } m > 2 \end{cases}$$



LIVELLO	ISTANZA	N° NODI	CONTR NODO	CONTR. LOCALE LIVELLO
0	m	1	m^2	m^2
1	$\sqrt{m}$	$2m$	$(\sqrt{m})^2 = m$	$2m \cdot m = 2m^2$
2	$\sqrt[4]{m}$	$2m \cdot 2\sqrt{m}$	$(\sqrt[4]{m}) = \sqrt{m}$	$2m \cdot 2\sqrt{m} \cdot \sqrt{m} = 4m^2 = 2^2 m^2$
i	$\sqrt[2^i]{m}$	$2m\, 2\sqrt{m} \cdots 2\sqrt[2^i]{m}$	$(\sqrt[2^i]{m})^2$	$2^i \cdot m^2$

Calcolo h: $\sqrt[2^i]{m} = 2 \Rightarrow m^{\frac{1}{2^i}} = 2 \Rightarrow \frac{1}{2^i} = \log_m 2 \Rightarrow i = \log_2(\log_2 m)$

Calcolo la sommatoria:

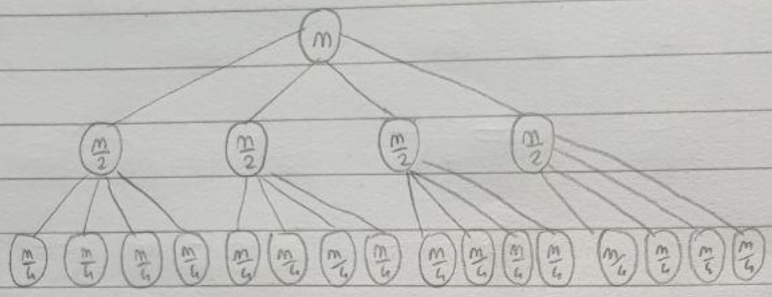
$$T(m) = \sum_{i=0}^{h} 2^i \cdot m^2 = \sum_{i=0}^{\log_2 \log_2 m} 2^i \cdot m^2 = m^2 \cdot \sum_{i=0}^{\log_2 \log_2 m} 2^i$$ serie geometrica con ragione > 1

$$= m^2 \cdot \frac{2^{h+1} - 1}{2 - 1} = m^2 \cdot \left(2^{\log_2 \log_2 m + 1} - 1\right) = m^2 \cdot \left(2^{\log_2 \log_2 m} \cdot 2 - 1\right)$$

$$= 2m^2 \log_2 m - m^2 \Rightarrow T(m) = \Theta(m^2 \log_2 m)$$

EQUAZIONE 14

$$T(n) = \begin{cases} 1 & \text{se } n \leq 1 \\ 4T\left(\frac{n}{2}\right) + n & \text{se } n > 1 \end{cases}$$



LIVELLO	CONTRIBUTO LIVELLO
0	n
1	$\frac{4}{2}n = 2n$
2	$\frac{16}{4}n = 4n = 2^2 \cdot n$
i	$2^i \cdot n$

Calcolo h: $\frac{n}{2^i} = 1 \Rightarrow 2^i = n \Rightarrow i = \log_2 n$

Calcolo la sommatoria:

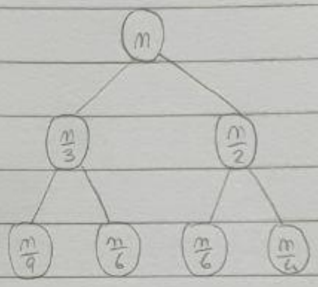
$$T(n) = \sum_{i=0}^{h} 2^i \cdot n = \sum_{i=0}^{\log_2 n} 2^i \cdot n = n \cdot \sum_{i=0}^{\log_2 n} 2^i$$ serie geometrica con ragione > 1

$$= n \cdot \frac{2^{h+1} - 1}{2 - 1} = n\left(2^{\log_2 n + 1} - 1\right) = n\left(2^{\log_2 n} \cdot 2 - 1\right) = n(n \cdot 2 - 1)$$

$$= 2n^2 - n \Rightarrow T(n) = \Theta(n^2)$$

EQUAZIONE 18

$$T(n) = \begin{cases} 1 & \text{se } n \leq 1 \\ T\left(\frac{n}{3}\right) + T\left(\frac{n}{2}\right) + n & \text{se } n > 1 \end{cases}$$



LIVELLO	CONTRIBUTO LIVELLO
0	n
1	$\frac{n}{3} + \frac{n}{2} = \frac{2n+3n}{6} = \frac{5}{6}n$
2	$\frac{n}{9} + \frac{n}{6} + \frac{n}{6} + \frac{n}{4} = \frac{4n+6n+6n+9n}{36} = \frac{25}{36}n = \left(\frac{5}{6}\right)^2 n$
i	$\left(\frac{5}{6}\right)^i \cdot n$

Calcolo h_1: $= \frac{n}{3^i} = 1 \Rightarrow 3^i = n \Rightarrow i = \log_3 n$

Calcolo h_2: $= \frac{n}{2^i} = 1 \Rightarrow 2^i = n \Rightarrow i = \log_2 n$

Calcolo la sommatoria:

$$T_{h_1}(n) = \sum_{i=0}^{h_1} \left(\frac{5}{6}\right)^i \cdot n = \sum_{i=0}^{\log_3 n} \left(\frac{5}{6}\right)^i \cdot n = n \cdot \sum_{i=0}^{\log_3 n} \left(\frac{5}{6}\right)^i$$ serie geometrica con ragione < 1

$$= n \cdot \frac{1}{1-\frac{5}{6}} = n \cdot \frac{1}{\frac{1}{6}} = 6n \Rightarrow T_{h_1}(n) = \Theta(n)$$

Calcolo la sommatoria:

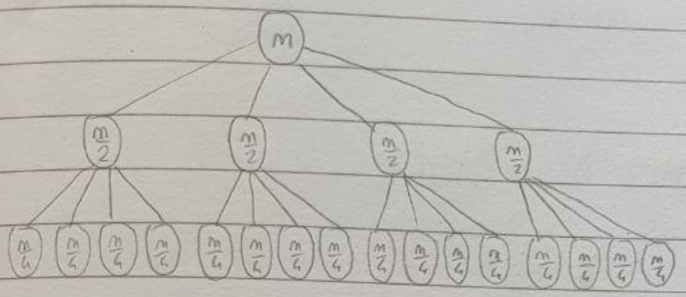
$$T_{h_2}(n) = \sum_{i=0}^{h_2} \left(\frac{5}{6}\right)^i \cdot n = \sum_{i=0}^{\log_2 n} \left(\frac{5}{6}\right)^i \cdot n = n \cdot \sum_{i=0}^{\log_2 n} \left(\frac{5}{6}\right)^i$$ serie geometrica con ragione < 1

$$= n \cdot \frac{1}{1-\frac{5}{6}} = n \cdot \frac{1}{\frac{1}{6}} = 6n \Rightarrow T_{h_2}(n) = \Theta(n)$$

Essendo $T_{h_1}(n) \leq T(n) \leq T_{h_2}(n)$, avremo che $T(n) = \Theta(n)$

EQUAZIONE 19

$$T(m) = \begin{cases} 1 & \text{se } m \leq 1 \\ 4T\left(\frac{m}{2}\right) + m^2 & \text{se } m > 1 \end{cases}$$



LIVELLO	CONTRIBUTO LIVELLO
0	m^2
1	$4\left(\frac{m}{2}\right)^2 = 4 \cdot \frac{m^2}{4} = m^2$
2	$16\left(\frac{m}{4}\right)^2 = 16 \cdot \frac{m^2}{16} = m^2$
i	$4^i\left(\frac{m}{2^i}\right)^2 = m^2$

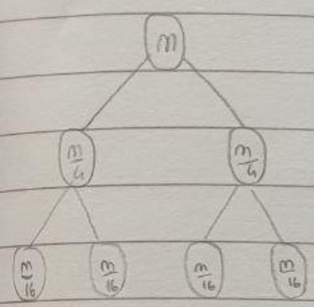
Calcolo h: $\frac{m}{2^i} = 1 \Rightarrow 2^i = m \Rightarrow i = \log_2 m$

Calcolo la sommatoria:

$$T(m) = \sum_{i=0}^{h} m^2 = \sum_{i=0}^{\log_2 m} m^2 = m^2 \cdot \sum_{i=0}^{\log_2 m} (1) \Rightarrow T(m) = \Theta(m^2)$$

EQUAZIONE 21

$$T(n) = \begin{cases} 1 & \text{se } n \leq 1 \\ 2T\left(\frac{n}{4}\right) + \sqrt{n} & \text{se } n > 1 \end{cases}$$



LIVELLO	CONTRIBUTO LIVELLO
0	$\sqrt{n}$
1	$\sqrt{\frac{n}{4}}+\sqrt{\frac{n}{4}} = \frac{\sqrt{n}}{2} + \frac{\sqrt{n}}{2} = \sqrt{n}$
2	$4\sqrt{\frac{n}{16}} = \cancel{4}\cdot\frac{\sqrt{n}}{\cancel{4}} = \sqrt{n}$
i	$2^i \cdot \sqrt{\frac{n}{4^i}} = \cancel{2^i}\cdot\frac{\sqrt{n}}{\cancel{2^i}} = \sqrt{n}$

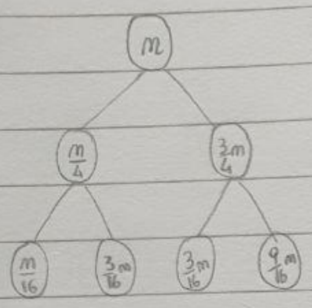
Calcolo $h := \frac{n}{4^i} = 1 \Rightarrow 4^i = n \Rightarrow \log_4 n$

Calcolo la sommatoria

$$T(n) = \sum_{i=0}^{h} \sqrt{n} = \sum_{i=0}^{\log_4 n} \sqrt{n} = \sqrt{n} \cdot \sum_{i=0}^{\log_4 n} 1 \Rightarrow T(n) = \Theta(\sqrt{n})$$

EQUAZIONE 22

$$T(n) = \begin{cases} 1 & \text{se } n \leq 1 \\ T\left(\frac{n}{4}\right) + T\left(\frac{3}{4}n\right) + n & \text{se } n > 1 \end{cases}$$



LIVELLO	CONTRIBUTO LIVELLO
0	n
1	$\frac{n}{4} + \frac{3}{4}n = n$
2	$\frac{n}{16} + \frac{3}{16}n + \frac{3}{16}n + \frac{9}{16}n = \frac{16}{16}n$
i	n

Calcolo h_1: $\frac{n}{4^i} = 1 \Rightarrow 4^i = n \Rightarrow i = \log_4 n$

Calcolo h_2: $\left(\frac{3}{4}\right)^i n = 1 \Rightarrow n = \left(\frac{4}{3}\right)^i \Rightarrow i = \log_{\frac{4}{3}} n$

Calcolo la sommatoria

$$T_{h_1}(n) = \sum_{i=0}^{h_1} n = \sum_{i=0}^{\log_4 n} n = n \cdot \sum_{i=0} 1 \Rightarrow T_{h_1}(n) = \Theta(n)$$

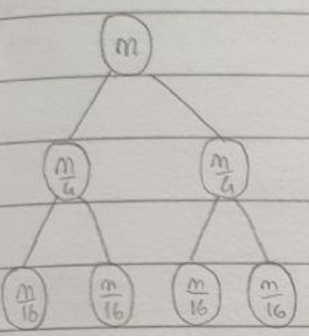
Calcolo la sommatoria

$$T_{h_2}(n) = \sum_{i=0}^{h_2} n = \sum_{i=0}^{\log_{\frac{4}{3}} n} n = n \cdot \sum_{i=0}^{\log_{\frac{4}{3}} n} 1 \Rightarrow T_{h_2}(n) = \Theta(n^2)$$

Essendo $T_{h_1}(n) \leq T(n) \leq T_{h_2}(n)$, avremo che $T(n) = \Theta(n)$

EQUAZIONE 23

$$T(n) = \begin{cases} 1 & \text{se } n \leq 1 \\ 2T\left(\frac{n}{4}\right) + n^3 & \text{se } n > 1 \end{cases}$$



LIVELLO	CONTRIBUTO LIVELLO
0	n^3
1	$2\left(\frac{n}{4}\right)^3 = 2 \cdot \frac{n^3}{\cancel{64}_{32}} = \frac{n^3}{32}$
2	$4\left(\frac{n}{16}\right)^3 = 4 \cdot \frac{n^3}{\cancel{4096}_{1024}} = \frac{n^3}{1024}$
i	$\frac{n^3}{32^i} = \left(\frac{1}{32}\right)^i \cdot n^3$

Calcolo h: $\frac{n}{4^i} = 1 \Rightarrow 4^i = n \Rightarrow i = \log_4 n$

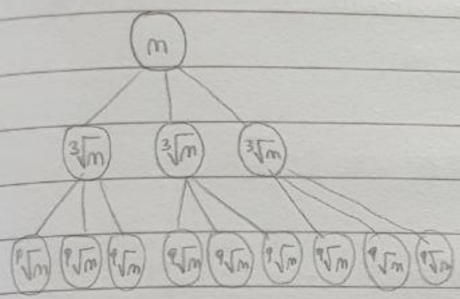
Calcolo la sommatoria:

$$T(n) = \sum_{i=0}^{h} \left(\frac{1}{32}\right)^i \cdot n^3 = \sum_{i=0}^{\log_4 n} \left(\frac{1}{32}\right)^i \cdot n^3 = n^3 \cdot \sum_{i=0}^{\log_4 n} \left(\frac{1}{32}\right)^i$$ serie geometrica con ragione < 1

$$= n^3 \cdot \frac{1}{1 - \frac{1}{32}} = n^3 \cdot \frac{1}{\frac{31}{32}} = \frac{32}{31} n^3 \Rightarrow T(n) = \Theta(n^3)$$

EQUAZIONE 27

$$T(n) = \begin{cases} 1 & \text{se } n \leq 3 \\ 3T(\sqrt[3]{n}) + \log_3 n & \text{se } n > 3 \end{cases}$$



LIVELLO	CONTRIBUTO LIVELLO
0	$\log_3 n$
1	$3 \log_3 \sqrt[3]{n} = 3 \log_3 n^{\frac{1}{3}} = 3 \cdot \frac{1}{3} \log_3 n$
2	$3^2 \cdot \log_3 \sqrt[9]{n} = 3^2 \cdot \log_3 n^{\frac{1}{9}} = 3^2 \cdot \frac{1}{9} \log_3 n$
i	$\log_3 n$

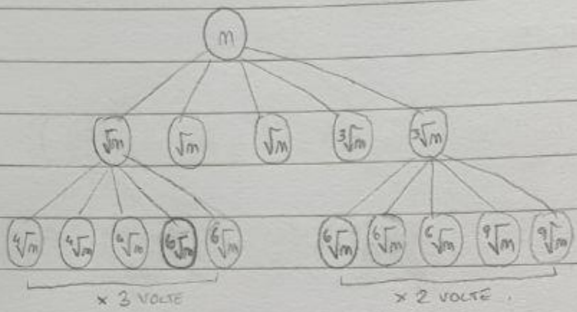
Calcolo h: $\sqrt[3^i]{n} = 3 \Rightarrow n^{\frac{1}{3^i}} = 3 \Rightarrow \frac{1}{3^i} = \log_n 3 \Rightarrow 3^i = \frac{1}{\log_n 3} \Rightarrow 3^i = \log_3 n \Rightarrow i = \log_3(\log_3 n)$

Calcolo la sommatoria:

$$T(n) = \sum_{i=0}^{h} \log_3 n = \sum_{i=0}^{\log_3 \log_3 n} \log_3 n = \log_3 n \cdot \sum_{i=0}^{\log_3 \log_3 n} 1 \Rightarrow T(n) = \Theta(\log_3 n)$$

EQUAZIONE 28

$$T(m) = \begin{cases} \text{costante} & \text{se } m \le x \\ 3T(\sqrt{m}) + 2T(\sqrt[3]{m}) + \log_x m & \text{se } m > x \end{cases}$$



LIVELLO	CONTRIBUTO LIVELLO
0	$\log_x m$
1	$3\log_x m^{\frac{1}{2}} + 2\log_x m^{\frac{1}{3}} = \left(\frac{3}{2}+\frac{2}{3}\right)\log_x m = \frac{13}{6}\log_x m$
2	$3\left(3m^{\frac{1}{4}}+2m^{\frac{1}{6}}\right)+2\left(3m^{\frac{1}{6}}+2m^{\frac{1}{9}}\right) = \left(\frac{169}{36}\right)\log_x m = \left(\frac{13}{16}\right)^2 \log_x m$
i	$\left(\frac{13}{16}\right)^i \cdot \log_x m$

Calcolo $h_1 := \sqrt[2^i]{m} = x \Rightarrow m^{\frac{1}{2^i}} = x \Rightarrow \frac{1}{2^i} = \log_m x \Rightarrow 2^i = \log_x m \Rightarrow i = \log_2(\log_x m)$

Calcolo $h_2 := \sqrt[3^i]{m} = x \Rightarrow m^{\frac{1}{3^i}} = x \Rightarrow \frac{1}{3^i} = \log_m x \Rightarrow 3^i = \log_x m \Rightarrow i = \log_3(\log_x m)$

Calcolo la sommatoria:

$T_{h_1}(m) = \sum_{i=0}^{h_1}\left(\frac{13}{16}\right)^i \cdot \log_x m = \sum_{i=0}^{\log_2(\log_x m)}\left(\frac{13}{16}\right)^i \cdot \log_x m = \log_x m \cdot \sum_{i=0}^{\log_2(\log_x m)}\left(\frac{13}{16}\right)^i$ serie geometrica con ragione < 1

$= \log_x m \cdot \frac{1}{1-\frac{13}{16}} = \log_x m \cdot \left(\frac{16}{3}\right) = \frac{16}{3}\log_x m \Rightarrow T_{h_1}(m) = \Theta(\log_x m)$

Calcolo la sommatoria:

$T_{h_2}(m) = \sum_{i=0}^{h_2}\left(\frac{13}{16}\right)^i = \sum_{i=0}^{\log_3(\log_x m)}\left(\frac{13}{16}\right)^i \cdot \log_x m = \log_x m \cdot \sum_{i=0}^{\log_3(\log_x m)}\left(\frac{13}{16}\right)^i$ serie geometrica con ragione < 1

$= \log_x m \cdot \frac{1}{1-\frac{13}{16}} = \frac{16}{3}\log_x m \Rightarrow T_{h_2}(m) = \Theta(\log_x m)$

Essendo $T_{h_1}(m) \le T(m) \le T_{h_2}$, avremo che $T(m) = \Theta(\log_x m)$