ガウスの消去法 (p.89 練習 1(1))

$$\begin{cases} x +2y +3z = 10 \\ 3x -3y +7z = 10 \\ -x +4y -9z = -4 \end{cases} \qquad \left(\begin{array}{ccc|c} 1 & 2 & 3 & 10 \\ 3 & -3 & 7 & 10 \\ -1 & 4 & -9 & -4 \end{array} \right) \qquad \begin{array}{l} \cdots ① \\ \cdots ② \\ \cdots ③ \end{array}$$

$$\downarrow \quad \begin{array}{l} ② + ① \times (-3) \\ ③ + ① \end{array}$$

$$\begin{cases} x +2y +3z = 10 \\ -9y -2z = -20 \\ 6y -6z = 6 \end{cases} \qquad \left(\begin{array}{ccc|c} 1 & 2 & 3 & 10 \\ 0 & -9 & -2 & -20 \\ 0 & 6 & -6 & 6 \end{array} \right)$$

$$\downarrow \quad ③ \div (6)$$

$$\begin{cases} x +2y +3z = 10 \\ -9y -2z = -20 \\ y -z = 1 \end{cases} \qquad \left(\begin{array}{ccc|c} 1 & 2 & 3 & 10 \\ 0 & -9 & -2 & -20 \\ 0 & 1 & -1 & 1 \end{array} \right)$$

$$\downarrow \quad \begin{array}{l} ① + ③ \times (-2) \\ ② + ③ \times (9) \end{array}$$

$$\begin{cases} x +5z = 8 \\ -11z = -11 \\ y -z = 1 \end{cases} \qquad \left(\begin{array}{ccc|c} 1 & 0 & 5 & 8 \\ 0 & 0 & -11 & -11 \\ 0 & 1 & -1 & 1 \end{array} \right)$$

$$\downarrow \quad ② \div (-11)$$

$$\begin{cases} x +5z = 8 \\ z = 1 \\ y -z = 1 \end{cases} \qquad \left(\begin{array}{ccc|c} 1 & 0 & 5 & 8 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & -1 & 1 \end{array} \right)$$

$$\downarrow \quad ②と③交換$$

$$\begin{cases} x +5z = 8 \\ y -z = 1 \\ z = 1 \end{cases} \qquad \left(\begin{array}{ccc|c} 1 & 0 & 5 & 8 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right) \qquad \cdots (上三角行列)$$

$$\downarrow \quad \begin{array}{l} ① + ③ \times (-5) \\ ② + ③ \end{array}$$

$$\begin{cases} x = 3 \\ y = 2 \\ z = 1 \end{cases} \qquad \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array} \right)$$

よって、 $\qquad x = 3, \ y = 2, \ z = 1 \ \cdots (答)$

ガウスの消去法 (p.89 練習 1(2))

$$\begin{cases} 2x +y -2z = -4 \\ 3x -y +3z = 1 \\ -x +y +4z = 11 \end{cases} \qquad \left(\begin{array}{ccc|c} 2 & 1 & -2 & -4 \\ 3 & -1 & 3 & 1 \\ -1 & 1 & 4 & 11 \end{array}\right) \qquad \begin{array}{l} \cdots ① \\ \cdots ② \\ \cdots ③ \end{array}$$

$$\downarrow \begin{array}{l} ① + ③ \times (2) \\ ② + ③ \times (3) \end{array}$$

$$\begin{cases} 3y +6z = 18 \\ 2y +15z = 34 \\ -x +y +4z = 11 \end{cases} \qquad \left(\begin{array}{ccc|c} 0 & 3 & 6 & 18 \\ 0 & 2 & 15 & 34 \\ -1 & 1 & 4 & 11 \end{array}\right)$$

$$\downarrow \; ① \div (3)$$

$$\begin{cases} y +2z = 6 \\ 2y +15z = 34 \\ -x +y +4z = 11 \end{cases} \qquad \left(\begin{array}{ccc|c} 0 & 1 & 2 & 6 \\ 0 & 2 & 15 & 34 \\ -1 & 1 & 4 & 11 \end{array}\right)$$

$$\downarrow \begin{array}{l} ② + ① \times (-2) \\ ③ + ① \times (-1) \end{array}$$

$$\begin{cases} y +2z = 6 \\ 11z = 22 \\ -x +2z = 5 \end{cases} \qquad \left(\begin{array}{ccc|c} 0 & 1 & 2 & 6 \\ 0 & 0 & 11 & 22 \\ -1 & 0 & 2 & 5 \end{array}\right)$$

$$\downarrow \; ② \div (11)$$

$$\begin{cases} y +2z = 6 \\ z = 2 \\ -x +2z = 5 \end{cases} \qquad \left(\begin{array}{ccc|c} 0 & 1 & 2 & 6 \\ 0 & 0 & 1 & 2 \\ -1 & 0 & 2 & 5 \end{array}\right)$$

$$\downarrow \; ③ \times (-1) \text{ を 1 行目, ① を 2 行目, ② を 3 行目}$$

$$\begin{cases} x -2z = -5 \\ y +2z = 6 \\ z = 2 \end{cases} \qquad \left(\begin{array}{ccc|c} 1 & 0 & -2 & -5 \\ 0 & 1 & 2 & 6 \\ 0 & 0 & 1 & 2 \end{array}\right) \qquad \cdots (\text{上三角行列})$$

$$\downarrow \begin{array}{l} ① + ③ \times (2) \\ ② + ③ \times (-2) \end{array}$$

$$\begin{cases} x = -1 \\ y = 2 \\ z = 2 \end{cases} \qquad \left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

よって, $\qquad x = -1, \; y = 2, \; z = 2 \; \cdots (\text{答})$

2 変数の場合 (p.90 練習 2(1))

$$\begin{cases} 3x -3y = 10 \\ -x +4y = -4 \end{cases} \qquad \left(\begin{array}{cc|c} 3 & -3 & 10 \\ -1 & 4 & -4 \end{array} \right) \qquad \cdots① \\ \cdots②$$

$$\downarrow \ ② \times (3)$$

$$\begin{cases} 3x \ -3y = \ 10 \\ -3x +12y = -12 \end{cases} \qquad \left(\begin{array}{cc|c} 3 & -3 & 10 \\ -3 & 12 & -12 \end{array} \right)$$

$$\downarrow \ ② + ①$$

$$\begin{cases} 3x -3y = 10 \\ 9y = -2 \end{cases} \qquad \left(\begin{array}{cc|c} 3 & -3 & 10 \\ 0 & 9 & -2 \end{array} \right) \qquad \cdots (上三角行列)$$

$$\downarrow \ ① \times (3)$$

$$\begin{cases} 9x -9y = 30 \\ 9y = -2 \end{cases} \qquad \left(\begin{array}{cc|c} 9 & -9 & 30 \\ 0 & 9 & -2 \end{array} \right)$$

$$\downarrow \ ① + ②$$

$$\begin{cases} 9x \ = 28 \\ 9y = -2 \end{cases} \qquad \left(\begin{array}{cc|c} 9 & 0 & 28 \\ 0 & 9 & -2 \end{array} \right)$$

$$\downarrow \ ① \div (9), \ ② \div (9)$$

$$\begin{cases} x \ = \dfrac{28}{9} \\ y = -\dfrac{2}{9} \end{cases} \qquad \left(\begin{array}{cc|c} 1 & 0 & \dfrac{28}{9} \\ 0 & 1 & -\dfrac{2}{9} \end{array} \right)$$

よって、 $\qquad x = \dfrac{28}{9}, \ y = -\dfrac{2}{9} \ \cdots (答)$

（注意）逆行列を利用して，検算する。

$\begin{cases} 3x -3y = \ 10 \\ -x +4y = -4 \end{cases}$ は，

$$\begin{pmatrix} 3 & -3 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ -4 \end{pmatrix} \cdots (*)$$

と表すことができる。

$\begin{pmatrix} 3 & -3 \\ -1 & 4 \end{pmatrix}$ の逆行列は，

$\begin{pmatrix} 3 & -3 \\ -1 & 4 \end{pmatrix}^{-1} = \dfrac{1}{9} \begin{pmatrix} 4 & 3 \\ 1 & 3 \end{pmatrix}$ である。これを式 $(*)$ の両辺に左からかけると，

$$\begin{pmatrix} x \\ y \end{pmatrix} = \dfrac{1}{9} \begin{pmatrix} 4 & 3 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 10 \\ -4 \end{pmatrix} = \dfrac{1}{9} \begin{pmatrix} 28 \\ -2 \end{pmatrix}$$

よって， $x = \dfrac{28}{9}, \ y = -\dfrac{2}{9} \ \cdots (答)$

ガウスの消去法 (p.90 練習 2(2))

$$\begin{cases} x & +y & +2z & -w = & 3 \\ 3x & +3y & -2z & +3w = & 1 \\ x & +y & +3z & +3w = & -6 \\ x & -y & +z & +2w = & -4 \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 2 & -1 & 3 \\ 3 & 3 & -2 & 3 & 1 \\ 1 & 1 & 3 & 3 & -6 \\ 1 & -1 & 1 & 2 & -4 \end{array}\right) \begin{array}{l} \cdots ① \\ \cdots ② \\ \cdots ③ \end{array}$$

$$\downarrow \begin{array}{l} ② + ① \times (-2) \\ ③ + ① \times (-1) \\ ④ + ① \times (-1) \end{array}$$

$$\begin{cases} x & +y & +2z & -w = & 3 \\ & y & -6z & +5w = & -3 \\ & & z & +4w = & -9 \\ & -2y & -z & +3w = & -7 \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 2 & -1 & 3 \\ 0 & 1 & -6 & 5 & -3 \\ 0 & 0 & 1 & 4 & -9 \\ 0 & -2 & -1 & 3 & -7 \end{array}\right)$$

$$\downarrow \begin{array}{l} ① + ② \times (-1) \\ ④ + ② \times (2) \end{array}$$

$$\begin{cases} x & +8z & -6w = & 6 \\ y & -6z & +5w = & -3 \\ & z & +4w = & -9 \\ & -13z & +13w = & -13 \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 8 & -6 & 6 \\ 0 & 1 & -6 & 5 & -3 \\ 0 & 0 & 1 & 4 & -9 \\ 0 & 0 & -13 & 13 & -13 \end{array}\right)$$

$$\downarrow \quad ④ \div (13)$$

$$\begin{cases} x & +8z & -6w = & 6 \\ y & -6z & +5w = & -3 \\ & z & +4w = & -9 \\ & -z & +w = & -1 \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 8 & -6 & 6 \\ 0 & 1 & -6 & 5 & -3 \\ 0 & 0 & 1 & 4 & -9 \\ 0 & 0 & -1 & 1 & -1 \end{array}\right)$$

$$\downarrow \begin{array}{l} ① + ④ \times (8) \\ ② + ④ \times (-6) \\ ③ + ④ \end{array}$$

$$\begin{cases} x & +2w = & -2 \\ y & -w = & 3 \\ & 5w = & -10 \\ & -z & +w = & -1 \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 2 & -2 \\ 0 & 1 & 0 & -1 & 3 \\ 0 & 0 & 0 & 5 & -10 \\ 0 & 0 & -1 & 1 & -1 \end{array}\right)$$

$$\downarrow \begin{array}{l} ③ \div (5) \\ ④ \div (-1) \end{array}$$

$$\begin{cases} x & +2w = & -2 \\ y & -w = & 3 \\ & w = & -2 \\ & z & -w = & 1 \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 2 & -2 \\ 0 & 1 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & -1 & 1 \end{array}\right)$$

$$\downarrow \begin{array}{l} ① + ③ \times (-2) \\ ② + ③ \\ ④ + ③ \end{array}$$

$$\begin{cases} x & = & 2 \\ y & = & 1 \\ & w = & -2 \\ z & = & -1 \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & -1 \end{array}\right)$$

$$\begin{cases} x & = & 2 \\ y & = & 1 \\ z & = & -1 \\ w & = & -2 \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -2 \end{array} \right)$$

よって,

$x = 2,\ y = 1,\ z = -1,\ w = -2\ \cdots(答)$