MAT 167: STATISTICS

MIDTERM EXAM

Instructor: Anthony Tanbakuchi

SUMMER 2008

Name:	
	Computer / Seat Number:

No books, notes, or friends. **Show your work.** You may use the attached equation sheet, R, and a calculator. No other materials. If you choose to use R, write what you typed on the test or copy and paste your work into a word document labeling the question number it corresponds to. When you are done with the test print out the document. Be sure to save often on a memory stick just in case. Using any other program or having any other documents open on the computer will constitute cheating.

You have until the end of class to finish the exam, manage your time wisely.

If something is unclear quietly come up and ask me.

If the question is legitimate I will inform the whole class.

Express all final answers to 3 significant digits. Probabilities should be given as a decimal number unless a percent is requested. Circle final answers, ambiguous or multiple answers will not be accepted. Show steps where appropriate.

The exam consists of 21 questions for a total of 66 points on 10 pages.

This Exam is being given under the guidelines of our institution's **Code of Academic Ethics**. You are expected to respect those guidelines.

Points Earned:	out of 66	total	points
Exam Score: $_$			

1. The following is a partial list of statistical methods that we have discussed:

mean
 median
 median
 histogram
 mode
 pareto chart
 standard deviation
 box plot

5. z-score 12. normal-quantile plot

6. percentile 13. confidence interval for a mean

7. coefficient of variation 14. confidence interval for a proportion

For each situation below, which method is most applicable? To get full points, give a short (1-3 sentence) description as to why your chosen method is appropriate!

• If it's a graphical method, also describe what you would be looking for.

• If it's a statistic, how susceptible to outliers is it?

(a) (2 points) A school board thinks that students might do better in morning math classes. They would like to compare the distribution of student math scores for 4 time categories: early morning classes, mid morning classes, early afternoon classes, and late afternoon classes.

(b) (2 points) A clothing manufacturer wants to determine how much variability there is in toddler waist sizes.

(c) (2 points) The Tucson realtors association is publishing their semi-annual newsletter and has an article discussing current housing prices. They would like to publish a single number

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		0 1		,			-
that is	representative	of the	typical	house	price :	in '	Tucson.

(d) (2 points) A political science researcher at the U of A wants to estimate the true percentage of Tucson residents who support Obama from a random survey of 1,000 people.

2. (1 point) Make a sketch of a normal distribution that has been positively skewed.

- 3. (1 point) What is the area under a positively skewed normal distribution?
- 4. (1 point) If the mean, median, and mode for a data set are not the same, what can you conclude about the data's distribution?

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5.	(2 points) Give an example of sampling error.
6.	In regards to \bar{x} and the Central Limit Theorem: (a) (2 points) What are the two conditions under which the CLT applies?
	(b) (2 points) If the conditions are met, what type of distribution will \bar{x} have?
7.	(2 points) Under what conditions can we approximate a binomial distribution as a normal distribution?
8.	(2 points) Which distribution (normal, binomial, both, or neither) would be appropriate for describing: The sampling distribution of the mean for a random sample from a uniformly distributed population using a sample size of 10.
9.	(2 points) Give a clear specific example of when you would use a population distribution.
10.	(2 points) Give an example of when you would use a sampling distribution.

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- 11. Super Fruity Chew candy has a mean weight of 500 g and a standard deviation of 10 g.
 - (a) (2 points) Construct an interval using the Empirical Rule which you would expect 95% of the weights to fall within.

(b) (2 points) Would you consider a candy with weight of 468 g unusual? (State why.)

12. Given the following frequency table summarizing data from a study:

age.years	frequency
0-9	5.00
10-19	8.00
20-29	12.00
30-39	2.00

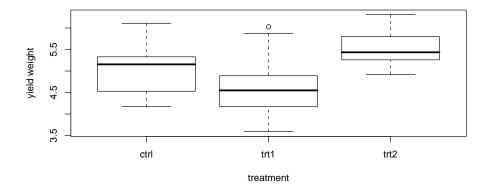
(a) (2 points) Construct a relative frequency table.

(b) (1 point) What is the probability of randomly selecting someone from the study who is between 20-39 years old?

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(c) (2 points) Calculate the approximate mean age of the subjects in the study from the data given.

13. Results from a randomized experiment to compare crop yields (as measured by dried weight of plants in grams) obtained under a control and two different treatment conditions are shown with a box plot of the data. The researcher who has developed the two new treatments hopes that at least one increases crop yield as compared to the control group.



- (a) (1 point) Did any groups contain an outlier? If so which ones?
- (b) (1 point) What group had the least variability?
- (c) (1 point) What was the approximate IQR for the control group (ctrl)?
- (d) (1 point) Which group had the highest median crop yield? What was the median value?

14. (2 points) You would like to conduct a study to estimate (at the 90% confidence level) the mean weight of brown bears with a margin of error of 5 lbs. A preliminary study indicates that bear weights are normally distributed with a standard deviation of 22 lbs, what sample size should you use for this study?

15. (2 points) A random sample of 5 men was conducted to determine the mean resting heart rate. Below is the study data in beats per minute.

Construct a 99% confidence interval for the true population mean resting heart rate for men using the above data. (Assume σ is unknown and the population is normally distributed.)

- 16. A bag of M&M's contains 18 red, 12 blue, 8 green, and 7 brown candies.
 - (a) (2 points) What is the probability of randomly selecting a red or brown M&M?

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Points earned: _____ / 8 points

	(b)	(2 points) If 8 M&M's are randomly selected with replacement, what is the probability of getting exactly 6 red M&M's?
	(c)	(1 point) Would it be unusual to observe 6 red out of 8 randomly selected M&M's? (Why)
17.	Ans	wer the following question for a couple who had 4 children.
		(1 point) Is the probability of a girl considered independent or dependent for each successive birth?
	(b)	(2 points) What is the probability that the couple has at least 1 girl?
	(c)	(2 points) What is the probability that all 4 children are born on different days of the year?

18. (2 points) With one method of a procedure called acceptance sampling, a sample of items is randomly selected without replacement and the entire batch is accepted if every item in the sample is okay. The Niko Electronics Company has just manufactured 10,000 CDs, and 500 are defective. If 10 of the CDs are randomly selected for testing without replacement, what is the probability that the entire batch will be accepted?

- 19. Engineers must consider the breadths of male heads when designing motorcycle helmets. Men have head breadths that are normally distributed with a mean of 6.0 in and a standard deviation of 1.0 in (based on anthropometric survey data from Gordon, Churchill, et al.).
 - (a) (2 points) If 1 man is randomly selected, find the probability that his head breadth is between 6.1 and 6.6 inches.

(b) (2 points) If 1 man is randomly selected, find the probability that his head breadth is less than 6.1 in.

(c) (2 points) If 30 men are randomly selected, find the probability that their mean head

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breadth is less than 6.1 in.

- 20. A craps table at a local casino has been losing more money than normal. It seems that bets involving a one on the face of the dice (such as "snake eyes") are appearing more than usual. The casino manager thinks that the dice have been weighted to cause the side with one to have a higher probability of occurring than a fair dice.
 - (a) (2 points) The casino manager takes one of the dice from the table and flips it 100 times, the side with a value of one appears 22 times. Construct a 95% confidence interval for the true probability of getting a one with this die.

- (b) (1 point) If the die is fair, what should the true probability be for getting a one?
- (c) (1 point) Based on the casino manager's experiment, does the die appear to be unfair? Why?

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21. (2 points) Given $x = \{4c, 2c, -2c\}$, where c is a constant, completely simplify the following expression:

$$\sqrt{\frac{\sum (x_i^2 - 2c)}{6c}}$$

End of exam. Reference sheets follow.

Basic Statistics: Quick	2.3 VISUAL	5 Continuous random variables	ı	6 Sampling distributions	
Reference & R Commands	All plots have optional arguments:	CDF $F(x)$ gives area to the left of x , $F^{-1}(p)$ expec	ts p	ď	
by Anthony Tanbakuchi. Version 1.7	• main="" sets title	is area to the left.		$\mu_{\bar{x}} = \mu$ $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{\sigma}}$	(57)
http://www.tanbakuchi.com	 xlab="", ylab="" sets x/y-axis label 	f(x): probability density	(34)	V	
ANTHONY@TANBAKUCHI-COM	 type="p" for point plot type="l" for line plot 	71771		$\mu_{\hat{p}} = p$ $\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}}$	(58)
Get R at: http://www.r-project.org	type="1" for ine piot type="b" for both points and lines	$E = \mu = \int_{-\infty}^{\infty} x \cdot f(x) dx$	(35)	·	
R commands: bold typewriter text	Ex: plot (x, y, type="b", main="My Plot")	7-m		7 Estimation	
1 Misc R	Plot Types:	$\sigma = \sqrt{\int_{-\infty}^{\infty} (x - \mu)^2 \cdot f(x) dx}$	(36)	7.1 CONFIDENCE INTERVALS	
To make a vector / store data: x=c (x1, x2,)	hist(x) histogram	V 2		7.1 CONFIDENCE INTERVALS	
Get help on function: ?functionName	stem(x) stem & leaf	()	(37)	proportion: $\hat{p} \pm E$, $E = z_{\alpha/2} \cdot \sigma_{\hat{n}}$	(59)
Get column of data from table:	boxplot(x) box plot	$F^{-1}(x)$: inv. cumulative prob. density	(38)		(60)
tableName\$columnName	plot(T) bar plot, T=table(x)	$F(x) = \int_{-x}^{x} f(x') dx'$	(39)		
List all variables: 1s()	plot (x, y) scatter plot, x, y are ordered vectors plot (t, y) time series plot, t, y are ordered vect		(39)	mean (σ unknown, use s): $\bar{x} \pm E$, $E = t_{\alpha/2} \cdot \sigma_{\bar{x}}$,	(61)
Delete all variables: rm(list=ls())	curve (expr, xmin, xmax) plot expr involving		(40)	df = n - 1	
			(41)	$(n-1)s^2$, $(n-1)s^2$	
$\sqrt{x} = \text{sqrt}(x)$ (1)	2.4 Assessing Normality	- (F)	(42)	variance: $\frac{(n-1)s^2}{\gamma_n^2} < \sigma^2 < \frac{(n-1)s^2}{\gamma_n^2}$,	(62)
$x^n = \mathbf{x}^{\wedge} \mathbf{n}$ (2)	Q-Q plot: qqnorm(x); qqline(x)			df = n - 1	
n = length(x) (3)		p = P(a < x < b) = F(b) - F(a)	(43)		
T = table(x) (4		l		2 proportions: $\Delta \hat{p} \pm z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n_1} + \frac{\hat{p}\hat{q}}{n_2}}$	(63)
7 - Cable (A)	Number of successes x with n possible outcomes.	5.1 UNIFORM DISTRIBUTION		- proportion - p - sa/2 V n ₁ · n ₂	(00)
2 Descriptive Statistics	(Don't double count!)	p = P(u < u') = F(u')		$s_1^2 - s_2^2$	
	_		(44)	2 means (indep): $\Delta \bar{x} \pm t_{\alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$,	(64)
2.1 NUMERICAL	$P(A) = \frac{x_A}{a}$	7)		$df \approx \min(m-1, m-1)$	
Let x=c (x1, x2, x3,)	$P(\bar{A}) = 1 - P(A)$	$u' = F^{-1}(p) = qunif(p, min=0, max=1)$	(45)		
$total = \sum_{i=1}^{n} x_i = sum(x)$ (5)		7		matched pairs: $\bar{d} \pm t_{\alpha/2} \cdot \frac{s_d}{\sqrt{n}}$, $d_i = x_i - y_i$,	(65)
i=1	I(A Of B) = I(A) + I(B) - I(A and B)	9) 5.2 NORMAL DISTRIBUTION		df = n - 1	
min = min(x) (6		(0) 1(x-y) ²		aj = n - 1	
max = max(x) (7	$P(A \text{ and } B) = P(A) \cdot P(B A)$	$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{1}{2} \frac{(x-\mu)^2}{\sigma^2}}$	(46)	7.2 CI CRITICAL VALUES (TWO SIDE	m)
six number summary : summary (x) (8	$P(A \text{ and } B) = P(A) \cdot P(B) \text{ if } A, B \text{ independent}$ ((2) $p = P(z < z') = F(z') = pnorm(z')$	(47)		-
	$n! = n(n-1) \cdots 1 = factorial(n)$ ((48)	$z_{\alpha/2} = F_z^{-1}(1 - \alpha/2) = \text{qnorm(1-alpha/2)}$	(66)
$\mu = \frac{\sum x_i}{N} = \text{mean}(\mathbf{x}) (9)$	$_{n}P_{k} = \frac{n!}{(n-k)!}$ Perm. no elem. alike ((48)	$t_{\alpha/2} = F_i^{-1}(1 - \alpha/2) = qt (1-alpha/2, df)$	(67)
$\bar{x} = \frac{\sum x_i}{\sum x_i} = \text{mean}(\mathbf{x})$ (10)				$\chi_l^2 = F_{\omega^2}^{-1}(\alpha/2) = \text{qchisq(alpha/2, df)}$	
$\bar{x} = \frac{\sum A_l}{n} = \text{mean}(\mathbf{x})$ (10)	$= \frac{n!}{n! m! \dots m!} \text{ Perm. } n_1 \text{ alike, } \dots ($	= pnorm(x', mean=\mu, sd=\sigma)	(49)	- A	
$\bar{x} = P_{50} = \text{median}(\mathbf{x})$ (11)	n1 n2 . ng .	" · (r)		$\chi_R^2 = F_{\chi^2}^{-1}(1 - \alpha/2) = \text{qchisq(1-alpha/2,}$	df)
$\sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{N}}$ (12)	${}_{n}C_{k} = \frac{n!}{(n-k)!k!} = \text{choose}(n,k)$ (eqnorm(p, mean=μ, sd=σ)	(50)		(69)
$\sigma = \sqrt{\frac{L(\lambda_i - \mu)}{N}}$ (12)	(n-k):k:	5.2 A PAGENTALISMON		7.3 REQUIRED SAMPLE SIZE	
\	4 Discrete Random Variables	5.3 t-distribution		7.3 REQUIRED SAMPLE SIZE	
$s = \sqrt{\frac{\sum (x_i - \vec{x})^2}{n-1}} = \operatorname{sd}(\mathbf{x}) (13)$	4 Distrete Raindoin variables	p = P(t < t') = F(t') = pt(t', df)	(51)	proportion: $n = \hat{p}\hat{q} \left(\frac{z_{\alpha/2}}{r}\right)^2$,	(70)
V n-1	$P(x_i)$: probability distribution ($t' = F^{-1}(p) = at(p, df)$	(52)	(E /	(70)
$CV = \frac{\sigma}{-} = \frac{s}{-}$ (14)	$E = \mu = \sum x_i \cdot P(x_i)$ (r = r (p) = qc (p, dr)	(32)	$(\hat{p} = \hat{q} = 0.5 \text{ if unknown})$	
μ \bar{x}	L- (-1)	5.4 ×2 DISTRIBUTION		(7-12-15) ²	
2.2 RELATIVE STANDING	$\sigma = \sqrt{\sum (x_i - \mu)^2 \cdot P(x_i)}$ (19)		mean: $n = \left(\frac{z_{\alpha/2} \cdot \hat{\sigma}}{F}\right)^2$	(71)
		$p = P(\chi^2 < \chi^{2'}) = F(\chi^{2'})$		(- /	
$z = \frac{x - \mu}{\sigma} = \frac{x - \bar{x}}{s}$ (15)	4.1 BINOMIAL DISTRIBUTION	= pchisq (X^2) , df)	(53)		
Percentiles:					
$P_k = x_i$, (sorted x)			(54)		
		31)			
$k = \frac{i - 0.5}{n} \cdot 100\%$ (16)	$P(x) = {}_{n}C_{n}p^{x}q^{(n-x)} = \text{dibinom}(\mathbf{x}, \mathbf{n}, \mathbf{p}) ($	(2) 5.5 F-DISTRIBUTION			
To find x_i given P_k , i is:		p = P(F < F') = F(F')			
10 ind x_i given P_k , t is: 1. $L = (k/100\%)n$	4.2 Poisson distribution				
 if L is an integer: i = L+0.5; otherwise i=L and 			(55)		
round up.	$P(x) = \frac{\mu^{x} \cdot e^{-\mu}}{x!} = \text{dpois}(x, \mu) \qquad ($	$F' = F^{-1}(p) = qf(p, df1, df2)$	(56)		
	•	•			

8 Hypothesis Tests Test statistic and R function (when available) are listed for each.

alternative="two_sided" can be: "two.sided". "less". "greater" conf.level=0.95 constructs a 95% confidence interval. Standard CI

only when alternative="two.sided". Optional arguments for power calculations & Type II error: alternative="two.sided" can be:

"two sided" or "one sided" sig.level=0.05 sets the significance level \alpha.

Optional arguments for hypothesis tests:

8.1. 1-SAMPLE PROPORTION

prop.test(x, n, p=p0, alternative="two.sided")

, alternative="two.sided")

$$z = \frac{\hat{p} - p_0}{\sqrt{m_0 m / n}}$$
(7)

8.2 1-SAMPLE MEAN (σ KNOWN)

$$H_0: u = u_0$$

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$
(73)

8.3 1-SAMPLE MEAN (σ UNKNOWN)

 $H_0: \mu = \mu_0$

Required Sample size:

 $t = \frac{\bar{x} - \mu_0}{2 / \sqrt{n}}, df = n - 1$

$$t = \frac{r \sigma}{s/\sqrt{n}}, df =$$

power.t.test(delta=h, sd =G, sig.level=0, power=1 β, type ="one.sample", alternative="two.sided")

8.4 2-SAMPLE PROPORTION TEST

 $H_0: p_1 = p_2$ or equivalently $H_0: \Delta p = 0$ prop.test(x, n, alternative="two.sided") where: $\mathbf{x}=\mathbf{c}(x_1, x_2)$ and $\mathbf{n}=\mathbf{c}(n_1, n_2)$

$$\mathbf{c}(x_1, x_2)$$
 and $\mathbf{n} = \mathbf{c}(n_1, n_2)$

$$z = \frac{\Delta \hat{p} - \Delta p_0}{\sqrt{\hat{p}_1^2 + \hat{p}_2^2}}, \quad \Delta \hat{p} = \hat{p}_1 - \hat{p}_2$$

$$\sqrt{\frac{\delta \tilde{q}}{n_1} + \frac{\delta \tilde{q}}{n_2}}$$

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2}, \quad \bar{q} = 1 - \bar{p}$$

power.prop.test(p1= p_1 , p2= p_2 , power= $1-\beta$, sig.level=q, alternative="two.sided")

8.5 2-SAMPLE MEAN TEST

 $H_0: \mu_1 = \mu_2$ or equivalently $H_0: \Delta \mu = 0$

t.test(x1, x2, alternative="two.sided") where: x1 and x2 are vectors of sample 1 and sample 2 data.

$$= \frac{\Delta \bar{x} - \Delta \mu_0}{\int_{\bar{x}_1^2 - \bar{x}_1^2}} df \approx \min(n_1 - 1, n_2 - 1), \quad \Delta \bar{x} = \bar{x}_1 - \bar{x}_2$$

$$\sqrt{\frac{x_1}{n_1} + \frac{x_2}{n_2}}$$

Required Sample size: power.t.test(delta=h, sd =0, sig.level=0, power=1 β, type ="two.sample", alternative="two.sided")

8.6 2-SAMPLE MATCHED PAIRS TEST $H_0: u_2 = 0$ t.test(x, y, paired=TRUE, alternative="two.sided")

 $t = \frac{\bar{d} - \mu_{d0}}{r \cdot / \sqrt{n}}, d_i = x_i - y_i, df = n - 1$

Required Sample size:
power.t.test(delta=
$$h$$
, sd = σ , siq.level= α , power=1 -

β, type ="paired", alternative="two.sided")

8.7 TEST OF HOMOGENEITY, TEST OF INDEPENDENCE

 $H_0: p_1 = p_2 = \cdots = p_n$ (homogeneity)

where: x and y are ordered vectors of sample 1 and sample 2 data.

Enter table: D=data.frame(c1, c2, ...), where c1, c2, ... are

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$
, $df = (\text{num rows - 1})(\text{num cols - 1})$ (79)

$$E_i = \frac{\text{(row total)(column total)}}{\text{(accord total)}} = np_i$$
 (80)

fisher.test(D, alternative="greater") (must specify alternative as greater)

9 Linear Regression

of raw categorical data

(74) 9.1 LINEAR CORRELATION

 $H_0 : \rho = 0$ cor.test(x, y)

where: x and v are ordered vectors.

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{(n-1)s_x s_y}, \quad t = \frac{r-0}{\sqrt{\frac{1-r^2}{n-2}}}, \quad df = n-2$$

R MODEL

9.2 MODELS IN R MODEL TYPE | FOUNTION

near 2 indep vars $y = b_0 + b_1x_1 + b_2x_2$ $y \sim x1 + x2$ inteaction $y = b_0 + b_1x_1 + b_2x_2 + b_{12}x_1x_2$ $y \sim x1 + x2 + x1 * x2$ polynomial $y = b_0 + b_1x_1 + b_2x_2^2$ $y \sim x1 + x(x2^{^{^{\prime}}}2)$	inear 1 indep var 0 intercept	$y = 0 + b_1x_1$	y~x1 y~0+x1
	inteaction	$y = b_0 + b_1x_1 + b_2x_2 + b_{12}x_1x_2$	

9.3 REGRESSION Simple linear regression steps:

- 1. Make sure there is a significant linear correlation.
- results=lm(v~x) Linear regression of v on x vectors 3. results View the results
 - plot(x, v): abline(results) Plot regression line on data
 - 5. plot (x, results\$residuals) Plot residuals

$$y = b_0 + b_1x_1$$

$$\sum (x_i - \bar{x})(y_i - \bar{y})$$
(82)

 $b_1 = \frac{\sum (x_i - \tilde{x})(y_i - \tilde{y})}{\sum (x_i - \tilde{x})^2}$ (83) (84) To predict v when x = 5 and show the 95% prediction interval with regression model in results:

predict (results, newdata=data.frame (x=5), int="pred")

10 ANOVA 10.1 ONE WAY A NOVA

results=aov(depVarColName~indepVarColName,

9.4 PREDICTION INTERVALS

data=tableName) Run ANOVA with data in TableName, factor data in indepVarColName column, and response data in depVarColName column. 2. summary (results) Summarize results

boxplot (depVarColName~indepVarColName, data=tableName) Boxplot of levels for factor To find required sample size and power see power.anova.test(...)

11 Loading external data

· Export your table as a CSV file (comma seperated file) from Excel.

. Import your table into MyTable in R using: