MAT 167: STATISTICS

Test I: Chapters 1-4

Instructor: Anthony Tanbakuchi

Fall 2008

Name:		
	Computer / Seat Number:	

No books, notes, or friends. **Show your work.** You may use the attached equation sheet, R, and a calculator. No other materials. Write your work in the provided space for each problem (including any R work if appropriate). You may not use personal computers, only use the classroom computer at your desk. Using any other program or having any other documents open on the computer will constitute cheating.

You have until the end of class to finish the exam, manage your time wisely.

If something is unclear quietly come up and ask me.

If the question is legitimate I will inform the whole class.

Express all final answers to 3 significant digits. Probabilities should be given as a decimal number unless a percent is requested. Circle final answers, ambiguous or multiple answers will not be accepted. Show steps where appropriate.

The exam consists of 8 questions for a total of 40 points on 7 pages.

This Exam is being given under the guidelines of our institution's **Code of Academic Ethics**. You are expected to respect those guidelines.

Points Earned:	_ out of 40	total	points
Exam Score:			

- 1. Provide **short succinct** written answers to the following conceptual questions.
 - (a) (1 point) Would ethnicity be classified as a nominal, ordinal, interval, or ratio level of measurement?
 - (b) (1 point) Which of the following measures of center is least susceptible to outliers: median, mean, midrange, mode
 - (c) (1 point) What percent of data is greater than Q_1 ?
 - (d) (1 point) If the mean, median, and mode for a data set are all the same, what can you conclude about the data's distribution?
 - (e) (1 point) If the mean is less than the mode for a data set, what can you conclude about the data's distribution?
 - (f) (1 point) What does the standard deviation represent conceptually **in words**? (Be concise but don't simply state the equation in words verbatim.)
 - (g) (1 point) To determine the proportion of support in Tucson for Vice Presidential candidate Palin, a researcher randomly samples 10 people (using an ideal simple random sample). From the sample data, 70% expressed their support. The researcher was suspicious of the result so she repeated the study randomly sampling 10 people 3 more times, the resultant statistics were 30% 70% and 0% respectively. The researcher thinks some sort of mistake has occurred causing the numbers to change each time the study is repeated. However, after carefully checking the procedures used, no mistake occurred. What type of error could we attribute to the variation between the sample results?
 - (h) (2 points) A histogram is a useful tool that can quickly communicate many traits about a set of data. List 4 useful pieces of information that an observer can easily assess using a histogram.

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- (i) (1 point) Your SAT results state that you scored in the 95th percentile. What does this mean?
- (j) (1 point) Why would a SAT percentile be preferred over a raw SAT score for college admissions committees?
- 2. A survey conducted in our class asked 24 students how many credits they were enrolled in this semester. Use the R output below to answer the following questions.

There are 24 data points stored in the variable x, below is the sorted data:

```
> sort(x)
```

The basic descriptive statistical analysis is as follows:

```
> summary(x)
```

```
Min. 1st Qu. Median Mean 3rd Qu. Max. 6.00 11.00 12.00 12.83 15.00 19.00
```

> var(x)

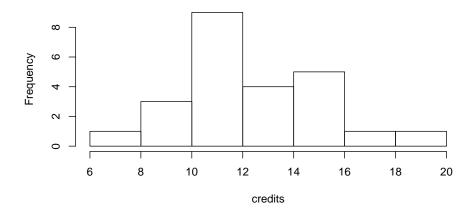
[1] 7.884058

> sd(x)

[1] 2.807856

> hist(x, xlab = "credits", main = "Student Enrollment Data")

Student Enrollment Data



(a) (1 point) Use the range rule of thumb to estimate the standard deviation. Is it close to the actual standard deviation?

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Points earned: _____ / 3 points

- (b) (1 point) What is P_{25} equal to?
- (c) (1 point) What is the IQR (inter quartile range) equal to?
- (d) (1 point) What percent of the data falls within the IQR?
- (e) (1 point) What is the percentile for the student who is taking 14 credits?
- (f) (1 point) What is the z-score for the student who is taking 10 credits?
- (g) (1 point) Is 6 credits an unusual (outlier) value based on its z score? (Why)
- (h) (1 point) Is the data positively skewed, negatively skewed, or symmetrical?
- (i) (1 point) Construct an interval using the Empirical Rule which you would expect 99.7% of the data to fall within.

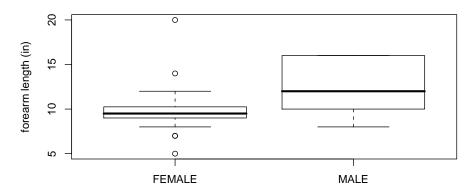
- 3. A researcher conducts a study in which 1,000 individuals in Tucson are randomly selected and asked if they prefer orange juice over grape juice. Eighty percent of the respondents preferred orange juice. The researcher concludes that "Study data indicates that eighty percent of Americans prefer orange juice over grape juice."
 - (a) (1 point) What is wrong with this conclusion?

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Points earned: _____ / 9 points

- (b) (1 point) Restate an appropriate conclusion that can be reached from the study.
- 4. Use the below box plot to answer the following questions.

Student forearm length verses gender



- (a) (1 point) Which gender has a higher median forearm length?
- (b) (1 point) What is the approximate median forearm length for the females?
- (c) (1 point) Which gender has a larger variation in forearm length for the middle 50% of individuals?
- (d) (1 point) How many outliers are there in this data set as indicated by the box plots?
- 5. Using the below table for our class to answer the following questions.

	BLACK	BLOND	BROWN
FEMALE	2	5	11
MALE	3	0	3

(a) (1 point) Find the probability of selecting a person with brown hair.

- (b) (1 point) Would it be unusual to randomly select a person with brown hair?
- (c) (1 point) Find the probability of randomly selecting three males without replacement.

(d) (1 point) If you randomly select 6 people with replacement, what is the probability that at least one has brown hair?

- (e) (1 point) Find the probability of selecting a female student or a student with brown hair.
- (f) (1 point) Find the probability of selecting a person with brown hair given that they are female.

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Points earned: _____ / 5 points

- 6. In the 1964 movie *Dr. Strangelove or: How I Learned to Stop Worrying and Love the Bomb*, Brigadier General Jack D. Ripper, a delusional commander of the United States Air Force, orders nuclear armed B-52 bomber planes to attack Russia. The planes use the CRM-114¹ discriminator, which, to prevent false or misleading orders from being received, is designed not to receive at all, unless the message is preceded by a three-letter code prefix.
 - (a) (1 point) To recall the planes before they bomb Russia, the US government must issue the correct recall code. How many possible codes are there if the code is composed of three characters, each character being one of the 26 letters in the alphabet?
 - (b) (1 point) If there are only 20 minutes before the planes drop the bombs and it takes 2 second to issue an individual code, how many random codes can be issued to guess the correct code?
 - (c) (1 point) What is the probability that the US government will issue the correct recall code by randomly guessing codes before it's too late?

(d) (1 point) Would it be unusual to issue the recall code in 20 minutes by guessing? (WHY)

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¹CRM114 is also the name of a computer program which uses a statistical approach for classifying data and is especially utilized for filtering email spam. It was named after the fictional device.

7. (2 points) When doing blood testing for HIV infections, the procedure can be made more efficient and less expensive by combining samples of blood specimens. If samples from five people are combined and the mixture tests negative, we know that all five individual samples are negative. Find the probability of a positive result for five samples combined into one mixture, assuming the probability of an individual blood sample testing positive is 10%. (Based on data from the NY State Health Department)

8. (2 points) Given $y = \{a, -2a, 4a\}$, completely simplify the following expression. Assume a is an unknown constant.

$$\left(\sum (y_i - 2a)\right)^2$$

D : 6/ // / O : 1		1					
Basic Statistics: Quick		2.3 VISUAL		5 Continuous random variables		6 Sampling distributions	
Reference & R Commands		All plots have optional arguments: • main="" sets title		CDF $F(x)$ gives area to the left of x , $F^{-1}(p)$ experies area to the left.	cts p	$\mu_{\bar{x}} = \mu$ $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{c}}$	(57)
by Anthony Tanbakuchi. Version 1.8 http://www.tanbakuchi.com		* xlab="", vlab="" sets x/v-axis label				$\mu_{\bar{x}} - \mu$ $G_{\bar{x}} - \sqrt{n}$	(37)
ANTHONY@TANBAKUCHI-COM		 type="p" for point plot 		f(x): probability density	(34)	$\mu_{\hat{p}} = p$ $\sigma_{\hat{p}} = \sqrt{\frac{pq}{q}}$	(58)
Get R at: http://www.r-project.org		• type="1" for line plot		$E = \mu = \int_{-\infty}^{\infty} x \cdot f(x) dx$	(35)	$\mu_p - \nu$ $\sigma_p - \sqrt{n}$	(50)
R commands: bold typewriter text		• type="b" for both points and lines Ex: plot(x, v, type="b", main="My Plot")		J		7 Estimation	
1 Misc R		Plot Types:		$\sigma = \sqrt{\int_{-\infty}^{\infty} (x - \mu)^2 \cdot f(x) dx}$	(36)	7.1 CONFIDENCE INTERVALS	
To make a vector / store data: x=c(x1, x2,)	hist(x) histogram stem(x) stem & leaf		F(x): cumulative prob. density (CDF)	(37)		
Get help on function: ?functionName		boxplot (x) box plot		$F^{-1}(x)$: inv. cumulative prob. density	(38)	proportion: $\hat{p} \pm E$, $E = z_{\alpha/2} \cdot \sigma_{\hat{p}}$	(59)
Get column of data from table: tableName\$columnName		plot(T) bar plot, T=table(x)				mean (σ known): $\bar{x} \pm E$, $E = z_{\alpha/2} \cdot \sigma_{\bar{x}}$	(60)
List all variables: 1s()		plot (x, y) scatter plot, x, y are ordered vectors	.	$F(x) = \int_{-\infty}^{x} f(x') dx'$	(39)	mean (σ unknown, use s): $\bar{x} \pm E$, $E = t_{\alpha/2} \cdot \sigma_{\bar{x}}$,	(61)
Delete all variables: rm(list=ls())		<pre>plot(t,y) time series plot, t, y are ordered vect curve(expr, xmin,xmax) plot expr involving</pre>		p = P(x < x') = F(x')	(40)	df = n - 1	
		Curve (expr., ameri, ameri, piot expr. informi	.	$y' = F^{-1}(n)$	(41)	$(n-1)s^2$, $(n-1)s^2$	
$\sqrt{x} = \mathtt{sqrt}(\mathbf{x})$	(1)	2.4 Assessing Normality		p = P(x > a) = 1 - F(a)	(42)	variance: $\frac{(n-1)s^2}{\gamma_p^2} < \sigma^2 < \frac{(n-1)s^2}{\gamma_1^2}$,	(62)
$x^n = \mathbf{x}^{\wedge} \mathbf{n}$	(2)	Q-Q plot: qqnorm(x); qqline(x)		p = F(x > a) = 1 - F(a) p = P(a < x < b) = F(b) - F(a)	(43)	df = n - 1	
n = length(x)	(3)			p = r(u < x < b) = r(b) = r(u)	(43)	/as as	
$T = \mathtt{table}(\mathbf{x})$	(4)	3 Probability		5.1 Uniform distribution		2 proportions: $\Delta \hat{p} \pm z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n_1} + \frac{\hat{p}\hat{q}}{n_2}}$	(63)
		Number of successes x with n possible outcomes.				7 2 2	
2 Descriptive Statistics		(Don't double count!)		p = P(u < u') = F(u')		2 means (indep): $\Delta \bar{x} \pm t_{\alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$,	(64)
2.1 Numerical		$P(A) = \frac{x_A}{a}$	(17)	= punif(u', min=0, max=1)	(44)	1	
Let x=c(x1, x2, x3,)		n n	(18)	$u' = F^{-1}(p) = qunif(p, min=0, max=1)$	(45)	$df \approx \min(n_1 - 1, n_2 - 1)$	
$total = \sum_{i=1}^{n} x_i = sum(x)$	(5)			5.2 NORMAL DISTRIBUTION		matched pairs: $\vec{d} \pm t_{\alpha/2} \cdot \frac{s_d}{\sqrt{n}}$, $d_i = x_i - y_i$,	(65)
i=1			(20)	5.2 NORMAL DISTRIBUTION		df = n - 1	
min = min(x)	(6)		(21)	$f(x) = \frac{1}{\sqrt{x^2 - x^2}} \cdot e^{-\frac{1}{2} \frac{(x-\mu)^2}{\sigma^2}}$	(46)	u) - n .	
max = max(x)	(7)	(/ (/ (/ /		√2πσ²		7.2 CI CRITICAL VALUES (TWO SIDE	ED)
six number summary : summary (x)	(8)	$P(A \text{ and } B) = P(A) \cdot P(B)$ if A, B independent		p = P(z < z') = F(z') = pnorm(z')	(47)	$z_{\alpha/2} = F_r^{-1}(1 - \alpha/2) = qnorm(1-alpha/2)$	
$\mu = \frac{\sum x_i}{N} = \text{mean}(\mathbf{x})$	(9)	$n! = n(n-1) \cdots 1 = factorial(n)$		$z' = F^{-1}(p) = qnorm(p)$	(48)		
. IV		$_{n}P_{k} = \frac{n!}{(n-k)!}$ Perm. no elem. alike	(24)	p = P(x < x') = F(x')		$t_{\alpha/2} = F_t^{-1}(1 - \alpha/2) = qt (1-alpha/2, df)$	
$\bar{x} = \frac{\sum x_i}{n} = \text{mean}(\mathbf{x})$	(10)	n!		$= pnorm(x', mean=\mu, sd=\sigma)$	(49)	$\chi_L^2 = F_{\chi^2}^{-1}(\alpha/2) = \text{qchisq(alpha/2, df)}$	(68)
$\tilde{x}=P_{50}=\mathtt{median}\left(\mathbf{x}\right)$	(11)	$= \frac{n!}{n_1!n_2!\cdots n_k!} \text{Perm. } n_1 \text{ alike, } \dots$	(25)	$x' = F^{-1}(p)$		$\chi_R^2 = F_{\chi^2}^{-1}(1 - \alpha/2) = \text{qchisq(1-alpha/2)}$	
$\sqrt{\nabla (r-u)^2}$		$_{n}C_{k} = \frac{n!}{(n-k)!k!} = \text{choose}(\mathbf{n}, \mathbf{k})$	(26)	= qnorm(p, mean=\mu, sd=\sigma)	(50)		(69)
$\sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{N}}$	(12)	(n - k):k:		5.3 t-DISTRIBUTION		7.3 REQUIRED SAMPLE SIZE	
V2		4 Discrete Random Variables					
$s = \sqrt{\frac{\sum (x_i - \vec{x})^2}{n-1}} = \operatorname{sd}(\mathbf{x})$	(13)	$P(x_i)$: probability distribution		p = P(t < t') = F(t') = pt(t'), df)	(51)	proportion: $n = \hat{p}\hat{q} \left(\frac{z_{\alpha/2}}{r}\right)^2$,	(70)
, , , , , , , , , , , , , , , , , , ,	(14)		(27)	$t' = F^{-1}(p) = qt(p, df)$	(52)	$(\hat{p} = \hat{q} = 0.5 \text{ if unknown})$	
$CV = \frac{\sigma}{\mu} = \frac{s}{\bar{x}}$	(14)	. 2	(28)	5.4 χ ² -distribution		* * * * * * * * * * * * * * * * * * * *	
2.2 RELATIVE STANDING		$\sigma = \sqrt{\sum (x_i - \mu)^2 \cdot P(x_i)}$	(29)	**		mean: $n = \left(\frac{z_{\alpha/2} \cdot \hat{\sigma}}{E}\right)^2$	(71)
r - n r - r		•		$p = P(\chi^2 < \chi^{2'}) = F(\chi^{2'})$		(- /	
0 7	(15)	4.1 BINOMIAL DISTRIBUTION		$= pchisq(X^2), df$	(53)		
Percentiles:		$u = n \cdot p$	(30)	$\gamma^{2'} = F^{-1}(p) = \operatorname{gchisg}(p, df)$	(54)		
$P_k = x_i$, (sorted x)			(31)	$\chi = r - (p) = qcnisq(p, di)$	(34)		
$k = \frac{i - 0.5}{\cdot 100\%}$	(16)		(32)	5.5 F-DISTRIBUTION			
n		r(x) - nCxp q = doinom(x, n, p)	(32)	warm with warmin			
To find x_i given P_k , i is: 1. $L = (k/100\%)n$		4.2 POISSON DISTRIBUTION		p = P(F < F') = F(F')			
 if L is an integer: i = L + 0.5; otherwise i=L and 		,x ,-p	(33)	= pf(F', df1, df2)	(55)		
round up.		$r(x) = \frac{1}{x!} = \text{dpois}(\mathbf{x}, \mu)$	(33)	$F' = F^{-1}(p) = qf(p, df1, df2)$	(56)		

8 Hypothesis Tests

Test statistic and R function (when available) are listed for each. Optional arguments for hypothesis tests: alternative="two_sided" can be: "two.sided". "less". "greater"

conf.level=0.95 constructs a 95% confidence interval. Standard CI only when alternative="two.sided". Optional arguments for power calculations & Type II error:

alternative="two.sided" can be: "two sided" or "one sided"

sig.level=0.05 sets the significance level α.

8 1 1-SAMPLE PROPORTION

prop.test(x, n, p=p0, alternative="two.sided")

$$z = \frac{\hat{p} - p_0}{\sqrt{p_0q_0/n}}$$
(7)

8.2 1-SAMPLE MEAN (σ KNOWN)

 $H_0: u = u_0$

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{a}}$$
(73)

8.3 1-SAMPLE MEAN (σ UNKNOWN)

 $H_0 : \mu = \mu_0$ t.test(x, mu=u0, alternative="two.sided")

Where
$$\mathbf{x}$$
 is a vector of sample data.

$$t = \frac{\bar{x} - \mu_0}{r / \sqrt{n}}, \quad df = n - 1$$

Required Sample size: power.t.test(delta=h, sd =G, sig.level=Q, power=1 β, type ="one.sample", alternative="two.sided")

8.4 2-SAMPLE PROPORTION TEST

 $H_0: p_1 = p_2$ or equivalently $H_0: \Delta p = 0$ prop.test(x, n, alternative="two.sided") where: $\mathbf{x} = \mathbf{c}(x_1, x_2)$ and $\mathbf{n} = \mathbf{c}(n_1, n_2)$

$$x_2$$
) and $\mathbf{n} = \mathbf{c} (n_1, n_2)$

$$z = \frac{\Delta \hat{p} - \Delta p_0}{\sqrt{\hat{p}_1^2 + \hat{p}_2^2}}, \quad \Delta \hat{p} = \hat{p}_1 - \hat{p}_2$$

$$\sqrt{\frac{\rho q}{n_1} + \frac{\rho q}{n_2}}$$
, $\Delta p = p_1 - p_2$

$$\bar{p} = \frac{x_1 + x_2}{x_1 + x_2}, \quad \bar{q} = 1 - \bar{p}$$
 (

$$n_1 + n_2$$

Required Sample size:

power.prop.test(p1= p_1 , p2= p_2 , power= $1-\beta$, sig.level=q, alternative="two.sided")

8.5 2-SAMPLE MEAN TEST

 $H_0: \mu_1 = \mu_2$ or equivalently $H_0: \Delta \mu = 0$

t.test(x1, x2, alternative="two.sided") where: x1 and x2 are vectors of sample 1 and sample 2 data.

$$-\frac{s_1^2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \quad \text{if } \sim \min(n_1 - 1, n_2 - 1), \quad \text{if } -x_1 - x_2$$

Required Sample size:

power.t.test(delta=h, sd =0, sig.level=0, power=1 β, type ="two.sample", alternative="two.sided")

8.6 2-SAMPLE MATCHED PAIRS TEST $H_0: u_i = 0$

t.test(x, v, paired=TRUE, alternative="two.sided")

Required Sample size

of raw categorical data

where: x and y are ordered vectors of sample 1 and sample 2 data.

$$t = \frac{\tilde{d} - \mu_{d0}}{s_d / \sqrt{n}}, d_i = x_i - y_i, df = n - 1$$
 (78)

power.t.test(delta=h, sd =G, siq.level=a, power=1 β, type ="paired", alternative="two.sided")

8.7 Test of homogeneity, test of independence $H_0: p_1 = p_2 = \cdots = p_n$ (homogeneity)

 $H_0: X$ and Y are independent (independence)

chisg.test(D)

Enter table: D=data.frame(c1, c2, ...), where c1, c2, ... are column data vectors. Or generate table: D=table (x1, x2), where x1, x2 are ordered vectors

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{r}$$
, $df = (\text{num rows} - 1)(\text{num cols} - 1)$ (79)

$$E_i = \frac{\text{(row total)(column total)}}{\text{(around total)}} = np_i$$
(80)

fisher.test(D, alternative="greater") (must specify alternative as greater)

9 Linear Regression

(74) 9.1 LINEAR CORRELATION

 $H_0: \rho = 0$ cor.test(x, y)

where: x and v are ordered vectors.

 $r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{(n-1)s_x s_y}, \quad t = \frac{r-0}{\sqrt{1-r^2}}, \quad df = n-2$ (81)

9.2 MODELS IN R MODEL TYPE | FOUNTION

linear 1 inden var $v = h_0 + h_1 v_1$... 0 intercept $y = 0 + b_1x_1$ v~0+x1 linear 2 indep vars $y = b_0 + b_1x_1 + b_2x_2$...inteaction $y = b_0 + b_1x_1 + b_2x_2 + b_{12}x_1x_2$ v~x1+x2+x1*x2 polynomial $y = b_0 + b_1x_1 + b_2x_2^2$ $v \sim x1 + I(x2^{\wedge}2)$

R MODEL

9.3 REGRESSION Simple linear regression steps:

- 1. Make sure there is a significant linear correlation. results=lm(v~x) Linear regression of v on x vectors
- 3. results View the results
- plot(x, v): abline(results) Plot regression line on data
- 5. plot(x, results\$residuals) Plot residuals

$$y = b_0 + b_1x_1$$
 (82)
 $b_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$ (83)
 $b_0 = \bar{y} - b_1\bar{x}$ (84)

To predict v when x = 5 and show the 95% prediction interval with regression

model in results: predict (results, newdata=data.frame (x=5), int="pred")

10 ANOVA

10.1 ONE WAY A NOVA

results=aov(depVarColName~indepVarColName,

9.4 PREDICTION INTERVALS

data=tableName) Run ANOVA with data in TableName, factor data in indepVarColName column, and response data in depVarColName column. 2. summary (results) Summarize results

boxplot (depVarColName~indepVarColName, data=tableName) Boxplot of levels for factor

To find required sample size and power see power, anovaltest(...)

11 Loading and using external data and tables 11.1 LOADING EXCEL DATA

1. Export your table as a CSV file (comma seperated file) from Excel.

2. Import your table into MyTable in R using: MvTable=read.csv(file.choose())

11.2 LOADING AN RDATA FILE You can either double click on the RData file or use the menu:

 Windows: File→Load Workspace... Mac: Workspace→Load Workspace File...

11.3 USING TABLES OF DATA

1. To see all the available variables type: 1s ()

2. To see what's inside a variable, type its name.

3. If the variable tableName is a table, you can also type names (tableName) to see the column names or type head (tableName) to see the first few rows of data. 4. To access a column of data type tableName\$columnName

An example demonstrating how to get the women's height data and find the mean:

> ls() # See what variables are defined [1] "women" "x"

> head(women) #Look at the first few entries

height weight 5.0

> names(women) # Just get the column names

[11 "height" "weight"

> womenSheight # Display the height data

[1] 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72

> mean(womenSheight) # Find the mean of the heights

f11 65