SOLUTIONS MAT 167: STATISTICS

Test I: Chapters 1-3

Instructor: Anthony Tanbakuchi

Fall 2007

Name:		
	Computer / Seat Number	

No books, notes, or friends. You may use the attached equation sheet, R, and a calculator. No other materials. If you choose to use R, copy and paste your work into a word document labeling the question number it corresponds to. When you are done with the test print out the document. Be sure to save often on a memory stick just in case. Using any other program or having any other documents open on the computer will constitute cheating.

You have until the end of class to finish the exam, manage your time wisely.

If something is unclear quietly come up and ask me.

If the question is legitimate I will inform the whole class.

Express all final answers to 3 significant digits. Probabilities should be given as a decimal number unless a percent is requested. Circle final answers, ambiguous or multiple answers will not be accepted. Show steps where appropriate.

The exam consists of 13 questions for a total of 40 points on 14 pages.

This Exam is being given under the guidelines of our institution's **Code of Academic Ethics**. You are expected to respect those guidelines.

Points Earned:	out of 40 total points
Exam Score:	

- 1. Given the following data collected from a random sample of individual's heights in cm: $170\ 185\ 155\ 168\ 162\ 164\ 280$
 - (a) (3 points) If you were asked to calculate the "average" height for this data, what measure of center (mean, median, mode, midrange) would you use? Give a clear reason why you choose this measure in terms of the data and what the measure represents.

Solution: Since this data set contains an outlier, using a measure of center that is resistant to outliers would give a better representation of the "average" height. Recall that the term "average" is non-specific, and all the measures of center are different types of averages. The most resistant measure of center would be the mode, but this is primarily for categorical data. Therefore, the median would be the correct choice for quantitative continuous data.

(b) (2 points) Compute the measure of center you recommended above for the data.

Solution: Depending on your answer above you should have gotten one of the following:

Median 168

Mean 183

Mode No mode for this data.

(c) (2 points) Using the Range Rule of Thumb, estimate the standard deviation for the data.

```
Solution:
> x
[1] 170 185 155 168 162 164 280
> s.approx = (max(x) - min(x))/4
> signif(s.approx, 3)
[1] 31.2
```

(d) (2 points) Compute the standard deviation for the data.

```
Solution: You should compute the sample standard deviation:

> s = sd(x)

> signif(s, 3)

[1] 43.6
```

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- 2. A certain type of mouse has a mean weight of 500g and a standard deviation of 30g.
 - (a) (2 points) Construct an interval using the Empirical Rule which you would expect 95% of the weights to fall within.

Solution: The Empirical Rule states that 95% of the data falls within $\mu \pm 2\sigma$. Therefore we would expect the data to fall within:

```
> s = 30
> mu = 500
> lower = mu - 2 * s
> upper = mu + 2 * s
> range.95 = c(lower, upper)
> signif(range.95, 3)
[1] 440 560
```

(b) (2 points) Find the z-score for a mouse with a weight of 445g.

```
Solution:

> x = 445

> z = (x - mu)/s

> signif(z, 3)

[1] -1.83
```

(c) (1 point) Would you consider a mouse with a weight of 445g unusual?

```
Solution: No, since |z| \ngeq 2.
```

3. (2 points) "38% of adults in the United States regularly visit a doctor". This conclusion was reached by a college student after she had questioned 520 randomly selected members of her college. What is wrong with her survey?

Solution: Here survey is only representative of students at her college but she is making conclusions about the US population. To make such conclusions she would need to randomly sample the whole US population, not just the students on her campus.

4. (2 points) Give an example of sampling error.

Solution: Sampling errors are errors caused by chance fluctuations. An example would be randomly sampling 10 people and computing their mean height. If you performed this random sample multiple times you would find that the mean height would vary somewhat from sample to sample. Sampling error is the natural fluctuation due to sampling, it is not a human error or misuse of statistics as a non-sapling error would be.

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5. (1 point) If the mean, median, and mode for a data set are all the same, what can you conclude about the data?

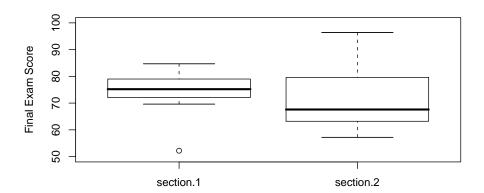
Solution: If all three measures of center are the same, the data is symmetrical with no outliers.

6. (2 points) Find $_{10}C_7$.

Solution: Using the combinations equation with n = 10, and r = 7: > factorial (10)/(factorial (10 - 7) * factorial (7)) [1] 120

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7. The following is a modified box plot of final exam scores for two different sections of a statistics class taking the same exam.



(a) (1 point) Which section had the highest score on the exam? Give an approximate value for the highest score.

Solution: Section 2, highest score was approximately 98.

(b) (1 point) Which section had the higher median score? Give the approximate median score for that section.

Solution: Section 1 had the higher median score of approximately 75.

(c) (1 point) A useful measure of variation is the inter-quartile range which is calculated as IQR = Q3 - Q1. Find the approximate IQR for section 2.

Solution: For section 2 the approximate values for Q1 and Q3 are:

$$> Q1 = 65$$

$$> Q3 = 80$$

$$> IQR = Q3 - Q1$$

(d) (1 point) What percent of data lies within the IQR?

Solution: Between the first and third quarter is 50% of the data since the quarters break the data into four equal parts.

8. (2 points) What is the probability of randomly selecting three people all born on the same day of the year? (Ignore leap years).

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Solution: Conceptually, we are happy with any birthday for the first person, for the second two they must have the same birthday as the first (a 1 in 365 chance).

```
> p = (365/365) * (1/365) * (1/365)
> signif(p, 3)
[1] 7.51e-06
```

- 9. If a couple has 5 children, what is the probability that:
 - (a) (1 point) They have exactly 2 girls and 3 boys (in that order).

Solution:

P(boy and boy and girl and girl) and the probability of a boy or a girl is 0.5.

```
> p = 0.5 * 0.5 * 0.5 * 0.5 * 0.5 
> p = 0.5^5 
> signif(p, 3) 
[1] 0.0312
```

(b) (2 points) They have at least one boy.

```
Solution: Easier to find compliment...

P(\text{at least one boy}) = 1 - P(\text{no boys}), \text{ where no boys is having 5 girls:}

P(\text{girl and girl and girl and girl and girl})

p = 1 - (0.5^5)

p = 1 - (0.5^5)

p = 1 - (0.5^6)

p = 1 - (0.5^6)
```

10. (3 points) In a blood testing procedure, blood samples from 5 people are combined into one mixture. The mixture will only test negative if all the individual samples are negative. If the probability that an individual sample tests positive is 0.20, what is the probability that the mixture will test positive?

Solution:

The sample will test positive if 1 or 2 or 3 or 4 or 5 of the people are positive or, more concisely, *one or more are positive*. So, the easy way to find the probability that one ore more are positive is to use the compliment:

P(1 or more positive) = 1 - P(all 5 are negative). The probability of a negative result is 1 - 0.20 = 0.80.

$$> p = 1 - (0.8)^5$$

 $> signif(p, 3)$
[1] 0.672

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11. (3 points) If a class consists of 12 freshmen and 8 sophomores, find the probability of randomly selecting three students in the following order: a sophomore then a sophomore then a freshman without replacement.

Solution:

P(sophomore and sophomore and freshman) these are dependent events so we need to carefully count the number available and total number for each trial.

```
> p = (8/20) * (7/19) * (12/18)
> signif(p, 3)
[1] 0.0982
```

12. (2 points) A cruise ship has 1000 people on it. Of the 1000 people, 25 are crew members. Find the probability of randomly selecting 10 people without replacement and none of them are crew members.

Solution: Since $n/N \le 0.05$, we can easily approximate the probability using independent events rather than calculate out all 10 probabilities.

 $P(10 \text{ non crew members}) \approx P(\text{non crew member})^{10}$

```
 > n = 10 
 > N = 1000 
 > n/N 
 [1] 0.01 
 > p.non.crew = 1 - 25/N 
 > p = p.non.crew^n 
 > signif(p, 3) 
 [1] 0.776
```

If you choose to do it without the approximation (more work), then you would have had:

```
\begin{array}{l} \frac{975}{1000} \frac{974}{999} \frac{973}{998} \cdots \frac{966}{991} = \frac{975!/965!}{1000!/990!} \\ > p = prod (975:966/1000:991) \\ > signif (p, 3) \\ [1] \quad 0.775 \end{array}
```

As you can see the two methods have very good agreement because our approximation is sufficiently accurate for $n/N \leq 0.05$

13. Given the following frequency table summarizing data from a study:

age.years	frequency
0-9	5.00
10-19	8.00
20-29	12.00
30-39	2.00

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(a) (1 point) Construct a relative frequency table.

Solution:	
age.years	relative.frequency.percent
0-9	18.52
10-19	29.63
20-29	44.44
30-39	7.41

(b) (1 point) What is the probability of randomly selecting someone from the study who is in the age range 10-29?

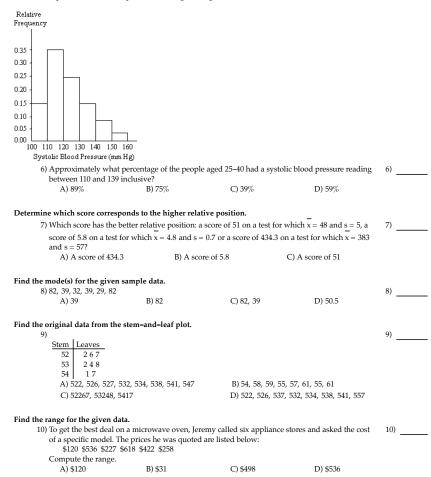
Solution:
>
$$p = (8 + 12)/(5 + 8 + 12 + 2)$$

> $signif(p, 3)$
[1] 0.741

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MAT67 Exam 1 Tanbakuchi				
Name				
Select the best answer from the choices provided.				
MULTIPLE CHOICE. Choose the one alternative that best complete	letes the statement or ar	nswers the question.		
Determine whether the given value is a statistic or a parameter. 1) A health and fitness club surveys 40 randomly selected members and found that the average				
weight of those questioned is 157 lb. A) Parameter	B) Statistic			
Determine which of the four levels of measurement (nominal, ordinal, interval, ratio) is most appropriate. 2) Nationalities of survey respondents. A) Ratio B) Nominal C) Interval D) Ordinal				
Identify the number as either continuous or discrete. 3) The number of stories in a Manhattan building is 22. A) Discrete	B) Continuous		3)	
Identify which of these types of sampling is used: random, stratified, systematic, cluster, convenience. 4) A tax auditor selects every 1000th income tax return that is received.				
A) Convenience B) Random C) Systematic D) Cluster E) Stratified				
Solve the problem. 5) On a test of 40 items, Ariel got 31 correct. What percent A) 129.032258% B) 77.5%	at were correct? C) 1.29032258%	D) 0.775%	5)	

A nurse measured the blood pressure of each person who visited her clinic. Following is a relative-frequency histogram for the systolic blood pressure readings for those people aged between 25 and 40. Use the histogram to answer the question. The blood pressure readings were given to the nearest whole number.



Solve the problem. 11) If the standard deviation of a set of data is zero, what can you conclude about the set of values? A) The sum of the values is zero. B) All values are equal to zero. C) The sum of the deviations from the mean is zero. D) All values are identical.				
Use the empirical rule to solve the p 12) At one college, GPA's are		a mean of 2.8 and a stan	dard deviation of	12)
0.6. What percentage of st				12)
A) 95.44%	B) 99.74%	C) 68.26%	D) 84.13%	
Answer the question, considering a 13) Assume that a study of 30 time. Is it "unusual" for a A) Yes	0 randomly selected scho			13)
Answer the question.				
14) Which of the following car	nnot be a probability?			14)
A) $\frac{1}{2}$	B) $\frac{3}{5}$	C) $\frac{5}{3}$	D) $\frac{2}{3}$	
Estimate the probability of the event. 15) In a certain class of students, there are 12 boys from Wilmette, 6 girls from Kenilworth, 9 girls from Wilmette, 7 boys from Glencoe, 5 boys from Kenilworth and 3 girls from Glencoe. If the teacher calls upon a student to answer a question, what is the probability that the student will be from Kenilworth?				
A) 0.333	B) 0.143	C) 0.262	D) 0.208	
Find the indicated probability. 16) A bag contains 4 red marbles, 3 blue marbles, and 5 green marbles. If a marble is randomly selected from the bag, what is the probability that it is blue?				
A) $\frac{1}{9}$	B) $\frac{1}{4}$	C) $\frac{1}{5}$	D) $\frac{1}{3}$	
17) A sample of 100 wood and wood and 13 graphite are the probability that the rad A) 0.565 B) 0.57 C) 0.135	I 100 graphite tennis rack defective and one racket	ets are taken from the wa is randomly selected from		17)

	Number of flights	Number of flights		
	which were on time	which were late		
Podunk Airlines	33	6		
Upstate Airlines	43	5		
		ed, find the probability that t	he flight selected arrived	
	it was an Upstate Airli	nes flight.		
A) $\frac{43}{87}$		B) 43/48		
A) $\frac{43}{87}$ C) $\frac{11}{76}$		D) None of the	above is correct.	
		pability that it will snow in a		19)
		given year it will not snow o		
A) 0.368	B) 3.717	C) 0.731	D) 1.269	

A) At least one of the textbooks is errorless. B) None of the textbooks are errorless. C) All of the textbooks are errorless.

Answer Key Testname: EXAM1MULTIPLECHOICE

- 1) B
- 2) B
- 3) A 4) C
- 5) B
- 6) B
- 7) B
- 8) C
- 9) A
- 10) C
- 11) D 12) C
- 13) B
- 14) C
- 15) C
- 16) B
- 17) A
- 18) B
- 19) C
- 20) A

Introductory Statistics Quick Reference & R Commands

by Anthony Tanbakuchi. Version 1.1 ANTHONY @TANKA KUCHLCOM

Get R at: http://www.r-project.org More R help & examples at:

http://tanbakuchi.com/Resources/R Statistics/RBasics.html R commands: bold text

1 Misc R

To make a vector / store data: x=c (x1, x2, ...) Get help on function: ?functionName

Get column of data from table: tableNameScolumnName List all variables: 1s()

Delete all variables: rm(list=ls())

 $\sqrt{x} = \text{sgrt}(x)$ v" — **v**^n n = length(x)T = table(x)

2 Descriptive Statistics

2.1 NUMERICAL

Let x=c (x1, x2, x3, ...)

total
$$=\sum_{i=1}^{n} x_i = sum(\mathbf{x})$$
 (5)
min = min(\mathbf{x}) (6)
max = max(\mathbf{x}) (7)
six number summary: $= summary(\mathbf{x})$ (8)

 $\mu = \frac{\sum x_i}{\sum x_i} = \text{mean}(\mathbf{x})$ $\bar{x} = \frac{\sum x_i}{\sum x_i} = \text{mean}(\mathbf{x})$ $\bar{x} = P_{\text{co}} = \text{median}(\mathbf{x})$ (9)

(10)

(11)

$$\sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{N}}$$

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{2}} = \operatorname{sd}(\mathbf{x})$$
(12)

$$s = \sqrt{\frac{L(x_t - X)}{n - 1}} = \operatorname{sd}(\mathbf{x})$$
 (13)
 $CV = \frac{s}{n} = \frac{s}{r}$ (14)

2.2 RELATIVE STANDING $z = \frac{x - \mu}{1 - \mu} = \frac{x - \bar{x}}{1 - \mu}$

Percentiles

 $P_r = r$ (sorted r) $k = \frac{i-0.5}{\cdot 100\%}$ (16)

To find x_i given P_k , i is:

 $1:L=\frac{k}{10000}\cdot n$

(17)2 · if I is an integer: i = I+0.5: otherwise i=I and round up

2.3 VISUAL All plots have optional arguments:

main=" " care title xlab="", ylab="" sets x/y-axis label type="p" for point plot

type="1" for line plot type="b" for both points and lines Ex: plot(x, v, type="b", main="My Plot")

Histogram: hist (x) Stem & leaf: stem(x) Box plot: boxplot (x)

Barnlot: plot (T) (where Tetable (x)) Scatter plot: plot (x, v) (where x, y are ordered vectors) Time series plot: plot (t, y) (where t, y are ordered vectors) Graph function: curve (expr. xmin, xmax) plot expr involving x

2.4 ASSESSING NORMALITY O-O plot: ganorm(x): galine(x)

3 Probability

Number of successes x with n possible outcomes. (Don't double count!)

 $P(A) = \frac{x_A}{}$

(18)

(24)

(25)

(26)

(27)

 $P(\bar{A}) = 1 - P(A)$ (19) P(A or B) = P(A) + P(B) - P(A and B)(20)

P(A or B) = P(A) + P(B)if A R mutually exclusive (21) $P(A \text{ and } B) = P(A) \cdot P(B|A)$ (22)(23) $P(A \text{ and } B) = P(A) \cdot P(B)$ if $A \cdot B$ independent $n! = n(n-1)(n-2)\cdots 2 \cdot 1 = factorial(n)$

 $_{n}P_{k} = \frac{n!}{(n-k)!}$ Perm. no elements alike

 $=\frac{n!}{n! + n!}$ Perm. n_1 alike, ...

 $_{n}C_{k} = \frac{n!}{(n-k)!k!} = \text{choose (n, k)}$

4 Random Variables

4.1 DISCRETE DISTRIBUTIONS

$$P(x_i)$$
: probability distribution
 $E = u = \sum x_i \cdot P(x_i)$

$$\sigma = \sqrt{\sum (x_i - \mu)^2 \cdot P(x_i)}$$
(30)

(28)

(29)

(51)

4.2 CONTINUOUS DISTRIBUTIONS

CDF F(x) gives area to the left of x, $F^{-1}(p)$ expects p is area to the left. f(x): probability density

$$E = \mu = \int_{-\infty}^{\infty} x \cdot f(x) dx \qquad (32)$$

 $\sigma = \sqrt{\int_{-\infty}^{\infty} (x - \mu)^2 \cdot f(x) dx}$ (33)

$$F(x)$$
: cumulative prob. density (CDF) (34)
 $F^{-1}(x)$: inv. cumulative prob. density (35)

 $F(x') = \int_{-x'}^{x'} f(x) dx$ (36)

$$p = P(x < x') = F(x')$$
 (37)
 $x' = F^{-1}(p)$ (38)

$$p = P(x > a) = 1 - F(a)$$
 (39)
 $p = P(a < x < b) = F(b) - F(a)$ (40)

4.3 SAMPLING DISTRIBUTIONS

$$\mu_{\bar{i}} = \mu$$
 $\sigma_{\bar{i}} = \frac{\sigma}{\sqrt{n}}$ (41)
 $\mu_{\bar{\nu}} = p$ $\sigma_{\bar{\nu}} = \sqrt{pq}$ (42)

4.4 BINOMIAL DISTRIBUTION

$$\mu = n \cdot p$$

$$\sigma = \sqrt{n \cdot p \cdot q}$$
(43)

$$P(x) = {}_{n}C_{x}p^{x}q^{(n-x)} = dbinom(x, n, p)$$
 (45)

$$P(x) = {}_{n}C_{x}p^{-}q^{n}$$
 $\xrightarrow{r} = \text{dbinom}(\mathbf{x}, \mathbf{n}, \mathbf{p})$ (45)

4.5 Poisson distribution $P(x) = \frac{\mu^x \cdot e^{-\mu}}{1} = \text{dpois}(\mathbf{x}, \mu)$

$$P(x) = \frac{\mu^{x} \cdot e^{-\mu}}{x!} = \text{dpois}(x, \mu)$$

4.6 NORMAL DISTRIBUTION

 $f(x) = \frac{1}{\sqrt{1 - x^2}} \cdot e^{-\frac{1}{2} \frac{(x-\mu)^2}{\sigma^2}}$ (47)n = P(z < z') = F(z') = pnorm(z')(48)

 $z' = F^{-1}(p) = \text{gnorm}(p)$ (49) $p = P(x < x') = F(x') = pnorm(x', mean=u, sd=\sigma)$ (50)

 $x' = F^{-1}(p) = \operatorname{qnorm}(p, \operatorname{mean} = \mu, \operatorname{sd} = \sigma)$

4.7 t-distribution		6 Hypothesis Tests	- 1
		alternative can be:	
		"two.sided", "less", "greater"	
p = P(t < t') = F(t') = pt(t'), df)	(52)	6.1 1-SAMPLE PROPORTION	
$t' = F^{-1}(p) = qt(p, df)$	(53)	H ₀ : $p = p_0$	
2		prop.test(x, n, p=p ₀ , alternative="two.sided")	
4.8 χ ² -distribution		[* * · · · · · * * · · · .	
		$z = \frac{\bar{p} - p_0}{\sqrt{p_0 q_0 / p}}$	(68)
$p = P(\gamma^2 < \gamma^{2'}) = F(\gamma^{2'}) = pchisq(X^2), df$	(EA)	,	
	(54)	0.2 1-SAMPLE MEAN (O KNOWN)	
$\chi^{2'} = F^{-1}(p) = \operatorname{qchisq}(\mathbf{p}, \operatorname{df})$	(55)	$H_0: \mu = \mu_0$	
4.9 F-DISTRIBUTION		$z = \frac{\bar{x} - \mu_0}{\sigma t / \pi}$	(69)
4.9 T-DISTRIBUTION		σ/\sqrt{n}	(0.7)
		6.3 1-SAMPLE MEAN (σ UNKNOWN)	
p = P(F < F') = F(F') = pf(F', df1, df2)	(56)		
$F' = F^{-1}(p) = qf(p, df1, df2)$	(57)	t.test(x, mu=µ0, alternative="two.sided")	
v, 14, . , . ,		Where x is a vector of sample data.	
5 Estimation		$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}, df = n - 1$	(70)
5.1 CONFIDENCE INTERVALS		s/\square	
proportion: $\hat{p} \pm E$, $E = z_{\alpha/2} \cdot \sigma_{\hat{p}}$	(58)		
mean (σ known): $\bar{x} \pm E$, $E = z_{\alpha/2} \cdot \sigma_{\bar{x}}$	(59)		
mean (σ unknown, use s): $\bar{x}\pm E$, $E=t_{\alpha/2}\cdot\sigma_{\bar{x}}$, $df=n-1$	(60)		
variance: $\frac{(n-1)s^2}{\chi_R^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_L^2}$, $df = n-1$	(61)		
5.2 CRITICAL VALUES			
J. Z. CKITICAL VALUES			
$z_{\alpha/2} = P(z > \alpha) = \text{qnorm(1-alpha/2)}$	(62)		
$t_{\alpha/2} = P(t > \alpha) = qt (1-alpha/2, df)$	(63)		
$\chi_I^2 = P(\chi^2 < \alpha) = \text{qchisq(alpha/2, df)}$	(64)		
$\chi_p^2 = P(\chi^2 > \alpha) = \text{qchisq}(1-\text{alpha/2}, df)$	(65)		
5.3 REQUIRED SAMPLE SIZE			
(7 , 2			
proportion: $n = \hat{p}\hat{q} \left(\frac{z_{\alpha/2}}{E}\right)^2$ $(\hat{p} = \hat{q} = 0.5 \text{ if unknown})$	(66)		
1 2 /			
mean: $n = \left(\frac{z_{\alpha/2} \cdot \hat{\sigma}}{E}\right)^2$	(67)		
. /			