Introductory Statistics Lectures

Permutations and Combinations

Probability III

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1 Permutations and Combinations

1.1 Arranging items and factorials

Question 1. If you have 10 items, how many ways can you arrange them? (Think of a tree diagram.)

FACTORIAL NOTATION x!.

Definition 1.1

Denote product of decreasing whole numbers from x to 1:

$$x! = x \cdot (x-1) \cdot (x-2) \cdot (x-3) \cdots 3 \cdot 2 \cdot 1 \tag{1}$$

Note: 0! = 1

FACTORIAL RULE. DEFINITION 1.2

R COMMAND

n items can be arranged in n! ways.

FACTORIAL:

factorial(x)

Finds x!

(There is a limitation on how large x can be.^a)

^aThe factorial function cannot compute values beyond $x \approx 170$ due to how it's implemented using the gamma function. The lfactorial(x) function can do larger numbers, it returns $\ln(x!)$.

Example 1. To find 10! in R:

R: factorial(10) [1] 3628800

1.2 Permutations and Combinations

PERMUTATIONS

Question 2. How many ways can you select k=4 students out of n=10 when order matters?

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MAT167

PERMUTATIONS. DEFINITION 1.3

The number of ways (permutations) that you can select k items from n total items (all unique) when **order** matters is:

$${}_{n}P_{k} = \frac{n!}{(n-k)!} \tag{2}$$

COMBINATIONS

Question 3. How many ways can you select k = 4 students out of n = 10 when order does not matter? (Hint: how many ways can you arrange 4 items?)

Definition 1.4

Combinations.

The number of combinations (when order does not matter) of k items selected from n different items without replacement is:

$${}_{n}C_{k} = \binom{n}{k} = \frac{n!}{(n-k)!k!} \tag{3}$$

We will need this for binomial probabilities!

COMBINATIONS:

choose(n, k)

Computes number of combinations of k items chosen from a total of n items.

Example 2. To find $_{10}C_4$, the number of **combinations** of 4 students chosen from 10:

1.3 Summary

- n items can be arranged in n! ways.
- Number ways you can select k of n items:
 - Permutations: when order matters

$$_{n}P_{k} = \frac{n!}{(n-k)!}$$

- Combinations: when order is unimportant

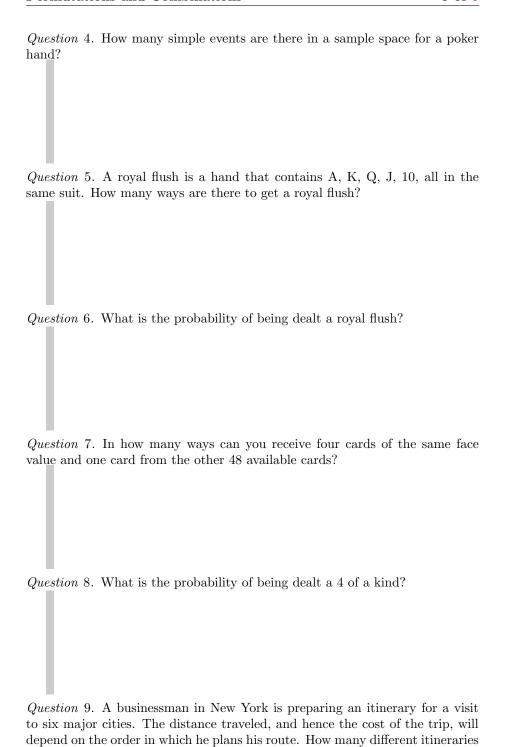
$$_{n}C_{k} = \binom{n}{k} = \frac{n!}{(n-k)!k!}$$

 $_{n}C_{k} = \text{choose(n,k)}$

1.4 Additional Examples

A Poker hand consists of 5 cards dealt from a deck of 52 cards. (A deck has 2-10, J, Q, K, A -13 difference valued cards, with 4 suits -4 of each face.)

R COMMAND



(and trip costs) are possible?

Question 10. In how many ways can you receive three cards of one face value and another two cards of another face value? (A full house)

Question 11. What is the probability of being dealt a full house?