

Minimax

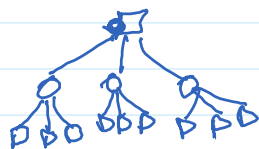


alpha-beta pruning :- technique to speed up minimax by cutting branches of the game tree

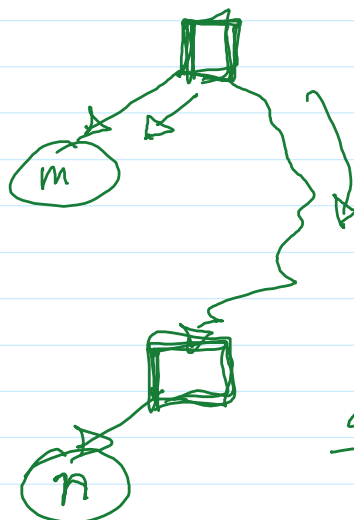
$$\text{minimax}(n) = \text{MAX} \left(\min(3, 4, 8), \min(2, ?, ?), \min(7, 3, ?) \right)$$

$$\quad \quad \quad (\quad 3 \quad , \quad \leq 2 \quad , \quad \leq 3)$$

$$= 3$$



Intuition



parameter α :- the value of the best choice along the path for MAX. (High value)
 parameter β :- the value of the best choice along the path for MIN. (Low value)

• The Alpha Beta Pruning Algorithm

```

PROCEDURE AlphaBetaSearch( s0 : state )
  v := maxValue( s0, -∞, +∞ )
  besta := action for which minimax value of result( s0, a ) equals v
  RETURN besta;
    
```

```

PROCEDURE maxValue( s : state,  $\alpha$  ,  $\beta$  )
  IF term( s ) THEN RETURN utility( s )
  v := - $\infty$ 
  FOREACH a in actions( s ) DO
    v := MAX( v, minValue( result( s, a ),  $\alpha$  ,  $\beta$  ) )
    if v >=  $\beta$  THEN RETURN v
   $\alpha$  := MAX( $\alpha$ , v)
  RETURN v

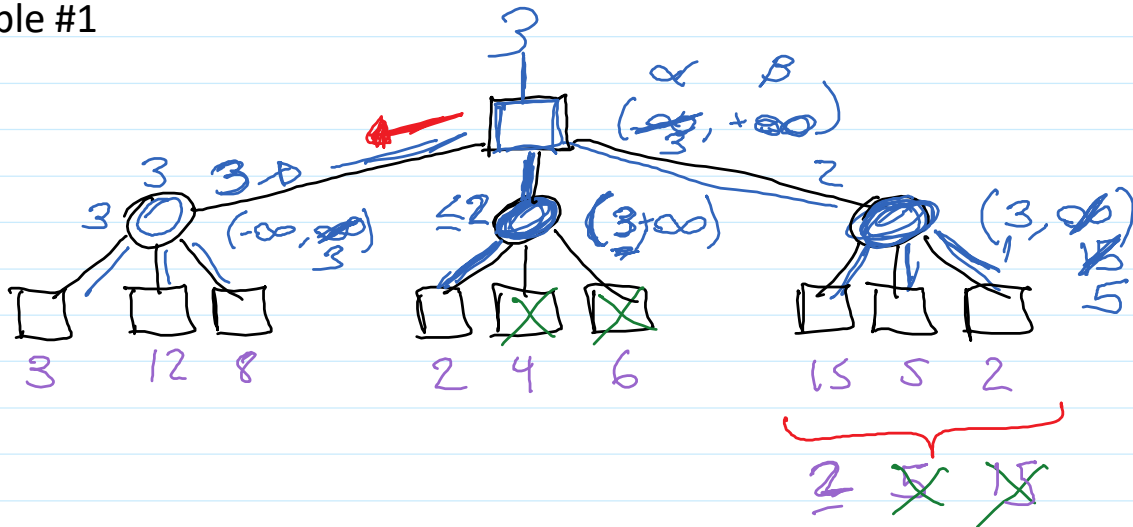
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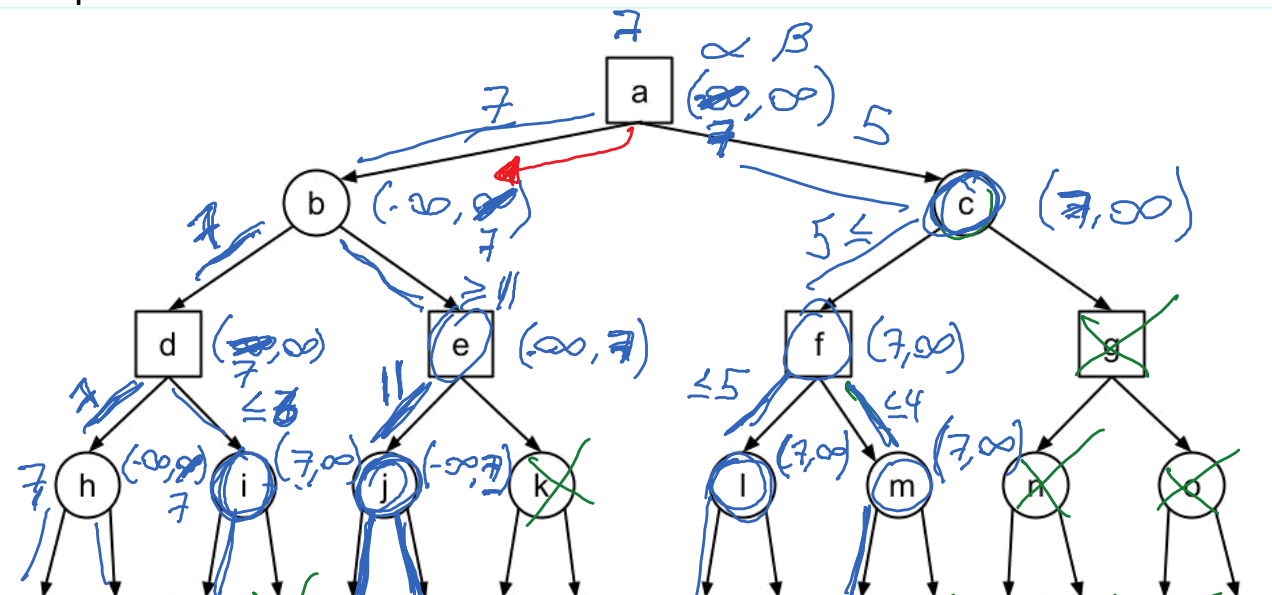
PROCEDURE minValue( s : state,  $\alpha$  ,  $\beta$  )
  IF term( s ) THEN RETURN utility( s )
  v :=  $\infty$ 
  FOREACH a in actions( s ) DO
    v := MIN( v, maxValue( result( s, a ),  $\alpha$  ,  $\beta$  ) )
    if v <=  $\alpha$  THEN RETURN v
   $\beta$  := MIN( $\beta$ , v)
  RETURN v

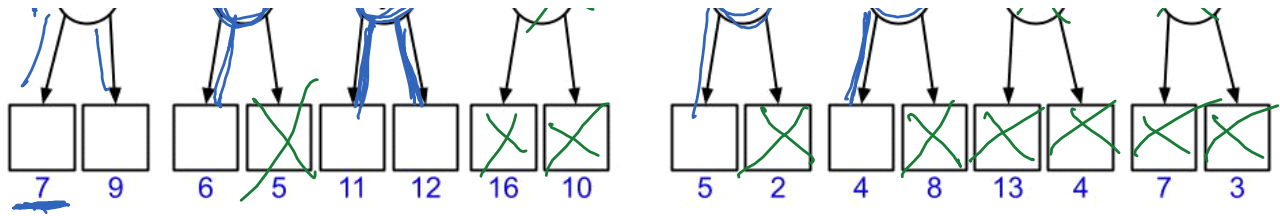
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• Example #1



• Example #2






• Move Ordering:

α - β - is sensitive to order of Moves.-

chess {
 - captures
 - threats
 - take position
 - backwards.

Minimax $O(b^m)$

m of  branching factor b

α - β $O(b^{m/2})$

\sqrt{b} best theoretical branching factor of α - β .

Chess $b=35$ $\sqrt{b} \approx 6$