$f(n) = n \cdot (f(n/2))^2$

$g(m) = f(2^m)$ $\quad \Rightarrow n = 2^m \Rightarrow m = \log_2(n)$

$h(m) = \log_2(g(m))$

- -

$h(m) = \log_2(g(m))$ // g ersetzen

$h(m) = \log_2(f(2^m))$ // f ersetzen

$h(m) = \log_2(2^m \cdot (f(\frac{2^m}{2}))^2)$ // Log Gesetz

$h(m) = \log_2(2^m) + \log_2(f(\frac{2^m}{2})^2)$ // Log Gesetz & Potenz gesetz

$h(m) = m + \log_2(f(2^{m-1})^2)$ // Binom

$h(m) = m + 2 \cdot \log_2(f(2^{m-1}))$ // f(n) ersetzen

$h(m) = m + 2 \cdot \log_2(g(m-1))$ // g(n) ersetzen

① $\boxed{h(m) = m + 2 \cdot h(m-1)}$ // Umformen

$\underline{1 h(m) - 2 \cdot h(m-1) = m}$

$a_0 f(m) + a_1 f(m-1) + \ldots + a_c f(m-c) = \sum_{i=1}^{k} o_i^m p_i(m)$

$a_0 = 1 \quad a_1 = -2 \quad c = 1$ $\qquad a_i^m = 1 \Rightarrow o_i = 1$

$\qquad\qquad\qquad\qquad\qquad\qquad p_i(m) = m^1$

$\qquad\qquad\qquad\qquad\qquad\qquad k = 1$

$(a_0 x^c + a_1 x^{c-1} + \ldots + a_c) \prod_{i=1}^{k}(x-o_i)^{c_i+1}$

$(x^1 - 2x^0)(x-1)^{1+1}$

$(x-2)^1 (x-1)^2$ $\qquad \Rightarrow r_1 = 2 \quad M_1 = 1 \Big\} \; d = 2$

$\qquad\qquad\qquad\qquad \Rightarrow r_2 = 1 \quad M_2 = 2$

$f(m) = \sum_{i=1}^{d} \sum_{j=1}^{M_i} b_{(i,j)} \, m^{j-1} r_i^m$

$h(m) = b_{(1,1)} \, m^0 \cdot 2^m + b_{(2,1)} \, m^0 \cdot 1^m + b_{(2,2)} \, m^1 \cdot 1^m$

$h(m) = b_{(1,1)} \, 2^m \qquad + b_{(2,1)} \, 1 \qquad + b_{(2,2)} \, m$

② $\Rightarrow h(m) = A \cdot 2^m \qquad + B \cdot 1 \qquad + C \cdot m$

② = ① // h(m-1) ersetzen

$\Rightarrow A \cdot 2^m + B + C m = m + 2 h(m-1)$

$A \cdot 2^m + B + C m = m + 2(A \cdot 2^{m-1} + B + C \cdot (m-1))$

$A \cdot 2^m + B + C m = m + 2A \cdot 2^{m-1} + 2B + 2C \cdot (m-1)$

$$A \cdot 2^m + B + C \cdot m = m + 2A2^{m-1} + 2B + 2Cm - 2c \qquad // -Cm$$

$$A \cdot 2^m + B = m + A2^m + 2B + Cm - 2c \qquad // -B$$

$$\cancel{A2^m} = m + \cancel{A2^m} + B + Cm - 2c$$

$$\Rightarrow \quad 0 = m + B + Cm - 2c$$

Raten von $C \Rightarrow \boxed{C = -1}$ (So dass m wegfällt)

$$\Rightarrow \quad 0 = m + B - m + 2$$

$$\Rightarrow \quad \boxed{-2 = B}$$

Einsetzen von B & C

$$\Rightarrow \quad h(m) = A \cdot 2^m - 2 - m \qquad // h(m) \rightarrow g(m)$$

$$g(m) = 2^{A \cdot 2^m - 2 - m} \qquad // g(m) \rightarrow f(n) \qquad n = 2^m \quad m = \log_2(n)$$

$$f(n) = 2^{A \cdot n - 2 - \log_2(n)}$$

$$f(n) = 2^{A \cdot n - 2} \cdot 2^{-\log_2(n)} \qquad // \text{Log Gesetz}$$

$$f(n) = 2^{An - 2} \cdot \frac{1}{n}$$

$$f(n) = 2^{An} \cdot \frac{1}{2^2} \cdot \frac{1}{n}$$

$$f(n) = 2^{An} \cdot \frac{1}{4n}$$

$$\boxed{f(n) = \frac{2^{An}}{4n}}$$