$$g(x) := e_a(x,t)e_b(x,t) = e^{ax-\frac{a^2t}{2}}e^{ax-\frac{b^2t}{2}} = e^{x(a+b)-\frac{t}{2}(a^2+b^2)} \tag{1}$$

$$\begin{aligned}
E(g(X)) &= \int_{-\infty}^{\infty} g(x)f_X(x)dx = \\
&= \int_{-\infty}^{\infty} e^{-\frac{t}{2}(a^2+b^2)}e^{x(a+b)}\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}dx \\
&= e^{-\frac{t}{2}(a^2+b^2)}\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}+x(a+b)}dx \\
&= e^{-\frac{t}{2}(a^2+b^2)}\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(x^2-2x(a+b))}dx \\
&= e^{-\frac{t}{2}(a^2+b^2)}\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}((x-(a+b))^2-(a+b)^2)}dx \\
&= e^{-\frac{t}{2}(a^2+b^2)}e^{\frac{1}{2}(a+b)^2}\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(x-(a+b))^2}dx \\
&= e^{\frac{1}{2}((a+b)^2-t(a^2+b^2))}\int_{-\infty}^{\infty}f_Z(x)dx \quad (Z\sim N(a+b,1)) \\
&= e^{\frac{1}{2}((a+b)^2-t(a^2+b^2))}
\end{aligned}$$

$$e^{ax-\frac{a^2t}{2}} = e^{a(x-\frac{at}{2})} = \sum_{n=0}^{\infty} \frac{a^n}{n!}(x - \frac{at}{2})^n = \sum_{n=0}^{\infty} \frac{a^n}{n!} h_n(x,t)$$

$$h_n(x,t) = (x - \frac{at}{2})^n$$

$$g(x) := h_n(x,t)h_n(x,t) = (x - \frac{at}{2})^{n+m} = \sum_{i=0}^{k} \binom{n+m}{i} x^i (-\frac{at}{2})^{n+m-i}$$

$$\begin{aligned}
E(g(X)) &= \int_{-\infty}^{\infty} g(x) f_X(x) dx \quad (k = m+n) \\
&= \int_{-\infty}^{\infty} \sum_{i=0}^{k} \binom{k}{i} x^i (-\frac{at}{2})^{k-i} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx \\
&= \sum_{i=0}^{k} \binom{k}{i} (-\frac{at}{2})^{k-i} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x^i e^{-\frac{x^2}{2}} dx \\
&= \sum_{i=0}^{k} \binom{k}{i} (-\frac{at}{2})^{k-i} \frac{2}{\sqrt{2\pi}} \int_{0}^{\infty} 2^{\frac{i}{2}} u^{\frac{i}{2}} e^{-u} \frac{du}{\sqrt{2u}}
\end{aligned}$$

tu smo uvedli

$$u = \frac{x^2}{2}$$

in upostevali, da je integral konvergenten in lih za lihe clene i, in zato enak 0, torej od sedaj naprej gledamo le sode i

$$\begin{aligned}
&= \sum_{i=0}^{k} \binom{k}{i} (-at)^{k-i} 2^{i-k+\frac{1}{2}} \frac{1}{\sqrt{\pi}} 2^{\frac{i-1}{2}} \int_{0}^{\infty} u^{\frac{i-1}{2}} e^{-u} du \\
&= \sum_{i=0}^{k} \binom{k}{i} (-at)^{k-i} 2^{\frac{3i}{2}-k} \frac{1}{\sqrt{\pi}} \Gamma(\frac{i+1}{2}) \\
&= \sum_{i=0}^{k} \binom{k}{i} (-at)^{k-i} 2^{i-k+\frac{1}{2}} (i-1)!!
\end{aligned}$$

$$\begin{aligned}
&\Gamma(\frac{i+1}{2}) = \frac{i-1}{2}\Gamma(\frac{i-1}{2}) = \frac{(i-1)(i-3)}{2^2}\Gamma(\frac{i-3}{2}) = \dots \\
&= \frac{(i-1)!!}{2^{\frac{i-1}{2}}}\Gamma(\frac{1}{2}) = 2^{\frac{i-1}{2}}(i-1)!!\sqrt{\pi}
\end{aligned} \tag{2}$$

$$
\begin{aligned}
&G(x,y) = (x, e^{a(x+cy)}) = (u,v)\\
&G^{-1}(u,v) = (u, \frac{ln(v)-ua}{ac}) = (x,y)\\
\\
&f_{(e^{a(X+cY)}/X)}(y/x) = \frac{(f_{X,e^{a(X+cY)})}(x,y)}{f_X(x)}\\
&f_{(X,e^{a(X+cY)})}(u,v) = \frac{f_{(X,Y)}(u, \frac{ln(v)-ua}{ac})}{|det(J_{(x,y)}(u, \frac{ln(v)-ua}{ac}))|}
\end{aligned}
\tag{3}
$$