Derive the steps that lead to the physically realistic equation of updating the u.
(i.e. u_new = u_old + (dt)(sqrt(2gh)/mag(dp/du))

assume: h_{max} is the maximum heigh of the track, h is the current height of the roller coaster.

g is the gravity constant

Δt is the time step

$\boxed{p \text{ is a function of } u \text{ that computes the position of the roller coaster at } u = u_{current}}$

$\frac{dp}{du}$ is the derivative of $p(u)$

$mag = \sqrt{x^2+y^2+z^2}$ (the size of vector (x,y,z))

$\left\|\frac{dp}{du}\right\|$ is the magnitude of the vector $\frac{dp}{du}$

$S = \frac{1}{2}at^2$, $a = \frac{v}{t} = \frac{\Delta h}{\Delta t}$ #

First, $H = \frac{1}{2}gt^2$ and $H = h_{max} - h$, t is the duration of falling to end

Second, derive both sides by t, $\frac{dH}{dt} = \frac{1}{2}g \times 2t \Rightarrow \frac{dH}{dt} = gt \Rightarrow dH = dt \times gt$

Third, add the velocity, $v = gt$ —— ①

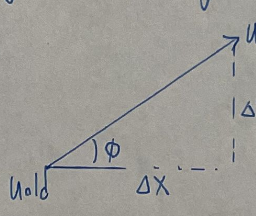
Fourth, $H = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2H}{g}}$ —— ②

Fifth, put t ② into ①, $v = g\sqrt{\frac{2H}{g}} = \sqrt{2gH}$ —— ③

Sixth, $dH = dt \times gt$, and ① = ③ = v, $dH = dt \times \sqrt{2gH}$

Seventh, $p(u)$ is the position of roller coaster, $\Delta u = u_{new} - u_{old}$, $p(\Delta u)$ is the distance between u

Eighth, use tangent vector to build the relation.



$\tan\phi = \frac{\Delta Y}{\Delta X}$

Ninth, $p(\Delta u) = p(u_{new}) - p(u_{old})$, $\left\| \text{vector } \frac{dp}{du} \right\| = \text{hypotenuse} = |\vec{u}|$

$v = \frac{dx}{dt}$ is the derivative of x $\Rightarrow$ $\frac{dH}{dt} = v = gt = \sqrt{2gH}$

$a = g = \frac{dv}{dt}$ is the derivative of v

Tenth, $\frac{d(\Delta u)}{dt} = v = \sqrt{2gH} \Rightarrow d(\Delta u) = dt \cdot \sqrt{2gH} = v \cdot dt$

Eleventh, Integral of $d(\Delta u)$ to get rid of d

$d(\Delta u) = v\,dt$, integrating , $\int d(\Delta u) = \int v\,dt \Rightarrow \Delta u = \int v\,dt = tv + C = \Delta t v + C$ (C is constant)

Twelfth, $p(u)$ computes the position of roller coaster u

$p'(u) = \frac{dp}{du}$ = velocity $\Rightarrow$ $\frac{v}{|\vec{v}|}$ = unit vector (so it will increase linearly)

Thus, according to 11th,

$\Delta u = \Delta t\, v \Rightarrow \Delta u = \Delta t \frac{v}{|\vec{v}|}$ and then put into $v = \sqrt{2gH}$, $|\vec{v}| = \text{mag}\left(\frac{dp}{du}\right)$, $H = h_{max} - h$

$\Delta u = u_{new} - u_{old}$

We can finally derive the formula

$$\Delta u = \Delta t \frac{\sqrt{2gH}}{\left\|\frac{dp}{du}\right\|} \Rightarrow u_{new} - u_{old} = \Delta t \frac{\sqrt{2g(h_{max}-h)}}{\left\|\frac{dp}{du}\right\|}$$

$$\Rightarrow u_{new} = u_{old} + \Delta t \frac{\sqrt{2g(h_{max}-h)}}{\left\|\frac{dp}{du}\right\|}$$

Also as $\Rightarrow u_{new} = u_{current} + \Delta t \frac{\sqrt{2g(h_{max}-h)}}{\left\|\frac{dp}{du}\right\|}$ #