# Supplementary Material of “An Efficient Dynamic Resource Allocation Framework for Evolutionary Bilevel Optimization”

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This is the supplementary material of the paper “An Efficient Dynamic Resource Allocation Framework for Evolutionary Bilevel Optimization”, and this document includes five sections. Section A provides the scalability test for the compared algorithms. Section B details the results of comparison study in terms of the number of total function evaluations (FEs). Section C presents the ablation study for different components of the proposed algorithm. Section D discusses the weights in selection probability update. Finally, Section E analyzes the comparison study on TP test problems.

## A. Comparison Study on Scalability

To investigate the scalability of DRC-CMA-ES, the experiments on dimension ($m$ = 10, $n$ = 10) and ($m$ = 30, $n$ = 30) are carried out and the statistical results in terms of accuracy are summarized in Tables S-I and S-II. As the dimension increases, the search space expands dramatically. It can be observed from the comparison between Tables II (in the manuscript) and the two tables that the accuracy of all algorithms deteriorates to different degrees, but DRC-CMA-ES still obtains the best results on most instances.

TABLE S-I
MEDIAN AND IQR RESULTS OF ACCURACY ($Acc_u$ AND $Acc_l$) ON SMD PROBLEMS WITH DIMENSION ($m$ = 10, $n$ = 10)

| ($m$ = 10, $n$ = 10) | | DRC-CMA-ES (ours) | BL-CMA-ES | TLEA-CMA-ES | NBLEA | BLEAQ-II |
|---|---|---|---|---|---|---|
| SMD1 | $Acc_u$ | 1.00E-06(0.00E+00) | 1.00E-06(0.00E+00)≈ | 1.00E-06(0.00E+00)≈ | 8.27E+01(3.85E+01)+ | 5.92E-03(9.91E-03)+ |
| | $Acc_l$ | 1.00E-06(0.00E+00) | 1.00E-06(0.00E+00)≈ | 1.00E-06(0.00E+00)≈ | 4.27E+01(2.01E+01)+ | 2.09E-03(2.68E-03)+ |
| SMD2 | $Acc_u$ | 1.00E-06(0.00E+00) | 1.00E-06(0.00E+00)≈ | 1.00E-06(0.00E+00)≈ | 4.58E+01(1.77E+01)+ | 5.58E-03(9.21E-03)+ |
| | $Acc_l$ | 1.52E-06(2.23E-06) | 1.34E-06(6.51E-07)≈ | 1.24E-06(1.02E-06)≈ | 3.05E+01(1.91E+01)+ | 6.78E-03(1.69E-02)+ |
| SMD3 | $Acc_u$ | 1.00E-06(0.00E+00) | 1.00E-06(0.00E+00)≈ | 1.00E-06(0.00E+00)≈ | 7.84E+01(2.95E+01)+ | 7.43E-03(9.10E-01)+ |
| | $Acc_l$ | 1.00E-06(2.63E-07) | 1.04E-06(3.08E-07)≈ | 1.01E-06(1.18E-07)≈ | 4.55E+01(3.08E+01)+ | 2.56E-03(9.55E-01)+ |
| SMD4 | $Acc_u$ | 1.00E-06(0.00E+00) | 1.00E-06(0.00E+00)≈ | 1.00E-06(0.00E+00)≈ | 1.46E+01(1.32E+01)+ | 1.46E-02(1.37E-01)+ |
| | $Acc_l$ | 1.06E-05(3.05E-04) | 1.76E-05(1.60E-05)+ | 1.64E-05(2.36E-05)+ | 1.29E+01(1.23E+01)+ | 4.04E-02(1.44E-01)+ |
| SMD5 | $Acc_u$ | 1.00E-06(0.00E+00) | 1.00E-06(0.00E+00)≈ | 1.00E-06(0.00E+00)≈ | 8.02E+01(3.38E+01)+ | 5.62E-03(2.15E-01)+ |
| | $Acc_l$ | 1.28E-05(1.87E-05) | 9.94E-06(8.94E-06)≈ | 7.67E-06(2.07E-05)≈ | 3.42E+01(3.41E+01)+ | 4.09E-03(5.13E-01)+ |
| SMD6 | $Acc_u$ | 1.00E-06(0.00E+00) | 1.00E-06(0.00E+00)≈ | 1.00E-06(0.00E+00)≈ | 4.67E-01(1.18E+00)+ | 1.00E-06(0.00E+00)≈ |
| | $Acc_l$ | 1.00E-06(0.00E+00) | 1.00E-06(0.00E+00)≈ | 1.00E-06(0.00E+00)≈ | 8.85E-02(9.36E-02)+ | 1.00E-06(0.00E+00)≈ |
| SMD7 | $Acc_u$ | 1.93E-01(2.19E-01) | 2.67E-01(2.92E-01)+ | 2.44E-01(1.69E-01)+ | 4.12E+00(4.21E+00)+ | 2.90E-01(3.01E-01)+ |
| | $Acc_l$ | 3.07E+02(4.79E+02) | 5.84E+02(6.40E+02)+ | 3.80E+02(4.42E+02)+ | 3.37E+02(5.17E+02)+ | 2.87E+02(5.24E+02)≈ |
| SMD8 | $Acc_u$ | 1.02E-01(9.75E-02) | 2.36E-03(1.05E-03)- | 2.69E-03(3.25E-03)- | 3.28E+01(6.36E+00)+ | 2.51E+00(6.20E+00)+ |
| | $Acc_l$ | 5.89E-02(6.12E-02) | 1.19E-03(2.05E-03)- | 1.39E-03(3.44E-03)- | 2.20E+01(1.18E+01)+ | 3.96E+00(7.12E+00)+ |
| SMD9 | $Acc_u$ | 3.69E-04(9.02E-01) | 2.46E-01(4.57E-01)+ | 2.63E-01(3.74E-01)+ | 5.58E+01(2.17E+01)+ | 4.55E+00(3.54E+00)+ |
| | $Acc_l$ | 3.31E-04(3.49E-01) | 4.49E-02(2.03E-01)+ | 6.66E-02(1.61E-01)+ | 3.44E+01(1.88E+01)+ | 7.90E+00(2.35E+01)+ |
| SMD10 | $Acc_u$ | 1.42E+01(2.36E+00) | 1.59E+01(1.18E+00)≈ | 1.53E+01(2.47E+00)≈ | 8.71E+01(3.72E+01)+ | 3.08E+01(2.91E+01)+ |
| | $Acc_l$ | 1.88E+01(2.42E+00) | 1.82E+01(1.50E+00)≈ | 1.81E+01(1.65E+00)≈ | 6.52E+01(5.93E+01)+ | 2.73E+01(1.21E+02)+ |
| SMD11 | $Acc_u$ | 9.82E-02(6.38E+01) | 1.14E+02(6.16E+01)+ | 3.23E-02(8.83E+01)≈ | 1.85E+02(1.43E+02)+ | 2.54E+02(8.96E+01)+ |
| | $Acc_l$ | 3.15E-02(5.97E+01) | 1.10E+02(6.15E+01)+ | 9.21E-03(8.45E+01)≈ | 1.82E+02(1.41E+02)+ | 2.49E+02(9.04E+01)+ |
| SMD12 | $Acc_u$ | 1.51E+01(2.08E+00) | 1.41E+01(2.15E+00)≈ | 1.41E+01(1.00E+00)≈ | 1.00E+02(3.58E+01)+ | 2.05E+01(5.95E+00)+ |
| | $Acc_l$ | 2.11E+01(1.67E+00) | 1.96E+01(1.79E+00)- | 1.99E+01(8.99E-01)- | 6.99E+01(4.60E+01)+ | 2.69E+01(3.94E+00)+ |
| +/-/≈ ($Acc_u$ & $Acc_l$) | | | 3/1/8 & 4/2/6 | 2/1/9 & 3/2/7 | 12/0/0 & 12/0/0 | 11/0/1 & 10/0/2 |

## B. Results of Comparison Study in Terms of the Number of Total Function Evaluations

The median results of the number of total FEs presented by all compared algorithms in 21 runs on different scales are summarized in Table S-III. This table provides the results on all test instances and complements Table IV in the manuscript.

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TABLE S-II

MEDIAN AND IQR RESULTS OF ACCURACY ($Acc_u$ AND $Acc_l$) ON SMD PROBLEMS WITH DIMENSION ($m$ = 30, $n$ = 30)

| ($m$ = 30, $n$ = 30) | | DRC-CMA-ES (ours) | BL-CMA-ES | TLEA-CMA-ES | NBLEA | BLEAQ-II |
|---|---|---|---|---|---|---|
| SMD1 | $Acc_u$ | 1.00E-06(0.00E+00) | 1.00E-06(0.00E+00)≈ | 1.00E-06(0.00E+00)≈ | 1.67E+02(7.36E+01)+ | 1.88E-01(1.82E-01)+ |
| | $Acc_l$ | 1.00E-06(0.00E+00) | 1.00E-06(0.00E+00)≈ | 1.00E-06(0.00E+00)≈ | 1.01E+02(3.59E+01)+ | 7.63E-02(8.18E-02)+ |
| SMD2 | $Acc_u$ | 1.00E-06(0.00E+00) | 1.00E-06(0.00E+00)≈ | 1.00E-06(0.00E+00)≈ | 2.00E+02(8.02E+01)+ | 8.79E+00(1.25E+01)+ |
| | $Acc_l$ | 5.07E-06(4.96E-06) | 5.49E-06(9.97E-06)+ | 6.28E-06(4.65E-06)+ | 1.36E+02(1.13E+02)+ | 3.15E+01(1.06E+01)+ |
| SMD3 | $Acc_u$ | 1.00E-06(0.00E+00) | 1.00E-06(0.00E+00)≈ | 1.00E-06(0.00E+00)≈ | 2.15E+02(6.06E+01)+ | 1.56E+01(6.77E+00)+ |
| | $Acc_l$ | 1.75E-06(3.59E-07) | 1.96E-06(3.21E-07)≈ | 1.82E-06(5.20E-07)≈ | 1.30E+02(4.35E+01)+ | 1.24E+01(5.71E+00)+ |
| SMD4 | $Acc_u$ | 1.00E-06(8.87E-01) | 1.00E-06(9.01E-01)≈ | 1.00E-06(0.00E+00)≈ | 1.72E+02(5.10E+01)+ | 1.14E-01(3.86E-01)+ |
| | $Acc_l$ | 2.98E-04(9.62E-01) | 3.14E-04(9.53E-01)≈ | 3.20E-04(2.32E-02)+ | 1.75E+02(5.00E+01)+ | 1.31E-01(8.72E-01)+ |
| SMD5 | $Acc_u$ | 6.13E-05(1.55E-04) | 1.00E-06(0.00E+00)- | 1.00E-06(0.00E+00)- | 2.06E+02(8.36E+01)+ | 3.27E+01(2.49E+01)+ |
| | $Acc_l$ | 5.42E-04(4.58E-04) | 1.91E-04(8.02E-04)- | 3.88E-04(8.40E-04)≈ | 4.79E+02(6.17E+02)+ | 3.45E+01(2.48E+01)+ |
| SMD6 | $Acc_u$ | 1.00E-06(4.29E-07) | 1.00E-06(0.00E+00)≈ | 1.00E-06(0.00E+00)≈ | 8.90E+01(3.17E+01)+ | 5.74E+00(3.00E+01)+ |
| | $Acc_l$ | 1.52E-06(6.40E-07) | 1.45E-06(6.94E-07)≈ | 1.34E-06(7.61E-07)≈ | 3.49E+01(1.84E+01)+ | 7.23E-03(3.99E-01)+ |
| SMD7 | $Acc_u$ | 1.11E+00(1.28E-01) | 3.45E-06(1.31E-05)- | 1.25E-05(9.75E-02)- | 1.55E+01(1.11E+01)+ | 3.14E+00(1.05E+01)+ |
| | $Acc_l$ | 1.19E+02(1.24E+02) | 2.17E-06(5.59E-06)- | 4.26E-06(1.16E+02)- | 2.26E+03(1.83E+03)+ | 3.77E+03(2.25E+03)+ |
| SMD8 | $Acc_u$ | 8.68E+00(4.07E+00) | 1.03E+01(3.30E+00)+ | 9.84E+00(1.60E+00)+ | 1.87E+03(6.86E+02)+ | 9.81E+00(1.36E+01)≈ |
| | $Acc_l$ | 3.27E+01(2.33E+01) | 4.29E+01(2.13E+01)+ | 4.10E+01(1.60E+01)+ | 2.38E+03(7.45E+02)+ | 4.04E+02(1.95E+02)+ |
| SMD9 | $Acc_u$ | 4.51E-01(3.42E-01) | 8.51E-01(7.53E-01)+ | 8.19E-01(5.07E-01)+ | 1.53E+02(8.77E+01)+ | 1.67E+01(5.72E+00)+ |
| | $Acc_l$ | 1.25E+00(5.00E-01) | 8.51E-01(1.11E+00)≈ | 7.48E-01(9.97E-01)- | 2.44E+02(1.05E+02)+ | 3.15E+01(4.78E+01)+ |
| SMD10 | $Acc_u$ | 3.79E+00(4.86E+00) | 4.40E+00(4.21E-01)≈ | 1.99E+00(3.37E+00)- | 5.00E+09(1.00E+10)+ | 7.85E+01(4.96E+01)+ |
| | $Acc_l$ | 2.39E+01(2.19E+00) | 2.61E+01(5.00E+00)≈ | 2.43E+01(4.58E+00)≈ | 5.00E+09(1.00E+10)+ | 2.11E+02(1.21E+02)+ |
| SMD11 | $Acc_u$ | 2.31E+02(9.59E+01) | 3.13E+02(1.30E+02)+ | 1.99E+02(1.19E+02)≈ | 8.18E+02(2.15E+02)+ | 5.43E+02(7.25E+02)+ |
| | $Acc_l$ | 2.18E+02(9.47E+01) | 3.01E+02(1.30E+02)+ | 1.87E+02(1.20E+02)≈ | 8.06E+02(2.16E+02)+ | 5.38E+02(7.24E+02)+ |
| SMD12 | $Acc_u$ | 2.47E+01(1.11E+01) | 7.29E+00(9.06E+00)- | 6.67E+00(5.09E+00)- | 2.62E+02(5.64E+01)+ | 1.57E+02(8.64E+01)+ |
| | $Acc_l$ | 3.01E+01(8.95E+00) | 2.60E+01(4.06E+00)≈ | 2.75E+01(5.01E+00)≈ | 1.70E+02(7.09E+01)+ | 1.46E+02(4.88E+01)+ |
| +/-/≈ ($Acc_u$ & $Acc_l$) | | | 3/3/6 & 3/2/7 | 2/4/6 & 3/2/7 | 12/0/0 & 12/0/0 | 11/0/1 & 12/0/0 |

TABLE S-III

MEDIAN RESULTS OF THE NUMBER OF TOTAL $FEs$ ON SMD PROBLEMS ON DIFFERENT SCALES

| Prob | Dimension | DRC-CMA-ES (ours) | BL-CMA-ES | TLEA-CMA-ES | NBLEA | BLEAQ-II |
|---|---|---|---|---|---|---|
| SMD1 | ($m$ = 2, $n$ =3) | 1.46E+04 | 2.07E+04(29.8%) | 2.07E+04(29.7%) | 5.23E+05(97.2%) | 5.44E+04(73.2%) |
| | ($m$ = 10, $n$ =10) | 9.95E+04 | 2.05E+05(51.6%) | 1.88E+05(47.2%) | 7.96E+05(87.5%) | 6.57E+05(84.8%) |
| | ($m$ = 30, $n$ =30) | 3.29E+05 | 6.42E+05(48.7%) | 6.49E+05(49.3%) | 3.16E+06(89.6%) | 5.61E+06(94.1%) |
| SMD2 | ($m$ = 2, $n$ =3) | 1.36E+04 | 1.99E+04(31.7%) | 2.03E+04(33.0%) | 6.20E+05(97.8%) | 7.37E+04(81.6%) |
| | ($m$ = 10, $n$ =10) | 9.86E+04 | 2.01E+05(50.9%) | 1.84E+05(46.4%) | 7.39E+05(86.7%) | 6.86E+05(85.6%) |
| | ($m$ = 30, $n$ =30) | 3.19E+05 | 6.54E+05(51.2%) | 6.52E+05(51.0%) | 1.90E+06(83.2%) | 3.38E+06(90.6%) |
| SMD3 | ($m$ = 2, $n$ =3) | 1.52E+04 | 2.16E+04(29.7%) | 2.10E+04(27.9%) | 5.82E+05(97.4%) | 6.27E+04(75.8%) |
| | ($m$ = 10, $n$ =10) | 1.02E+05 | 1.68E+05(39.7%) | 1.73E+05(41.2%) | 8.76E+05(88.4%) | 8.59E+05(88.2%) |
| | ($m$ = 30, $n$ =30) | 3.40E+05 | 5.22E+05(34.9%) | 5.40E+05(37.0%) | 3.62E+06(90.6%) | 5.81E+06(94.1%) |
| SMD4 | ($m$ = 2, $n$ =3) | 1.56E+04 | 2.24E+04(30.1%) | 2.13E+04(26.8%) | 5.67E+05(97.2%) | 5.53E+04(71.7%) |
| | ($m$ = 10, $n$ =10) | 1.15E+05 | 1.96E+05(41.1%) | 1.95E+05(40.7%) | 8.18E+05(85.9%) | 2.45E+05(52.9%) |
| | ($m$ = 30, $n$ =30) | 4.22E+05 | 7.18E+05(41.2%) | 6.81E+05(38.0%) | 1.13E+06(62.6%) | 2.69E+06(84.3%) |
| SMD5 | ($m$ = 2, $n$ =3) | 1.56E+04 | 2.19E+04(28.7%) | 2.07E+04(24.5%) | 6.17E+05(97.5%) | 8.66E+04(81.9%) |
| | ($m$ = 10, $n$ =10) | 1.37E+05 | 2.37E+05(42.0%) | 2.25E+05(39.0%) | 8.19E+05(83.2%) | 6.07E+05(77.4%) |
| | ($m$ = 30, $n$ =30) | 4.38E+05 | 7.89E+05(44.5%) | 7.74E+05(43.5%) | 1.68E+06(74.0%) | 2.98E+06(85.3%) |
| SMD6 | ($m$ = 2, $n$ =3) | 2.09E+04 | 2.44E+04(14.4%) | 2.38E+04(12.1%) | 3.49E+04(40.2%) | 3.45E+03(-83.5%) |
| | ($m$ = 10, $n$ =10) | 1.20E+05 | 2.30E+05(47.9%) | 2.09E+05(42.8%) | 5.06E+05(76.4%) | 5.49E+04(-54.0%) |
| | ($m$ = 30, $n$ =30) | 3.90E+05 | 7.45E+05(47.6%) | 7.49E+05(47.9%) | 2.76E+06(85.8%) | 2.45E+06(84.1%) |
| SMD7 | ($m$ = 2, $n$ =3) | 1.48E+04 | 2.39E+04(38.3%) | 2.65E+04(44.3%) | 5.48E+05(97.3%) | 8.97E+04(83.6%) |
| | ($m$ = 10, $n$ =10) | 1.26E+05 | 1.87E+05(32.7%) | 1.93E+05(34.7%) | 9.00E+05(86.0%) | 7.59E+05(83.4%) |
| | ($m$ = 30, $n$ =30) | 3.53E+05 | 1.47E+06(76.0%) | 1.47E+06(76.0%) | 1.60E+06(77.9%) | 3.08E+06(88.5%) |
| SMD8 | ($m$ = 2, $n$ =3) | 4.21E+04 | 6.35E+04(33.6%) | 6.54E+04(35.5%) | 8.11E+05(94.8%) | 1.26E+05(66.6%) |
| | ($m$ = 10, $n$ =10) | 1.55E+05 | 6.81E+05(77.3%) | 6.56E+05(76.4%) | 8.36E+05(81.5%) | 9.55E+05(83.8%) |
| | ($m$ = 30, $n$ =30) | 4.35E+05 | 1.41E+06(69.1%) | 1.46E+06(70.2%) | 1.96E+06(77.8%) | 3.25E+06(86.6%) |
| SMD9 | ($m$ = 2, $n$ =3) | 1.43E+04 | 2.01E+04(28.7%) | 2.04E+04(29.6%) | 1.92E+06(99.3%) | 1.24E+05(88.4%) |
| | ($m$ = 10, $n$ =10) | 1.43E+05 | 2.30E+05(38.0%) | 2.10E+05(31.9%) | 9.43E+05(84.9%) | 3.75E+05(61.9%) |
| | ($m$ = 30, $n$ =30) | 4.31E+05 | 1.43E+06(69.9%) | 1.64E+06(73.7%) | 1.95E+06(77.9%) | 3.45E+06(87.5%) |
| SMD10 | ($m$ = 2, $n$ =3) | 4.28E+04 | 6.98E+04(38.7%) | 7.46E+04(42.7%) | 1.58E+06(97.3%) | 1.06E+05(59.5%) |
| | ($m$ = 10, $n$ =10) | 1.59E+05 | 3.96E+05(59.8%) | 3.97E+05(59.9%) | 5.00E+06(96.8%) | 4.55E+05(65.0%) |
| | ($m$ = 30, $n$ =30) | 4.64E+05 | 1.09E+06(57.5%) | 1.10E+06(57.8%) | 8.08E+06(94.3%) | 3.26E+06(85.8%) |
| SMD11 | ($m$ = 2, $n$ =3) | 4.52E+04 | 1.29E+05(65.1%) | 1.25E+05(63.7%) | 1.31E+06(96.5%) | 1.78E+06(97.5%) |
| | ($m$ = 10, $n$ =10) | 1.42E+05 | 3.50E+05(59.3%) | 4.12E+05(65.4%) | 3.07E+06(95.4%) | 5.88E+05(75.7%) |
| | ($m$ = 30, $n$ =30) | 3.97E+05 | 1.04E+06(61.8%) | 1.05E+06(62.2%) | 5.51E+06(92.8%) | 3.55E+06(88.8%) |
| SMD12 | ($m$ = 2, $n$ =3) | 3.62E+04 | 4.79E+04(24.4%) | 4.54E+04(20.4%) | 1.69E+06(97.9%) | 2.25E+05(83.9%) |
| | ($m$ = 10, $n$ =10) | 1.56E+05 | 4.18E+05(62.6%) | 4.12E+05(62.1%) | 3.53E+06(95.6%) | 4.67E+05(66.5%) |
| | ($m$ = 30, $n$ =30) | 4.56E+05 | 1.18E+06(61.3%) | 1.19E+06(61.7%) | 1.05E+07(95.7%) | 3.99E+06(88.6%) |
| SMD-Avg | ($m$ = 2, $n$ =3) | 2.42E+04 | 4.05E+04(40.1%) | 4.04E+04(40.0%) | 9.00E+05(97.3%) | 2.32E+05(89.6%) |
| | ($m$ = 10, $n$ =10) | 1.29E+05 | 2.92E+05(55.6%) | 2.88E+05(55.0%) | 1.57E+06(91.7%) | 5.59E+05(76.8%) |
| | ($m$ = 30, $n$ =30) | 3.98E+05 | 9.74E+05(59.2%) | 9.96E+05(60.1%) | 3.65E+06(89.1%) | 3.63E+06(89.0%) |

The values in parentheses indicate the relative percentage reduction of $FEs$ that DRC-CMA-ES consumed, and SMD-Avg denotes the average value over all problems.

### C. Ablation Study

In the proposed algorithm, two strategies are of the essence: the dynamic resource competition mechanism and the knowledge cooperation mechanism between competing tasks. To investigate the effectiveness of these two components, ablation experiments are conducted and three variants are considered:

- DRC-CMA-ES-v1: the dynamic resource competition mechanism is deactivated. Instead of being selected based on competitiveness evaluation, one task is randomly selected each time to perform a lower-level optimization iteration.
- DRC-CMA-ES-v2: the knowledge cooperation mechanism between competing tasks is removed.
- DRC-CMA-ES-v3: both the resource competition and cooperation mechanism are removed from DRC-CMA-ES.

The comparison between DRC-CMA-ES and the above variants in terms of upper-level accuracy and the number of total FEs is presented in Table S-IV. The results of the Wilcoxon test indicate a significant difference between DRC-CMA-ES and its variants. As can be observed, DRC-CMA-ES-v1 is inferior to DRC-CMA-ES in both accuracy and number of evaluations, indicating that the selection probability update mechanism based on task performance can effectively prioritize resource to promising tasks. On the contrary, DRC-CMA-ES-v1 equalizes the selection probability, which consumes more FEs. Even worse, optimization can be misled by random task selection, resulting in poor accuracy.

As distinct from the resource competition mechanism, the effect of knowledge cooperation mechanism seems to be problem-specific. On simple problems such as SMD1-SMD3, removal of the mechanism leads to more resource consumption. On complex problems, DRC-CMA-ES-v2 executes less FEs, but the accuracy deteriorates appreciably. The results indicate that the knowledge cooperation can promote task convergence in the parallel framework on simple problems, while on complex problems, the main role of knowledge cooperation is to help tasks escape from the local optima and prevent them from premature convergence. Nevertheless, the two impacts may work simultaneously on specific problems, and the respective contribution can not be accurately assessed. By reason of removing two crucial components, the performance of the DRC-CMA-ES-v3 is further deteriorated compared to the previous two variants.

TABLE S-IV
MEDIAN RESULTS OF UPPER-LEVEL ACCURACY ($Acc_u$) AND THE NUMBER OF TOTAL $FEs$ ACHIEVED BY DRC-CMA-ES AND ITS VARIANTS REGARDING KEY COMPONENTS WITH DIMENSION ($m$ = 30, $n$ = 30)

| ($m$ = 30, $n$ = 30) | | DRC-CMA-ES | DRC-CMA-ES-v1 | DRC-CMA-ES-v2 | DRC-CMA-ES-v3 |
|---|---|---|---|---|---|
| SMD1 | $Acc_u$ | 1.00E-06 | 2.31E-05+ | 4.86E-03+ | 6.77E-02+ |
| | $FEs$ | 3.29E+05 | 4.16E+05+ | 3.59E+05+ | 3.60E+05+ |
| SMD2 | $Acc_u$ | 1.00E-06 | 2.30E-05+ | 8.37E-03+ | 4.60E-02+ |
| | $FEs$ | 3.19E+05 | 4.14E+05+ | 3.51E+05+ | 3.57E+05+ |
| SMD3 | $Acc_u$ | 1.00E-06 | 5.73E-06+ | 4.11E-05+ | 6.69E-03+ |
| | $FEs$ | 3.40E+05 | 4.48E+05+ | 4.05E+05+ | 4.03E+05+ |
| SMD4 | $Acc_u$ | 1.00E-06 | 5.07E-05+ | 1.65E-03+ | 5.20E-03+ |
| | $FEs$ | 4.22E+05 | 4.62E+05+ | 3.94E+05- | 3.95E+05- |
| SMD5 | $Acc_u$ | 6.13E-05 | 5.49E-04+ | 3.54E-03+ | 5.11E-02+ |
| | $FEs$ | 4.38E+05 | 4.64E+05+ | 3.95E+05- | 3.97E+05- |
| SMD6 | $Acc_u$ | 1.00E-06 | 1.53E-03+ | 9.25E-02+ | 1.47E+00+ |
| | $FEs$ | 3.90E+05 | 4.21E+05+ | 3.43E+05- | 3.43E+05- |
| SMD7 | $Acc_u$ | 1.11E+00 | 1.53E+00+ | 1.52E+00+ | 1.74E+00+ |
| | $FEs$ | 3.53E+05 | 3.74E+05+ | 3.06E+05- | 3.07E+05- |
| SMD8 | $Acc_u$ | 8.68E+00 | 1.13E+01+ | 1.05E+01+ | 1.08E+01+ |
| | $FEs$ | 4.35E+05 | 4.59E+05+ | 3.93E+05- | 3.99E+05- |
| SMD9 | $Acc_u$ | 4.51E-01 | 1.36E+01+ | 9.45E-01+ | 3.39E+01+ |
| | $FEs$ | 4.31E+05 | 4.71E+05+ | 3.92E+05- | 4.51E+05- |
| SMD10 | $Acc_u$ | 3.79E+00 | 8.04E+00+ | 4.63E+00+ | 8.80E+00+ |
| | $FEs$ | 4.64E+05 | 4.91E+05+ | 4.45E+05- | 4.46E+05- |
| SMD11 | $Acc_u$ | 2.31E+02 | 2.42E+02+ | 2.83E+02+ | 2.66E+02+ |
| | $FEs$ | 3.97E+05 | 4.18E+05+ | 3.58E+05- | 3.57E+05- |
| SMD12 | $Acc_u$ | 2.47E+01 | 5.01E+01+ | 4.25E+01+ | 7.44E+01+ |
| | $FEs$ | 4.56E+05 | 4.88E+05+ | 4.41E+05- | 4.41E+05- |

### D. Discussion on the Weights in Selection Probability Update

In the selection probability update process of the proposed algorithm, the three sub-probabilities are merged with weights of 0.1, 0.7 and 0.2, respectively. Among them, the base probability serves as a basic assurance for each task with a low weight, and the performance probability is considered to reflect the competitiveness of the tasks and is therefore assigned the highest weight. We observed various weight settings to investigate the rationality of such allocation. The weight of the base probability is fixed at 0.1, while the weight of the performance probability is set at 0.1, 0.3, 0.5 and 0.9 as variants. Variants with different weights were tested on the SMD problems with dimension ($m$ = 30, $n$ = 30), and the results of upper-level accuracy and total FEs are presented in Table S-V. Considering both indicators, DRC-CMA-ES achieves the best performance on most instances.

As the weight of the performance probability decreases, both accuracy and the number of FEs deteriorate noticeably. When the weight of the performance probability is set to 0.1, the algorithm consumed the most FEs but achieved the worst accuracy, which illustrates the dominant of performance probability in guiding resources to be allocated appropriately. When the weight of the performance probability is set to 0.9 (i.e., the potential probability $w_{pt}$ is removed), the algorithm consumes less FEs than DRC-CMA-ES does on some instances. However, on most instances, the accuracy achieved by DRC-CMA-ES-$w_{pf0.9}$ is inferior to that of DRC-CMA-ES. This indicates that although the potential probability may lead to a little more iterations, its resource allocation preference for task evolving potential can provide diversity preservation for the algorithm and help the algorithm achieve higher accuracy.

TABLE S-V

MEDIAN RESULTS OF UPPER-LEVEL ACCURACY ($Acc_u$) AND THE NUMBER OF TOTAL $FEs$ ACHIEVED BY DRC-CMA-ES AND ITS VARIANTS REGARDING THE PARAMETER $w_{pf}$ WITH DIMENSION ($m$ = 30, $n$ = 30)

| ($m$ = 30, $n$ = 30) | | DRC-CMA-ES | DRC-CMA-ES-$w_{pf0.1}$ | DRC-CMA-ES-$w_{pf0.3}$ | DRC-CMA-ES-$w_{pf0.5}$ | DRC-CMA-ES-$w_{pf0.9}$ |
|---|---|---|---|---|---|---|
| SMD1 | $Acc_u$ | 1.00E-06 | 4.51E-06+ | 1.00E-06≈ | 1.00E-06≈ | 1.00E-06≈ |
| | $FEs$ | 3.29E+05 | 4.08E+05+ | 3.75E+05+ | 3.45E+05+ | 3.15E+05≈ |
| SMD2 | $Acc_u$ | 1.00E-06 | 8.57E-06+ | 1.00E-06≈ | 1.00E-06≈ | 1.00E-06≈ |
| | $FEs$ | 3.19E+05 | 4.09E+05+ | 3.78E+05+ | 3.50E+05+ | 3.05E+05≈ |
| SMD3 | $Acc_u$ | 1.00E-06 | 2.15E-06+ | 1.00E-06≈ | 1.00E-06≈ | 1.00E-06≈ |
| | $FEs$ | 3.40E+05 | 4.41E+05+ | 3.92E+05+ | 3.60E+05+ | 3.25E+05- |
| SMD4 | $Acc_u$ | 1.00E-06 | 1.87E-05+ | 1.00E-06≈ | 8.84E-01+ | 8.91E-01+ |
| | $FEs$ | 4.22E+05 | 4.44E+05+ | 4.35E+05+ | 4.36E+05+ | 4.38E+05+ |
| SMD5 | $Acc_u$ | 6.13E-05 | 3.14E-04+ | 2.24E-04+ | 4.68E-05- | 8.67E-05+ |
| | $FEs$ | 4.38E+05 | 4.45E+05+ | 4.43E+05+ | 4.40E+05≈ | 4.33E+05≈ |
| SMD6 | $Acc_u$ | 1.00E-06 | 6.50E-04+ | 3.15E-05+ | 3.82E-06+ | 3.47E-06+ |
| | $FEs$ | 3.90E+05 | 4.09E+05+ | 4.02E+05+ | 3.96E+05+ | 3.71E+05- |
| SMD7 | $Acc_u$ | 1.11E+00 | 1.43E+00+ | 1.27E+00+ | 1.14E+00≈ | 1.31E+00+ |
| | $FEs$ | 3.53E+05 | 3.61E+05+ | 3.57E+05+ | 3.57E+05+ | 3.52E+05≈ |
| SMD8 | $Acc_u$ | 8.68E+00 | 1.10E+01+ | 9.68E+00+ | 9.82E+00+ | 8.70E+00≈ |
| | $FEs$ | 4.35E+05 | 4.40E+05+ | 4.41E+05+ | 4.38E+05+ | 4.33E+05≈ |
| SMD9 | $Acc_u$ | 4.51E-01 | 8.32E+00+ | 1.59E+00+ | 4.94E-01+ | 5.46E-01+ |
| | $FEs$ | 4.31E+05 | 4.61E+05+ | 4.22E+05≈ | 4.35E+05≈ | 4.31E+05≈ |
| SMD10 | $Acc_u$ | 3.79E+00 | 4.04E+00≈ | 3.00E+00≈ | 2.54E+00≈ | 2.68E+00≈ |
| | $FEs$ | 4.64E+05 | 4.69E+05+ | 4.66E+05+ | 4.65E+05≈ | 4.61E+05≈ |
| SMD11 | $Acc_u$ | 2.31E+02 | 2.73E+02+ | 2.68E+02+ | 2.70E+02+ | 2.44E+02+ |
| | $FEs$ | 3.97E+05 | 4.01E+05+ | 3.98E+05≈ | 3.97E+05≈ | 3.99E+05≈ |
| SMD12 | $Acc_u$ | 2.47E+01 | 4.91E+01+ | 5.00E+01+ | 3.56E+01+ | 2.54E+01≈ |
| | $FEs$ | 4.56E+05 | 4.64E+05+ | 4.61E+05+ | 4.58E+05+ | 4.54E+05≈ |

## *E. Comparison Study on TP Test Problems*

The results for accuracy and the number of FEs on the TP test suite are presented in Tables S-VI and S-VII. The TP instances are extracted from real-world problems, and most of the functions are linear or quadratic. The number of variables for these test instances is fixed.

The proposed DRC-CMA-ES also shows a competitive performance on the TP problems and achieves 12 best accuracy results out of 20, which is comparable to BLEAQ-II and slightly better than BL-CMA-ES and TLEA-CMA-ES. Compared with the results in the SMD problem set, the accuracy of BLEAQ-II is greatly improved. This is attributed to the implementation of quadratic approximation in BLEAQ-II, which is consistent with the properties of functions in TP problems, thus facilitating the search process.

Similar to the comparison of number of FEs on the SMD problems, DRC-CMA-ES consumes more FEs at the upper level and less FEs at the lower level compared to BL-CMA-ES and TLEA-CMA-ES. These extra upper-level FEs are strategically invoked to guide the allocation of resources at the lower level, thereby reducing the total FEs consumption. NBLEA appears to consume less FEs, but its accuracy in both upper- and lower-levels is significantly worse than other algorithms, so NBLEA is likely to fall into premature convergence. BLEAQ-II achieves an overwhelming advantage in the number of FEs on almost all instances, because the deterministic approximation allows the population to quickly approach the optimal region. The performance of BLEAQ-II on the TP problems illustrates that when the pre-set deterministic approximate model matches the problem property, the accuracy and efficiency of the algorithm can be effectively enhanced. However, approximation introduces errors. When the pre-set model does not match the problem property, or when the data sampled for approximation is biased, the search process may be misguided. For instance, BLEAQ-II achieved poor accuracy on TP2 and TP8.

It is worth noting that, for algorithms employing quadratic approximations that are consistent with the function properties, the TP instances are not considered as ”black-box” problems. The prior knowledge to some extent compromises the fairness of the experiments. Comparatively, DRC-CMA-ES demonstrates notable effectiveness when tackling unknown problems.

TABLE S-VI
MEDIAN AND IQR RESULTS OF ACCURACY ($Acc_u$ AND $Acc_l$) ON TP PROBLEMS

| TP instances | | DRC-CMA-ES (ours) | BL-CMA-ES | TLEA-CMA-ES | NBLEA | BLEAQ-II |
|---|---|---|---|---|---|---|
| TP1 | $Acc_u$ | 1.00E-06(9.62E-06) | 1.00E-06(7.98E-06)≈ | 1.00E-06(6.41E-06)≈ | 4.56E-01(3.53E-01)+ | 1.33E-06(2.61E-06)+ |
| | $Acc_l$ | 1.50E-05(4.71E-05) | 1.85E-05(4.66E-05)≈ | 1.34E-05(6.61E-05)≈ | 7.56E-01(6.74E-01)+ | 1.00E-06(0.00E+00)- |
| TP2 | $Acc_u$ | 5.22E-06(1.26E-03) | 1.00E-06(2.16E-04)- | 7.56E-06(2.96E-04)+ | 1.00E-06(1.02E-03)- | 1.00E-06(0.00E+00)- |
| | $Acc_l$ | 1.51E-03(1.18E-02) | 1.06E-03(2.45E+01)- | 1.94E-04(3.53E-03)- | 1.00E+02(1.00E+02)+ | 1.00E+02(4.01E-08)+ |
| TP3 | $Acc_u$ | 1.15E-02(1.08E-02) | 4.52E-02(4.80E-02)+ | 2.85E-02(2.23E-02)+ | 5.62E-03(2.56E-02)- | 5.74E-04(6.81E-05)- |
| | $Acc_l$ | 8.13E-04(3.23E-03) | 2.21E-02(7.02E-02)+ | 4.69E-03(2.34E-02)+ | 1.27E-03(5.52E-03)+ | 8.49E-05(7.03E-06)- |
| TP4 | $Acc_u$ | 1.00E-06(0.00E+00) | 1.00E-06(0.00E+00)≈ | 1.00E-06(0.00E+00)≈ | 2.76E-02(7.90E-02)+ | 1.03E-04(1.68E-02)+ |
| | $Acc_l$ | 1.00E-06(0.00E+00) | 1.00E-06(0.00E+00)≈ | 1.00E-06(0.00E+00)≈ | 5.33E-03(1.53E-02)+ | 2.04E-05(3.25E-03)+ |
| TP5 | $Acc_u$ | 3.65E-01(1.12E+00) | 1.10E+00(1.12E+00)+ | 4.21E-01(1.12E+00)+ | 3.70E-04(6.41E-04)- | 1.39E-04(1.04E-03)- |
| | $Acc_l$ | 6.17E-01(4.92E+00) | 3.35E+00(5.62E+00)+ | 1.05E+00(5.70E+00)+ | 1.85E-03(2.05E-03)- | 4.60E-04(4.44E-03)- |
| TP6 | $Acc_u$ | 1.00E-06(0.00E+00) | 1.00E-06(0.00E+00)≈ | 1.00E-06(0.00E+00)≈ | 3.80E-06(9.34E-06)+ | 6.59E-06(6.52E-08)+ |
| | $Acc_l$ | 2.10E-05(5.30E-06) | 2.25E-05(9.35E-06)≈ | 2.51E-05(8.20E-06)+ | 2.12E-05(5.15E-05)≈ | 4.13E-06(2.44E-07)- |
| TP7 | $Acc_u$ | 6.38E-04(1.88E-05) | 6.72E-04(2.31E-05)≈ | 6.75E-04(2.26E-05)≈ | 1.30E-03(7.14E-04)+ | 6.76E-04(5.45E-05)≈ |
| | $Acc_l$ | 6.38E-04(1.88E-05) | 6.72E-04(2.31E-05)≈ | 6.75E-04(2.26E-05)≈ | 1.30E-03(7.14E-04)+ | 6.76E-04(5.45E-05)≈ |
| TP8 | $Acc_u$ | 1.00E-06(9.86E-05) | 1.70E-06(6.59E-05)+ | 1.00E-06(1.23E-05)≈ | 1.00E-06(3.61E-05)≈ | 1.00E-06(5.00E+00)≈ |
| | $Acc_l$ | 5.06E-05(1.01E-04) | 1.00E-04(3.88E-04)+ | 5.36E-05(6.37E-04)≈ | 1.00E+02(4.00E-08)+ | 1.00E+02(1.41E-05)+ |
| TP9 | $Acc_u$ | 1.00E-06(0.00E+00) | 1.00E-06(0.00E+00)≈ | 1.00E-06(0.00E+00)≈ | 2.95E-05(1.32E-05)+ | 1.00E-06(0.00E+00)≈ |
| | $Acc_l$ | 1.00E-06(0.00E+00) | 1.00E-06(0.00E+00)≈ | 1.00E-06(0.00E+00)≈ | 1.00E-06(0.00E+00)≈ | 1.00E-06(0.00E+00)≈ |
| TP10 | $Acc_u$ | 1.00E-06(0.00E+00) | 1.00E-06(0.00E+00)≈ | 1.00E-06(0.00E+00)≈ | 2.18E+00(3.39E+00)+ | 1.00E-06(0.00E+00)≈ |
| | $Acc_l$ | 1.00E-06(0.00E+00) | 1.00E-06(0.00E+00)≈ | 1.00E-06(0.00E+00)≈ | 1.00E-06(1.08E-05)+ | 1.00E-06(0.00E+00)≈ |
| +/-/≈ ($Acc_u$ & $Acc_l$) | | | 3/1/6 & 3/1/6 | 3/0/7 & 3/1/6 | 6/3/1 & 7/1/2 | 3/3/4 & 3/4/3 |

TABLE S-VII
MEDIAN AND IQR RESULTS OF NUMBER OF FES ($FEs_u$ AND $FEs_l$) ON TP PROBLEMS

| TP instances | | DRC-CMA-ES (ours) | BL-CMA-ES | TLEA-CMA-ES | NBLEA | BLEAQ-II |
|---|---|---|---|---|---|---|
| TP1 | $FEs_u$ | 3.41E+03(9.83E+02) | 1.82E+03(3.98E+02)- | 1.83E+03(3.13E+02)- | 7.97E+02(2.50E+02)- | 1.86E+02(1.65E+02)- |
| | $FEs_l$ | 6.53E+04(1.93E+04) | 9.17E+04(1.78E+04)+ | 9.25E+04(1.22E+04)+ | 1.40E+04(4.26E+03)- | 1.23E+03(1.84E+03)- |
| TP2 | $FEs_u$ | 2.74E+03(8.91E+02) | 1.58E+03(5.80E+02)- | 1.64E+03(5.64E+02)- | 3.75E+02(3.96E+02)- | 3.12E+02(6.63E+02)- |
| | $FEs_l$ | 5.58E+04(2.03E+04) | 8.30E+04(2.24E+04)+ | 8.36E+04(2.03E+04)+ | 6.60E+03(7.06E+03)- | 2.29E+03(4.23E+03)- |
| TP3 | $FEs_u$ | 5.00E+03(6.00E+00) | 5.00E+03(5.00E+00)≈ | 5.00E+03(5.00E+00)≈ | 8.48E+02(4.75E+02)- | 7.19E+02(4.12E+02)- |
| | $FEs_l$ | 8.41E+04(2.43E+03) | 2.62E+05(6.11E+04)+ | 2.63E+05(4.64E+04)+ | 1.49E+04(8.39E+03)- | 4.26E+03(2.25E+02)- |
| TP4 | $FEs_u$ | 2.73E+03(1.14E+03) | 1.00E+03(1.42E+02)- | 1.14E+03(2.41E+02)- | 8.29E+02(3.28E+02)- | 7.42E+02(2.68E+02)- |
| | $FEs_l$ | 5.67E+04(1.08E+04) | 6.23E+04(5.61E+03)+ | 6.54E+04(1.38E+04)+ | 6.23E+04(8.88E+03)+ | 8.82E+03(4.34E+03)- |
| TP5 | $FEs_u$ | 2.33E+03(1.96E+03) | 1.55E+03(1.55E+03)- | 1.52E+03(1.14E+03)- | 6.77E+02(2.15E+02)- | 8.58E+02(3.75E+02)- |
| | $FEs_l$ | 5.01E+04(3.87E+04) | 7.69E+04(6.58E+04)+ | 7.34E+04(4.93E+04)+ | 1.19E+04(3.72E+03)- | 6.27E+03(1.79E+03)- |
| TP6 | $FEs_u$ | 1.65E+03(2.05E+02) | 5.94E+02(1.57E+02)- | 5.25E+02(8.60E+01)- | 8.89E+02(2.22E+02)- | 9.34E+02(1.49E+02)- |
| | $FEs_l$ | 2.97E+04(3.86E+03) | 3.86E+04(7.19E+03)+ | 3.29E+04(4.91E+03)+ | 3.27E+04(3.99E+03)+ | 7.12E+03(8.01E+02)- |
| TP7 | $FEs_u$ | 4.77E+03(1.08E+03) | 1.95E+03(1.20E+03)- | 1.90E+03(6.22E+02)- | 9.46E+02(2.68E+02)- | 8.42E+02(4.59E+02)- |
| | $FEs_l$ | 8.24E+04(1.72E+04) | 1.12E+05(5.09E+04)+ | 1.04E+05(3.63E+04)+ | 2.51E+05(6.95E+04)+ | 5.39E+04(2.50E+04)- |
| TP8 | $FEs_u$ | 3.02E+03(1.13E+03) | 1.69E+03(4.39E+02)- | 1.47E+03(4.43E+02)- | 3.88E+02(4.47E+02)- | 7.05E+02(7.12E+02)- |
| | $FEs_l$ | 5.95E+04(2.16E+04) | 8.60E+04(2.22E+04)+ | 7.47E+04(1.94E+04)+ | 6.83E+03(7.91E+03)- | 5.20E+03(4.90E+03)- |
| TP9 | $FEs_u$ | 2.66E+03(1.02E+02) | 1.05E+03(5.20E+01)- | 1.07E+03(6.00E+01)- | 5.97E+02(1.84E+02)- | 2.48E+02(2.20E+01)- |
| | $FEs_l$ | 5.17E+04(2.38E+03) | 7.51E+04(4.14E+03)+ | 7.45E+04(4.77E+03)+ | 3.01E+05(4.64E+04)+ | 2.69E+04(5.48E+02)- |
| TP10 | $FEs_u$ | 4.77E+03(1.40E+02) | 2.22E+03(7.50E+01)- | 2.25E+03(7.90E+01)- | 1.47E+03(6.60E+02)- | 8.87E+02(1.89E+02)- |
| | $FEs_l$ | 1.07E+05(2.87E+03) | 1.39E+05(5.62E+03)+ | 1.41E+05(3.63E+03)+ | 8.22E+05(2.09E+05)+ | 1.39E+05(2.11E+04)+ |
| +/-/≈ ($FEs_u$ & $FEs_l$) | | | 0/9/1 & 10/0/0 | 0/9/1 & 10/0/0 | 0/10/0 & 5/5/0 | 0/10/0 & 1/9/0 |