

Assignment - 4

Bayesian Networks

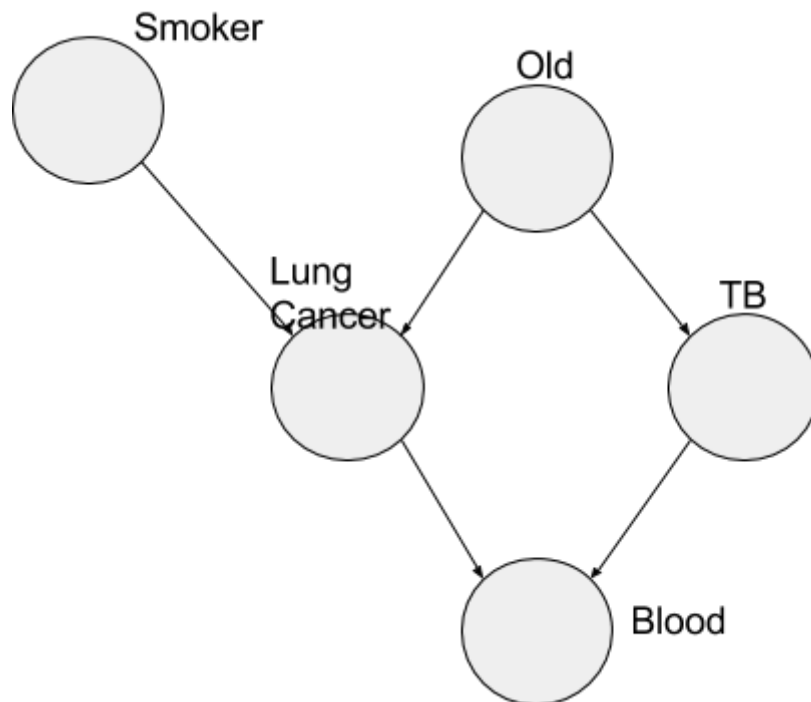
Submission Deadline - November 3rd, 11.55 PM

Name-

Roll No -

1. What is the maximum number of edges in a Bayesian network with n nodes?

2.



Consider the above graph and the following given probabilities and answer the following questions.

$$P(\text{Smoker}) = 0.1$$

$$P(\text{old}) = 0.2$$

$$P(\text{tb} \mid \text{old}) = 0.05$$

$$P(\text{tb} \mid \sim \text{old}) = 0.01$$

$$P(\text{lungcancer} \mid \text{old} \wedge \text{smoker}) = 0.3$$

$$P(\text{lungcancer} \mid \sim \text{old} \wedge \text{smoker}) = 0.1$$

$$P(\text{lungcancer} \mid \text{old} \wedge \sim \text{smoker}) = 0.2$$

$$P(\text{lungcancer} \mid \sim \text{old} \wedge \sim \text{smoker}) = 0.01$$

$$P(\text{blood} \mid \text{lungcancer} \wedge \text{tb}) = 1.0$$

$$P(\text{blood} \mid \text{lungcancer} \wedge \sim \text{tb}) = 0.9$$

$$P(\text{blood} \mid \sim \text{lungcancer} \wedge \text{tb}) = 0.8$$

$$P(\text{blood} \mid \sim \text{lungcancer} \wedge \sim \text{tb}) = 0.01$$

- i) What is $P(\text{Smoker} \mid \text{lungcancer})$?
- ii) What is $P(\text{blood} \mid \text{Smoker})$?
- iii) What is $P(\text{lungcancer} \mid \text{old} \wedge \text{smoker} \wedge \text{blood})$?

3. For each of the networks given in Figure 1 (a,b), do the following statements hold? Please explain your reasoning.

- a) $A \perp C \mid B, D$
- b) $B \perp D \mid A, C$

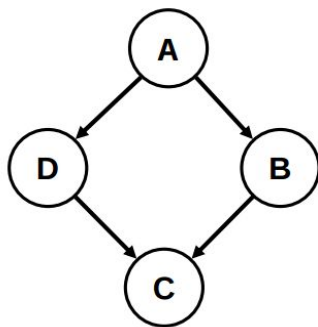


Fig 1 a

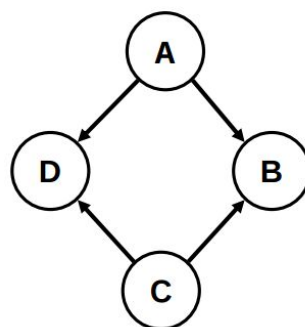
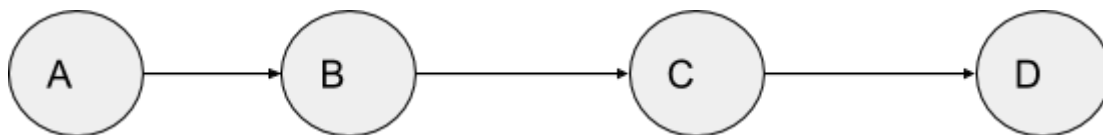


Fig 1 b

4. Construct two different Bayesian network which encode exactly the following conditional independence assumptions

- $$\begin{aligned} A &\perp C \mid B \\ A &\perp D \mid B \\ C &\perp D \mid B \end{aligned}$$

5. Assume the following Bayes net and the corresponding distributions over the variables in the Bayes net



$$\begin{aligned} P(+a) &= \frac{1}{4} \\ P(-a) &= \frac{3}{4} \end{aligned}$$

$$\begin{aligned} P(-b \mid -a) &= \frac{2}{3} \\ P(+b \mid -a) &= \frac{1}{3} \\ P(-b \mid +a) &= \frac{4}{5} \\ P(+b \mid +a) &= \frac{1}{5} \end{aligned}$$

$$\begin{aligned} P(-d \mid -c) &= \frac{1}{8} \\ P(+d \mid -c) &= \frac{7}{8} \\ P(-d \mid +c) &= \frac{5}{6} \\ P(+d \mid +c) &= \frac{1}{6} \end{aligned}$$

$$P(-c \mid -b) = \frac{1}{4}$$

$$P(+c \mid -b) = \frac{3}{4}$$

$$P(-c \mid +b) = \frac{1}{2}$$

$$P(+c \mid +b) = \frac{1}{2}$$

a) You are given the following samples

+a +b -c -d
 +a -b +c -d
 -a +b +c -d
 -a -b +c -d
 +a -b -c +d
 +a +b +c -d
 -a +b -c +d
 -a -b +c -d

- i) Assume that these samples came from performing Prior Sampling, calculate the sample estimate of $P(+c)$
- ii) Which samples would not be used when doing Rejection sampling for the task estimating $P(+c \mid +a, -d)$. What is the sample of estimate of $P(+c \mid +a, -d)$

b) Using Likelihood Weighting Sampling to estimate $P(-a \mid +b, -d)$ the following samples were obtained. Calculate the weight of each sample

- i) -a +b +c -d
- ii) +a +b +c -d
- iii) +a +b -c -d
- iv) -a +b -c -d

c) From the weighted samples in the previous question estimate $P(-a \mid +b, -d)$

d) Which query is better suited for likelihood weighting, $P(D|A)$ or $P(A|D)$? Justify.

e) Recall that during Gibbs Sampling, samples are generated through an iterative process. Assume that the only evidence that is available is $A=+a$. Which of the following sequences below could have been generated by Gibbs sampling. Justify.

- i) Sequence 1:
 - 1) +a -b -c +d
 - 2) +a -b -c +d
 - 3) +a -b +c +d
- ii) Sequence 2:
 - 1) +a -b -c +d
 - 2) +a -b -c -d
 - 3) -a -b -c +d
- iii) Sequence 3:
 - 1) +a -b -c +d
 - 2) +a -b -c -d
 - 3) +a +b -c -d

iv) Sequence 4:

1) $+a -b -c +d$

2) $+a -b -c -d$

3) $+a +b -c +d$