

+ a) Assignment - 3  
 Pred Propositional & Predicate Logic

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 15CS30024

E(1). Predicates are as follows.

SC: Let  $m(x, y)$  denote  $x$  murdered  $y$

CC: Let  $l(x)$  denote person was in Kharagpur

V: Let  $k(x, y)$  denote  $x$  has key to  $y$ 's house

F: Let  $e(x, y)$  denote  $x$  has entry access to  $y$ 's house

$F_1$ : Let  $t(x, y)$  denote  $y$  trusts  $x$ .

$$F_1: (m(\text{Animesh}, \text{Heera}) \wedge \neg m(\text{Heera}, \text{Mahesh}) \wedge \neg m(\text{Prem}, \text{Mahesh}))$$

$$(\neg m(\text{Animesh}, \text{Mahesh}) \wedge m(\text{Heera}, \text{Mahesh}) \wedge \neg m(\text{Prem}, \text{Mahesh}))$$

$$(\neg m(\text{Animesh}, \text{Mahesh}) \wedge \neg m(\text{Heera}, \text{Mahesh}) \wedge m(\text{Prem}, \text{Mahesh}))$$

$$F_2: \forall x (m(x, \text{Mahesh}) \rightarrow (e(x, \text{Mahesh}) \wedge l(x)))$$

$$F_3: \forall x [(t(x, \text{Mahesh}) \vee k(x, \text{Mahesh})) \rightarrow e(x, \text{Mahesh})]$$

$$\neg [\neg (t(x, \text{Mahesh}) \vee k(x, \text{Mahesh})) \rightarrow \neg e(x, \text{Mahesh})]$$

$$F_4: t(\text{Animesh}, \text{Mahesh}) \wedge \neg t(\text{Heera}, \text{Mahesh}) \wedge \neg t(\text{Prem}, \text{Mahesh})$$

$$F_5: k(\text{Heera}, \text{Mahesh}) \wedge \neg k(\text{Animesh}, \text{Mahesh}) \wedge \neg k(\text{Prem}, \text{Mahesh})$$

$$F_6: \neg l(\text{Animesh}) \wedge l(\text{Heera}) \wedge l(\text{Prem})$$

$$G: m(\text{Heera}, \text{Mahesh})$$

$$\text{Simplification of } F_3: \forall x (\neg (t(x, \text{Mahesh}) \vee k(x, \text{Mahesh})) \vee e(x, \text{Mahesh}))$$

$$(\neg (t(x, \text{Mahesh}) \vee k(x, \text{Mahesh})) \vee e(x, \text{Mahesh}))$$

$$F_7: \forall x (\neg m(x, \text{Mahesh}) \vee (e(x, \text{Mahesh}) \wedge l(x)))$$

From  $F_2$  and  $F_6$ ,  $\neg m(\text{Anirudh, Mahesh})$

From  $F_3, F_4$  and  $F_5$

$C_1: \neg t(\text{Prem, Mahesh}), \neg k(\text{Prem, Mahesh}), \neg e(\text{Prem, Mahesh})$

$C_2: \neg t(\text{Heera, Mahesh}), \neg k(\text{Heera, Mahesh}), \neg e(\text{Heera, Mahesh})$

From  $C_1, F_2$  and  $F_6$ ,  $\neg m(\text{Prem, Mahesh})$

From  $C_2, F_2$  and  $F_6$ ,  $\neg m(\text{Heera, Mahesh})$

$C_0, C_3, C_4$  and  $F_1$  all are consistent

Hence from  $C_4: m(\text{Heera, Mahesh})$

Hence proved.

2 a)  $\forall x ((P(x) \wedge L(z, x)) \rightarrow \neg E(x))$

b)  $\forall x \exists y (P(y) \wedge L(x, y))$

c) All even primes are less than 100.

3 Let  $p$  denote "the investigation continues"

$q$  denote "new evidence is brought to light"

$r$  denote "several leading citizens are implicated"

$s$  denote "the newspapers stop publicising the case"



$$F_1: p \rightarrow q ; \neg p \vee q$$

$$F_2: q \rightarrow r ; \neg q \vee r$$

$$F_3: r \rightarrow s ; \neg r \vee s$$

$$F_4: ((p \rightarrow s) \rightarrow (q \rightarrow p)) ; (p \wedge \neg s) \vee (\neg q \vee p) ;$$

$$(p \vee \neg q) \wedge (p \vee \neg q \vee \neg s)$$

$$F_5: \neg p$$

$$G: \neg q$$

$$\neg G: q$$

Resolutions

$$1. \neg p \vee q$$

$$2. \neg q \vee r$$

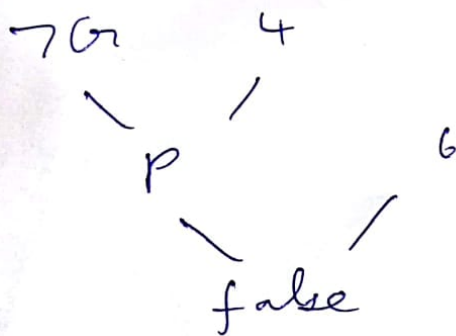
$$3. \neg r \vee s$$

$$4. p \vee \neg q$$

$$5. p \vee \neg q \vee \neg s$$

$$6. \neg p$$

$$\neg G: q$$



Hence stmt is valid  
Hence proved.

4. a)

Predicates

$E(x)$  -  $x$  entered country

$S(x, y)$  -  $x$  is searched by  $y$ .

$C(x)$  -  $x$  is customs official

$V(x)$  -  $x$  is VIP

$P(x)$  -  $x$  is drug pusher

$$F_1: \forall x [ (E(x) \wedge \neg V(x)) \rightarrow (\exists y (S(x, y) \wedge C(y))) ]$$

$$\forall x [ \neg (E(x) \wedge \neg V(x)) \vee (\exists y (S(x, y) \wedge C(y))) ]$$

$$\forall x [ (\neg E(x) \vee V(x)) \vee (\exists y (S(x, y) \wedge C(y))) ]$$

$$(\neg E(x) \vee V(x)) \vee (S(x, f(x)) \wedge C(f(x)))$$

$$[\neg E(x) \vee V(x) \vee S(x, f(x))] \wedge [\neg E(x) \vee V(x) \vee C(f(x))]$$

$$F_2: \exists x [ P(x) \wedge E(x) \wedge \forall y (\neg S(x, y) \vee P(y)) ]$$

$$P(a) \wedge E(a) \wedge (\neg S(a, y) \vee P(y))$$

$$F_3: \forall x [ P(x) \rightarrow \neg V(x) ]$$

$$\neg P(x) \vee \neg V(x)$$

$$F_4: \neg \exists x [ C(x) \wedge P(x) ]$$

$$\neg (C(x) \vee \neg P(x))$$

Resolution

$$1a: \neg E(x) \vee V(x) \vee S(x, f(x))$$

$$1b: \neg E(x) \vee V(x) \vee C(f(x))$$

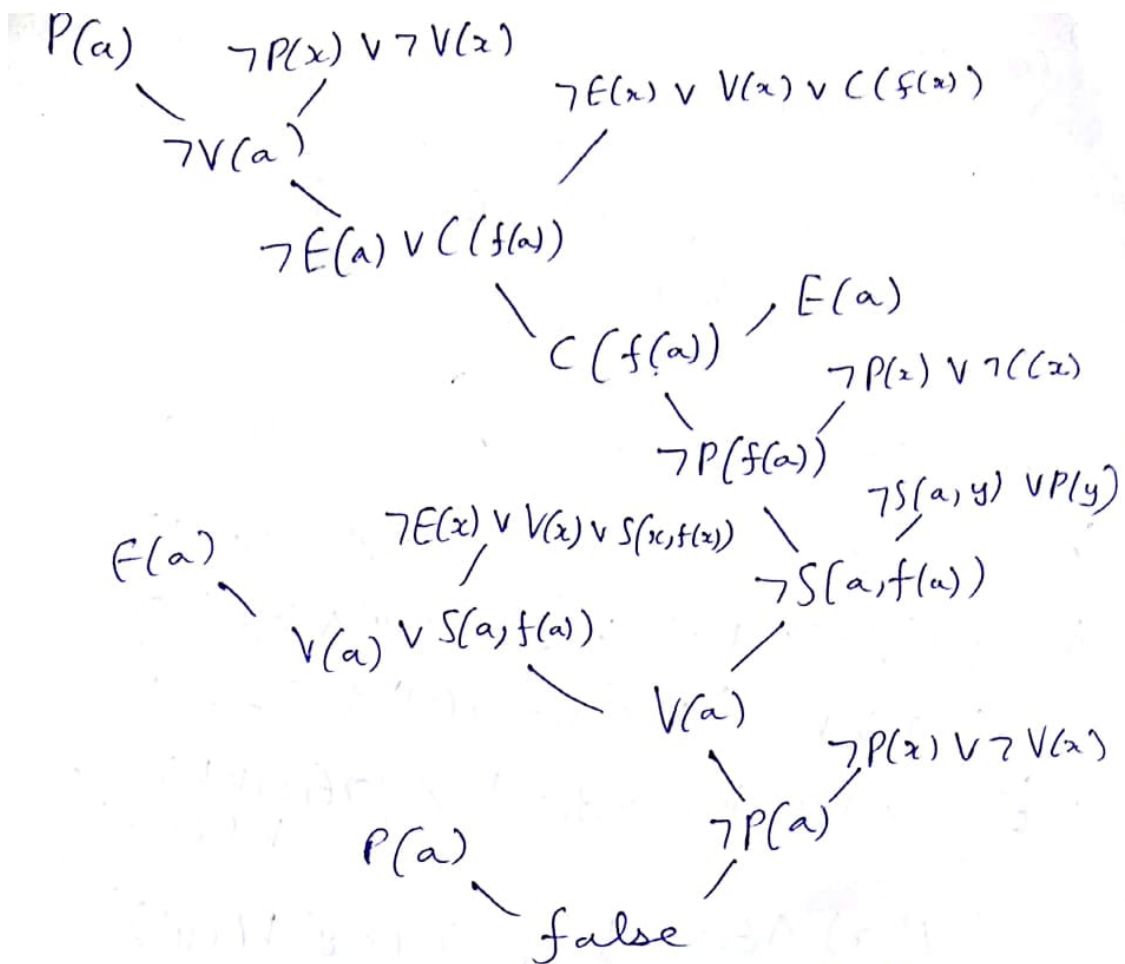
$$2a: P(a)$$

$$2b: E(a)$$

$$2c: \neg S(a, y) \vee P(y)$$

$$3: \neg P(x) \vee \neg V(x)$$

$$4: \neg C(x) \vee \neg P(x)$$



Hence stmt is valid

4.6) Predicates are as follows.

$l(x) = x$  likes George

$C(x, y)$  -  $x$  chooses  $y$  for team

$f(x, y)$  -  $x$  and  $y$  are friends

$$F_1: \forall x (l(x) \rightarrow c(x, \text{Nick}))$$

$$\forall x (\neg l(x) \vee c(x, \text{Nick}))$$

$$\neg L(x) \vee C(x, \text{Nick})$$

$$F_2 = \forall x (f(x, \text{Mike}) \rightarrow \neg f(x, \text{Nick}))$$

$$\forall x ( \neg f(x, \text{Mike}) \vee \neg f(x, \text{Nick}) )$$

$$\neg f(x, Mike) \vee \neg f(x, Nick)$$

$$F_3: \forall x (C(\text{Jay}, x) \rightarrow f(x, \text{Ken}))$$

$$\forall x ( \neg C(\text{Jay}, x) \vee f(x, \text{Ken}) )$$

$$\neg C(\text{Jay}, x) \vee f(x, \text{Ken})$$



$$G: f(\text{Ken}, \text{Mike}) \rightarrow \neg l(\text{Jay})$$

$$G: \neg f(\text{Ken}, \text{Mike}) \vee \neg l(\text{Jay}) \quad \neg G: f(\text{Ken}, \text{Mike}) \wedge l(\text{Jay})$$

Resolutions are

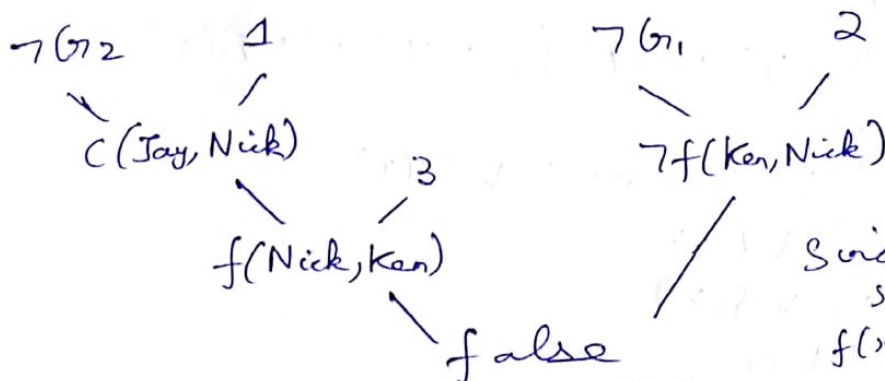
$$\neg l(x) \vee c(x, \text{Nick})$$

$$1. \neg f(x, \text{Mike}) \vee \neg f(x, \text{Nick})$$

$$2. \neg c(\text{Jay}, x) \vee f(x, \text{Ken})$$

$$\neg G_1: f(\text{Ken}, \text{Mike})$$

$$\neg G_2: l(\text{Jay})$$



Hence stmt is valid.

4.c) Predicates are

$A(x)$  -  $x$  is author

$S(x)$  -  $x$  is successful

$W(x)$  -  $x$  is well read

$I(x)$  -  $x$  is intellectual

$$F_1: \forall x [(A(x) \wedge S(x)) \rightarrow W(x)] \wedge (W(x) \rightarrow (A(x) \wedge S(x)))$$

$$F_2: \forall x [A(x) \rightarrow I(x)]$$

$$F_3: \exists x [A(x) \wedge \neg W(x)]$$

$$G: \forall x [I(x) \rightarrow A(x)]$$

From  $F_1$

$$\forall x [(A(x) \wedge S(x)) \vee W(x) \wedge (\neg W(x) \vee (A(x) \wedge S(x)))]$$

$$\forall x [(\neg A(x) \vee \neg S(x) \vee W(x)) \wedge (\neg W(x) \vee (A(x) \wedge S(x)))]$$

$$\forall x [(\neg A(x) \vee \neg S(x) \vee W(x)) \wedge (\neg W(x) \vee A(x)) \wedge (\neg W(x) \vee S(x))]$$

$$\neg A(x) \vee \neg S(x) \vee W(x); \neg W(x) \vee A(x); \neg W(x) \vee S(x)$$

From  $F_2$

$$\forall x [\neg A(x) \vee I(x)]; \neg A(x) \vee I(x)$$

$$\text{From } F_3 \quad A(a) \wedge \neg W(a)$$

From  $F_4$

$$\forall x [\neg I(x) \vee A(x)]; \neg I(x) \vee A(x)$$

Resolution

$$1. \neg A(x) \vee \neg S(x) \vee W(x)$$

$$2. \neg W(x) \vee A(x)$$

$$3. \neg W(x) \vee S(x)$$

$$4. \neg A(x) \vee I(x)$$

$$5. A(a) \vee \neg W(a)$$

$$G: \neg I(x) \vee A(x)$$

If stmt is not valid, if we prove that  $(F_1 \wedge F_2 \wedge F_3 \wedge G)$  is unsatisfiable then it is proved.

$$G: \neg I(x) \vee A(x)$$

Let  $A(x)$  be false i.e.  $\neg A(x)$  be true

From 4 and  $c_1$ , we get  $I(x)$  is true.

From  $G$  and  $c_2$ , since we get  $A(x)$  is true

Hence  $A(x) \wedge \neg A(x)$  gives false.

$\therefore$  It is unsatisfiable for  $A(x)$  is false.

There is atleast one instance in which  $F_1 \wedge F_2 \wedge F_3 \wedge G$  is unsatisfiable.

Hence  $G$  is not valid. Hence disproved.