Assignment 3: Propositional and Predicate Logic

1. Encode the following sentences in predicate (first-order) logic.
   Either Animesh or Heera or Prem murdered Mahesh in Kharagpur yesterday. The murderer would have to be in Kharagpur and enter Mahesh's house. One can enter Mahesh's house if he or she either has the key or is trusted by Mahesh. Out of the three suspects Mahesh only trusts Animesh. Heera has the only extra key to Mahesh's house. Animesh was not in Kharagpur yesterday but Heera and Prem were in Kharagpur yesterday.
   Prove that "*Heera killed Mahesh*".

2. Using the universe of integers for all quantifiers, express the following English sentences in predicate logic using the predicates P, E and L. Where $P(x)$ means 'x is prime', $E(x)$ means 'x is even' and $L(x, y)$ means '$x < y$'. In the following replace * with the appropriate entry.

   (a) "No prime greater than 2 is even" can be expressed as $\forall x((*\wedge *) \rightarrow *)$.

   (b) "For every number, there is a larger prime number" can be expressed as $\forall x * (* \wedge *)$.

   (c) The English language translate for the expression $\forall x((E(x) \wedge P(x)) \rightarrow L(x, 100))$ is *.

3. Encode the following sentences in propositional logic and use resolution refutation technique to prove the validity of the statements.

   > If the investigation continues then new evidence is brought to light. If new new evidence is brought to light then several leading citizens are implicated. If several leading citizens are implicated then newspapers stop publicizing the case. If continuation of the investigation implies that the newspapers stop publicizing the case, then the bringing to light new evidence implies that the investigation continues. The investigation does not continue. Therefore, new evidence is not brought to light.

4. Encode the following arguments so that their validity can be shown and prove their validity:

   (a) The custom officials searched everyone who entered this country and who was not a VIP. Some of the drug pushers entered this country and they were only searched by drug pushers. No drug pusher was a VIP. Therefore, some of the officials were drug pushers.

   (b) Anyone who likes George will choose Nick for his team. Nick is not a friend of anyone who is a friend of Mike. Jay will choose no one but a friend of Ken for his team. Therefore, if Ken is a friend of Mike, then Jay does not like George.

(c) Any authors are successful iff they are well read. All authors are intellectuals. Some authors are successful but not well read. Therefore, all intellectuals are authors. (Note: This may not be a valid argument.)