4 a) Assignment - 3

Pred Propositional & Predicate Logic

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15CS30024

E(a) 1. Predicates are as follows.

SC Let $m(x,y)$ denote x murdered y

C(Let $l(x)$ denote person was in Kharagpur

V Let $k(x,y)$ denote x has key to y's house.

F Let $e(x,y)$ denote x has entry access to y's house

F_1

Let $t(x,y)$ denote y trusts x.

F_1 : $(m(\text{Animesh}, \text{Mahesh}) \wedge \neg m(\text{Heera}, \text{Mahesh}) \wedge \neg m(\text{Prem}, \text{Mahesh}))$
$\vee$
$(\neg m(\text{Animesh}, \text{Mahesh}) \wedge m(\text{Heera}, \text{Mahesh}) \wedge \neg m(\text{Prem}, \text{Mahesh}))$
$\vee$
$(\neg m(\text{Animesh}, \text{Mahesh}) \wedge \neg m(\text{Heera}, \text{Mahesh}) \wedge m(\text{Prem}, \text{Mahesh}))$

F_2 : $\forall x (m(x, \text{Mahesh}) \rightarrow (e(x, \text{Mahesh}) \wedge l(x)))$

F_3 : $\forall x \Big[(t(x, \text{Mahesh}) \vee k(x, \text{Mahesh})) \rightarrow e(x, \text{Mahesh}) \Big]$
$\vee$
$\Big[\neg (t(x, \text{Mahesh}) \vee k(x, \text{Mahesh})) \rightarrow \neg e(x, \text{Mahesh}) \Big]$

F_4 : $t(\text{Animesh}, \text{Mahesh}) \wedge \neg t(\text{Heera}, \text{Mahesh}) \wedge \neg t(\text{Prem}, \text{Mahesh})$

F_5 : $k(\text{Heera}, \text{Mahesh}) \wedge \neg k(\text{Animesh}, \text{Mahesh}) \wedge \neg k(\text{Prem}, \text{Mahesh})$

F_6 : $\neg l(\text{Animesh}) \wedge l(\text{Heera}) \wedge l(\text{Prem})$

G : $m(\text{Heera}, \text{Mahesh})$

Simplification of F_3 : $\forall x (\neg (t(x, \text{Mahesh}) \vee k(x, \text{Mahesh})) \vee e(x, \text{Mahesh})$
$\wedge$
$((t(x, \text{Mahesh}) \vee k(x, \text{Mahesh})) \vee \neg e(x, \text{Mahesh}))$

F_7 : $\forall x (\neg m(x, \text{Mahesh}) \vee (e(x, \text{Mahesh}) \wedge l(x)))$

From F_2 and F_6, C_0: $\neg m(\text{Animal}, \text{Mahesh})$

From F_3, F_4 and F_5

C_1: $\neg t(\text{Prem}, \text{Mahesh})$, $\neg k(\text{Prem}, \text{Mahesh})$, $\neg e(\text{Prem}, \text{Mahesh})$

C_2: $\neg t(\text{Heera}, \text{Mahesh})$, $k(\text{Heera}, \text{Mahesh})$, $e(\text{Heera}, \text{Mahesh})$

From C_1, F_2 and F_6, C_3: $\neg m(\text{Prem}, \text{Mahesh})$

From C_2, F_2 and F_6, C_4: $m(\text{Heera}, \text{Mahesh})$

C_0, C_3, C_4 and F_1 all are consistent

Hence from C_4: $m(\text{Heera}, \text{Mahesh})$

Hence proved.

2 a) $\forall x \, ((P(x) \land L(2,x)) \rightarrow \neg E(x))$

b) $\forall x \, \exists y \, (P(y) \land L(x,y))$

c) All even primes are less than 100.

3 . Let p denote "the investigation continues"

q denote "new evidence is brought to light"

r denote "several leading citizens are implicated"

s denote "the newspapers stop publicising the case"

$F_1: p \to q$; $\neg p \vee q$

$F_2: q \to r$; $\neg q \vee r$

$F_3: r \to s$; $\neg r \vee s$

$F_4: ((p \to s) \to (q \to p))$; $(p \wedge \neg s) \vee (\neg q \vee p)$;
$(p \vee \neg q) \wedge (p \vee \neg q \vee \neg s)$

$F_5: \neg p$

$G: \neg q$

$\neg G: q$

Resolutions

1. $\neg p \vee q$
2. $\neg q \vee r$
3. $\neg r \vee s$
4. $p \vee \neg q$
5. $p \vee \neg q \vee \neg s$
6. $\neg p$

$\neg G: q$

$$
\begin{tikzcd}
\neg G \arrow[dr, dash] & & 4 \arrow[dl, dash] & \\
 & p \arrow[dr, dash] & & 6 \arrow[dl, dash] \\
 & & \text{false} &
\end{tikzcd}
$$

Hence stmt is valid
Hence proved.

4. a)

Predicates

$E(x)$ - x entered country

$S(x,y)$ - x is searched by y.

$C(x)$ - x is customs official

$V(x)$ - x is VIP

$P(x)$ - x is drug pusher

F_1: $\forall x\left[(E(x) \wedge \neg V(x)) \rightarrow (\exists y(S(x,y) \wedge C(y)))\right]$

$\forall x\left[\neg(E(x) \wedge \neg V(x)) \vee (\exists y(S(x,y) \wedge C(y)))\right]$

$\forall x\left[(\neg E(x) \vee V(x)) \vee (\exists y(S(x,y) \wedge C(y)))\right]$

$(\neg E(x) \vee V(x)) \vee (S(x,f(x)) \wedge C(f(x)))$

$\left[\neg E(x) \vee V(x) \vee S(x,f(x))\right] \wedge \left[\neg E(x) \vee V(x) \vee C(f(x))\right]$

F_2: $\exists x\left[P(x) \wedge E(x) \wedge \forall y(\neg S(x,y) \vee P(y))\right]$

$P(a) \wedge E(a) \wedge (\neg S(a,y) \vee P(y))$

F_3: $\forall x\left[P(x) \rightarrow \neg V(x)\right]$

$\neg P(x) \vee \neg V(x)$

~~F_4~~: $\neg \exists x\left[C(x) \wedge P(x)\right]$

F_4 $\qquad \neg C(x) \vee \neg P(x)$

Resolution

1 a: $\neg E(x) \vee V(x) \vee S(x,f(x))$

1 b: $\neg E(x) \vee V(x) \vee C(f(x))$

2 a: $P(a)$

~~3~~ 2b: $E(a)$

2 c: $\neg S(a,y) \vee P(y)$

3: $\neg P(x) \vee \neg V(x)$

4: $\neg C(x) \vee \neg P(x)$

$P(a)$ $\quad$ $\neg P(x) \vee \neg V(x)$

$\neg V(a)$ $\quad$ $\neg E(x) \vee V(x) \vee C(f(x))$

$\neg E(a) \vee C(f(a))$ $\quad$ $E(a)$

$C(f(a))$ $\quad$ $\neg P(x) \vee \neg C(x)$

$\neg P(f(a))$ $\quad$ $\neg S(a,y) \vee P(y)$

$F(a)$ $\quad$ $\neg E(x) \vee V(x) \vee S(x, f(x))$ $\quad$ $\neg S(a, f(a))$

$V(a) \vee S(a, f(a))$

$V(a)$ $\quad$ $\neg P(x) \vee \neg V(x)$

$P(a)$ $\quad$ $\neg P(a)$

false

Hence stmt is valid

4.b) Predicates are as follows.

$l(x)$ - x likes George

$C(x,y)$ - x chooses y for team

$f(x,y)$ - x and y are friends

F_1: $\forall x (l(x) \rightarrow c(x, Nick))$

$\forall x (\neg l(x) \vee c(x, Nick))$

$\neg l(x) \vee c(x, Nick)$

F_2: $\forall x (f(x, Mike) \rightarrow \neg f(x, Nick))$

$\forall x (\neg f(x, Mike) \vee \neg f(x, Nick))$

$\neg f(x, Mike) \vee \neg f(x, Nick)$

F_3: $\forall x (c(Jay, x) \rightarrow f(x, Ken))$

$\forall x (\neg C(Jay, x) \vee f(x, Ken))$

$\neg C(Jay, x) \vee f(x, Ken)$

G : $f(Ken, Mike) \rightarrow \neg l(Jay)$

G : $\neg f(Ken, Mike) \vee \neg l(Jay)$ $\neg G$: $f(Ken, Mike) \wedge l(Jay)$

Resolutions are

1. $\neg l(x) \vee c(x, Nick)$
2. $\neg f(x, Mike) \vee \neg f(x, Nick)$
3. $\neg c(Jay, x) \vee f(x, Ken)$

~~G~~ $\neg G_1$: $f(Ken, Mike)$

$\neg G_2$: $l(Jay)$

$\neg G_2$ 1

$c(Jay, Nick)$ 3

$f(Nick, Ken)$

$\neg G_1$ 2

$\neg f(Ken, Nick)$

false

Since f is symmetric $f(x,y) = f(y,x)$

Hence stmt is valid.

4.c) Predicates are

$A(x)$ - x is author

$S(x)$ - x is successful

$W(x)$ - x is well read

$I(x)$ - x is intellectual

F_1: $\forall x \left[\left((A(x) \wedge S(x)) \rightarrow W(x) \right) \wedge \left(W(x) \rightarrow (A(x) \wedge S(x)) \right) \right]$

F_2: $\forall x \left[A(x) \rightarrow I(x) \right]$

F_3: $\exists x \left[A(x) \wedge \neg W(x) \right]$

G: $\forall x \left[I(x) \rightarrow A(x) \right]$

From F_1

$\forall x [(\neg(A(x) \wedge S(x)) \vee W(x)) \wedge (\neg W(x) \vee (A(x) \wedge S(x)))]$

$\forall x [(\neg A(x) \vee \neg S(x) \vee W(x)) \wedge (\neg W(x) \vee (A(x) \wedge S(x))]$

$\forall x [(\neg A(x) \vee \neg S(x) \vee W(x)) \wedge (\neg W(x) \vee A(x)) \wedge (\neg W(x) \vee S(x))]$

$\neg A(x) \vee \neg S(x) \vee W(x)$; $\neg W(x) \vee A(x)$; $\neg W(x) \vee S(x)$

From F_2

$\forall x [\neg A(x) \vee I(x)]$; $\neg A(x) \vee I(x)$

From F_3 $A(a) \wedge \neg W(a)$

From F_4

$\forall x [\neg I(x) \vee A(x)]$; $\neg I(x) \vee A(x)$

Resolution

1. $\neg A(x) \vee \neg S(x) \vee W(x)$
2. $\neg W(x) \vee A(x)$
3. $\neg W(x) \vee S(x)$
4. $\neg A(x) \vee I(x)$
5. $A(a) \vee \neg W(a)$

G : $\neg I(x) \vee A(x)$

If stmt is not valid, if we prove that $(F_1 \wedge F_2 \wedge F_3 \wedge G)$ is unsatisfiable then it is proved.

G : $\neg I(x) \vee A(x)$

Let $A(x)$ be false i.e $c_1: \neg A(x)$ be true

From 4 and c_1, we get $c_2: I(x)$ is true.

From G and c_2, ~~subs~~ we get $A(x)$ is true

Hence $A(x) \wedge \neg A(x)$ gives false.

$\therefore$ It is unsatisfiable for $A(x)$ is false.

There is atleast one instance in which $F_1 \wedge F_2 \wedge F_3 \wedge G$ is unsatisfiable.

Hence G is not valid. Hence disproved