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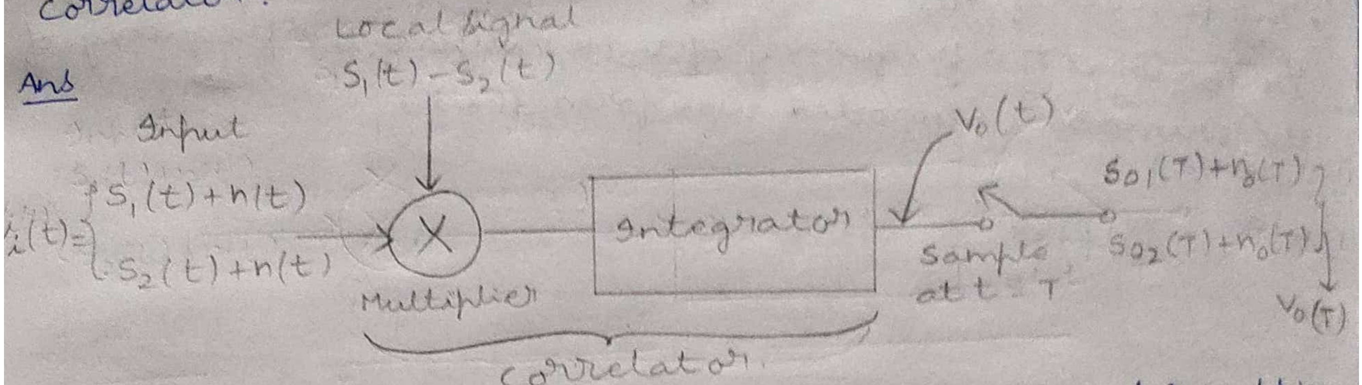
3rd year - ECE

DC Assignment - 2

1Q. Explain about optimum filter Realization using correlator?

Ans



- The above figure is a coherent system of signal reception.
- As shown in figure, the input is a binary data waveform $S_1(t)$ or $S_2(t)$ corrupted by noise $n(t)$. The bit length is T.
- The received signal plus noise $V_i(t)$ is multiplied by a locally generated waveform $S_1(t) - S_2(t)$.
- The output of the multiplier is passed through an integrator whose output is sampled at $t = T$.
- As before, immediately after each sampling, at the beginning of each new bit interval, all energy-storing elements in the integrator are discharged.
- This type of receiver is called a correlator, since we are correlating the received signal and noise with the waveform $S_1(t) - S_2(t)$.
- The output signal and noise of the correlator shown in above figure are,

$$S_0(T) = \frac{1}{\tau}\int_0^T S_i(t)\left[S_1(t) - S_2(t)\right] dt \quad \text{—} ①$$

$$n_0(T) = \frac{1}{\tau}\int_0^T n(t)\left[S_1(t) - S_2(t)\right] dt \quad \text{—} ②$$

where $S_i(t)$ is either $S_1(t)$ or $S_2(t)$ and where τ is the constant of the integrator i.e the output is $1/\tau$ times the integral of its input

- Now we compare these outputs with the matched filter outputs.

• If $h(t)$ is the impulsive response of the matched filter, then the output of the matched filter $v_o(t)$ can be found using the convolution integral we have,

$$v_o(T) = \int_{-\infty}^{\infty} v_i(\lambda)\, h(t-\lambda)\, d\lambda = \int_{0}^{T} v_i(\lambda)(t-\lambda)\, d\lambda \quad \text{--- (3)}$$

• The limits on the integral have been changed to 0 and T since we are interested in the filter response to a bit which extends only over that integral.

• $h(t)$ of matched filter is,

$$h(t) = \frac{2K}{\eta}\left[S_1(T-t) - S_2(T-t)\right] \quad \text{--- (4)}$$

$$h(t-\lambda) = \frac{2K}{\eta}\left[S_1(T-t+\lambda) - S_2(T-t+\lambda)\right] \quad \text{--- (5)}$$

Substituting eq (5) in eq (3) we get,

$$v_o(t) = \frac{2K}{\eta}\int_0^T v_i(\lambda)\left[S_1(T-t+\lambda) - S_2(T-t+\lambda)\right] d\lambda \quad \text{--- (6)}$$

since $v_i(\lambda) = S_i(\lambda) + n(\lambda)$ and $v_o(t) = S_o(t) + n_o(t)$, setting $t = T$ yields,

$$S_o(T) = \frac{2K}{\eta}\int_0^T S_i(\lambda)\left[S_1(\lambda) - S_2(\lambda)\right] d\lambda \quad \text{--- (7)}$$

where $S_i(\lambda)$ is equal to $S_1(\lambda)$ or $S_2(\lambda)$. Similarly we get,

$$n_o(t) = \frac{2K}{\eta}\int_0^T n(\lambda)\left[S_1(\lambda) - S_2(\lambda)\right] d\lambda \quad \text{--- (8)}$$

• Thus $S_o(t)$ and $n_o(t)$ as calculated from eq (1) and (2) for the correlation receiver, and as calculated from eq (7) and eq (8) for the matched filter receiver, are identical.

• Hence the performance of the two systems are identical.

• Matched filter and correlator are two techniques of synthesizing the optimum filter $h(t)$.

2Q. Derive Probability of error for FSK?

Ans • In frequency shift keying (FSK) the received signal is either,

$$S_1(t) = A\cos(\omega_0 + \Omega)t \quad \text{--- (1)}$$

$$\text{(or)}\ S_2(t) = A\cos(\omega_0 - \Omega)t \quad \text{--- (2)}$$

• In FSK the required local waveform is,

$$S_1(t) - S_2(t) = A\cos(\omega_0 + \Omega)t - A\cos(\omega_0 - \Omega)t \quad \text{--- (3)}$$

- To calculate the probability of error for FSK, we return to a point in the derivation of matched filter and assume that $S_1(t) = -S_2(t)$.

we know,
$$\left[\frac{P_o^2(T)}{\sigma_o^2}\right]_{max} = \frac{2}{\eta}\int_0^T [S_1(t) - S_2(t)]^2 dt \quad \text{--- (4)}$$

- Substituting eq (1) and eq (2) in eq (4) and performing the indicated integration we have,

$$\left[\frac{P_o^2(T)}{\sigma_o^2}\right]_{max} = \frac{2A^2T}{\eta}\left[1 - \frac{\sin 2\Omega T}{2\Omega T} + \frac{1}{2}\frac{\sin[2(\omega_0+\Omega)T]}{2(\omega_0+\Omega)T} - \frac{1}{2}\frac{\sin[2(\omega_0-\Omega)T]}{2(\omega_0-\Omega)T} - \frac{\sin 2\omega_0 T}{2\omega_0 T}\right] \quad \text{--- (5)}$$

- If we ~~offset~~ assume that the offset angular frequency Ω is very small in comparison with the carrier angular frequency ω_0, then the last 3 terms in eq (5) each have the form $(\sin 2\omega_0 T)/2\omega_0 T$.
- This ratio approaches zero as $\omega_0 T$ increases. we further assume that $\omega_0 T >> 1$.
- So we neglect the last 3 terms and we get,

$$\left[\frac{P_o^2(T)}{\sigma_o^2}\right]_{max} = \frac{2A^2T}{\eta}\left(1 - \frac{\sin 2\Omega T}{2\Omega T}\right) \quad \text{--- (6)}$$

- The quantity $[P_o^2(T)/\sigma_o^2]_{max}$ in eq (6) attains its largest value when Ω is selected so that $2\Omega T = 3\pi/2$. For this value of Ω we find,

$$\left[\frac{P_o^2(T)}{\sigma_o^2}\right]_{max} = 2.42\frac{A^2T}{\eta} = 4.84\frac{(A^2/2)T}{\eta} \quad \text{--- (7)}$$

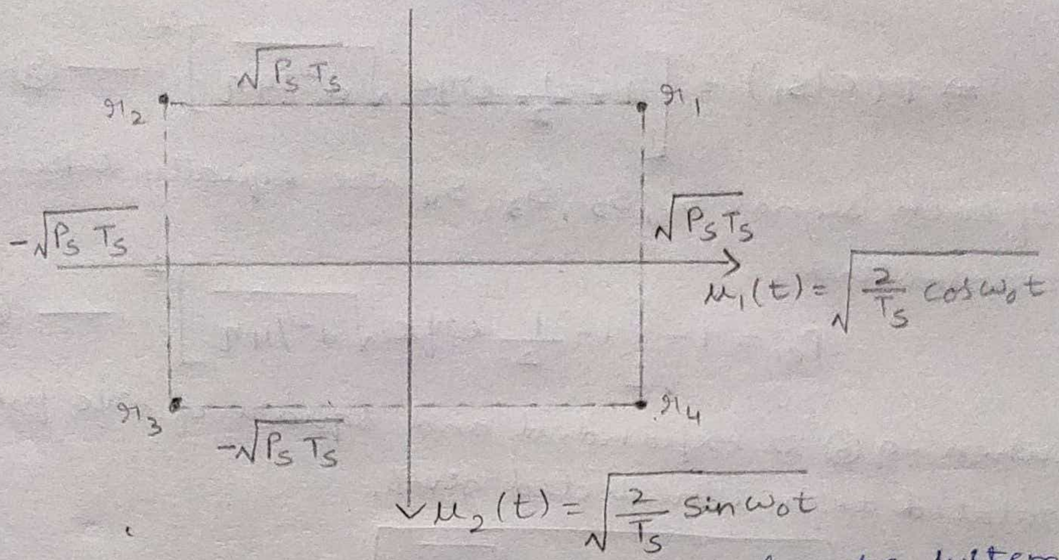
- The probability of error calculated and is given by,

$$P_e = \frac{1}{2}\operatorname{erfc}\left\{\frac{1}{8}\left[\frac{P_o^2(T)}{\sigma_o^2}\right]_{max}\right\}^{1/2} = \frac{1}{2}\operatorname{erfc}\left(0.6\frac{E_s}{\eta}\right)^{1/2} \quad \text{--- (8)}$$

where the signal energy is $E_s = A^2T/2$.

3Q. Derive the Probability of error for QPSK?

Ans • The signal space for QPSK is shown below,



• The unit vectors which establish the coordinate system are,

$$u_1(t) = \sqrt{\frac{2}{T_s}} \cos \omega_0 t \quad \text{— (1)}$$

$$u_2(t) = \sqrt{\frac{2}{T_s}} \sin \omega_0 t \quad \text{— (2)}$$

where $T_s = 2T_b$, T_b being the bit time.

• The Relevant Noise is,

$$n(t) = n_1 u_1(t) + n_2 u_2(t) \quad \text{— (3)}$$

where n_1 and n_2 are independent, gaussian random variables of variance $\eta/2$.

• In the present case, it is more convenient to calculate not the error probability P_e but rather the probability P_c that the decision is correct.

• Thereafter, we can calculate P_e since $P_e = 1 - P_c$.

• If the transmitted signal is s_1 then a determination will be correct provided that the noise does not move r_1 out of the 1st quadrant.

• If this is the case we require that both n_1 and n_2 be in the range from $-\sqrt{P_s T_s}$ to infinity.

• Accordingly, since $d/2 = \sqrt{P_s T_s}$, we have that the Probability of a correct decision, given that s_1 was transmitted is,

$$P(C|s_1) = P\left(n_1 > \frac{-d}{2}, n_2 > \frac{-d}{2}\right) = \int_{-d/2}^{\infty} \frac{e^{-n_1^2/\eta}}{\sqrt{\pi\eta}} dn_1 = \int_{-d/2}^{\infty} \frac{e^{-n_2^2/\eta}}{\sqrt{\pi\eta}} dn_2$$

$$\Rightarrow P(C|S_1) = \left[\frac{1}{\sqrt{\pi \eta}} \int_{-d/2}^{\infty} e^{-n_1^2/\eta}\, dn_1\right]^2 \quad \text{--- (4)}$$

$$\Rightarrow P(C|S_1) = \left[1 - \frac{1}{2}\, \text{erfc}\sqrt{d^2/4\eta}\right]^2 \quad \text{--- (5)}$$

- If each signal S_1, S_2, S_3, S_4 are equally likely to be transmitted,

$$P_e = 1 - \left[1 - \frac{1}{2}\, \text{erfc}\sqrt{d^2/4\eta}\right]^2 \quad \text{--- (6)}$$

- when eq (6) is expanded and square of erfc function related term is neglected gives,

$$\boxed{P_e = \frac{1}{2}\, \text{erfc}\sqrt{\frac{d^2}{4\eta}}}$$

4Q. Derive the Probability of Error for MSK?

Ans
- MSK employs synchronous detection to seperate the even and odd bits streams $b_e(t)$ and $b_o(t)$.
- Furthermore, the correlation interval is $2T_b$ as in QPSK.
- Thus to all intents and Purposes, the synchronous detector (correlator) could be processing a QPSK signal rather than an MSK signal.
- Hence we would expect that the probability of error of MSK be the same as the Probability of error of QPSK:

$$P_{eb}(MSK) = P_{eb}(QPSK) = \frac{1}{2}\, \text{erfc}\sqrt{\frac{E_b}{\eta}} \quad \text{--- (1)}$$

- Let us refer to the signal space representation of MSK from that $d^2 = 4E_b$.
- Therefore the symbol error rate of MSK is,

$$P_e(MSK) = 2 \cdot \frac{1}{2}\, \text{erfc}\sqrt{\frac{d^2}{4\eta}} = \text{erfc}\sqrt{\frac{E_b}{\eta}} \quad \text{--- (2)}$$

- Hence the bit error of MSK is,

$$P_{eb}(MSK) = \frac{1}{2} P_e(MSK) = \frac{1}{2}\, \text{erfc}\sqrt{\frac{E_b}{\eta}}.$$