HW Question 3

for i=0 to log(n)
    for j=1 to n/(2^i)
        do work

- The total runtime is O(n) since the inner for loop runs more times than the outer for loop, making n/(2^i) the dominant term.

$n + n/2 + n/4 + \ldots + n/(2^{\wedge}\log(n))$

$$\sum_{i=0}^{\log n} \left(\frac{n}{2^i}\right) =$$ using the geometric sum formula

$$= \frac{a(1-r^n)}{1-r}$$

$$= \frac{1(1-(1/2)^{\log(n+1)})}{1-1/2}$$ ← counting for the next term

$$= \frac{1-\left(\frac{1}{n}\cdot\frac{1}{2}\right)}{\frac{1}{2}}$$

$$= \frac{1-\frac{1}{2n}}{\frac{1}{2}}$$

$$= 2 - \frac{1}{n}$$

taking account the first for loop → $= n\left(2 - \frac{1}{n}\right)$

$= 2n - 1$

after ignoring constants → $\approx O(n)$