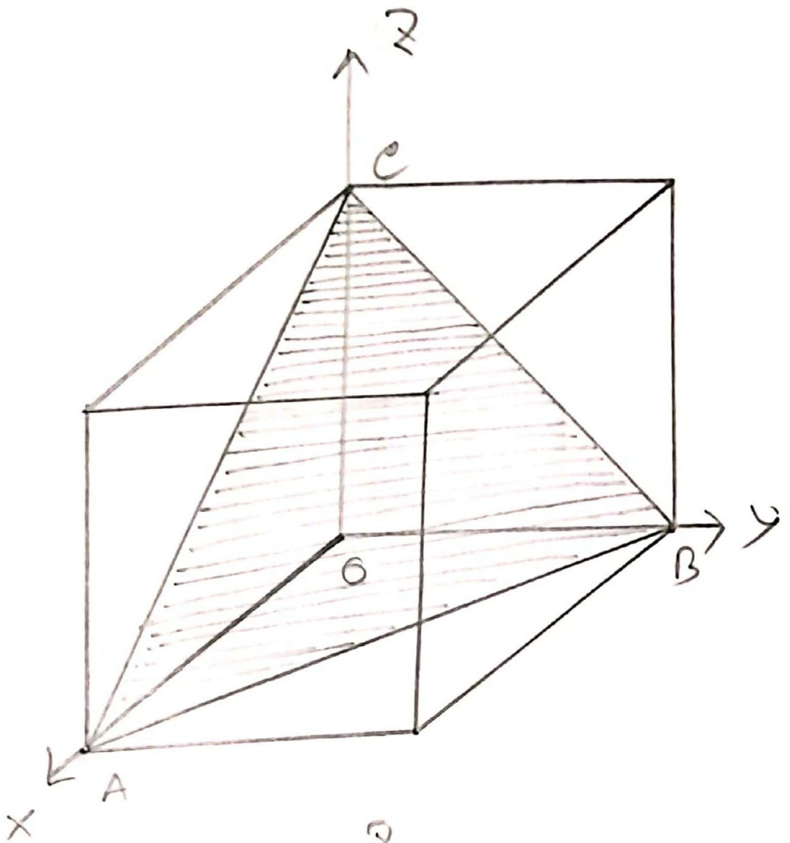
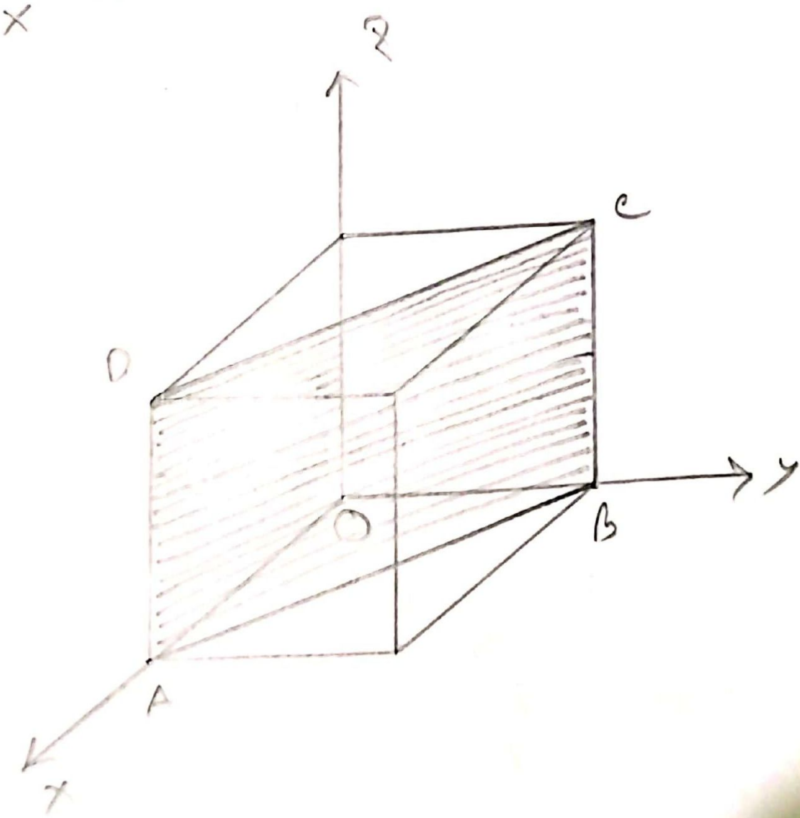
① Sketch the [1,1,1], [110], [211], [112], [332], [103], [101] planes in a simple cubic cell.



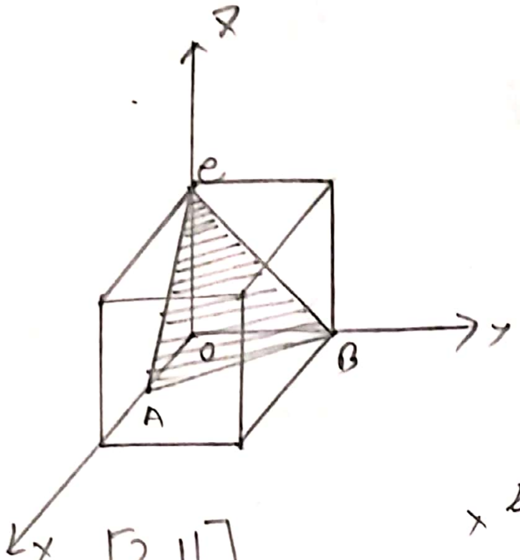
[111]

Intercepts $\frac{1}{1}, \frac{1}{1}, \frac{1}{1}$

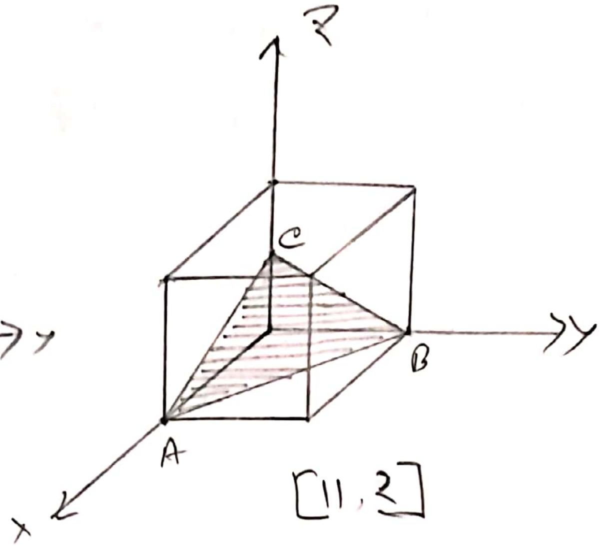


[110]

Intercepts $\frac{1}{1}, \frac{1}{1}, \frac{1}{0}$



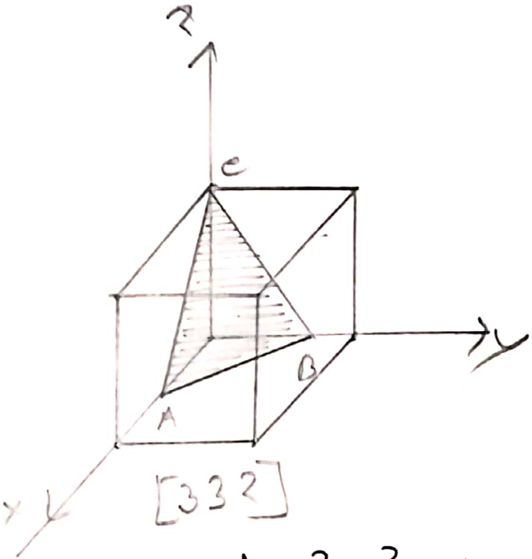
[2 1 1]



[1 1 2]

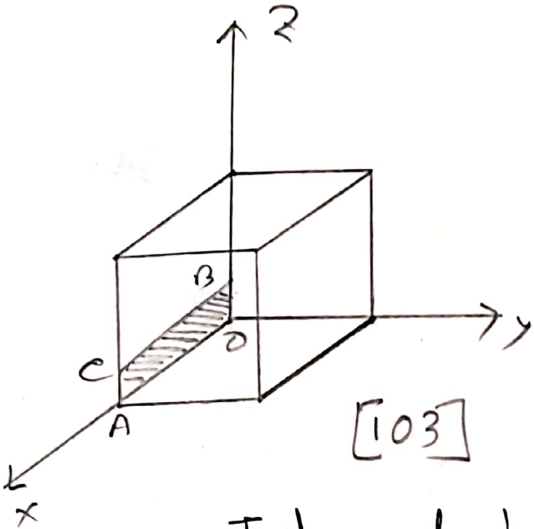
Intercepts $\frac{1}{2}, \frac{1}{1}, \frac{1}{1}$

Intercepts $\frac{1}{1}, \frac{1}{1}, \frac{1}{2}$



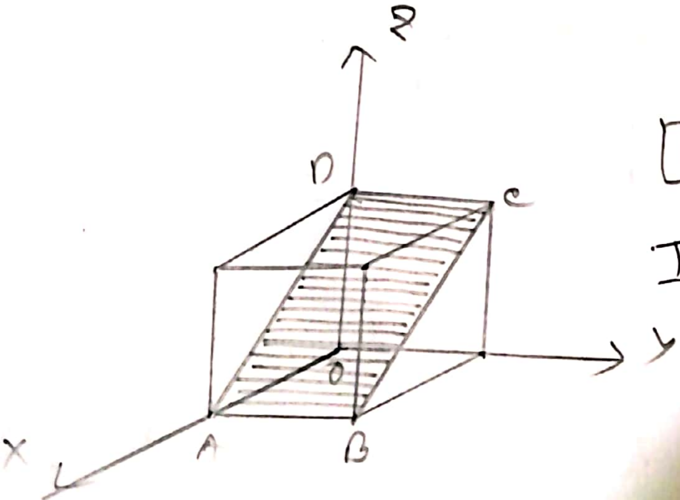
[3 3 2]

Intercepts $\frac{3}{3}, \frac{3}{3}, 1$



[1 0 3]

Intercepts $\frac{1}{1}, \frac{1}{0}, \frac{1}{3}$



[1 0 1]

Intercepts $\frac{1}{1}, \frac{1}{0}, \frac{1}{1}$

(2) The lattice constant of a cubic lattice is 4Å. Calculate the speacing between (211), (111), (001), (110), (123), (234)

Here,

lattice constant $a = 4\mathring{A}$

The specing $d = ?$

we know,

$$d = \frac{a}{\sqrt{h^2+k^2+l^2}}$$

for (211), $d = \frac{4}{\sqrt{2^2+1^2+1^2}} = 1.633\mathring{A}$

for (111), $d = \frac{4}{\sqrt{1^2+1^2+1^2}} = 2.3094\mathring{A}$

for (001), $d = \frac{4}{\sqrt{1^2+0^2+0^2}} = 4\mathring{A}$

for (110), $d = \frac{4}{\sqrt{1^2+1^2+0}} = 2.8284\mathring{A}$

for (123), $d = \frac{4}{\sqrt{1^2+2^2+3^2}} = 1.069\mathring{A}$

for (234), $d = \frac{4}{\sqrt{2^2+3^2+4^2}} = 0.7428\mathring{A}$

(3) Calculate the number of atoms per unit cell for a fcc lattice of cupper crystal. Given $a = 3.8 \mathring{A}$, atomic weight of cupper = 8.9.

Given that,

$a = 3.8 \mathring{A}$

$= 3.8 \times 10^{-8}$ cm

$M = 63.5$

$\rho = 8.9$

$N = 6.023 \times 10^{23}$

$n = ?$

We know,

$$n = \frac{a^3 \rho N}{M}$$

$$= \frac{(3.8 \times 10^{-8})^3 \times 8.9 \times 6.02 \times 10^{23}}{63.5}$$

$$= 4.63 \approx 4$$

$\therefore$ The number of atoms per unit cell for a fcc lattice of cupper crystal is $\boxed{4}$

(4) a) For a cubic (fcc) crystal, latice constant $a = \left[\frac{4M}{\rho N}\right]^{1/3}$, where M is the gm moleculer weight of molecules a lattice points, ρ is the density of crystal

and N is the Avogadro's number.

b) A substance has fcc lattice, molecular weight 60.2 and density 6250 kg/m³ calculate the lattice constant 'a'

Ans (a)

where

fcc, $n = 4$

$M = 60.2$

$\rho = 6250$ kg/m³

$N = 6.023 \times 10^{26}$

$a = ?$

$$a = \sqrt[3]{\frac{nM}{\rho N}}$$

$$= \sqrt[3]{\frac{4 \times 60.2}{6250 \times 6.023 \times 10^{26}}}$$

$$= 4 \times 10^{-10} \text{ m}$$

$$= 4 \text{ Å}$$

∴ Lattice constant of the substance is 4 Å

5) A KCl crystal which has fcc lattice structure has a density of 1.97×10^3 kg/m³. It's molecular weight is 74.5. Find the distance between

adjacent atoms.

$\Rightarrow$

Given that,

fcc, $n=4$

$M = 74.5$

$\rho = 1.98 \times 10^{3}\ kg/m^{3}$

$N = 6.023 \times 10^{26}$

$$a = \sqrt[3]{\frac{nM}{\rho N}}$$

$$= \sqrt[3]{\frac{4 \times 74.5}{1.98 \times 10^{3} \times 6.023 \times 10^{26}}}$$

$$= 6.3 \times 10^{-10}\ m$$

$$= 6.3\ \mathring{A}$$

the distance between adjacent atoms of the same kind. the distance between K and Cl is

$$d = \frac{a}{2}$$

$$= \frac{6.3}{2}$$

$$= 3.15\ \mathring{A}$$

$\therefore$ The distance between adjacent atoms is $3.15\ \mathring{A}$.

⑥ In a unit cell of sc structure, find the angle between the normal to pair of planes whose Miller indices are (i) [111] & [101] (ii) [112] & [011] (iii) [212] & [101] & (iv) [110] & [211]

⇒

$$\theta = \cos^{-1} \frac{u_1 u_2 + v_1 v_2 + w_1 w_2}{\sqrt{u_1^2 + v_1^2 + w_1^2}\ \sqrt{u_2^2 + v_2^2 + w_2^2}}$$

i) $$\theta = \cos^{-1} \frac{1\times1 + 1\times0 + 1\times1}{\sqrt{1^2+1^2+1^2}\ \sqrt{1^2+0^2+1^2}}$$

$= 35.26^\circ$

ii) $$\theta = \cos^{-1} \frac{1\times0 + 1\times1 + 2\times1}{\sqrt{1^2+1^2+2^2}\ \sqrt{0^2+1^2+1^2}}$$

$= 30^\circ$

iii) $$\theta = \cos^{-1} \frac{2\times1 + 1\times0 + 2\times1}{\sqrt{2^2+1^2+2^2}\ \sqrt{1^2+0^2+1^2}}$$

$= 19.47^\circ$

iv) $$\theta = \cos^{-1} \frac{1\times2 + 1\times1 + 0\times1}{}$$

(7) The orthorhombic crystal has axial units in the ratio of 0.424 : 1 : 0.367. Find the Miller indecies of the crystal face whose intercepts are in the ratio 0.212 : 1 : 0.183

$\Rightarrow$ axial units are $a : b : c = 0.424 : 1 : 0.367$

Now

$pa = 0.212$

$\Rightarrow p \times 0.424 = 0.212$

$\therefore p = \frac{1}{2}$

again

$qb = 1$

$\Rightarrow q \times 1 = 1$

$\therefore q = 1$

again

$rc = 0.183$

$\Rightarrow r = \frac{0.183}{0.367}$

$\therefore r = \frac{1}{2}$

Hence numerical parameters of this plane are

$\frac{1}{2}, 1, \frac{1}{2}$

$\therefore$ Miller indices $= \left(\frac{1}{1/2}, \frac{1}{1}, \frac{1}{1/2}\right)$

$= (2, 1, 2)$

(8) The primitives of a crystal are 1.24 Å, 1.82 Å and 2 Å along whose Miller indices [211] cut intercepts 1.2 Å along x-axis. What will be the lengths of intercept along Y & Z axes?

$\Rightarrow$ $h:k:l = 2:1:1$

$\therefore \frac{1}{p} : \frac{1}{q} : \frac{1}{r} = h:k:l = 2:1:1$

$\therefore p:q:r = 1:2:2$

$\therefore pa : qb : rc = (1 \times 1.24) : (2 \times 1.82) : (2 \times 2)$

$= 1.24 : 3.64 : 4$

$$\frac{L_x}{L_y} = \frac{1.24}{3.64}$$

The length of intercepts along Y axes,

$$L_y = \frac{3.64}{1.24} \times 1.24 = 3.64 \text{ Å}$$

Again

$$\frac{L_x}{L_z} = \frac{1.24}{4}$$

The lengths of intercepts Z-axes

$$L_z = \frac{4}{1.24} \times 1.24 = 4 \text{ Å}$$

(9) Calculate the packing fraction in cristal for (i) Sc (ii) bcc & (iii) fcc structure treating the atoms as spherical.

⇒ i) Sc:

Number of atoms per cube is one

volume of one atom $= \frac{4}{3}\pi r^3$

atomic radious, $r = \frac{a}{2}$

∴ Volume occupied by the atom in the Sc unit cell

$$v = 1 \times \frac{4}{3}\pi \times \left(\frac{a}{2}\right)^3 = \frac{\pi a^3}{6}$$

Volume of unit cell, $V = a^3$

$$\therefore PF = \frac{v}{V} = \frac{\pi a^3/6}{a^3} = \frac{\pi}{6} = 0.52 = 52\%$$

ii) bcc:

Number of atoms per cube is one

Volume of atom $= \frac{4}{3}\pi r^3$

atomic radious, $r = \frac{\sqrt{3}}{4} a$

$\therefore$ Volume occupied by the atom in the bcc unitcell $v = 2 \times \frac{4}{3}\pi\left(\frac{\sqrt{3}}{4}a\right)^3$

$$= \frac{\sqrt{3}}{8}\pi a^3$$

Volume of the unit cell, $V = a^3$

$$\therefore PF = \frac{v}{V} = \frac{\frac{\sqrt{3}\pi a^3}{8}}{a^3} = \frac{\sqrt{3}}{8}\pi = 0.68 = 68\%$$

iii) fcc :

Number of atoms per cube is 4

Atomic radious, $r = \frac{a}{2\sqrt{2}}$

$\therefore$ Volume occupied by the atom in the fcc unit cell $v = 4 \times \frac{4}{3}\pi\left(\frac{a}{2\sqrt{2}}\right)^3$

$$= \frac{\pi a^3}{3\sqrt{2}}$$

Volume of the unit cell, $V = a^3$

$$\therefore PF = \frac{v}{} = \frac{\frac{\pi a^3}{3\sqrt{2}}}{} \quad \pi$$

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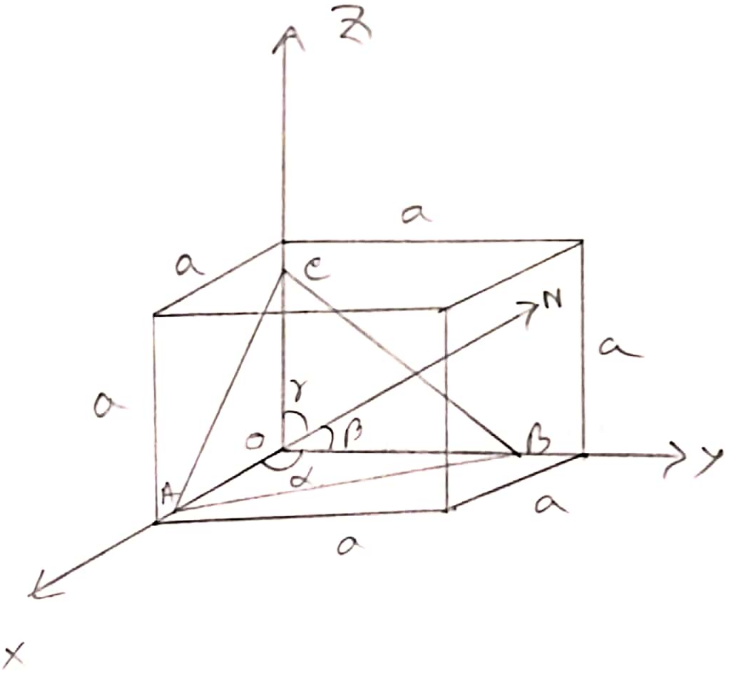
(10) Show that spacing d of plane [hkL] in a simple cubic lattice of side a is $d = \frac{a}{\sqrt{h^2+k^2+L^2}}$

⇒ We can think the plane shown in the following figure belongs in fon ob plane where Miller indices are [hkL]



The perpendicular ON from origin to the plane represents the interplaner spacing d of the family of planes, Let the direction cosines of ON be $\cos\alpha, \cos\beta, \cos\gamma$ as shown.

The intercepts of the plane on the three axes area

$OA = \frac{a}{h}$, $OB = \frac{a}{k}$, $OC = \frac{a}{l}$;

where a is the length of the cubic edge.

$$\cos\alpha' = \frac{d}{OA} = \frac{d}{a/h} = \frac{dh}{a}$$

$$\cos\beta' = \frac{d}{OB} = \frac{d}{a/k} = \frac{dk}{a}$$

$$\cos\gamma' = \frac{d}{OC} = \frac{d}{a/l} = \frac{dl}{a}$$

where d = ON perpendicular distance between adjacent members of the same family of planes,

Now,

$$\cos^2\alpha' + \cos^2\beta' + \cos^2\gamma' = 1$$

$$\Rightarrow \left(\frac{dh}{a}\right)^2 + \left(\frac{dk}{a}\right)^2 + \left(\frac{dl}{a}\right)^2 = 1$$

$$\Rightarrow d^2 = \frac{a^2}{h^2+k^2+l^2}$$

$$\Rightarrow d = \frac{a}{\sqrt{h^2+k^2+l^2}}$$ [showed]

(11) The interplanar spacing d_{111} in a Fcc metal is 0.25 mm. Calculate its lattice constant and atomic radius.

Given that

$d = 0.25$ mm

$= 2.5$ Å

$a = ?$

$r = ?$

$\Rightarrow$ $$d = \frac{a}{\sqrt{h^2+k^2+l^2}}$$

$$\Rightarrow a = d\sqrt{h^2+k^2+l^2}$$

$$= 2.5 \times \sqrt{1^2+1^2+1^2}$$

$$= 4.33 \text{ Å}$$

For Fcc,

$$r = \frac{a}{2\sqrt{2}} = \frac{4.33}{2\sqrt{2}} = 1.53 \text{ Å}$$

(12) How many atoms per mm^2 surface area are there in (i) [110] plane (ii) [111] and (iii) [211] plane for copper which has fcc structure and a lattice constant 'a' = 3.5×10^{-10} m

Given that

$a = 3.5 \times 10^{-10}$ m

(i) $\Rightarrow$ $$n^3 = d$$

$$\therefore n = \sqrt[3]{d}$$

ii) $n = \left(\dfrac{3.5\times10^{-10}}{\sqrt{1^2+1^2+1^2}} \right)^{-3}$

$= 1.2\times10^{29}$ atom/m^2

$= 1.2\times10^{35}$ atom/mm^2

iii) $n = \left(\dfrac{3.5\times10^{-10}}{\sqrt{2^2+1^2+1^2}} \right)^{-3}$

$= 3.4\times10^{29}$ atom/m^2

$= 3.4\times10^{35}$ atom/mm^2

(13) A certain orthorhombic crystals has axial units a:b:c of 0.424:1:0.367. Find the Miller indices of crystal faces whose intercepts are:

a. 0.212 : 1 : 0.183

b. 0.848 : 1 : 0.732

c. 0.424 : ∞ : 0.123

Here axial units are $a:b:c = 0.424:1:0.367$

(a) ⇒

$pa = 0.212$

$\Rightarrow p \times 0.424 = 0.212$

$\therefore p = \frac{1}{2}$

$qb = 1$

$\Rightarrow q \times 1 = 1$

$\therefore q = 1$

Again

$rc = 0.183$

$\Rightarrow r \times 0.367 = 0.183$

$\therefore r = \frac{1}{3}$

The numerical parameters of this plane are $1/2, 1, 1/2$

$\therefore$ Miller indices $= \left(\frac{1}{1/2} : \frac{1}{1} : \frac{1}{1/2} \right)$

$= (2\ 1\ 2)$

b) $\Rightarrow$

$pa = 0.848$

$\Rightarrow p \times 0.424 = 0.848$

$\therefore p = 2$

$qb = 1$

$\Rightarrow q \cdot 1 = 1$

$\Rightarrow q = 1$

again

$rc = 0.732$

$\Rightarrow r \times 0.367 = 0.732$

$\therefore r = 2$

The numerical parameters of this plane are 2, 1, 2

$\therefore$ Miller indices $= \left(\frac{1}{2} : \frac{1}{1} : \frac{1}{2} \right)$

$= (1\ 2\ 1)$

c) $\Rightarrow$

$pa = 0.424$

$\Rightarrow p \times 0.424 = 0.424$

$\therefore p = 1$

$qb = \alpha$

$\Rightarrow q = \alpha$

$\therefore q = \alpha$

$rc = 0.123$

$r \times 0.367 = 0.123$

$\therefore r = \frac{1}{3}$

The numerical parameters of this plane are $1, \infty, \frac{1}{3}$

$\therefore$ Miller indices $= \left(\frac{1}{1} : \frac{1}{\infty} : \frac{1}{1/3}\right) = (1\,0\,3)$

(14) Calculate the maximum phonon frequency generated by scattering of visible light of wavelength $\lambda = 4800$ Å. Given that velocity of sound in medium is 4.8×10^5 cm/sec and refractive index 1.5.

$\Rightarrow$ We know

$$\Omega = \frac{2V_s \omega \mu}{c} \sin \frac{\phi}{2}$$

$$\omega = 2\pi\nu = 2\pi \frac{c}{\lambda}$$

Given that

$V_s = 4.8 \times 10^5$ cm/sec

$\lambda = 4800$ Å

$= 4800 \times 10^{-8}$ cm

$\mu = 1.5$

For maximum phonon frequency

$\sin \frac{\phi}{2} = 1$

$$\Omega_{max} = \frac{2V_s \, 2\pi \frac{c}{\lambda} \cdot \mu}{c}$$

$$= \frac{4 \times 3.14 \times 4.8 \times 10^5 \times 1.5}{4800 \times 10^{-8}}$$

$$= 1.88 \times 10^{11} \text{ rad } s^{-1}$$

(15) Compare the frequency of sound waves of wavelength $\lambda = 10^{-8}$ cm for (i) a homogeneous line (ii) acoustic waves on a linear lattice containing two identical atoms per primitive cell of inter-atomic ~~sapa~~ spacing 2.5 Å and (iii) light waves of the same wavelength, given that $v_0 = 10^5$ cm/sec

i) => We know

$v_0 = 10^5$ cm/sec

$\lambda = 10^{-8}$ cm

$$\omega = k\sqrt{c/\rho}$$

$$= \frac{2\pi}{\lambda} v_0$$

$$= \frac{2\pi}{10^{-8}} \times 10^5$$

$$= 2\pi \times 10^{13} \text{ rad/sec}$$

ii) => $a = 2.5$ Å $= 2.5 \times 10^{-8}$ cm

$$\omega = \frac{v_0}{a}$$

$$= \frac{10^5}{2.5 \times 10^{-8}}$$

$$= 4 \times 10^{13} \text{ rad/sec}$$

iii) $\Rightarrow$ $\omega = 2\pi \nu = 2\pi \frac{c}{\lambda}$

$= 2\pi \times \frac{3\times10^{10}}{10^{-7}}$

$= 6\pi \times 10^{17}$ rad/sec

$c = 3\times10^{10}$ cm/sec

(16) Calculate the conductivity of germanium given mobilities of electrons and holes in a sample germanium at room temperature are 0.56 $m^2 V^{-1} s^{-1}$ and 0.19 $m^2 V^{-1} s^{-1}$ respectively. Assuming that electron and hole density are each equal to 3.62×10^{19} per m^3. If a potential difference of 2 V is applied across the germanium plate of thickness 0.2 mm and area $1 cm^2$, calculate the current produced in the plate.

$\Rightarrow$ Conductivity of semiconductor

$\sigma = n_I e (\mu_e + \mu_h)$

$= 3.62\times10^{19} \times 1.60\times10^{-19}$

Here

$\mu_e = 0.56$ $m^2 V^{-1} s^{-1}$

$\mu_h = 0.19$ $m^2 V^{-1} s^{-1}$

$= 4.344 \, mho/m$

$n_I = 3.62 \times 10^{19}$ per m^3

Current

$I = n_I e (\mu_e + \mu_n) E A$

$= n_I e (\mu_e + \mu_n) \frac{VA}{l}$

$= 3.62 \times 10^{19} \times (0.56 + 0.19) \times 1.60 \times 10^{-19} \times \left(\frac{2 \times 10^{-4}}{0.2 \times 10^{-3}} \right)$

$= 4.344 \, A$

$V = 2\,V$

$A = 1\,cm^2$

$l = 0.2 \times 10^{-3}$ m

(12) Assuming that each atom of copper contributes free electron. Calculate the drift velocity of free electrons in the copper conductor of cross section area 10^{-4} m^2 carrying a current of 2000 mA.

We kn

=> We know

$I = n A V_d e$

$\Rightarrow V_d = \frac{I}{nAe}$

Here

$I = 200 \times 10^{-3}$ A

$A = 10^{-4} \, m^2$

$V_d = ?$

$n = \frac{N}{V} = \frac{N_d}{m}$

$= 8.24 \times 10^{25}$

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$$= \frac{200 \times 10^{-3}}{1 \times 10^{-4} \times 1.60 \times 10^{-19}} \times \frac{1}{8.4 \times 10^{25}}$$

$$= 1.5 \times 10^{-4} \ m s^{-1}$$

(18) The intrinsic carrier density of Ge at 27°C is $2.5 \times 10^{17} \ m^{-3}$. Calculate its intrinsic resistance, if the electron & hole mobilities are $0.35 \ m^2 V^{-1} s^{-1}$

$\Rightarrow$ Resistivity $\rho = \dfrac{1}{n_i e (\mu_e + \mu_n)}$

Here,
$\mu_e = \mu_n$
$= 0.35 \ m^2 V^{-1} s^{-1}$
$n_i = 2.5 \times 10^{17} \ m^{-2}$

$$= \frac{1}{2.5 \times 10^{17} \times 1.6 \times 10^{-19} \times (2 \times 0.35)}$$

$$= 35.71 \ \Omega m$$

(19) A small current of 1.1 mA exists in a copper wire of diameter 2.1 cm. Compute drift velocity.

$I = nAV_d e$

$n = \frac{dN_A}{M}$

$= \frac{8920 \times 6.023 \times 10^{23}}{64}$

$= 8.4 \times 10^{25} m^{-3}$

$I = 1.1 \times 10^{-3} A$

$D = 2.1 \times 10^{-3} m$

$r = \frac{D}{2}$

$= \frac{2.1 \times 10^{-3}}{2} m$

$A = 4\pi r^2$

$= 1.38 \times 10^{-5} m^2$

$V_d = \frac{I}{nAe}$

$= \frac{1.1 \times 10^{-3}}{8.4 \times 10^{25} \times 1.38 \times 10^{-5} \times 1.6 \times 10^{-19}}$

$= 5.93 \times 10^{-6} ms^{-1}$

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(20) Compute free electrons per unit volume n and electron mobility μ for copper if its atomic weight is 63.54 kg kmol^{-1}, density 8960 kg m^{-3}. Velocity (rms) 1.6×10^6 m s^{-1} and electrical conductivity $6.5 \times 10^7 (\Omega m)^{-1}$

$$\Rightarrow \quad n = \frac{d N_0}{M} = \frac{8960 \times 6.023 \times 10^{26}}{63.54}$$

$$= 8.49 \times 10^{28} \text{ electrons } m^{-3}$$

$$\mu_e = \frac{\sigma_e}{n_e} = \frac{6.5 \times 10^7}{8.49 \times 10^{28} \times 1.6 \times 10^{-19}}$$

$$= 4.785 \times 10^{-3} \; m \Omega s^{-1}$$

(21) i) Calculate Einstein temperature given Einstein frequency as 9.5×10^{11} Hz.

ii) Calculate the frequency of Einstein oscillator for $\theta_E = 248$ k

Given $k_B = 1.38 \times 10^{-23}$ J and $h = 6.63 \times 10^{-34}$ J-s

i) ⇒ Einstein temperature

$$\theta_E = \frac{h\nu}{k_B} = \frac{6.63\times10^{-34}\times9.5\times10^{4}}{1.38\times10^{-23}}$$

$$= 45.64\,k$$

ii) ⇒ $\theta_E = \dfrac{h\nu}{k_B}$

$$\therefore \nu = \frac{\theta_E k_B}{h} = \frac{248\times1.38\times10^{-23}}{6.63\times10^{-34}}$$

$$= 5.16\times10^{12}\ Hz$$

(22) The Debye temperature of carbon (diamond structure) is 1840k. Calculate the specific heat per kmole for diamond at 25k. Aloso compute the highest lattice frequency involved in the Debye theory.

⇒ $C_V = 234R\left(\dfrac{T}{T_D}\right)^3$ | $T_D = 1840\,k$, $T = 25k$

$$= 234\times8.316\times\left(\frac{25}{1840}\right)^3$$

$$= 4.88\times10^{-3}\ Jk^{-1}\ mol^{-1}$$

Again

$$T_D = \frac{h\nu_D}{T}$$

$$\therefore \nu_D = \frac{T_D T}{h}$$

$$= \frac{1840 \times 25}{6.63 \times 10^{-34}}$$

$$= 6.938 \times 10^{37} \ Hz$$

(23) Show that average kinetic energy of a free electron at 0 k is $3/5 \, E_f$ where E_f is Fermi energy and average speed is $3/4 \, V_f$ where V_f is the velocity at Fermi surface

$\Rightarrow$ Average K.E of an electron is given

$$\overline{E}_0 = \frac{1}{N} \int_0^{\infty} E \, g^{(E)} f(E) \, dE$$

substituting the value of $g(E)$ and using the result $f(E) = 1$ for $E > E_{f(0)}$ and $f(E) = 0$ for $E > E_{f(0)}$ at $T = 0° k$, we get

$$\bar{E}_0 = \frac{1}{N}\int_0^{E_{f(0)}} E \cdot CE^{1/2}\left[dE + \frac{1}{N}\int_{E_{f_0}}^{\alpha} E\cdot CE^{1/2} \cdot 0 \cdot dE\right]$$

$$= \frac{3}{5}\frac{C}{N} E_{f(0)}^{5/2}$$

Now,

$$CE_{f(0)}^{1/2} = g\{E_{f(0)}\} \text{ and } g\{E_{f(0)}\} = \frac{3}{2}\frac{N}{E_{f(0)}}$$

Combining the two we, get,

$$CE_{(f_0)}^{1/2} = 3/2 \times \frac{N}{E_{f(0)}} \quad \text{or} \quad \frac{C}{N} = \frac{3}{2} \cdot \frac{1}{E_{f(0)}^{3/2}}$$

$$\therefore \bar{E}_0 = \frac{2}{5} \cdot \frac{3}{2} \cdot \frac{1}{E_{f(0)}^{3/2}} \cdot E_{f(0)}^{5/2}$$

$$\therefore \bar{E}_0 = \frac{3}{5} E_{f(0)}$$

Average kinetic energy of a free electrone at 0K is $3/5\, E_F$

Average speed,

The amplitude of the wave vector is not the fermi surface, $k_f = \left(\frac{3\pi^2 N}{V}\right)^{1/3}$

V_f is the velocity of the electron atom of the fermi surface (level)

(24) Consider silver in the metallic state with one free electron per atom. Calculate the Fermi energy and given density of silver is $10.5\ gm \cdot cm^{3}$ and atomic weight 108.

=> The concentration of conduction electrons,

$$n = \frac{N}{V} = \frac{N\rho}{M}$$

$$= \frac{6.023 \times 10^{23} \times 1.05 \times 10^{3}}{108}$$

$$= 5.85 \times 10^{25}\ kg\,m^{-3}$$

$$\rho = 10.5\ gm/cc$$

$$= \frac{10.5 \times 10^{-3}}{10^{-6}}\ kg\,m^{-3}$$

$$= 10.5 \times 10^{3}\ kg\,m^{-3}$$

Fermi vector

$$k_f = \left(\frac{3\pi^2 N}{V}\right)^{1/3}$$

$$= (3\pi^2 n)^{1/3}$$

$$= (3\pi^2 \times 5.85 \times 10^{25})^{1/3}\ m^{-1}$$

$$= (1.73 \times 10^{27})^{1/3}\ m^{-1}$$

$$= 1.2 \times 10^{9}\ m^{-1}$$

(25) Aluminum metal crystallizes in fcc structure. If each contributes single electron as free electron and the lattice constant as free electron Fermi gas

i) Fermi energy (E_f) and Fermi vector (k_f)

ii) total kinetic energy of free electron gas per unit volume at 0K.

[$\hbar = 1.054 \times 10^{-27}$ erg-sec, Electron rest mass $= 9.81 \times 10^{-28}$ gm]

i) $k_f = \left(3\pi^2 \times \frac{N}{V}\right)^{1/3}$

$$= \left(3\pi^2 \times \frac{6.023 \times 10^{23}}{(4 \times 10^{-10})^3}\right)^{1/3}$$

$$= 6.53 \times 10^{18}$$

$$V_f = \frac{h k_f}{2\pi m}$$

$$= \frac{6.63 \times 10^{-34} \times 6.53 \times 10^{18}}{2\pi \times 9.11 \times 10^{-31}} = 7.56 \times 10^{14} \text{ m/s}$$

$$E_f = \frac{1}{2} m V_f^2$$

$$\frac{1}{2} \times 9.11 \times 10^{-31} \times (7.56 \times 10^{14})^2$$

$= 0.26\ J = 1.629 \times 10^{8}\ eV$

ii) Total kinetic energy $= \frac{3}{5} E_f$

$= \frac{3}{5} \times 1.629 \times 10^{18}\ eV$

$= 9.77 \times 10^{17}\ eV$

(26) Copper has a mass density $\rho_m = 8.9\ gm/cm^3$ and an electrical resistivity $\rho = 1.66 \times 10^{-8}$ Ω-m at room temperature. Calculate

i) The fermi energy (E_f)

ii) The concentration of the electrons

iii) The mean free time τ

iv) The fermi velocity V_f and

v) The mean free path λ_f at fermi level

⇒ We know,

$$\rho = \frac{M}{V}$$

$$\therefore V = \frac{M}{\rho_m} = \frac{63.5}{8900} = 7.13 \times 10^{-3}\ m^3$$

The concentration of the conduction electrons,

$$n = \frac{N}{V} = \frac{6.023 \times 10^{26}}{7.13 \times 10^{-3}} = 8.442 \times 10^{28} m^{-3}$$

The mean free time, $\tau = \frac{m_e}{ne^2 \rho}$

$$= \frac{9.1 \times 10^{-31}}{8.442 \times 10^{28} \times (1.6 \times 10^{-19})^2 \times 1.66 \times 10^{-8}}$$

$$= 2.535 \times 10^{-14} s$$

Fermi vecton $k_f = \left(3\pi^2 \times \frac{N}{V}\right)^{1/3}$

$$= \left(3\pi^2 \times 8.442 \times 10^{28}\right)^{1/3}$$

$$= 1.352 \times 10^{14}$$

The fermi velocity $v_f = \frac{h k_f}{2\pi m^2}$

$$= \frac{6.63 \times 10^{-34} \times 1.352 \times 10^{10}}{2\pi \times 9.1 \times 10^{-31}}$$

$$= 1.52 \times 10^{6} m s^{-1}$$

The fermi energy $= E_f = \frac{1}{2} m_e v_f^2$

$$= \frac{1}{2} \times 9.1 \times 10^{-31} \times (1.52 \times 10^{6})^2$$

$$= 1.121 \times 10^{-18} J$$

The mean free path, $\lambda_f = V_f \tau$

$= 1.52 \times 10^6 \times 2.535 \times 10^{-14}$

$= 3.98 \times 10^{-8}$ m

(22) The inter collision time in copper 2.6×10^{-14} s calculate its thermal conductivity at 30k.

$\tau = 2.6 \times 10^{-14}$ s

$T = 310$ k

$d = 8960$ kg m^{-3}

$N = 6.023 \times 10^{26}$ mole m^{3}

$\Rightarrow \quad n = \dfrac{Nd}{M}$

$= \dfrac{6.023 \times 10^{26} \times 8966}{63.5}$

$= 8.499 \times 10^{28}$ m^{-3}

$\sigma = \dfrac{ne^2\tau}{m} = \dfrac{8.499 \times 10^{28} \times (1.6 \times 10^{-19})^2 \times 2.6 \times 10^{-14}}{9.1 \times 10^{-31}}$

$= 6.2 \times 10^7$

~~$k = 3\rho$~~ $k = 3\sigma \left(\dfrac{k}{e}\right)^2 T$

$= 3 \times 6.2 \times 10^7 \times \left(\dfrac{1.38 \times 10^{-23}}{1.6 \times 10^{-19}}\right) \times 310$

$= 4.29 \times 10^{2}\ W m^{2} k^{-1}$

(28) A copper wire of length 0.5 m & diameter 0.3 mm has a resistance $0.11\ \Omega$ at $20°C$. If the thermal conductivity of copper at $20°C$ is 390 $Wm^{-1}k^{-1}$, calculate lorents number. Compare this value with the value predicted by a classical free electron theory.

(30) Calculate the inter collision time at room temperature and drift velocity in a field of 100 V m⁻¹ in sodium, where conductivity is $2.16 \times 10^{7} \Omega^{-1} m^{-1}$

$\Rightarrow$

$$n = \frac{Nd}{M} = \frac{6.023 \times 10^{23} \times 0.92 \times 10^{3}}{23 \times 10^{-3}}$$

$$= 2.54 \times 10^{28} \ m^{-3}$$

$$\sigma_e = n e \mu_e$$

$$\therefore \mu_e = \frac{\sigma_e}{ne} = \frac{2.16 \times 10^{7}}{2.54 \times 10^{28} \times 1.6 \times 10^{-19}}$$

$$= 5.31 \times 10^{-3} \ m^2 V^{-1} s^{-1}$$

$$v_d = \mu_e E = 5.31 \times 10^{-3} \times 100 = 0.531 \ ms^{-1}$$

$$\tau = \frac{\mu_e m}{e} = \frac{5.31 \times 10^{-3} \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19}}$$

$$= 3.02 \times 10^{-14} \ s$$

(31) The density of states function for electrons in a metal is given by $Z(E)dE = 13.6\,E^{1/2}\,dE$. Calculate the Fermi level at a room temperature a few degrees above absolute zero for sodium which has 2.3×10^{28} electrons per cubic meter.

$$\Rightarrow E_{f(0)} = \frac{h^2}{2m}\left(\frac{3n}{8\pi}\right)^{1/3}$$

$$= \frac{(6.63 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31}} \times \left(\frac{3 \times 2.3 \times 10^{28}}{8\pi}\right)^{1/3}$$

$$= 3.38 \times 10^{-27}$$

Again

$$\int Z(E)\,dE = \int 13.6\,E^{1/2}\,dE$$

$$\Rightarrow (f) = 13.6\,\frac{V}{2\pi^2}\left(\frac{2m}{h^2}\right)^{3/2} E_{f(0)}^{1/2}$$

$$= 13.6 \times \frac{1}{2(3.1416)^2} \times \left[\frac{2 \times 9.1 \times 10^{-34}}{(6.63 \times 10^{-34})^2}\right]^{3/2}$$

$$\times 1.84 \times 10^{-14}$$

$$= 1.0627 \times 10^{41}$$

(32) Show that if the mean free path is independent of the velocity, the electrical conductivity of Maxwell-Boltzmann free electron gas may expressed by the relation $\sigma = \frac{4ne^2\lambda}{3\sqrt{2\pi kTm}}$

when γ and λ are independent of velocity, the Maxwell-Boltzmann distribution gives average value of J as $\bar{J} = \frac{\lambda(v)}{v^2}$

$\Rightarrow$ $\frac{1}{2}m(v^2) = \frac{3}{2}kT$

$\therefore v^2 = \frac{3kT}{m}$

Hence $\bar{V} = \frac{\lambda\bar{c}^2}{3c^-}\left(\frac{m}{kT}\right)$

But $\bar{c}^2 = \frac{8kT}{\pi m}$

$v^2 = \frac{8\lambda}{3\pi c}$

The electrical conductivity is given by

$$\sigma = \frac{ne^2\bar{V}}{m} = \frac{8\lambda ne^2}{8\pi\bar{c}m} = \frac{8\lambda ne^2}{3\pi m\sqrt{\frac{8kT}{\pi m}}}$$

$$= \frac{8ne^2\lambda}{3.2\sqrt{2\pi k T M}}$$

[showed]

(33) Electrical resistivity of copper and Ni Nickel at room temperature are 1.65×10^{-3} and $1.4\times10^{-8}\ \Omega\cdot m$ respectively. If were mechanical treatment of - Fronz law applis to these materials, find the electronic contributions to the thermal conductivities of these materials

$$\Rightarrow \quad \frac{\sigma_c}{\sigma_n} = \frac{k_c}{k_n} = \frac{1.65\times10^{-3}}{14\times10^{-8}} = \frac{82500}{7}$$

$$\therefore \sigma_c : \sigma_n = 82500 : 7$$

(34) Find the mobility of electrons in copper assuming that each atom contributes one free electron for conduction. For Cu, resistivity $= 1.7\times10^{-6}$ Ω-cm. $d = 8.9$ gm/cm^3, atomic weight $= 63.5$ and $N = 6.02\times10^{23}$ /gm-mole

$\Rightarrow R_H = \dfrac{M}{d N_e} = \dfrac{63.5}{8.9 \times 6.023 \times 10^{23} \times 1.6 \times 10^{-19}}$

$= 7.4 \times 10^{-5}$

$\mu_e = \dfrac{R_H}{n} = \dfrac{7.4 \times 10^{-5}}{1.7 \times 10^{-6}} = 43.529.$

(35) Find the Hall co-efficient & electron mobility for Ge if for a given sample [length 1cm, thickness 1mm] 1.5V supply develops a hall voltage 20mV across at 0.45 wb/m²

$\Rightarrow$ Resistance, $\rho = \dfrac{RA}{L}$

$= 0.12 \Omega m$

Resistance of specimen

$R = \dfrac{V}{I} = \dfrac{1.5}{5 \times 10^{-3}} \Omega$

$= 300 \Omega$

Again Hall field,

$E_y = \dfrac{V_y}{\text{thickness}} = \dfrac{20 \times 10^{-3}}{1 \times 10^{-3}}$

$= 20$

Given that

$A = 4 \times 10^{-6} m^2$

$l = 1 \times 10^{-2} m$

$V_y = 20 \times 10^{-3} V$

$I = 5 \times 10^{-3} A$

$B = 0.45 \, wb/m^3$

$V = 1.5 V$

We know,

$$\frac{1}{ne} = \frac{E_y}{BJ} = \frac{E_y}{\frac{BI}{A}}$$

$$\Rightarrow \frac{1}{ne} = \frac{20 \times 4 \times 10^{-6}}{0.45 \times 10^{-3}} = 0.035 \ m^3/coulomb$$

Hall co-efficient, $R_H = \frac{3\pi}{8} \times \frac{1}{ne}$

$$= \frac{3\pi}{8} \times 0.035$$

$$= 0.0412 \ m^3/coulomb$$

Electron mobility, $\mu_e = \frac{R_H}{\rho} = \frac{0.0412}{0.12}$

$$= 0.343 \ m^2/volt/sec$$

(36) A copper strip 2.25 cm wide & 1.15 mm thick is placed in a magnetic field with $B = 1.75 \ wb/m^2$ perpendicular to the strip. If a current of 220 mA is set up in the strip, what Hall potential difference appears across the strip?

$\Rightarrow$ We know, Number of electrons per unit volume

Given that $d = 8.9 \times 10^3$ kg/m^3

$M = 64 \times 10^{-3}$ kg/mol

$$n = \frac{N_A d}{M} = \frac{6.023 \times 10^{23} \times 8.9}{64}$$

$$= 8.47 \times 10^{22} \text{ electrons/cm}^3$$

$$= 8.47 \times 10^{28} \text{ electrons/m}^3$$

Again Hall potential, $V_H = \frac{BI}{net}$

$$= \frac{1.75 \times 220 \times 10^{-3}}{8.47 \times 10^{28} \times 1.60 \times 10^{-19} \times 1.15 \times 10^{-3}}$$

$$= 2.47 \times 10^{-8} \text{ V}$$

$$= 24.7 \text{ nV}$$

37) Find the Hall co-efficient & electron mobility for Germanium if for a given sample [length 1cm, breadth 3mm & thickness 1mm] a current of 4.2 mA flown from a 1.5 volts supply develops a Hall voltage 20mV across the specimen in a magnetic field of

0.45 wb/m3

$\Rightarrow$ Resistance, $R = \frac{V}{I}$

$= \frac{1.5}{4.2 \times 10^{-3}}$

$= 357.14 \Omega$

$A = b \times d$

$= 3 \times 1 \times 10^{-3} \times 10^{-3}$

m^2

Resistivity $\rho = \frac{RA}{l}$

$= \frac{357.14 \times 3 \times 10^{-3} \times 1 \times 10^{-3}}{1 \times 10^{-2}}$

Hall field,

$E_y = \frac{V_y}{d} = \frac{20 \times 10^{-3}}{1 \times 10^{-3}} = 20$

we know,

$\frac{1}{ne} = \frac{E_y}{B_J} = \frac{A \cdot E_y}{BI}$

$= \frac{3 \times 10^{-6} \times 20}{0.45 \times 4.2 \times 10^{-3}}$

$= 0.032$ m³/coulomb

Hall co-efficient,

$$R_H = \frac{3\pi}{8} \times \frac{1}{ne} = 0.0377\ m^3/coulomb$$

Electron mobility, $\mu_e = \frac{R_H}{\rho}$

$$= \frac{124.67}{0.107} = 0.352\ m^2V^{-1}s^{-1}$$

(38) A semiconductor slice is 2mm thick & carries a current of 0.22 A, when a magnetic field of flux density 0.52 wb/m² is applied. If the hall voltage developed is 8.2 mV find the charges per m³. given, $e = 1.60 \times 10^{-19}$ C

$$\Rightarrow V_H = \frac{BI}{ne\,l} = \frac{0.52 \times 0.22}{n \times 1.60 \times 10^{-19} \times 2 \times 10^{-3}}$$

$$V_H = 8.2 \times 10^{-3}\ V$$

$$n = \frac{BI}{V_H\, e\, l} = 4.36 \times 10^{22}\ m^{-3}$$

$B = 0.52\ wb/m^2$

$I = 0.22\ A$

$n = ?$

$l = 2 \times 10^{-3}\ m$

(39) In case of a metal strip of thickness 1mm, when a magnetic field of 1 Tesla is applied, a current of 12A flows through the strip. Determine the magnetic flux density. if a Hall voltage of 0.65 μV is developed.

$e = 1.60 \times 10^{-19}$ C

$t = 1 \times 10^{-3}$ m

$I = 12$ A

$V_H = 0.65 \times 10^{-6}$ V

$n = 10^{29}$ m^{-3}

$B = 1$ T

$= 1$ wb/m^2

$$V_H = \frac{BI}{ne t}$$

$$\Rightarrow B = \frac{V_H\, n e t}{I}$$

$$= \frac{0.65 \times 10^{-6} \times 10^{29} \times 1.60 \times 10^{-19}}{12}$$

$$= 0.87 \text{ wb/m}^2$$

(40) A LASER beam has a wavelength of 8.6×10^{-7} m and aperture 5.4×10^{-3} m. the LASER beam is sent to moon is 4×10^5 km from the earth. Calculate

i) the angular spread of the beams

ii) the areal spread when it reaches the moon

i) $d\theta = \frac{\lambda}{d} = \frac{8.6\times10^{-8}}{5.4\times10^{-5}} = 1.59\times10^{-4}$ radian

ii) The areal spread $= (D\,d\theta)^2$

$= (4\times10^{8} \times 1.59\times10^{-4})^2$

$= 4.04\times10^{9}\ m^2$

(41) The coherence length for sodium light is 2.9×10^{-2} m. The wave length of sodium light is 5896 Å. Calculate
i) the number of oscillation corresponding to the co-herence length and
ii) the coherence time.

i) $n = \frac{L}{\lambda} = \frac{2.9\times10^{-2}}{5896\times10^{-10}} = 4.92\times10^{4}$

ii) $t_e = \frac{L}{c} = \frac{2.9\times10^{-2}}{3\times10^{8}}$

$= 9.67\times10^{-11}$ s

(42) A LASER beam $\lambda = 5890\,\mathring{A}$ on earth is focused by a lens (or mirror) of diameter 2 m on the crater on the moon. The distance of the moon is 4×10^8 m How big is the spot on the moon?

=>

i) Angular spread $d\theta = \frac{\lambda}{d} = \frac{5890 \times 10^{-10}}{2}$

$= 3054 \times 10^{-10}$ radian

ii) Area spread, $A = (D d\theta)^2$

$= (4 \times 10^8 \times 3054 \times 10^{-10})^2$

$= 1.5 \times 10^4 \, m^2$

(43) A LASER beam has a power of 75 mW. If it has an aperture of 5×10^{-3} m and it emits light of wavelength $6000\,\mathring{A}$. The beam of focused with a lens of focused with a lens of focal length 0.16 m calculate the area and the intensity of the image

$$\Rightarrow r = \frac{d}{2} = \frac{4\lambda f}{2\pi d}$$

$$= \frac{4 \times 6000 \times 10^{-10} \times 0.16}{2\pi \times 5 \times 10^{-3}}$$

$$= 1.22 \times 10^{-5} m$$

$P = 25 \times 10^{-3} W$

$d = 5.0 \times 10^{-3} m$

$\lambda = 6000 \times 10^{-10} m$

$f = 0.16 m$

$$A = \pi r^2 = \pi \times (1.22 \times 10^{-5})^2$$

$$= 4.676 \times 10^{-10} m^2$$

$$I = \frac{P}{A} = \frac{25 \times 10^{-3}}{4.676 \times 10^{-10}} = 1.6 \times 10^{8} \ Wm^{-2}$$

(44) The coherence length for the red cadmium line of wavelength $6.4 \times 10^{-7} m$ is 30 cm. Calculate

i) the number of oscillation corresponding to the coherence length and

ii) the coherence time.

$$\Rightarrow \text{i)} \ n = \frac{L}{\lambda} = \frac{30 \times 10^{-2}}{6.4 \times 10^{-7}} = 4.69 \times 10^{5}$$

$$\text{ii)} \ t = \frac{L}{c} = \frac{30 \times 10^{-2}}{3 \times 10^{8}} = 1 \times 10^{-9} S$$

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(45) A laser beam has a power of 60 mW. It has an aperture of 5×10^{-3} m and it emits light of wavelength 7200 Å. The beam is focused with a lens of focal length 0.1 m. Calculate the area and the ~~i~~ intensity of the image.

$$\Rightarrow r = \frac{4\lambda f}{2\pi d} = \frac{4 \times 7200 \times 10^{-10} \times 0.1}{2\pi \times 5 \times 10^{-3}} = 9.167 \times 10^{-6}\,m$$

$$A = \pi r^2 = \pi \times (9.167 \times 10^{-6})^2 = 2.64 \times 10^{-10}\,m$$

$$I = \frac{P}{A} = \frac{60 \times 10^{-3}}{2.64 \times 10^{-10}} = 2.27 \times 10^{8}\,W m^{-2}$$

(46) A 8 kW LASER emits light of 650 nm wavelength. Calculate the number of photon emitted by the laser every second.

$$\Rightarrow n = \frac{E}{} = \frac{E\lambda}{} = \frac{8 \times 10^{3} \times 650 \times 10^{-9}}{}$$

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(47) Find the increase in the relative population of lasing levels of He-Ne LASER when the temperature is increased from 300k to 1000k. Given the Boltzman constant is 1.38×10^{-23} JK^{-1}

(48) Line width of a commercial LASER is 22 kHz. Find its coherence length

⇒ We know,

LASER line width $\Delta\nu = \frac{c}{\lambda L}$

$$\Rightarrow L = \frac{c}{\lambda \Delta\nu} = \frac{3 \times 10^{8}}{22 \times 10^{3} \times \lambda}$$

$$= 4340.59 \text{ m}$$

Co-herence time, $t_c = \frac{L}{c} = \frac{4340.59}{3 \times 10^{8}}$

$$= 1.45 \times 10^{-5} s$$

(49) A beam has a power of 0.25 watt and has an aperture of 1 mm. It emits light of wavelength 6000 Å. If it is focused by a lens of f.l. 80cm calculate the area and intensity of the image.

$$\Rightarrow r = \frac{4\lambda f}{2\pi d} = \frac{4 \times 6000 \times 10^{-10} \times 80 \times 10^{-2}}{2 \times \pi \times 1 \times 10^{-3}}$$

$$= 3.056 \times 10^{-4} m$$

$$A = \pi r^2 = \pi (3.056 \times 10^{-4})^2 = 2.93 \times 10^{-7} m^2$$

$$I = P/A = \frac{0.25}{2.93 \times 10^{-7}} = 8.53 \times 10^{5} \text{ watt } m^{-2}$$

(50) The coherence length of sodium D_2 line is 2.5 cm. Deduce the (i) coherence time (ii) spectral width of line and (iii) quality factor. Given λ = 5890 Å

$\Rightarrow$

$$\text{i) } t = \frac{L}{c} = \frac{2.5 \times 10^{-2}}{3 \times 10^{8}} = 8.33 \times 10^{-11} s$$

$$\text{ii) } \Delta\nu = \frac{c}{\pi L} = \frac{3 \times 10^{8}}{\pi \times 2.5 \times 10^{-2}} = 3.82 \times 10^{9} s^{-1}$$

$$\text{iii) } L = Q\lambda$$

$$Q = \frac{L}{\lambda} = \frac{2.5 \times 10^{-2}}{5890 \times 10^{-10}} = 4.24 \times 10^{4}$$

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