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CSE , KUET

SOLID STATE PHYSICS

SOLUTION

1. Copper has fcc structure and the atomic radius is 1.16A°. Calculate its density. Atomic weight of copper = 63.4.

Given that,

$r = 1.16\,A^\circ = 1.16 \times 10^{-8}$ cm

$M = 63.6$

for fcc structure, $n = 4$

We know that, the lattice constant for fcc is,

$a = r 2\sqrt{2}$

$= 1.16 \times 10^{-8} \times 2\sqrt{2}$

$= 3.2 \times 10^{-8}$ cm

Now, we know,

$$\rho = \frac{nM}{a^3 N}$$

$$= \frac{4 \times 63.6}{(3.2 \times 10^{-8})^3 \times 6.023 \times 10^{23}}$$

$= 12.89\ g/cm^3$ Ans:

Question no 2:

Given,

$\rho = 6250\ kg/m^3$

$M = 60.4$

$N = 6.02 \times 10^{26}\ kg.mole^{-1}$

$n =$ For fcc structure $= 2$

$a = ?$

We know,

$$a^3 \rho = \frac{nM}{N}$$

$\therefore$ lattice constant,

$$a = \sqrt[3]{\frac{nM}{N\rho}}$$

$$\Rightarrow a = \sqrt[3]{\frac{4 \times 60.4}{6250 \times 6.02 \times 10^{26}}}$$

$\Rightarrow a = 4 \times 10^{-10}$ m

$\therefore a = 4A^\circ$ Ans:

2. A substance with fcc lattice has density 6250kg/m³ and molecular weight 60.4. Calculate the lattice constant a. Given Avogadro's number = 6.02×10^{26}kg-mole^{-1}.

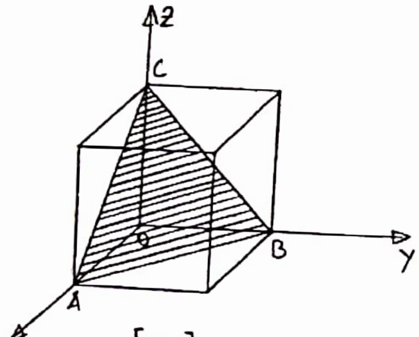
5. *Sketch the* [111], [110], [100], [101], [211], [221] & [211].

Question no 05:

A plane whose Miller indices are [111] has the following intercepts on the three axes,

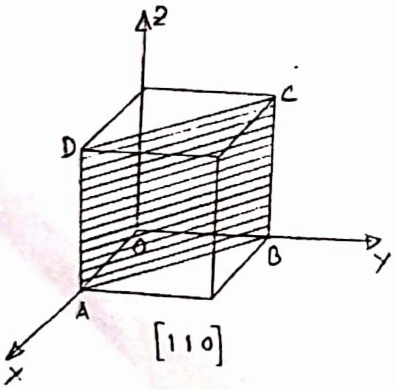
$$\frac{1}{1}, \frac{1}{1}, \frac{1}{1}$$

This plane is sketched in the following figure,



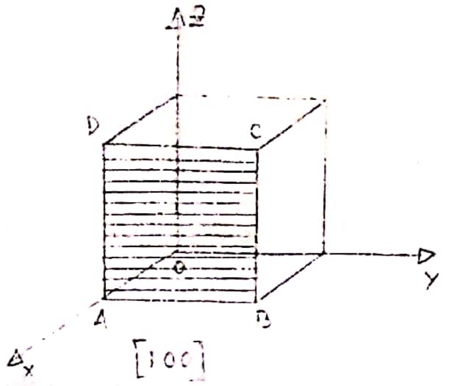
[111]

Similarly, for the plane [110] the intercepts are, $\frac{1}{1}, \frac{1}{1}, \frac{1}{0}$.
This plane is sketched in the following figure,



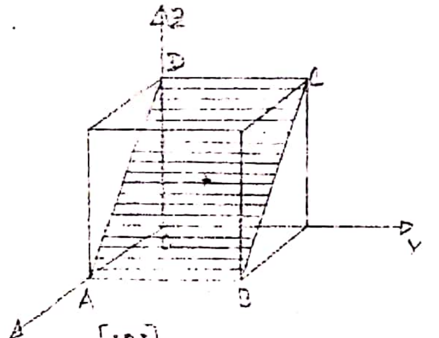
[110]

Similarly, for the plane [100] the intercepts are $\frac{1}{1}, \frac{1}{0}, \frac{1}{0}$
This pplane is sketched in the following figure.



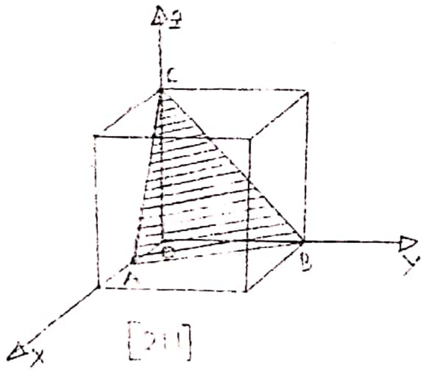
[100]

Similarly for the plane [101] the intercepts are, $\frac{1}{1}, \frac{1}{0}, \frac{1}{1}$
This plane is sketched in the following figure.



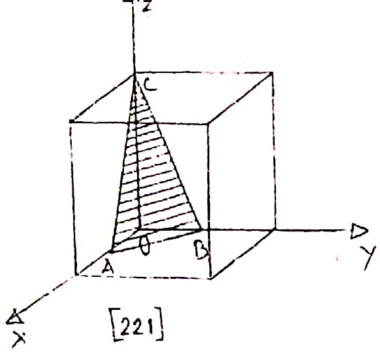
[101]

Similarly for the plane [211] the intercepts are, $\frac{1}{2}, \frac{1}{1}, \frac{1}{1}$
This plane is sketched in the following figure.



[211]

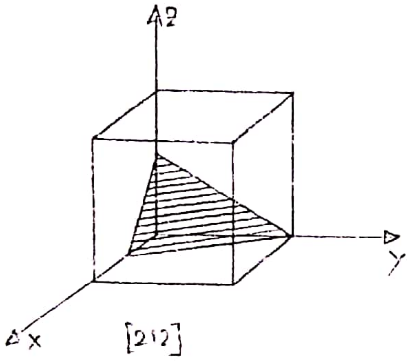
Similarly for the plane [221] the intercepts are $\frac{1}{2}, \frac{1}{2}, \frac{1}{1}$
This plane is sketched in the following figure.



[221]

Similarly, for the plane [212] the intercepts are, $\frac{1}{2}, \frac{1}{1}, \frac{1}{2}$

This plane is sketched in the following figure,



[212]

Question no 06:

Given, $r = 1.74 A^{\circ} = 1.74 \times 10^{-10} m$

We know, the interplaner spacing is given by,

$$d = \frac{a}{\sqrt{h^2+k^2+l^2}}$$

and, lattice constant of the unit cell of lead (fcc) is,

$$a = r 2\sqrt{2}$$
$$= 1.74 \times 2\sqrt{2}$$
$$= 4.92 A^{\circ}$$

6. Lead is fcc with an atomic radius of $r = 1.74A^{\circ}$. Find the spacing of (i) [202] planes, (ii) [111] planes (iii) [222] planes & (iv) [100] planes.

i) For [202] plane $h=2, k=0, l=2$

$$\therefore d_{202} = \frac{4.92}{\sqrt{2^2+0^2+2^2}} = 1.74\ A^\circ$$

ii) $$d_{111} = \frac{4.92}{\sqrt{1^2+1^2+1^2}} = 2.84\ A^\circ$$

iii) $$d_{222} = \frac{4.92}{\sqrt{2^2+2^2+2^2}} = 1.42\ A^\circ$$

iv) $$d_{100} = \frac{4.92}{\sqrt{1^2+0^2+0^2}} = 4.92\ A^\circ$$

Ans:

Question no 07:

The angle between the two normals can be found by the relation.

$$\cos\theta = \frac{u_1u_2+v_1v_2+w_1w_2}{\sqrt{u_1^2+v_1^2+w_1^2}\ \sqrt{u_2^2+v_2^2+w_2^2}}$$

$\therefore$ i) $$\cos\theta = \frac{1\times1+1\times1+1\times0}{\sqrt{1^2+1^2+1^2}\ \sqrt{1^2+1^2+0^2}} = 0.82$$

$$\therefore \theta = \cos^{-1}(0.82) = 35.26^\circ$$

ii) $$\cos\theta = \frac{1\times1+2\times1+1\times1}{\sqrt{1^2+2^2+1^2}\ \sqrt{1^2+1^2+1^2}} = 0.94$$

$$\therefore \theta = \cos^{-1}(0.94) = 19.47^\circ$$

iii) $$\cos\theta = \frac{2\times0+2\times1+0\times1}{\sqrt{2^2+2^2+0^2}\ \sqrt{0^2+1^2+1^2}} = 0.5$$

$$\therefore \theta = \cos^{-1}(0.5) = 60^\circ$$

Ans:

7. In a unit cell of simple cubic structure. find the angle between the normal to pair of planes whose Miller indices are (i) [111] & [110] (ii) [121] & [111] and (iii) [220] & [011].

8. The orthorhombic crystal has axial units in the ratio of 0.424:1:0.367. Find the Miller indices of crystal face whose *intercepts are in the ratio* 0.212:1:0.183.

Question no 08:

Here, axial units are $a:b:c = 0.424:1:0.367$

Now, $pa = 0.212$

$\Rightarrow p \times 0.424 = 0.212$

$\Rightarrow p = \frac{1}{2}$

Similarly,

$qb = 1$

$\Rightarrow q \times 1 = 1$

$\therefore q = 1$

also. $rc = 0.183$

$\Rightarrow r \times 0.367 = 0.183$

$\therefore r = \frac{1}{2}$

Here, numerical parameters of this plane are $\frac{1}{2}, 1, \frac{1}{2}$

$\therefore$ Miller indices $= \left(\frac{1}{\frac{1}{2}} \cdot \frac{1}{1} \cdot \frac{1}{\frac{1}{2}}\right) = [2\,1\,2]$ Ans:

9. Calculate the packing fraction in crystal for (i) s.c. (ii) b.c.c. and (iii) f.c.c. structures, treating the atoms as spherical.

Packing fraction:

i) For S.c :

In this case number of atoms per cube is one.

Volume of one atom $= \frac{4}{3}\pi r^3$

Now, atomic radius, $r = \frac{a}{2}$

$\therefore$ Volume occupied by the atom in the S.c unit cell. $v = 1 \times \frac{4\pi}{3} \times \left(\frac{a}{2}\right)^3$

$= \frac{\pi a^3}{6}$

Volume of the unit cell. $V = a^3$

$\therefore P.F = \frac{v}{V} = \frac{\frac{\pi a^3}{6}}{a^3} = \frac{\pi}{6} = 0.54 = 54\%$

ii) For B.c.c

ii) For B.c.c :

In this case number of atoms per cube is 2.

Atomic radius, $r = \frac{a\sqrt{3}}{4}$

$\therefore$ Volume occupied by the atoms in the B.c.c unit cell $v = 2 \times \frac{4\pi}{3}\left(\frac{a\sqrt{3}}{4}\right)^3$

$= \frac{\sqrt{3}}{8}\pi a^3$

Volume of the unit cell, $V = a^3$

$\therefore P.F = \frac{v}{V} = \frac{\sqrt{3}\pi a^3}{8a^3} = 0.68 = 68\%$

iii) For F.c.c :

In this case number of atoms per cube is 4.

Atomic radius, $r = \frac{a}{2\sqrt{2}}$

$\therefore$ Volume occupied by the atoms in the F.c.c unit cell. $v = 4 \times \frac{4\pi}{3}\left(\frac{a}{2\sqrt{2}}\right)^3$

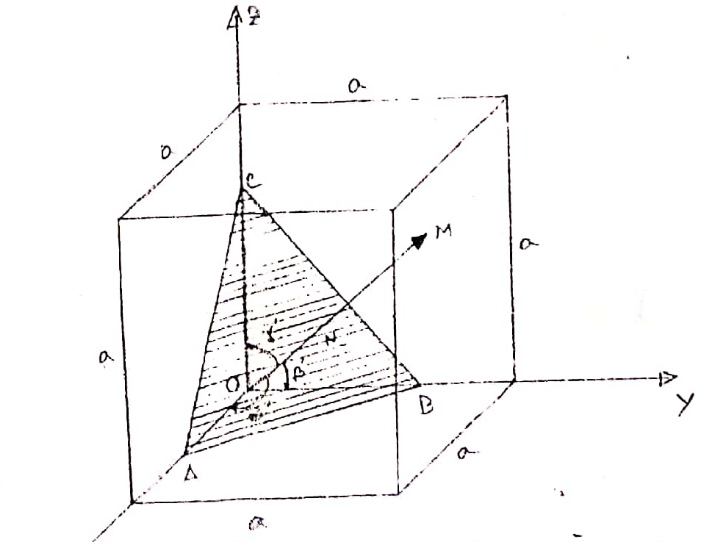
$= \frac{\pi a^3}{3\sqrt{2}}$

Volume of the unit cell, $V = a^3$

$\therefore P.F = \frac{v}{V} = \frac{\pi a^3}{3\sqrt{2}\, a^3} = 0.74 = 74\%$

Question no 10 :

Suppose that the plane shown in the following figure belongs to a family of planes whose Miller indices are <hkl>.



10. Show that spacing d of plane {hkl} in a simple cubic lattice of side a is $d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$

The perpendicular ON from origin to the plane represents the inter-planar spacing d of this family of planes. Let the direction cosines of ON be $\cos\alpha'$, $\cos\beta'$ and $\cos\gamma'$ as shown. The intercepts of the plane on the three axes are.

$OA = \frac{a}{h}$; $OB = \frac{a}{k}$; $OC = \frac{a}{l}$; where a is the length of the cube edge

$\cos\alpha' = \frac{d}{OA} = \frac{d}{a/h} = \frac{dh}{a}$; $\cos\beta' = \frac{d}{OB} = \frac{dk}{a}$; $\cos\gamma' = \frac{d}{OC} = \frac{dl}{a}$

where, d = ON perpendicular distance between adjacent members of the same family of planes.

Now, $\cos^2\alpha' + \cos^2\beta' + \cos^2\gamma' = 1$

$\Rightarrow \left(\frac{dh}{a}\right)^2 + \left(\frac{dk}{a}\right)^2 + \left(\frac{dl}{a}\right)^2 = 1$

$\Rightarrow \frac{d^2}{a^2}(h^2 + k^2 + l^2) = 1$

$\Rightarrow d^2 = \frac{a^2}{h^2 + k^2 + l^2}$

$\therefore d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$ [Showed]

Question no 11 : 11. The primitives of a crystal are 1.26A°, 1.82A° and 2.22A° along whose Miller indices [111] cut intercepts 1.4A° along x-axis. What will be the lengths of intercept along y-axis and z-axis.

Given,

Miller indices of the crystal [111]

Length of intercept along x-axis = 1.4 A°

So, h : k : l = 1 : 1 : 1

$\therefore \frac{1}{p} : \frac{1}{q} : \frac{1}{r} = h : k : l = 1 : 1 : 1$

$\therefore p : q : r = 1 : 1 : 1$

The primitives of the crystal are 1.26 A°, 1.82 A° and 2.22 A°

$\therefore pa : qb : rc = (1 \times 1.26) : (1 \times 1.82) : (1 \times 2.22)$

$= 1.26 : 1.82 : 2.22$

Therefore

Therefore.

$l_x : l_y : l_z = 1.26 : 1.82 : 2.22$

$$\therefore \frac{l_x}{l_y} = \frac{1.26}{1.82}$$

∴ length of intercept along x-axis. $l_y = \frac{1.82}{1.26} \times 1.4 = 2.022\ A^\circ$

again,

$$\frac{l_y}{l_z} = \frac{1.82}{2.22}$$

∴ length of intercept along z-axis. $l_z = \frac{2.22}{1.82} \times 2.022 = 2.47\ A^\circ$

Ans:

12. Show that for a crystal of cubic symmetry the direction [hkl] is perpendicular to the plane (hkl).

Let, OC be the direction of [hkl]

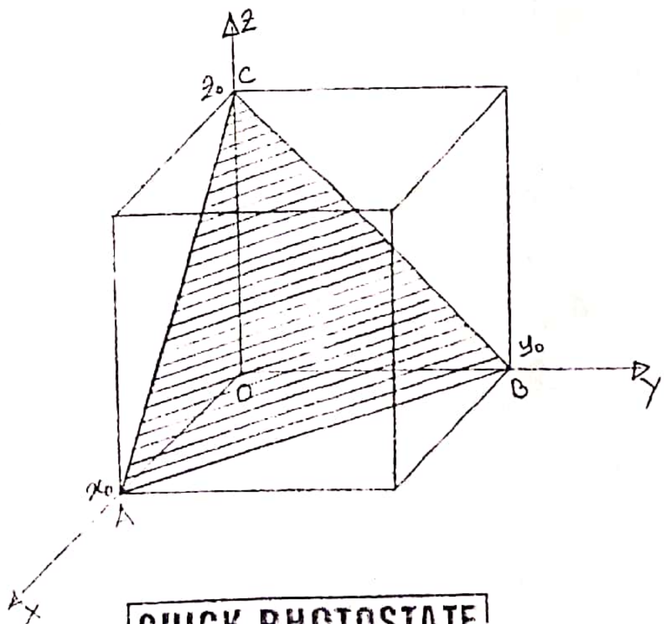
Let, ABC be the plane of [hkl]

Let, x_0, y_0, z_0 be the intercepts of the planes with x, y, z axes.

Miller indices are $\frac{1}{x_0}, \frac{1}{y_0}, \frac{1}{z_0}$

and reduced indices become $\frac{g}{x_0}, \frac{g}{y_0}, \frac{g}{z_0}$

Where g in an integer.



For cubic lattice $h=k=l$ or $x_0=y_0=z_0=\alpha$

And the direction, $\vec{OP} = \left(i\frac{g}{\alpha} + j\frac{g}{\alpha} + k\frac{g}{\alpha}\right)$

$$\vec{OA} + \vec{OB} = \vec{OR}$$

$$\vec{AB} = \vec{OB} - \vec{OA} = \left(j\frac{g}{\alpha} - i\frac{g}{\alpha}\right)$$

$$\vec{OP}.\vec{AB} = -\frac{g^2}{\alpha^2} + \frac{g^2}{\alpha^2} = 0$$

Similarly, we may prove,

$$\vec{OP}.\vec{BC} = 0$$

$$\vec{OP}.\vec{CA} = 0$$

Hence, dot product of $\vec{OP}$ with $\vec{AB}, \vec{BC}$ and $\vec{CA}$ is zero.

Hence $\vec{OP}$ is perpendicular to $\vec{AB}, \vec{BC}$ or $\vec{CA}$

Hence, [h, k, l] is normal or perpendicular to plane [hkl]

[Proved]

13. **Calculate the maximum phonon frequency generated by scattering of visible light of wavelength λ = 4000A^0. Given that velocity of sound in medium is 5×10^5 cm/sec and refractive index is 1.5.**

The frequency of the phonon generated during the inelastic scattering of photon is given by,

$$\Omega = \frac{2 V_s \omega \mu}{c} \sin\frac{\phi}{2}$$

Where, V_s is the velocity of sound $= 5\times10^5$ cm/sec.

ω is the angular frequency. $\omega = 2\pi\nu = 2\pi\frac{c}{\lambda}$

Where, c is the velocity of light

λ is the wave length of incident radiation (photon) $= 4\times10^{-5}$ cm

μ is the refractive index of the medium = 1.5

ϕ is the scattering angle.

For Ω to be maximum, $\sin\phi/2 = +1$

$$\Omega_{max} = \frac{2V_s \cdot 2\pi c}{\lambda c}$$

$$= \frac{2\times5\times10^5\times2\times3.14}{4\times10^{-5}}$$

$$= 1.57\times10^{11} \text{ rad/sec}$$

Ans.

14. Compare the frequencies of sound waves of wavelength $\lambda = 10^{-7}$ cm for (a) a homogeneous line (b) acoustic waves, on a linear lattice atoms per primitive cell of interatomic spacing 2.5A° and (c) light waves of the same wavelength, given that $v_0 = 10^5$ cm/sec.

Question on 14:

(a) Frequency in case of homogeneous line is given by.

$$\omega = k\sqrt{\frac{c}{\rho}} = V_0 k$$

Where $V_0 = \sqrt{\frac{c}{\rho}}$ and $k = \frac{2\pi}{\lambda}$

$$\therefore \omega = 10^5 \times \frac{2\pi}{10^{-7}} = 2\pi \times 10^{12} \text{ rad/sec} \quad \text{Ans}$$

(b) For acoustic waves in a diatomic lattice the frequency varies from $\omega = 0$ for $k = 0$ to $\omega = \sqrt{(2c/M)}$ for $k = (2\pi/a)$

In case of diatomic lattice.

$$V_0 = \sqrt{\frac{2c}{M}} \cdot a$$

Where, c is force constant, M is mass of the particle and a the interatomic distance.

Thus we have,

$$\frac{V_0}{a} = \sqrt{\frac{2c}{M}}$$

$$\omega = \frac{V_0}{a} = \frac{10^5}{2.5 \times 10^{-8}} = 4 \times 10^{12} \text{ rad/sec} \quad \text{Ans}$$

(c) For light waves of wavelength 10^{-7}, the velocity $c = 3 \times 10^{10}$ cm/sec.

We have, $\omega = 2\pi\nu = 2\pi\frac{c}{\lambda}$

$$= \frac{2\pi \times 3 \times 10^{10}}{10^{-7}} = 6\pi \times 10^{17} \text{ rad/sec} \quad \text{Ans}$$

$$\omega = V_0 k = \frac{V_0}{a} = 2\pi f$$

$$= \sqrt{\frac{c}{\rho}} \times \frac{2\pi}{\lambda} = \sqrt{\frac{2c}{M}}$$

15 Show that the heat capacity of a monoatomic lattice in one dimension in the Debye approximation is proportional to $\frac{T}{\Theta}$ for low temperature $T \ll \Theta$, where Θ is the effective Debye temperature in one dimension defined as $\Theta = \frac{\hbar\omega}{k} = \frac{\hbar\pi C_s}{ka}$, where k is Boltzmann constant and a is the interatomic separation.

According to Debye approximation, the internal energy is given by,

$$E = \int_0^{\omega_m} [\text{No. of energy states per unit frequency range}] \times \frac{\hbar\omega}{\left[\exp\left(\frac{\hbar\omega}{kT}\right) - 1\right]}$$

No of energy states per unit frequency range in one dimension is given by,

$$= \frac{2L}{C_s} \cdot \frac{d\omega}{2\pi} = \frac{L}{\pi C_s} \cdot d\omega$$

$$\therefore E = \frac{L}{\pi C_s} \int_0^{\omega_m} \frac{\hbar\omega}{\exp\left[\left(\frac{\hbar\omega}{kT}\right) - 1\right]} \cdot d\omega$$

Let, $\frac{\hbar\omega}{kT} = x$ or $\omega = \frac{kT}{\hbar} x$ or $d\omega = \frac{kT}{\hbar} dx$

$$\therefore E = \frac{L}{\pi C_s} \cdot \frac{(kT)^2}{\hbar} \int_0^{x_m} \frac{x}{e^x - 1} \cdot dx$$

Since $L = Na$ we get, $E = \frac{Na(kT)^2}{\pi C_s \hbar} \int_0^{x_m} \frac{x}{e^x - 1} \cdot dx$

Given that, $\Theta = \frac{\hbar\pi C_s}{k.a}$, we get,

$$E = \frac{N.a(kT)^2}{\Theta.k.a} \int_0^{x_m} \frac{x}{e^x - 1} \cdot dx = N.k\,T\left(\frac{T}{\Theta}\right) \int_0^{x_m} \frac{x}{e^x - 1} \cdot dx$$

For low temperature, $T \ll \Theta$, $x \to \infty$ we get,

$$E = N.k.T.\left(\frac{T}{\Theta}\right) \int_0^{x_\infty} x.e^{-x}.dx$$

$$= N.k.T\left(\frac{T}{\Theta}\right)\left[x.e^{-x}\right]_0^{\infty} + \int_0^{\infty} e^{-x}.dx = N.k.T\left(\frac{T}{\Theta}\right).\left[e^{-x}\right]_0^{\infty}$$

$$= N.k.T\left(\frac{T}{\Theta}\right).1$$

$$\therefore E = N.k.T.\left(\frac{T}{\Theta}\right)$$

$$C_v = \frac{dE}{dT} = 2Nk.\left(\frac{T}{\Theta}\right)$$

$$\therefore C_v \propto \left(\frac{T}{\Theta}\right)$$

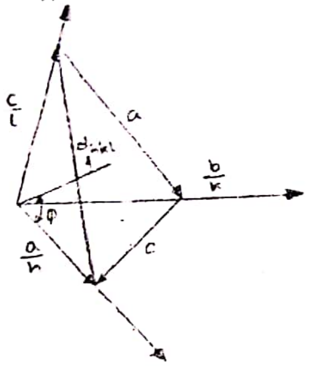
[Showed]

16 Show that every reciprocal lattice vector is normal to a lattice plane of the direct crystal lattice

Question no 16:

'Every reciprocal lattice vector is normal to a lattice plane of the crystal lattice'. In order to prove that σ_{hkl} is normal to the crystal plane (hkl) we will have to show that the scaler product of σ_{hkl} and vectors lying in the (hkl) plane vanish. The plane (hkl) shown in the figure intercepts a at a/h, b at b/k and c at c/l. Considering the vector

$$c = \frac{a}{h} - \frac{b}{k}$$



lying in this plane. The scaler product of c with σ_{hkl} is,

$$c \cdot \sigma_{hkl} = \left(\frac{a}{h} - \frac{b}{k}\right) \cdot (ha^* + kb^* + lc^*)$$

$$= \frac{a}{h}(ha^* + kb^* + lc^*) - \frac{b}{k}(ha^* + kb^* + lc^*)$$

$$= \left(\frac{h}{h} + 0 + 0\right) - \left(0 + \frac{k}{k} + 0\right)$$

$$= 1 - 1$$

$$= 0 \quad \text{——— (1)}$$

Similarly, the scaler product of a with σ_{hkl} vanishes also,

$$a \cdot \sigma_{hkl} = \left(\frac{b}{k} - \frac{c}{l}\right)(ha^* + kb^* + lc^*)$$

$$= \left(0 + \frac{k}{k} + 0\right) - \left(0 + 0 + \frac{l}{l}\right)$$

$$= 0 \quad \text{——— (2)}$$

Since according to (1) & (2) σ_{hkl} is normal to c & a, it is normal to the plane containing c and a that is to the plane (hkl)

17. Calculate the spacing between adjacent lattice plane of a crystal in terms of reciprocal space lattice.

Question no 17 :

In view of this $\hat{n}$. the unit vector normal to (hkl), is parallel to G_{hkl} so that we can write.

$$|G_{hkl}| = \hat{n} = ha^* + kb^* + lc^*$$

$$\Rightarrow \hat{n} = \frac{ha^* + kb^* + lc^*}{|G_{hkl}|}$$

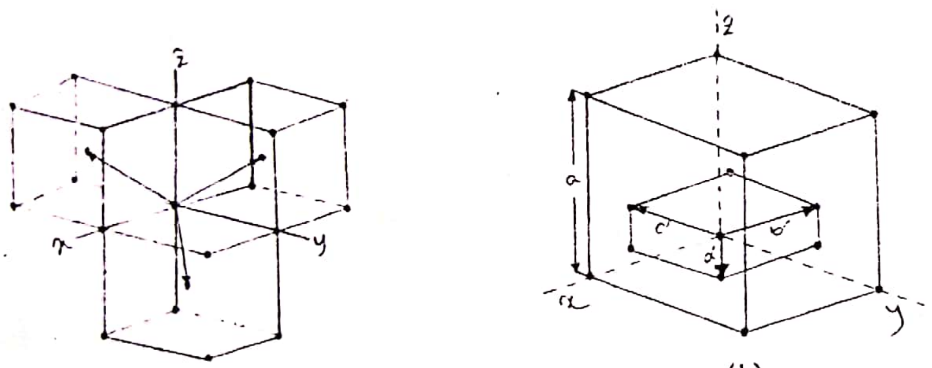
The length of the interplaner spacing of the plane is obviously.

$$d_{hkl} = \frac{a}{h}\cos\phi = \frac{a}{h}\cdot\hat{n} = \frac{a}{h}\cdot\frac{(ha^* + kb^* + lc^*)}{|G_{hkl}|} = \frac{1}{|G_{hkl}|}$$

Question no 18 :

Reciprocal lattice to b.c.c lattice:

The primitive translation vectors of b.c.c lattice as shown in figure are



(a) (b)

Show that (i) the sc reciprocal lattice is itself a simple cubic lattice constant $\frac{2\pi}{a}$ (ii) the bcc lattice is reciprocal lattice of fcc lattice and (iii) the fcc lattice is reciprocal lattice of bcc lattice with lattice constant $\frac{2\pi}{a}$.

$$a' = (a/2)(\hat{x} + \hat{y} - \hat{z})$$

$$b' = (a/2)(-\hat{x} + \hat{y} + \hat{z})$$

$$c' = (a/2)(\hat{x} - \hat{y} + \hat{z})$$

Where a is the side of the conventional unit cube and $\hat{x}, \hat{y}, \hat{z}$ are orthogonal unit vectors parallel to the cube edges. The volume of the primitive cell is.

$$V = |a' \cdot b' \times c'| = \frac{1}{2}a^3$$

Question no 19:

A harmonic oscillator may have energies given by an expression

$$E_n = n\hbar\omega$$

The zero point energy has been neglected in order to avoid non essential complications.

The probability for a given level n to be occupied is given by an expression.

$$\langle n \rangle = \frac{\sum_n n \cdot e^{-E_n/kT}}{\sum_n e^{-E_n/kT}} = \frac{\sum_n n \cdot e^{-n\hbar\omega/kT}}{\sum_n e^{-n\hbar\omega/kT}}$$

$$= \frac{-\frac{d}{d(\hbar\omega/kT)}\sum_{n=0}^{\infty} e^{-n\hbar\omega/kT}}{\sum_{n=0}^{\infty} e^{-n\hbar\omega/kT}} = \frac{e^{-\hbar\omega/kT}\left(1-e^{-\hbar\omega/kT}\right)^{-2}}{\left(1-e^{-\hbar\omega/kT}\right)^{-1}}$$

$$= \frac{1}{e^{\hbar\omega/kT}-1}$$

Since each molecule has three modes of vibration, hence we get.

$$\langle E \rangle = \frac{3N\hbar\omega}{e^{\hbar\omega/kT}-1}$$

At high temperatures. $E = 3NkT = 3nRT$, where n is the number of moles and R is the gas constant.

At low temperatures, $E = 3N\hbar\omega e^{-\hbar\omega/kT}$

Hence, at high temperatures. $C = \frac{1}{n} \cdot \frac{\delta E}{\delta T} = 3R \approx 6$ cal/mole.deg

At low temperatures, $C = 3R.\left(\frac{\hbar\omega}{kT}\right)^2 . e^{-\hbar\omega/kT}$

At high temperatures. the specific heat is in agreement with the empirical law of Dulong and Petit; at low temperatures $c \to 0$. in agreement with observation. However the temperature dependence of c for small T predicted here is not in agreement with experiment. Calculations based on Debye model give better agreement.

19. A crystalline body in a state of thermally excited elastic vibration may be treated as a system of N distinguishable independent quantum – harmonic oscillators of the same angular frequency ω (Einstein's model). Give the expression for the distribution law of the system. Compute the average energy of the system at high and low temperatures; find the molar specific heat c in those limiting cases, and discuss the validity of the model in those cases.

But reciprocal lattice vector, $a^* = \dfrac{\vec{b}\times\vec{c}}{\vec{a}\,\vec{b}\times\vec{c}}$

$$\Rightarrow a^* = \frac{(a/2)(-\hat{x}+\hat{y}+\hat{z})\times(a/2)(\hat{x}-\hat{y}+\hat{z})}{\frac{1}{2}a^3}$$

$$= \frac{1}{2a}\left[\hat{x}\hat{y}-\hat{x}\hat{z}+\hat{y}\hat{x}+\hat{y}\hat{z}+\hat{z}\hat{x}-\hat{z}\hat{y}\right]$$

Since cross product of same vector is zero.

$$= \frac{1}{2a}\left[\hat{x}\hat{y}+\hat{z}\hat{x}-\hat{x}\hat{y}+\hat{y}\hat{z}+\hat{z}\hat{x}+\hat{y}\hat{z}\right]$$

as vectors $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$

$$= \frac{1}{2a}\left[\hat{z}+\hat{y}-\hat{z}+x+\hat{y}+\hat{x}\right]$$

as $i\times j = k$ $j\times k = i$ and $k\times i = j$

or $a^* = \frac{1}{a}(\hat{x}+\hat{y})$, Similarly $b^* = \frac{1}{a}(\hat{y}+\hat{z})$, and $c^* = \frac{1}{a}(\hat{x}+\hat{z})$

By comparison with Fig (b) they are just the primitive vectors of f.c.c lattice. Thus the f.c.c lattice is the reciprocal lattice of the b.c.c lattice

Reciprocal lattice to f.c.c lattice:

The primitive translation vectors of the f.c.c lattice are,

$$a' = (a/2)(\hat{x}+\hat{y}),\ b' = (a/2)(\hat{y}+\hat{z}),\ c' = (a/2)(\hat{z}+\hat{x})$$

- Thus the primitive translation vector a^* of reciprocal lattice.

$$a^* = \frac{\vec{b}\times\vec{c}}{\vec{a}.\vec{b}\times\vec{c}} = \frac{\frac{a^2}{4}(\hat{y}+\hat{z})\times(\hat{z}+\hat{x})}{\frac{a^3}{8}(\hat{x}+\hat{y}).\left[(\hat{y}+\hat{z})+(\hat{z}+\hat{x})\right]}$$

or. $a^* = \frac{1}{a}(\hat{x}+\hat{y}-\hat{z})$

similarly. $b^* = \frac{1}{a}(-\hat{x}+\hat{y}+\hat{z})$

and $c^* = \frac{1}{a}(\hat{x}-\hat{y}+\hat{z})$

- These are primitive translation vectors of b.c.c lattice so that b.c.c lattice is the reciprocal lattice of f.c.c lattice.

20. Show that the

20. Show that the kinetic energy of a three dimensional gas of N free electrons at 0°K is $E_0 = \frac{3}{5} N E_{f(0)}$

Question no 20:

Average K.E of an electron $\bar{E}_0$ is given by an expression.

$$\bar{E}_0 = \frac{1}{N}\int_0^{\infty} E \cdot g(E) \cdot f(E) dE$$

Substituting the value of $g(E)$ and using the result, $f(E) = 1$ for $E > E_{f(0)}$ and $f(E) = 0$ for $E > E_{f(0)}$ at $T = 0°K$, we get.

$$\bar{E}_0 = \frac{1}{N}\int_0^{E_{f(0)}} E \cdot CE^{1/2} \cdot 1 dE + \frac{1}{N}\int_{E_{f(0)}}^{\infty} E \cdot CE^{1/2} \, 0 \cdot dE$$

$$= \frac{2}{5} \cdot \frac{C}{N} E_{f(0)}^{5/2}$$

Now, $CE_{f(0)}^{1/2} = g\{E_{f(0)}\}$ and $g\{E_{f(0)}\} = \frac{3}{2} \cdot \frac{N}{E_{f(0)}}$

Combining the two, we get,

$$CE_{f(0)}^{1/2} = \frac{3}{2} \cdot \frac{N}{E_{f(0)}} \quad \text{or,} \quad \frac{C}{N} = \frac{3}{2} \cdot \frac{1}{E_{f(0)}^{3/2}}$$

$$\therefore \bar{E}_0 = \frac{2}{5} \cdot \frac{3}{2} \cdot \frac{1}{E_{f(0)}^{3/2}} E_{f(0)}^{5/2}$$

$$\Rightarrow \bar{E}_0 = \frac{3}{5} E_{f(0)}$$

$\therefore$ Kinetic energy of N free electrons $= \frac{3}{5} N \cdot E_{f(0)}$

[Showed]

21. Aluminum metal crystalline in fcc structure. If each atom contributes single electron as free electron and the lattice constant a is 3.8A°, calculate treating conduction electrons as free electron Fermi gas (i) Fermi energy (E_f) and Fermi vector (K_f) (ii) total kinetic energy of free electron gas per unit volume at 0K.

Given,

$a = 3.8\,Å = 3.8 \times 10^{-10}\,m$

i) We know, K

Fermi vector $K_f = \left(3\pi^2 \times \frac{N}{V}\right)^{1/3}$

$= \left(29.58 \times \frac{6.023 \times 10^{23}}{(3.8 \times 10^{-10})^3}\right)^{1/3}$

Fcc structure
$V = a^3$

$= 6.87 \times 10^{17}\,m^{-1}$ Ans

Fermi velocity $V_f = \frac{hk_f}{2\pi m}$

$= \frac{6.63 \times 10^{-34} \times 6.87 \times 10^{17}}{2 \times 3.14 \times 9.1 \times 10^{-31}}$

$= 7.97 \times 10^{13}\,ms^{-1}$

∴ Fermi energy $E_f = \frac{1}{2} m v_f^2$

$= \frac{1}{2} \times 9.1 \times 10^{-31} \times (7.97 \times 10^{13})^2$

$= 2.89 \times 10^{-3}\,J$

$= 1.8 \times 10^{16}\,eV$ Ans

ii) Total Kinetic energy $= \frac{3}{5} E_f$

$= \frac{3}{5} \times 1.8 \times 10^{16}$

$= 1.08 \times 10^{16}\,eV$ Ans

22. Copper has a mass density ρ_m = 8.8gm/cc and an electrical resistivity $\rho = 1.5 \cdot 10^{-8}$ ohm-m at room temperature. Calculate (i) the concentration of the conduction electrons (ii) the mean free time (iii) the Fermi energy (iv) the Fermi velocity & (v) the mean free path at Fermi level.

1 gm mole of copper weights 64 gms and its volume is $\frac{64}{\rho_m}$

i) The concentration of conduction electrons

$$= \frac{N}{V} = \frac{6.023 \times 10^{26} \times 8800}{64}$$

$$= 8.28 \times 10^{28} \ m^{-3} \quad \underline{\text{Ans:}}$$

$\rho_m = 8.8 \ gm/cm^3$
$= 8800 \ kg/m^3$

ii) Mean free time. $\tau = \frac{m}{ne^2\rho}$

$$= \frac{9.1 \times 10^{-31}}{8.28 \times 10^{28} \times (1.6 \times 10^{-19})^2 \times 1.5 \times 10^{-8}}$$

$$= 2.86 \times 10^{-14} \ sec \quad \underline{\text{Ans:}}$$

iv) Fermi velocity. $v_f = \frac{\hbar k_f}{m} = \frac{h k_f}{2\pi m}$

But, Fermi vector, $k_f = (3\pi^2 \times \frac{N}{V})^{1/3}$

$$= (29.58 \times 8.28 \times 10^{28})^{1/3}$$

$$= 1.35 \times 10^{10} \ m^{-1}$$

$\therefore$ Fermi velocity. $v_f = \frac{6.63 \times 10^{-34} \times 1.35 \times 10^{10}}{2 \times 3.14 \times 9.1 \times 10^{-31}}$

$$= 1.56 \times 10^{6} \ ms^{-1} \quad \underline{\text{Ans:}}$$

iii) Fermi energy. $E_f = \frac{1}{2} m v_f^2$

$$= \frac{1}{2} \times 9.1 \times 10^{-31} \times (1.56 \times 10^{6})^2$$

$$= 1.11 \times 10^{-18} \ J$$

$$= 6.95 \ eV \quad \underline{\text{Ans:}}$$

v) The mean free path, $\lambda_f = v_f \tau$

$$= 1.56 \times 10^{6} \times 2.86 \times 10^{-14}$$

$$= 4.46 \times 10^{-8} \ m \quad \underline{\text{Ans:}}$$

24. Find the Hall coefficient and electron mobility for germanium if for a given sample (length 1cm, breadth 5mm & thickness 1mm) a current of 5milliamperes flown from a 1.4volts supply develops a Hall voltage of 25millivolts across the specimen in a magnetic field of 0.48wb/m².

We know,

R = resistance of specimen

$$= \frac{1 \cdot 4}{5\times10^{-3}}$$

$$= 280\,\Omega$$

Also,

Resistivity,

$$\rho = \frac{Ra}{L}$$

$$= \frac{280 \times 5\times10^{-6}}{1\times10^{-2}}$$

$$= 0 \cdot 14\,\Omega\text{-m}$$

Here,

a = area of cross section

$$= 1\times10^{-3} \times 5\times10^{-3} = 5\times10^{-6}\,m^2$$

$L = 1\times10^{-2}\,m$

$V_y = 25\times10^{-3}\,V$

$I = 5\times10^{-3}\,A$

$B = 0{\cdot}48\,wb/m^2$

$V = 1{\cdot}4\,V$

$R_H = ?$

$\mu_e = ?$

Again,

$$\text{Hall field, } E_y = \frac{V_y}{\text{thickness}}$$

$$= \frac{25\times10^{-3}}{1\times10^{-3}} = 25$$

We know,

$$\frac{1}{ne} = \frac{E_y}{BJ} = \frac{E_y}{\frac{BI}{a}}$$

$$\Rightarrow \frac{1}{ne} = \frac{25 \times 5\times10^{-6}}{0{\cdot}48 \times 5\times10^{-3}} = 0{\cdot}052\ m^3/\text{coulomb}$$

$$\therefore \text{Hall co-efficient, } R_H = \frac{3\pi}{8} \times \frac{1}{ne}$$

$$= \frac{3\pi}{8} \times 0{\cdot}052$$

$$\therefore R_H = 0{\cdot}061\ m^3/\text{coulomb}$$ Ans.

and,

$$\text{Electron mobility, } \mu_e = \frac{R_H}{\rho}$$

$$= \frac{0{\cdot}061}{0{\cdot}14}$$

$$\therefore \mu_e = 0{\cdot}438\ m^2/\text{volt sec}$$

25. A coppe
field wit
200amp.
appears

25. A copper strip 2cm wide and 1mm thick is placed in a magnetic field with $B = 1.5 wb/m^2$ perpendicular to the strip. If a current of 200amp. Is set up in the strip. What Hall potential difference appears across the strip?

We know,

number of electrons per unit volume,

$$n = \frac{dN_0}{M}$$

$$= \frac{9 \times 6.023 \times 10^{23}}{64}$$

$$= 8.47 \times 10^{22} \text{ electrons}/cm^3$$

$$\therefore n = 8.47 \times 10^{28} \text{ electrons}/m^3$$

Again,

Hall potential,

$$V_H = \frac{IB}{net}$$

$$= \frac{200 \times 1.5}{8.47 \times 10^{28} \times 1.6 \times 10^{-19} \times 1.0 \times 10^{-3}}$$

$$= \cancel{2.2133} \times 10^{-5} \text{ volts}$$

$$\therefore V_H = \cancel{2.213}\ 22.13\ \mu V$$

Ans!

Here,

$I = 200 A$

$B = 1.5\ wb/m^2$

$t = 1.0 \times 10^{-3} m$

$N_0 = 6.023 \times 10^{23}$

d = density = $9.0\ gm/cm^3$

$M = 64\ gm/mole$

$n = ?$

$V_H = ?$

26. In a Hall Effect experiment on silver a potential of $60\mu V$ is developed across a foil of thickness 0.05mm when a current of 30mA is passed in a direction perpendicular to a magnetic field of 1.2T. Calculate the Hall coefficient for silver.

We know,

Hall co-efficient,

$$R_H = \frac{V_H t}{BI}$$

$$= \frac{60 \times 10^{-6} \times 0.05 \times 10^{-3}}{1.2 \times 30 \times 10^{-3}}$$

$$\therefore R_H = 8.3 \times 10^{-8}\ m^3/\text{coulomb}$$

Ans:

Given,

$V_H = 60\ \mu V = 60 \times 10^{-6} v$

$t = 0.05\ mm = 0.05 \times 10^{-3} m$

$I = 30\ mA = 30 \times 10^{-3} A$

$B = 1.2\ T$

$R_H = ?$

27. Calculate the inter collision time at room temperature and drift velocity in a field of 110Vm^{-1} in sodium. whose conductivity is $2.2 \times 10^7 \Omega^{-1} m^{-1}$.

Question no 27:

We know,

The mass density of sodium, $d = 0.97 \times 10^3 \, kg/m^3$

Atomic weight, $M = 23 \times 10^{-3} \, kg/mole$

electron concentration, $n = \frac{dN}{M}$

$$= \frac{0.37 \times 10^3 \times 6.023 \times 10^{23}}{23 \times 10^{-3}}$$

$$= 2.54 \times 10^{28} \, m^{-3}$$

electrical conductivity, $\sigma_e = ne\mu_e$

$\therefore$ electron mobility, $\mu_e = \frac{\sigma_e}{ne}$

$$= \frac{2.17 \times 10^{17}}{2.54 \times 10^{28} \times 1.6 \times 10^{-19}}$$

$$= 5.34 \times 10^{-3} \, m^2/V\text{-}s$$

$\therefore$ Drift velocity, $V_d = \mu_e E$

$$= 5.34 \times 10^{-3} \times 100$$

$$= 0.534 \, ms^{-1}$$ Ans!

$\therefore$ inter collision time, $\tau = \frac{\mu_e m}{e}$

$$= \frac{5.34 \times 10^{-3} \times 9.1 \times 10^{-3}}{1.6 \times 10^{-19}}$$

$$= 3.04 \times 10^{-14} \, sec$$ Ans!

28. The inter collision time in copper $2.4 \cdot 10^{-14}$s. Calculate its thermal conductivity at 300K.

Question no 28:

Given,

inter collision time, $\tau = 2.4 \times 10^{-14}$ s

mass density of copper, $\rho_m = 8.92$ gm/cm^3 = 8920 kg/m^3

Now,

electron concentration, $n = \frac{Nd}{M} = \frac{6.023 \times 10^{23} \times 8920}{64}$

$= 8.4 \times 10^{24}$ m^{-3}

$\therefore \sigma = \frac{ne^2\tau}{m} = \frac{8.4 \times 10^{24} \times (1.6 \times 10^{-19})^2 \times 2.4 \times 10^{-14}}{9.1 \times 10^{-31}}$

$\therefore \sigma = 5.67 \times 10^{3}\ \Omega^{-1} m^{-1}$

$\therefore$ Thermal conductivity, $K = 3\sigma \left(\frac{K}{e}\right)^2 T$

$= 3 \times 5.67 \times 10^{3} \times \left(\frac{1.38 \times 10^{-23}}{1.6 \times 10^{-19}}\right)^2 \times 300$

$= 1.84 \times 10^{9}$ Wm^{-1}K^{-1} Ans:

Question no 29:

We know,

Number of electrons in a cubic meter of sodium,

$n = \frac{Nd}{M}$

$= \frac{6.023 \times 10^{23} \times 0.97 \times 10^{3}}{23 \times 10^{23}}$

$\therefore n = 2.54 \times 10^{28}$ m^{-3} Ans:

Given,

$M = 23 \times 10^{-23}$ kg/mole.

$d = 0.97 \times 10^{3}$ kg/m^3

$n = ?$

29. The atomic weight of sodium is 23 and its density is 0.97×10^3Kg/cm^3. Calculate the number of electrons in a cubic meter of sodium.

Question no 30:

From Kinetic theory and law of equipartion of energy give.

$$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} KT$$

$$\therefore \langle v^2 \rangle = \frac{3KT}{m}$$

Hence, $\bar{\tau} = \frac{\lambda \bar{c}}{\frac{3KT}{m}} = \frac{\lambda \bar{c}^2}{(3KT/m)\bar{c}}$

But, $\bar{c}^2 = \frac{8KT}{\pi m}$

$$\therefore \bar{\tau} = \frac{\lambda 8KT/m\pi}{(3KT/m)\bar{c}} = \frac{8\lambda}{3\pi\bar{c}}$$

The electrical conductivity is given by.

$$\sigma = \frac{ne^2\bar{\tau}}{m}$$

$$= \frac{ne^2 8\lambda}{3\pi m\bar{c}}$$

$$= \frac{ne^2 8\lambda}{3\pi m\sqrt{\frac{8KT}{\pi m}}}$$

$$= \frac{8ne^2\lambda}{3\times 2\sqrt{2\pi KTm}}$$

$$\therefore \sigma = \frac{4}{3}\cdot\frac{ne^2\lambda}{\sqrt{2\pi KTm}}$$

[Showed]

30. Show that if the mean free path is independent of the velocity, the electrical conductivity of Maxwell-Boltzmann free electron gas may be expressed by the relation $\sigma = \frac{4ne^2\lambda}{3\sqrt{2\pi mKT}}$, where λ and τ are independent of velocity, the Maxwell-Boltzmann distribution gives average value of τ as $\bar{\tau} = \frac{\lambda\langle v \rangle}{\langle v^2 \rangle}$.

31. A 5kW LASER emits light of 650nm wavelength. Calculate the number of photons emitted by the LASER every second.

Question no 31:

Given.

Energy of the laser : $E = 5\,kW = 5000\,W$

Wavelength, λ : $650\,nm = 650 \times 10^{-9}\,m$

We know,

number of photons emitted by the laser every second.

$$n = \frac{E}{h\nu} = \frac{E\lambda}{hc}$$

$$= \frac{5000 \times 650 \times 10^{-9}}{6.63 \times 10^{-34} \times 3 \times 10^{8}}$$

$$\therefore n = 1.633 \times 10^{22}\,s^{-1}$$ Ans

Question no 32:

i) Let, $d\theta$ be the angular spread,

$$d\theta = \frac{\lambda}{d}$$

Here, $\lambda = 8 \times 10^{-7}\,m$

$d = 5 \times 10^{-3}\,m$

$$\therefore d\theta = \frac{8 \times 10^{-7}}{5 \times 10^{-3}} = 1.6 \times 10^{-4}\ \text{radian}$$ Ans

ii) Axial spread $= (D\,d\theta)^2$

Here, $D = 4 \times 10^{5}\,km = 4 \times 10^{8}\,m$

$\therefore$ Axial spread $= (4 \times 10^{8} \times 1.6 \times 10^{-4})^2$

$= 4.096 \times 10^{9}\,m^2$ Ans

32. A laser beam has a wavelength of 8000A^0 and aperture 5×10^{-3}m. The laser beam is sent to moon. The distance of the moon is 4×10^5km from the earth. Calculate (i) the angular spread of the beam and (ii) the axial spread when it reaches the moon.

33. The coherence length for sodium light is 3×10^{-2}m. The wavelength of sodium light is 5893A°. Calculate (i) the number of oscillations corresponding to the coherence length and (ii) the coherence time.

Question no 33:

Here.

$\lambda = 5893\times10^{-10}$ m

$C = 3\times10^{8}\,ms^{-1}$

Coherence length, $L = 3\times10^{-2}$ m

i) Number of oscillations in length L,

$$n = \frac{L}{\lambda}$$

$$= \frac{3\times10^{-2}}{5893\times10^{-10}} = 5.09\times10^{4}$$ Ans

ii) Coherence time $= \frac{L}{C}$

$$= \frac{3\times10^{-2}}{3\times10^{8}} = 1\times10^{-10}\ sec$$ Ans:

Question no 34:

Here,

$\lambda = 6000\ A^{\circ} = 6\times10^{-7}$ m

$d = 2$ m

$D = 4\times10^{8}$ m

i) Angular spread, $d\theta = \frac{\lambda}{d} = \frac{6\times10^{-7}}{2} = 3\times10^{-7}$ radian

ii) Areal spread i.e. area of the spot on the moon,

$$A = (D\,d\theta)^2$$

$$= (4\times10^{8}\times3\times10^{-7})^2$$

$$\therefore A = 1.44\times10^{4}\,m^2$$ Ans:

34. A laser beam λ = 6000A° on earth is focused by a lens (or mirror) of diameter 2m on the crater on the moon. The distance of the moon is 4×10^{8}m. How big is the spot on the moon? Neglect the effect of earth's atmosphere.

Question-35

Copper has F

its density. At

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Question-35

Copper has FCC structure and the atomic radius is 1.25A°. Calculate its density. Atomic weight of copper = 63.6.

Given that,

$r = 1.25 \text{Å} = 1.25 \times 10^{-8}$ cm.

$M = 63.6$.

for $\boxed{\text{FCC structure } n=4.}$

we know that, the lattice constant for FCC structure is;

$$a = r\,2\sqrt{2}$$
$$= 1.25\times10^{-8} \times 2\sqrt{2}$$
$$= 3.54 \times 10^{-8} \text{ cm.}$$

Now, we know,

$$\rho = \frac{nM}{a^3N}$$
$$= \frac{4\times63.6}{(3.54\times10^{-8})^3\times6.023\times10^{23}}$$
$$= 8.56 \text{ g/cm}^3 \text{ (Ans).}$$

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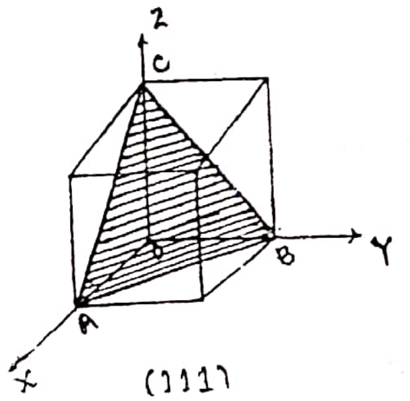
Question-37

Sketch the [111], [110], [010], [101], [212], [100], [001] and [211] planes in a simple cubic cell.

A plane whose Miller indices are (111) has the following intercepts on the three axes

$$\frac{1}{1}, \frac{1}{1}, \frac{1}{1}$$

this plane is sketched in the following figure;

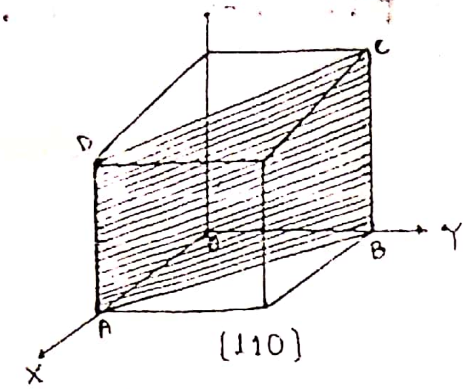


(111)

Similarly, for the plane (110), the axial intercept of the (110) are

$$\frac{1}{1}, \frac{1}{1}, \frac{1}{0}$$

This plane is sketched in the following figure.

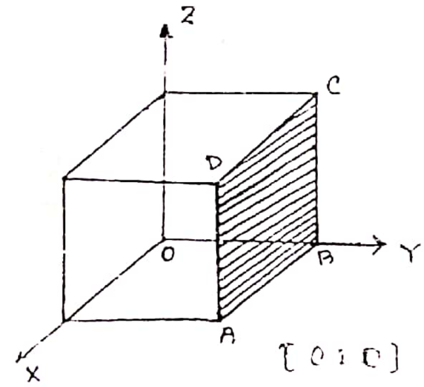


[110]

⇒ Similarly, for [010] plane the axial intercepts of the (0 1 0) are,

$$\frac{1}{0}, \frac{1}{1}, \frac{1}{0}$$

This plane is sketched in the following figure;

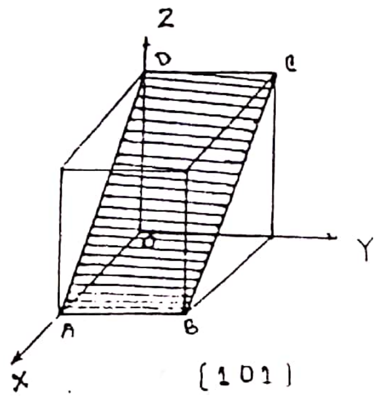


[010]

⇒ Similarly, for [101] plane the axial intercepts of the (1 0 1) plane are

$$\frac{1}{1}, \frac{1}{0}, \frac{1}{1}$$

This plane is sketched in the following figure;

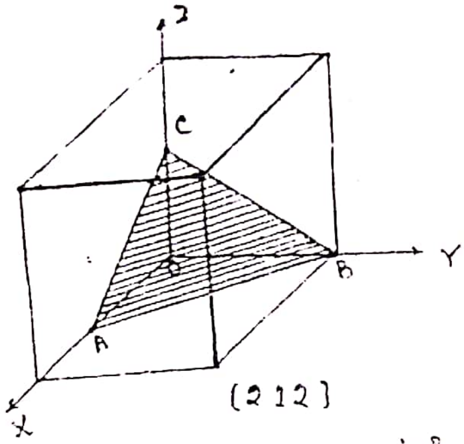


[101]

⇒ Similarly for [2 1 2] plane the axial intercepts of the (2 1 2) plane are

$$\frac{1}{2}, \frac{1}{1}, \frac{1}{2}$$

This plane is sketched in the following figure:

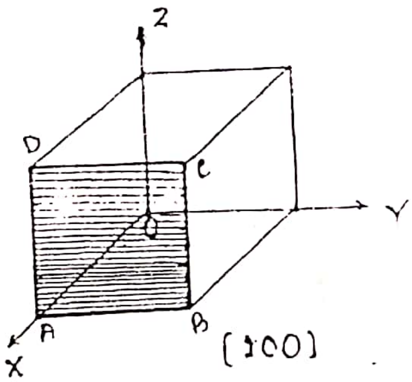


(2 1 2)

⇒ Similarly for (1 0 0) plane, the axial intercepts of the (1 0 0) plane are,

$$\frac{1}{1}, \frac{1}{0}, \frac{1}{0}$$

This plane is sketched in the following figure:

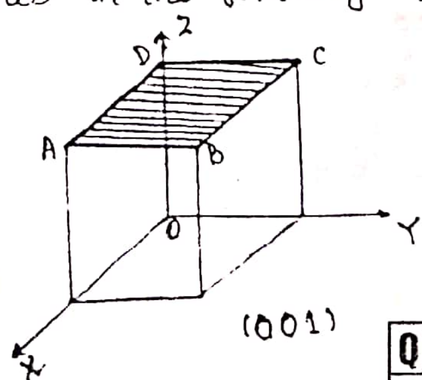


(100)

⇒ Similarly, for (0 0 1) plane the axial intercepts of the plane (0 0 1) is.

$$\frac{1}{0}, \frac{1}{0}, \frac{1}{1}$$

This plane is sketched in the following figure:

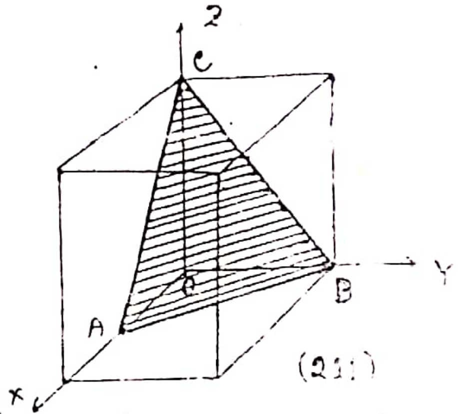


(0 0 1)

⇒ Similarly for the plane (211), the axial intercepts of the plane (211) are

$$\frac{1}{2}, \frac{1}{1}, \frac{1}{1}.$$

This plane is sketched in the following figure.



(211)

Question-38

Lead is face-centred cubic with an atomic radius of r = 1.65A°. Find the spacing of (i) [210] planes. (ii) [222] planes. (iii) [111] planes. (iv) [110] planes and (v) [121] planes.

Given, $r = 1.65 A^\circ = 1.65 \times 10^{-10}$ m.

We know, the interplanar spacing is given by,

$$d = \frac{a}{\sqrt{h^2+k^2+l^2}}$$

and lattice constant of the unit cell of lead (FCC) is,

$$a = r\,2\sqrt{2}$$
$$= 1.65 \times 2\sqrt{2} = 4.67 A.$$

i) For (210) planes, $h=2,\ k=1,\ l=0$

$$\therefore d_{210} = \frac{4.67}{\sqrt{2^2+1^2+0}} = 2.088 A^\circ.$$

(Ans)

Similarly,

ii) $$d_{222} = \frac{4.67}{\sqrt{2^2+2^2+2^2}} = 1.348 A^\circ.$$ (Ans)

iii) $$d_{111} = \frac{4.67}{\sqrt{1^2+1^2+1^2}} = 2.696 A^\circ.$$ (Ans).

iv) $$d_{110} = \frac{4.67}{\sqrt{1^2+1^2+0^2}} = 3.30 A^\circ$$ (Ans).

v) $d_{121} = \frac{4.67}{\sqrt{1^2+2^2+1^2}} = 1.907\ A^\circ$. (Ans)

Que-39:

In a unit cell of simple cubic structure. find the angle between the normal to pair of planes whose Miller indices are (i) [100] and [010] (ii) [101] and [121].

The directions of the two normals are [100], [010] and [111], [121] respectively.

The angle between the two normals can be found by the relation,

$$\cos\theta = \frac{u_1u_2 + v_1v_2 + w_1w_2}{\sqrt{u_1^2+v_1^2+w_1^2}\sqrt{u_2^2+v_2^2+w_2^2}}$$

∴ i) $\cos\theta = \frac{1\times0 + 0\times1 + 0\times0}{\sqrt{1^2+0+0}\sqrt{0+1^2+0}} = 0$.

∴ $\theta = \cos^{-1}(0^\circ) = 90^\circ$. (Ans).

ii) $\cos\theta = \frac{1\times1 + 1\times2 + 1\times1}{\sqrt{1^2+1^2+1^2}\sqrt{1^2+2^2+1^2}} = 0.943$

∴ $\theta = \cos^{-1}(.943) = 19^\circ 28' 16.39'$ (Ans).

Que-40:

Calculate the number of atoms per unit cell for a face centred cubic lattice of copper crystal. Given a=3 60A^0, atomic weight of copper = 63.6 and density of copper = 8.86.

We know,

$\rho = \frac{nM}{a^3N}$

$\Rightarrow n = \frac{a^3\rho N}{M}$

$\Rightarrow n = \frac{(3.60\times10^{-8})^3\times8.86\times6.023\times10^{23}}{63.6}$

∴ $n = 3.96 \approx 4$

∴ Number of atoms per unit cell for a face centred cubic lattice of copper crystal is 4. (Ans)

Given,

$a = 3.60\ A^\circ$

$= 3.60\times10^{-8}$ cm.

$M = 63.6$

$\rho = 8.86$ g/cm^3

$n = ?$

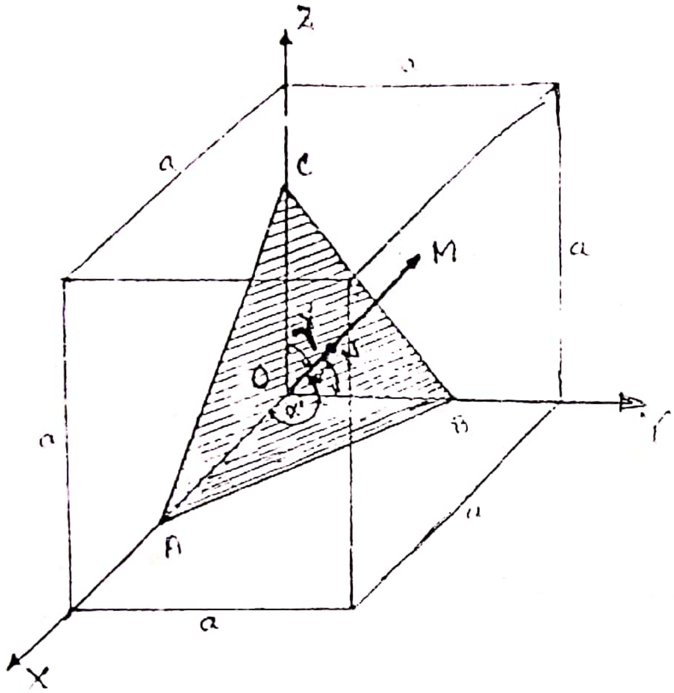
$N = 6.023\times10^{23}$ per gm-mole

Que-41:

Show that spacing d of plane [hkl] in a simple cubic lattice of side a is

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

Suppose, that the plane shown in the following figure belongs to a family of planes whose Miller indices are <hkl>. The perpendicular ON from origin to the plane



represents the interplaner spacing d of the family of planes.
let, the direction cosines of ON be $\cos\alpha'$, $\cos\beta'$ and $\cos\gamma'$ as shown.
The intercepts of the plane on the three axes are.

$$OA = \frac{a}{h} \; ; \; OB = \frac{a}{k} \; ; \; OC = \frac{a}{l}$$; where a is the length of the cube edge.

$$\cos\alpha' = \frac{d}{OA} = \frac{d}{a/h} = \frac{dh}{a} \; ; \; \cos\beta' = \frac{d}{OB} = \frac{d}{a/k} = \frac{dk}{a} \; ; \; \cos\gamma' = \frac{d}{OC} = \frac{d}{a/l} = \frac{dl}{a}$$

Where, d = ON - perpendicular distance between adjacent members of the same family of planes.

Now, $\cos^2\alpha' + \cos^2\beta' + \cos^2\gamma' = 1$

$$\Rightarrow \left(\frac{dh}{a}\right)^2 + \left(\frac{dk}{a}\right)^2 + \left(\frac{dl}{a}\right)^2 = 1$$

$$\Rightarrow \frac{d^2}{a^2}(h^2 + k^2 + a^2) = 1.$$

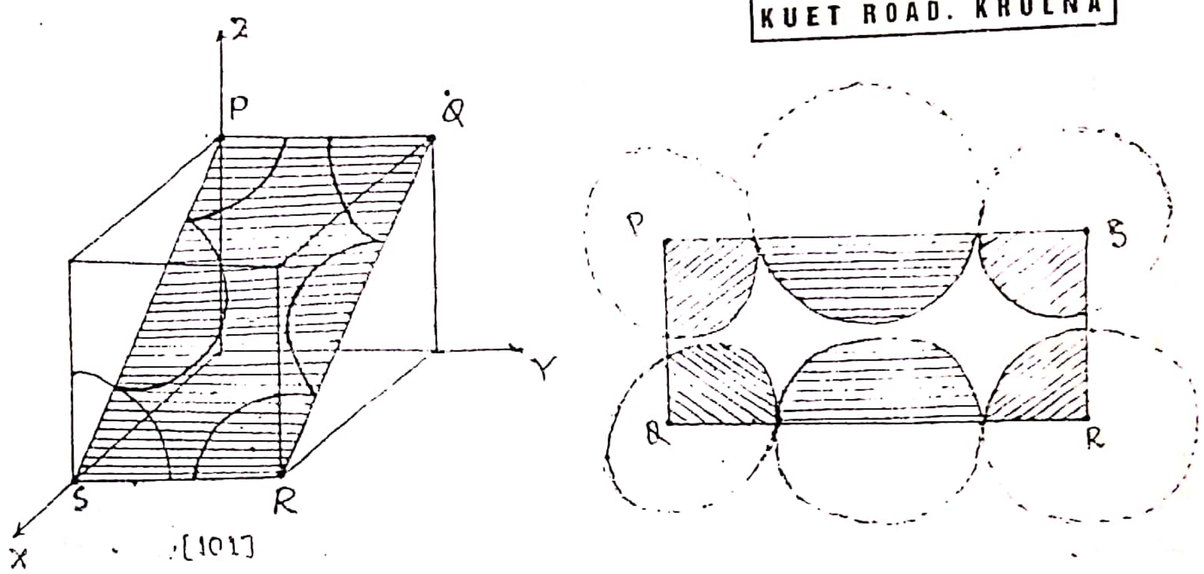
$$\Rightarrow d^2 = \frac{a^2}{h^2 + k^2 + l^2}$$

$$\therefore d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$ [Showed]

Question-42:

The lattice constant of a unit cell of lead is 4.93A°. Find the number of atoms/mm^2 of [101], [111] and [101] planes.

i) Two views of the [101] planes are shown in the following figure.



From the figure it is seen that [101] plane is PQRS. As seen, it contains four quarter atoms and two half atoms.

$\therefore$ Total number of atoms $= 4\times\frac{1}{4} + 2\times\frac{1}{2}$

$= 2.$

Area of the plane $= PQ \times QR$

$= a \times \sqrt{2}.a$

$= \sqrt{2}\, a^2.$

$= \sqrt{2} \times (4.93\times10^{-7})^2$; [Given, $a = 4.93 A^\circ$]

$= 3.44\times10^{-13}$ mm^2.

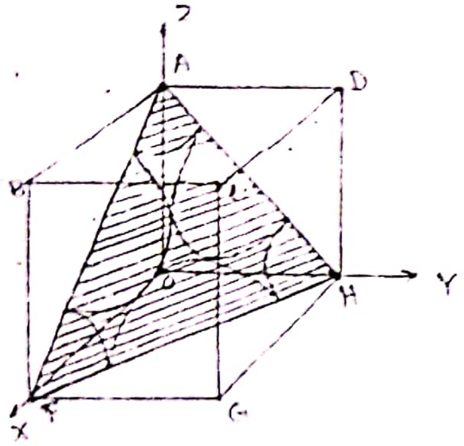
$\therefore$ No. of atoms/mm^2 $= \dfrac{2}{3.44\times10^{-13}} = 5.82\times10^{12}$ atoms/mm^2.

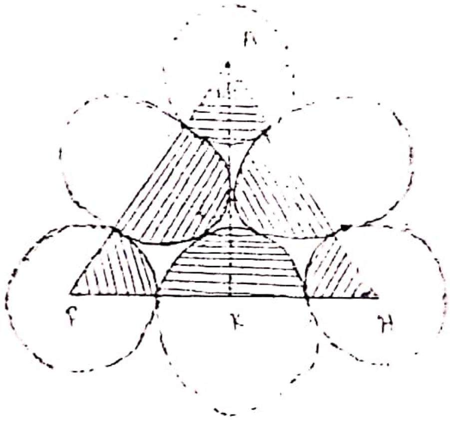
(Ans).

ii) The two views of the (111) plane are shown in the following figure. As seen, it is an equilateral triangle having each side equal to the face diagonal of the cube.

$AF = AH = FH = a\sqrt{2}$

$AK = \frac{\sqrt{3}}{2} AH = \frac{\sqrt{3}}{2} \times \sqrt{2}\, a$





$\therefore$ AREA of $\Delta AFH = \frac{1}{2} FH \times AK$

$= \frac{1}{2} \times a\sqrt{2} \times \frac{\sqrt{3}}{2}\sqrt{2}\, a$

$= \frac{\sqrt{3}}{2} a^2$

$= \frac{\sqrt{3}}{2} (4.93 \times 10^{-7})^2$

$= 2.10 \times 10^{-13}$ mm².

As seen from figure, it contains three one sixths of an atom and three one halfs. Hence number of atoms

$= 3 \times \frac{1}{6} + 3 \times \frac{1}{2} = 2.$

$\therefore$ No. of atoms/mm² $= \frac{2}{2.10 \times 10^{-13}}$

$= 9.50 \times 10^{12}$ atoms/mm².

(Ans.

Question- 43:

Calculate the packing fraction in crystals for (i) s.c. (ii)b.c.c and (iii) f.c.c., structures, treating the atoms as spherical.

i) Packing fraction for simple cube structure : In this case, number of atoms per cube is one.

volumn of the one atom $= \frac{4}{3}\pi r^3$

Now, atomic radius, $\boxed{r = \frac{a}{2}}$

∴ Volume occupied by the atom in the S.C unit cell, $v = 1 \cdot \frac{4\pi}{3}\left(\frac{a}{2}\right)^3$

$= \frac{4\pi}{3} \cdot \frac{a^3}{8}$

$= \frac{\pi a^3}{6}$

Volume of the unit cell, $V = a^3$.

$$\therefore P.F = \frac{v}{V} = \frac{\frac{\pi a^3}{6}}{a^3} = \frac{\pi}{6} = 0.52 = 52\%$$

ii) Packing fraction for Body-Centred Cube Structure : In this case, number of atoms per cube is 2.

atomic radius, $\boxed{r = \frac{a\sqrt{3}}{4}}$

∴ Volume occupied by the atoms in the b.c.c. unit cell,

$$v = 2 \times \frac{4\pi}{3}\left(\frac{a\sqrt{3}}{4}\right)^3 = \frac{\sqrt{3}}{8}\pi a^3.$$

Volume of the unit cell, $V = a^3$.

$$\therefore P.F = \frac{v}{V} = \frac{\sqrt{3}\pi a^3}{8a^3} = \frac{\sqrt{3}}{8}\pi = 0.68 = 68\%.$$

iii) Packing fraction for Face-Centred Cube Structure : In this case, there are 4 atoms per cube.

atomic radius, $\boxed{r = \frac{a}{2\sqrt{2}}}$

∴ Volume occupied by the atoms in f.c.c. unit cell,

$$v = 4 \cdot \frac{4\pi}{3}\left(\frac{a}{2\sqrt{2}}\right)^3 = \frac{\pi a^3}{3\sqrt{2}}$$

Volume of the unit cell, $V = a^3$.

$$\therefore P.F = \frac{v}{V} = \frac{\pi a^3}{3\sqrt{2}a^3} = \frac{\pi}{3\sqrt{2}} = 0.74 = 74\%.$$

$$P.F. = \frac{\text{Volume occupied by the atom in the unit cell}}{\text{Volume of the unit cell (always } a^3)}$$

Que- 44:

X-rays of wavelength 0.72A° are reflected from the [111] plane of a rocksalt crystal (a = 2.84A°). Calculate the glancing angle corresponding to second order reflection.

We know, interplaner spacing for [111] plane is,

| Given,
| $\lambda = 0.72 A^\circ$
| $a = 2.84 A^\circ$
| $n =$ second order $= 2$
| $\theta = ?$

$$d_{111} = \frac{2.84}{\sqrt{1^2+1^2+1^2}}$$

$$= 1.64 A^\circ.$$

Now we know

$$2d \sin\theta = n\lambda$$

$\therefore$ glancing angle, $\theta = \sin^{-1}\left(\frac{n\lambda}{2d}\right)$

$$\therefore \theta = 26.05^\circ.$$

(Ans).

Question no:45:

We know, the lattice constant for FCC structure is

| Given,
| $r = 1.16 \times 10^{-8}$ cm
| $M = 63.6$
| $N = 6.023 \times 10^{23}$ /mol
| $n = 4$ (for FCC structure).
| $\rho = ?$

$$a = r\,2\sqrt{2}$$

$$= 1.16 \times 10^{-8} \times 2\sqrt{2}$$

$$= 3.28 \times 10^{-8} \text{ cm}$$

Now,

$$\rho = \frac{nM}{a^3 N}$$

$$= \frac{4 \times 63.6}{(3.28 \times 10^{-8})^3 \times 6.023 \times 10^{23}}$$

$$= 11.97 \text{ g/cm}^3.$$

(Ans).

: Copper has FCC structure and the atomic radius is 1.16A°. Calculate its density. Atomic weight of copper = 63.15.

Que – 46:

. Nickel has face-centred cubic structure and its atomic radius is 1.35×10^{-10}m. Calculate (a) the number of atoms/mm^2 and (b) inter-planar spacing.

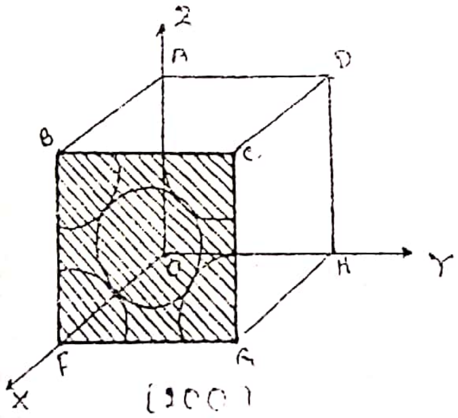
a) Given,

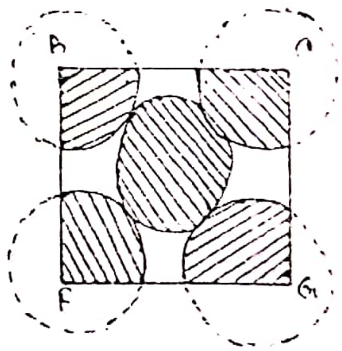
$$r = 1.35\times10^{-10} m = 1.35\times10^{-7} mm.$$

We know, that the lattice constant for FCC structure is,

$$a = r\,2\sqrt{2}$$
$$= 1.35\times10^{-7} \times 2\sqrt{2}$$
$$= 3.82\times10^{-7} mm.$$

Two views of the (100) plane are shown in the following figure.





area of the face BCGF is $a^2 = (3.82\times10^{-7})^2 = 1.459\times10^{-13} mm^2$.

As seen from the figure, the (100) plane includes one full atom and four quarter atoms.

∴ Total number of atoms included $= 1 + 4\cdot\frac{1}{4} = 2$.

∴ Number of atoms/mm^2 $= \dfrac{2}{1.459\times10^{-13}}$

$$= 1.37\times10^{13} \text{ atoms/mm}^2.$$

(Ans)

b) interplaner spacing for the plane (100) is,

$$d_{100} = \frac{3.82\times10^{-7}}{\sqrt{1^2+0+0}}$$

$$= 3.82\times10^{-7} mm = 3.82 A^\circ \quad \text{(Ans)}$$

Question- 47:

. Calculate the lattice constant for a rock salt crystal (density equal to $2.19 gm/cm^3$). Given that the crystal has f.c.c lattice and molecular weight = 58.45.

Given.

$\rho = 2.19 \ gm/cm^3$

$M = 58.45$.

$N = 6.023 \times 10^{23}$ per gm mol.

n = for fcc structure = 4

$a = ?$

We know,

$$\rho = \frac{nM}{a^3 N}$$

$\therefore$ lattice constant

$$a = \sqrt[3]{\frac{nM}{\rho N}}$$

$$= \sqrt[3]{\frac{4 \times 58.45}{2.19 \times 6.023 \times 10^{23}}}$$

$\therefore a = 5.62 \times 10^{-8}$ cm.

(Ans).

48: The orthorhombic crystal has axial units in the ratio of 0.424:1:0.367. Find the Miller indices of a crystal face whose intercepts are in the ratio 0.212:1:0.183.

Here, axial units are $a:b:c$ :: $0.424:1:0.367$

Now, $pa = 0.212$

$\Rightarrow p \times 0.424 = 0.212$

$\Rightarrow p = \frac{.212}{.424}$ $\therefore p = \frac{1}{2}$

Similarly, $qb = 1$

$\Rightarrow q \times 1 = 1$

$\therefore q = 1$.

also, $rc = .183$

$\Rightarrow r \times .367 = .183$

$\therefore r = \frac{.183}{.367} = \frac{1}{2}$.

Hence, numerical parameters of this plane are $\frac{1}{2}$, 1, $\frac{1}{2}$.

$\therefore$ Miller indices $= \left(\frac{1}{\frac{1}{2}} : \frac{1}{1} : \frac{1}{\frac{1}{2}}\right) = (212)$

(Ans)

Question-49:

The primitives of a crystal are 1.2A^0, 1.8A^0 and 2A^0 along whose Miller indices [231] cut intercepts 1.2A^0 along x-axis. What will be the lengths of intercept along y and z axes?

Given,

Miller indices of the crystal [231]

length of intercepts along x-axis $l_x = 1.2A^\circ$.

So, $h : k : l = 2 : 3 : 1$.

$\therefore \frac{1}{p} : \frac{1}{q} : \frac{1}{r} = h : k : l = 2 : 3 : 1$.

$\therefore p : q : r = \frac{1}{2} : \frac{1}{3} : \frac{1}{1} = 3 : 2 : 6$

The primitives of a crystal are $1.2A^\circ$, $1.8A^\circ$ and $2A^\circ$.

$\therefore pa : qb : rc = (3 \times 1.2) : (2 \times 1.8) : (6 \times 2)$

$= 3.6 : 3.6 : 12$

Therefore, $l_x : l_y : l_z = 3.6 : 3.6 : 12$.

$\therefore \frac{l_x}{l_y} = \frac{3.6}{3.6}$

$\therefore$ length of intercepts along Y-axis, $l_y = \frac{3.6}{3.6} \times 1.2 = 1.2A^\circ$. (Ans).

again.

$\frac{l_y}{l_z} = \frac{3.6}{12}$

$\therefore$ length of intercepts along z axes, $l_z = \frac{12}{3.6} \times 1.2 = 4A^\circ$. (Ans)

Question no:

We know,

$a^3\rho = \frac{nM}{N}$

$\therefore$ lattice constant,

$a = \sqrt[3]{\frac{nM}{\rho N}}$

$\Rightarrow a = \sqrt[3]{\frac{4 \times 60.2}{6250 \times 6.02 \times 10^{26}}}$

$\Rightarrow a = 4 \times 10^{-10}$ m.

$\therefore a = 4A^\circ$. (Ans).

Given,

$\rho = 6250$ kg/m^3.

$M = 60.2$

$N = 6.02 \times 10^{26}$ kg. mole^{-1}

n = for FCC Structure = 4

$a = ?$

Question no: 50:

Given,

Density of added impurity atoms. $N_d = \frac{4.2\times10^{28}}{10^6}$

$= 4.2\times10^{22}$ atoms/m^3.

As seen, it is extremely large as compared to the intrinsic charge carrier density of $2.6\times10^{19}/m^3$ which will, therefore be neglected.

Now, we know,

$\sigma_n = N_d e \mu_e$

$= 4.2\times10^{22} \times 1.6\times10^{-19} \times .35$

$= 2.352\times10^{3}$ mho/m.

A specimen of pure germanium at 300°K has a density of charge carriers of $2.61\times10^{19}/m^3$. It is doped with donor impurity atoms at the rate one impurity atom for every 10^{10} atoms of germanium. All impurity atoms may be supposed to be ionized. The density of germanium atoms is 4.22×10^{28} atoms/m^3. Find the resistivity of the doped germanium if electron mobility is $0.45 m^3/V$-s.

$\therefore$ resistivity, $\rho = \frac{1}{\sigma_n} = \frac{1}{2.352\times10^{3}} = 0.423\times10^{-3}\ \Omega$-m.

(Ans)

Question no: 51:

Given, $d = 8.96$ g/cc, $M = 63.5$, $N = 6.023\times10^{23}$/gm-mole.

$\rho = 1.7\times10^{-6}\ \Omega$-cm.

Electron concentration in copper,

$n = \frac{Nd}{M}$

$= \frac{6.023\times10^{23}\times8.96}{63.5}$

$= 8.50\times10^{22}$ atoms.

It represents the number of Cu atoms as well as electrons per c.c i.e. $n = 8.50\times10^{22}$ per c.c.

Now, mobility of electron.

$\mu_e = \frac{1}{ne\rho}$

$= \frac{1}{8.50\times10^{22}\times1.6\times10^{-19}\times1.7\times10^{-6}}$

$\therefore \mu_e = 43.3\ cm^2/V$-s. Ans

Find the mobility of electrons in copper assuming that each atom contributes one free electrons for conduction. For Cu, resistivity = 1.72×10^{-6} ohm-cm, density = 8.9g/cc and atomic weight = 63.3. Avogadro's number = 6.02×10^{23}/gm-mole.

Question - 52:

Show that for a crystal of cubic symmetry the direction [hkl] is perpendicular to the plane [hkl].

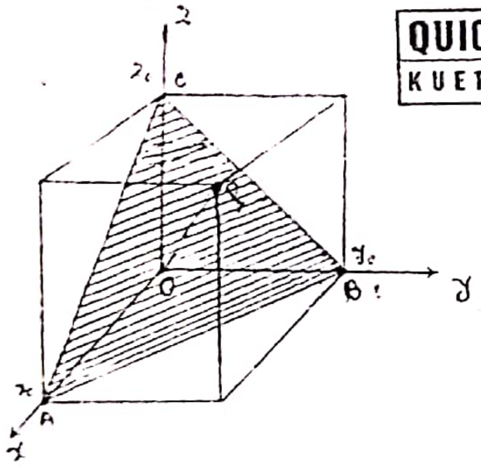
Let CP be the direction of (h, k, l).
Let ABC be the plane of (h, k, l).

Let, x_c, y_c, z_c be the intercepts of the planes with x, y, z axes.

Miller indices are $\frac{1}{x_c} : \frac{1}{y_c} : \frac{1}{z_c}$

and reduced indices become $\frac{g}{x_c}, \frac{g}{y_c}, \frac{g}{z_c}$

where, g is an integer.



For cubic lattice h = k = l or $x_c = y_c = z_c = \alpha$.

And the direction $\vec{OP} = \left(i\frac{g}{\alpha} + j\frac{g}{\alpha} + k\frac{g}{\alpha}\right)$

$$\vec{OA} + \vec{AB} = \vec{OB}$$

$$\vec{AB} = \vec{OB} - \vec{OA} = \left(j\frac{g}{\alpha} - i\frac{g}{\alpha}\right)$$

$$\vec{OP} \cdot \vec{AB} = -\frac{g^2}{\alpha^2} + \frac{g^2}{\alpha^2} = 0$$

Similarly, we may prove

$$\vec{OP} \cdot \vec{BC} = 0$$

$$\vec{OP} \cdot \vec{CA} = 0$$

Hence dot product of $\vec{OP}$ with $\vec{AB}$, $\vec{BC}$ and $\vec{CA}$ is zero.

Hence, $\vec{OP}$ is perpendicular to $\vec{AB}$, $\vec{BC}$ or $\vec{CA}$.

Hence [h, k, l] is normal or perpendicular to plane (h, k, l).

[Proved]

QU-53: A copper strip 2.0cm wide and 1.0 mm thick is placed in a magnetic field with B = 1.5 weber/meter^2 perpendicular to the strip. If a current of 220 amp. Is set up in the strip, what Hall potential difference appears across the strip?

Here,

$I = 220$ amp.

$B = 1.5$ weber/meter^2.

$t = 1.0 \times 10^{-3}$ m.

$N_0 = 6.023 \times 10^{23}$ atoms/mole

d = density = 9.0 gm/cm^3.

$m = 64$ gms/mole

$n = ?$

$V_H = ?$

number of electrons per unit volume

$$n = \frac{d N_0}{m}$$

$$= \frac{9.0 \times 6.023 \times 10^{23}}{64}$$

$$\therefore n = 8.4 \times 10^{22} \text{ electrons/cm}^3$$

$$= 8.4 \times 10^{28} \text{ electrons/m}^3.$$

Again,

Hall potential,

$$V_H = \frac{IB}{net}$$

$$\Rightarrow V_H = \frac{220 \times 1.5}{8.4 \times 10^{28} \times 1.6 \times 10^{-19} \times 1 \times 10^{-3}}$$

$$\therefore V_H = 24.55 \times 10^{-6} \text{ v} = 24.55\ \mu\text{V}.$$

(Ans)

QU-54: Find the Hall coefficient and electron mobility for germanium if for a given sample (length 1cm. breadth 5mm and thickness 1mm) a current of 6 milliamperes flown from a 1.38 volts supply develops a Hall voltage of 25millivolts across the specimen in a magnetic field of 0.48 webers/meter^2.

Here,

a = area of cross section

$= 5 \times 10^{-3} \times 1 \times 10^{-3} = 5 \times 10^{-6}$ m^2

$L = 1 \times 10^{-2}$ m.

$V_H = 25 \times 10^{-3}$ v.

$I = 6 \times 10^{-3}$ A.

$B = .48$ weber/m^2.

$V = 1.38$

$R_H = ?$

$\mu_e = ?$

We know,

R = resistance of specimen

$$= \frac{1.38}{6 \times 10^{-3}}$$

Z

Also,

Resistivity,

$$\rho = \frac{Ra}{L}$$

$$= \frac{1.38}{6 \times 10^{-3}} \times \frac{5 \times 10^{-6}}{1 \times 10^{-2}}$$

$$\therefore \rho = 0.115\ \Omega\text{-m}.$$

Again,

Hall field, $E_y = \frac{V_y}{\text{thickness}}$

$= \frac{2.5 \times 10^{-3}}{1 \times 10^{-3}}$

We know,

$\frac{1}{ne} = \frac{E_y t}{BJ} = \frac{E_y t}{\frac{BI}{a}}$

$\Rightarrow \frac{1}{ne} = \frac{25 \times 10^{-3} \times 1 \times 10^{-3}}{.48 \times \frac{6 \times 10^{-3}}{5 \times 10^{-6}} \times 10^{-3}}$

$\Rightarrow \frac{1}{ne} = 0.043\ m^3/\text{coulomb}.$

$\therefore$ Hall coefficient, $R_H = \frac{3\pi}{8} \times \frac{1}{ne}$

$= \frac{3\pi}{8} \times .043$

$\therefore R_H = 0.051\ m^3/\text{coulomb}.$ (Ans)

and,

Electron mobility,

$\mu_e = \frac{R_H}{\rho}$

$= \frac{0.051}{.115}$

$\therefore \mu_e = 0.445\ m^2/\text{Volt sec.}$

(Ans)

QUICK PHOTOSTATE
KUET ROAD. KHULNA

Question no: — 55:

From kinetic theory and law of equipartition of energy give,

$$\frac{1}{2}m\langle v^2\rangle = \frac{3}{2}kT$$

$$\therefore \langle v^2\rangle = \frac{3kT}{m}$$

Hence. $$\bar{\tau} = \frac{\lambda\bar{c}}{3kT/m} = \frac{\lambda\bar{c}^2}{(3kT/m)\,\bar{c}}$$

But, $$\bar{c}^2 = \frac{8kT}{\pi m}$$

$$\tau = \frac{\lambda\bar{c}^2}{\langle v^2\rangle\bar{c}} = \frac{\lambda\left(\frac{8kT}{\pi m}\right)}{\left(\frac{3kT}{m}\right)\bar{c}}$$

$$\therefore \bar{\tau} = \frac{\lambda 8kT/m\pi}{(3kT/m)\,\bar{c}} = \frac{8\lambda}{3\pi\bar{c}}$$

The electrical conductivity is given by.

$$\sigma = \frac{ne^2\bar{\tau}}{m}$$

$$\sigma = \frac{ne^2\bar{\tau}}{m}$$

$$= \frac{ne^2 8\lambda}{3\pi m\bar{c}}$$

$$= \frac{ne^2 8\lambda}{3\pi m\sqrt{\frac{8kT}{\pi m}}}$$

$$= \frac{8ne^2\lambda}{3\times2\sqrt{2\pi kTm}}$$

$$\therefore \sigma = \frac{4}{3}\cdot\frac{ne^2\lambda}{\sqrt{2\pi kTm}}$$

[Proved]

Question-56:

. Show that if the mean free path is independent of the velocity, the electrical conductivity of Maxwell-Boltzmann free electron gas may be expressed by the relation

$$\sigma = \frac{4ne^2\lambda}{3\sqrt{2\lambda mkT}}$$

when λ and τ are independent of velocity, the Maxwell-Boltzmann distribution gives average value of τ as $\bar{\tau} = \frac{\lambda\langle v\rangle}{\langle v^2\rangle}$.

Question - 57:

1 gm mole of copper weighs 64 gms and its volume is $\frac{64}{\rho_m}$.

a) The concentration of conduction electrons

$$= \frac{N}{V} = \frac{6.023\times10^{26}\times 8950}{64} \quad [\because \rho_m = 8.95\ gm/cm^3 = 8.95\times10^3\ kg/m^3 = 8950\ kg/m^3]$$

$$= 8.42\times10^{28}/m^3.$$

(Ans)

d) Fermi velocity, $v_F = \frac{\hbar k_f}{m} = \frac{h k_f}{2\pi m}$

But, fermi vector, $k_f = (3\pi^2 \times \frac{N}{V})^{1/3}$

$$= (29.61\times 8.42\times10^{28})^{1/3}$$

$$= 1.356\times10^{10}\ m^{-1}.$$

$\therefore$ Fermi velocity, $v_f = \frac{6.63\times10^{-34}\times 1.356\times10^{10}}{2\times 3.1416\times 9.1\times10^{-31}} = 1.576\times10^6\ m/s$

(Ans)

c) Fermi energy, $E_f = \frac{1}{2}mv_f^2$

$$= \frac{1}{2}\times 9.1\times10^{-31}\times(1.576\times10^6)^2$$

$$= 1.13\times10^{-18}\ J.$$

$\therefore E_f = 7.06\ eV.$

Ans.

b) Mean free time, $\tau = \frac{m}{ne^2\rho}$

$$= \frac{9.1\times10^{-31}}{8.42\times10^{28}\times(1.6\times10^{-19})^2\times 1.55\times10^{-8}}$$

$$= 2.72\times10^{-14}\ sec.$$ (Ans)

e) The mean free path, $\lambda_f = v_f \tau$

$$= 1.576\times10^6 \times 2.72\times10^{-14}$$

$$= 4.30\times10^{-8}\ m$$

(Ans)

Copper has a mass density ρ_m = 8.95 gm/cm^3 and an electrical resistivity ρ = 1.55×10^{-8}ohm-m at room temperature. Calculate-(a) The concentration of the conduction electrons (b) The mean free time τ (c) The Fermi energy E_f (d) The Fermi velocity v_f and (e) The mean free path λ_f at Fermi level.

Question no:- 58:

Given,

$a = 4A^\circ = 4 \times 10^{-10}$ m.

i) We know,

Fermi vector $k_f = \left(3\pi^2 \times \frac{N}{V}\right)^{1/3}$

$= \left(29.61 \times \frac{6.023 \times 10^{23}}{(4 \times 10^{-10})^3}\right)^{1/3}$. [fcc structure $V = a^3$]

$= 6.53 \times 10^{17}$ m^{-1}. (Ans)

Fermi velocity, $v_f = \frac{h k_f}{2\pi m}$

$= \frac{6.63 \times 10^{-34} \times 6.53 \times 10^{17}}{2\pi \times 9.1 \times 10^{-31}}$

$= 7.57 \times 10^{13}$ ms^{-1}

$\therefore$ Fermi energy, $E_f = \frac{1}{2} m v_f^2$

$= \frac{1}{2} \times 9.1 \times 10^{-31} \times (7.57 \times 10^{13})^2$

$= 2.61 \times 10^{-3}$ J

$= 1.63 \times 10^{16}$ eV. (Ans)

ii) Total kinetic energy $= \frac{3}{5} E_f$

$= \frac{3}{5} \times 1.63 \times 10^{16}$

$= 9.78 \times 10^{15}$ eV.

(Ans).

Aluminium metal crystalline in fcc structure. If each atom contributes single electron as free electron and the lattice constant a is $4A^0$, calculate treating conduction electrons as free electron Fermi gas (i) Fermi energy (E_f) and Fermi vector (K_f) (ii) Total kinetic energy of free electron gas per unit volume at 0K.

Question 59:

Question 59: Calculate the inter collision time at room temperature, and drift velocity in a field of $100 Vm^{-1}$ in sodium. whose conductivity is $2.17\times10^{7}\Omega^{-1}m^{-1}$.

We know,

The mass density of sodium, $\delta = 0.97\times10^{3}\ kg/m^3$.

Atomic weight, $M = 23\times10^{-3}\ kg/mole$

electrons concentration, $n = \frac{\delta N}{M}$

$= \frac{0.97\times10^{3}\times6.023\times10^{23}}{23\times10^{-3}}$

$= 2.54\times10^{28}/m^3$

Electrical conductivity, $\sigma_e = ne\mu_e$

$\therefore$ electron mobility, $\mu_e = \frac{\sigma_e}{ne}$

$= \frac{2.17\times10^{7}}{2.54\times10^{28}\times1.6\times10^{-19}}$

$= 5.34\times10^{-3}\ m^2/V\text{-}s$

$\therefore$ Drift velocity, $V_d = \mu_e E$

$= 5.34\times10^{-3}\times100$

$= 0.534\ ms^{-1}$ Ans.

$\therefore$ inter collision time, $\tau = \frac{\mu_e m}{e}$

$= \frac{5.34\times10^{-3}\times9.1\times10^{-31}}{1.6\times10^{-19}}$

$= 3.04\times10^{-14}$ sec. Ans.

Question-60:

The inter collision time in copper 2.3×10^{-14}s. Calculate its thermal conductivity at 300K.

Given, intercollision time, $\tau = 2.3\times10^{-14}$s.

mass density of copper, $\rho_m = 8.92\ gm/cm^3 = 8920\ kg/m^3$

Now, electron concentration, $n = \frac{N\delta}{M} = \frac{6.023\times10^{26}\times8920}{64}$

$= 8.4\times10^{24}/m^3$

$\therefore \sigma = \frac{ne^2\tau}{m} = \frac{8.4\times10^{24}\times(1.6\times10^{-19})^2\times2.3\times10^{-14}}{9.1\times10^{-31}}$

$= 54.32\times10^{6}\ \Omega^{-1}m^{-1}$.

$\therefore$ Thermal conductivity, $K = 3\sigma\left(\frac{k}{e}\right)^2 T$

$= 3\times54.32\times10^{6}\times\left(\frac{1.38\times10^{-23}}{1.6\times10^{-19}}\right)^2\times300$

$= 363.88\ Wm^{-1}K^{-1}$ (Ans.)

QU-61: In a Hall effect experiment on silver a potential of 59μV is developed across a foil of thickness 0.05mm when a current of 28mA is passed in a direction perpendicular to a magnetic field of 1.25T. Calculate the Hall coefficient for silver.

Given,

$V_H = 59\,\mu V = 59 \times 10^{-6}\,V$

$t = 0.05\,mm = 0.05 \times 10^{-3}\,m$

$I = 28\,mA = 28 \times 10^{-3}\,A$

$B = 1.25\,T$

$R_H = ?$

Hall coefficient,

$$R_H = \frac{V_H t}{BI}$$

$$= \frac{59\times10^{-6} \times .05\times10^{-3}}{1.25 \times 28\times10^{-3}}$$

$$\therefore R_H = 8.4 \times 10^{-8}\ m^3/c.$$

(Ans)

Question-62:

A piece of gallium arsenide is doped with $1.0\times10^{24}m^{-3}$ donor atoms; estimate its conductivity.

Given,

$n = 1.0\times10^{24}\ m^{-3}$

$\mu = 0.88$

$e = 1.6\times10^{-19}$ coulomb.

$\sigma = ?$

We know,

Conductivity,

$$\sigma = ne\mu$$

$$= 1.0\times10^{24} \times 1.6\times10^{-19} \times 0.88$$

$$\therefore \sigma = 140.8 \times 10^{3}\ \Omega^{-1} m^{-1}.$$

(Ans)

Question-63:

A rod of p-type germanium 1.2cm long, 1mm wide and 0.5mm thick has a resistance of 240Ω; calculate the impurity concentration.

Given,

$R = 240\,\Omega$ $\mu = 363\ m^2/v$

$A = 1\times10^{-3} \times 0.5\times10^{-3}$

$= 0.5\times10^{-6}\ m^2.$

$L = 1.2\times10^{-2}\ m.$

We know,

$$\rho = \frac{RA}{L}$$

$$= \frac{240 \times 0.5\times10^{-6}}{1.2\times10^{-2}}$$

$$= 0.01\ \Omega\text{-}m.$$

also,

$$\sigma = \frac{1}{\rho} = \frac{1}{0.01} = 100\ \Omega^{-1} m^{-1}$$

Again we know,

$$2\sigma = N_d \mu e$$

$\therefore$ impurity concentration, $N_d = \frac{2\sigma}{\mu e}$

$$= \frac{2\times100}{363\times1.6\times10^{-19}}$$

$$= 3.44\times10^{21}\ /m^3$$

(Ans)

QU-64: Aluminium is trivalent with atomic weight 27 and density 2.7×10^3K/gm^3, whilst the mean free time between electron collisions is 4×10^{-4}s. Calculate the current flowing through an aluminium wire 10m long and 1mm^2 cross sectional area when a potential of 2V is applied to its ends.

We kno

electron concentration,

$n = \frac{Nd}{M}$

$= \frac{6.023\times10^{23}\times 2.7\times10^{+3}\times 3}{27\times10^{-3}}$

[∵ electron concentration (valency 3) in aluminium]

$= 1.81\times10^{29}$ /m^3.

Given,

$M = 27\times10^{-3}$ kg/mole.

$d = 2.7\times10^{3}$ kg/m^3.

$\tau = 4\times10^{-4}$ sec

$L = 10$m

$A = 1$mm^2 $= 1\times10^{-6}$ m^2.

$V = 2$V.

$I = ?$

also. Resistivity, $\rho = \frac{m}{ne^2\tau}$

$= \frac{9.1\times10^{-31}}{1.81\times10^{29}\times(1.6\times10^{-19})^2\times4\times10^{-4}}$

$= 4.91\times10^{-19}$ Ω-m.

∴ $R = \frac{\rho l}{A} = \frac{4.91\times10^{-19}\times10}{1\times10^{-6}}$

$= 4.91\times10^{-12}$ Ω.

∴ Current flowing through aluminium wire $I = \frac{V}{R}$

$= \frac{2}{4.91\times10^{-12}}$

∴ $I = 4.07\times10^{11}$ ampere.

(Ans)

Question no: 65:

We know,

Number of electrons in a cubic meter of sodium,

$n = \frac{Nd}{M}$

$= \frac{6.023\times10^{23}\times0.97\times10^3}{23\times10^{-3}}$

∴ $n = 2.54\times10^{28}$ /m^3.

(Ans)

Given,

$M = 23\times10^{-23}$ kg/mole.

$d = 0.97\times10^3$ kg/m^3.

$n = ?$

. The atomic weight of sodium is 23 and its density is 0.97×10^3K/gm^3. Calculate the number of electrons in a cubic metre of sodium.

QUE-66 Show that the kinetic energy of a three dimensional gas of N free electrons at 0^0K is $E_0 = \frac{3}{5} N E_{f(0)}$.

We can know that, the average K.E. of an electron is given by an expression

$$\bar{E}_0 = \frac{1}{N} \int_0^\infty E \cdot g(E) \cdot f(E)\, dE$$

Substituting the value of $g(E)$ from equation (i) and using the result, $f(E) = 1$ for $E < E_{f(0)}$ and $f(E) = 0$ for $E > E_{f(0)}$ at $T = 0$, we get,

$$E_0 = \frac{1}{N} \int_0^{E_{f(0)}} E \cdot C E^{1/2} \cdot 1\, dE + \frac{1}{N} \int_{E_{f(0)}}^\infty E \cdot C E^{1/2} \cdot 0 \cdot dE$$

$$= \frac{2}{5} \cdot \frac{C}{N} E_{f(0)}^{5/2}$$

[where, $g(E) = C E^{1/2}$ — (i)]

Now,

$$C E_{f(0)}^{1/2} = g(E_{f(0)}) \quad \text{and} \quad g(E_{f(0)}) = \frac{3}{2} \cdot \frac{N}{E_{f(0)}}$$

Combining the above two equations, we get

$$C E_{f(0)}^{1/2} = \frac{3}{2} \cdot \frac{N}{E_{f(0)}}$$

or, $$\frac{C}{N} = \frac{3}{2} \cdot \frac{1}{E_{f(0)}^{3/2}}$$

$$\therefore \bar{E}_0 = \frac{2}{5} \cdot \frac{3}{2} \cdot \frac{1}{E_{f(0)}^{3/2}} \cdot E_{f(0)}^{5/2}$$

$$\therefore E_0 = \frac{3}{5} \cdot E_{f(0)}$$

$\therefore$ Kinetic energy of N free electrons $= \frac{3}{5} N \cdot E_{f(0)}$

[Proved]

A 5.5Kw laser emits light of 655nm wavelength Calculate the number
Question 67: of photons emitted by the laser every second

Given,

Energy of the laser, $E = 5.5\,kW$
$= 5500\,W$

Wavelength, $\lambda = 655\,nm$
$= 655 \times 10^{-9}\,m$

We know, number of photons emitted by the laser every second,

$$\boxed{n = \frac{E}{h\nu}} = \frac{E\lambda}{hc}$$

$$= \frac{5500 \times 655 \times 10^{-9}}{6.63 \times 10^{-34} \times 3 \times 10^{8}}$$

$$\therefore n = 1.81 \times 10^{22}\,/sec$$

(Ans)

Question no: 68

i) let, $d\theta$ be the angular spread.

$$\boxed{d\theta = \frac{\lambda}{d}}$$

Here, $\lambda = 8 \times 10^{-7}\,m$
$d = 5 \times 10^{-3}\,m$

$$\therefore d\theta = \frac{8 \times 10^{-7}}{5 \times 10^{-3}} = 1.6 \times 10^{-4}\ radian$$

(Ans)

ii) Areal spread $= \boxed{(D\,d\theta)^2}$

Here, $D = 4 \times 10^{5}\,km = 4 \times 10^{8}\,m.$

$$\therefore \text{Areal spread} = (4 \times 10^{8} \times 1.6 \times 10^{-4})^2$$
$$= 4.096 \times 10^{9}\,m^2$$

(Ans)

A laser beam has a wavelength of 8×10^{-7}m and aperture 5×10^{-3}m. The laser beam is sent to moon. The distance of the moon is 4×10^{5}Km from the earth. Calculate (i) the angular spread of the beam and (ii) the axial spread when it reaches the moon.

Question-69 :

The coherence length for sodium light is 2.95×10^{-2}m. The wavelength of sodium light is 5893A°. Calculate (i) the number of oscillations corresponding to the coherence length and (ii) the coherence time.

Here,

$\lambda = 5893\times10^{-10}$ m

$c = 3\times10^{8}$ m/s

Coherence length, $L = 2.95\times10^{-2}$ m.

i) Number of oscillations in length,

$$n = \frac{L}{\lambda}$$

$$= \frac{2.95\times10^{-2}}{5893\times10^{-10}}$$

$$= 5\times10^{4}$$

(Ans)

ii) Coherence time, $= \frac{L}{c}$

$$= \frac{2.95\times10^{-2}}{3\times10^{8}}$$

$$= 9.83\times10^{-11}\text{ s}$$

(Ans)

Question no: 70 :

Here $\lambda = 6000A° = 6\times10^{-7}$ m

$d = 2$m

$D = 4\times10^{8}$ m.

i) Angular spread, $d\theta = \frac{\lambda}{d} = \frac{6\times10^{-7}}{2} = 3\times10^{-7}$ radian

ii) Areal spread i.e. area of the spot on the moon,

$A = (D\times d\theta)^2$

$= (4\times10^{8}\times3\times10^{-7})^2$

$\therefore A = 1.44\times10^{4}$ m²

(Ans)

A laser beam λ = 6000A° on earth is focussed by a lens (or mirror) of diameter 2m on the crater on the moon. The distance of the moon is 4×10^{8}m. How big is the spot on the moon? Neglect the effect of earth's atmosphere.