PHY 1107 Solution

Prepared by

Fuad Hossain Shawon

CSE , KUET

Inspired by

Tanvir Ahmmed

CSE , KUET

Hardcopy Project Credit

Md. Tasnin Tanvir

CSE , KUET

CSE-1

50

KHULNA UNIVERSITY OF ENGINEERING & TECHNOLOGY
B.Sc. Engineering 1st Year 1st Term Examination, 2018
Department of Computer Science and Engineering
PHY 1107
Physics

TIME: 3 hours — FULL MARKS: 210

N.B. i) Answer **ANY THREE** questions from each section in separate scripts.
ii) Figures in the right margin indicate full marks.

SECTION A

(Answer **ANY THREE** questions from this section in Script A)

1. a) What is ultra violet catastrophe? What is the significance of it in the development of modern physics? (10)

   b) Show that the spectral distribution of black body radiation derived by Plank is (15)

   $$n(\upsilon)d\upsilon = \frac{8\pi h\upsilon^3}{c^3}\frac{1}{\exp(\frac{h\upsilon}{kT}-1)}d\upsilon$$

   Derive Wein's law and Rayleigh-Jeans law as a function of frequency for high and low frequency limit respectively.

   c) The longest wavelength of light that will cause photo emission from sodium is approximately 540nm. (10)
      i) Find the work function of sodium.
      ii) Find the maximum kinetic energy for photo electrons emitted when light of wave length 400nm strikes a sodium plate.

2. a) Why was a change in the Bohr model of atom required? Mention the quantum numbers associate with vector atom model. (10)

   b) Explain Compton effect and show that Compton shift depends only on the angle of scattering and it is independent of the wavelength of the incident photons. (15)

   c) In a Compton effect experiment in which the incident X-ray have wavelength of 10pm, the scattered X-ray at a certain angle have a wavelength of 10.5pm. Find the momentum (magnitude & direction) of the corresponding recoil electrons. (10)

   $\theta = 37.436^\circ$ $p = h/\lambda = 1.3254\times10^{-21}$

   $p = \frac{h}{mv}$

3. a) What is quantum mechanics? What are the old and new 'quantum mechanics'? (10)

   b) What is the need of Schrodinger's wave equation? Develop time-dependent and time-independent Schrodinger's wave equations. (15)

   c) Find the first excited state of the harmonic oscillator. (10)

4. a) What are the common defects in the images produced by a single lens? Obtain the conditions for achromatism of two thin lenses of the same materials placed at a distance apart. (12)

   b) Why, in Young's double slit experiment, the slit-slit separation should be small and the slit-screen separation large? (13)

   c) The radius of the 10th dark ring in Newton's ring apparatus changes from 60 to 50 mm when a liquid is introduced between the lens and the plate. Calculate the refractive index of the liquid. (10)

SECTION B

(Answer **ANY THREE** questions from this section in Script B)

5. a) Define the terms (i) Unit cell (ii) Polycrystalline (iii) Basis (iv) Miller index and (v) Amorphous solid. (15)

   b) Define atomic packing fraction. Find the atomic packing fraction in a crystals for (i) Simple cubic, (ii) Body centered cubic and face centered cubic structures, treating the atoms as spherical. (10)

   c) In Bragg's law set up, X-rays were diffracted by a fcc crystal having lattice constant 0.407nm at an angle $2\theta = 26.697^\circ$ from the (220) planes. Find the wavelength of X-rays. (Assume first order diffraction). (10)

6. a) Discuss the inelastic scattering photons by phonons and obtain an expression for the frequency of phonon emitted in the process. (10)

b) Following Einstein theory deduce an expression for the lattice heat capacity. Discuss the successes and failures of this model. (15)

c) The Debye temperature of carbon (diamond structure) is 1850K. Calculate the specific heat per k-mole for diamond at 20K. Also compute the highest lattice frequency involved in the Debye theory. (10)

7. a) What are the outstanding properties of metals in the case of free electron model? Obtain an expression for the thermal conductivity from the free electron theory of metals. (15)

b) Derive the expression for Fermy energy of a free electron in the three dimensions. (10)

c) Shaw that the average kinetic energy of an electron at 0K is $\frac{3}{5}E_f$; where E_f is the Fermy energy at Fermy surface. (10)

8. a) What are temporal and spatial coherence? Which properties of the LASER they lead to? (10)

b) Explain the terms (i) Induced absorption (ii) Spontaneous emission and (iii) Stimulated emission. Describe the working principle of Ruby LASER with suitable diagram. (15)

c) A LASER beam has a power of 50 mw. It has an aperture of 5.1×10^{-3} m and it emits light of wavelengths 7200 A. The beam is focused with a lens of focal length 0.1m. Calculate the area and the intensity of the image. (10)

1. a) Ans: The ultra-violet catastrophe, also called the Rayleigh-Jeans catastrophe, was the prediction of late 19th century classical phisics that an ideal black body at thermal equilibrium will emit radiation in all frequency ranges, emitting more energy as the frequency increases.

Ans:
The significance: The ultra violet catastrophe A black body is an idealized object which absorbs and emits all frequencies. Classical physics can be used to derive an equation which describes the intensity of black body radiation as a function of frequency for a fixed temperature - the result is knowns as the Rayleigh jeans law.

c). Ans:
(i) given,

$\lambda = 540nm = 540 \times 10^{-9}m$

$E = ?$

we know

$E = h\nu = h\frac{c}{\lambda}$

Here,
$h = 6.63 \times 10^{-34} Js$
$c = 3 \times 10^{8} ms^{-1}$

(2)

$$\Rightarrow \phi = \frac{6.63\times10^{-34}\times 3\times10^{8}}{540\times10^{-9}}$$

$$= 3.683\times10^{-19}\,J = 2.30\,eV$$

(ii) given,

$\lambda = 400nm = 400\times10^{-9}m$

$KE = ?$ and $\phi = 3.683\times10^{-19}$ (From (i))

we know,

$$E = KE + \phi$$

$$\Rightarrow \frac{hc}{\lambda} = KE + \phi$$

$$\Rightarrow \frac{6.63\times10^{-34}\times 3\times10^{8}}{400\times10^{-9}} = KE + 3.683\times10^{-19}$$

$$\Rightarrow KE = 1.2895\times10^{-19}\,J = 0.8059\,eV$$

-o-

b). Ans:

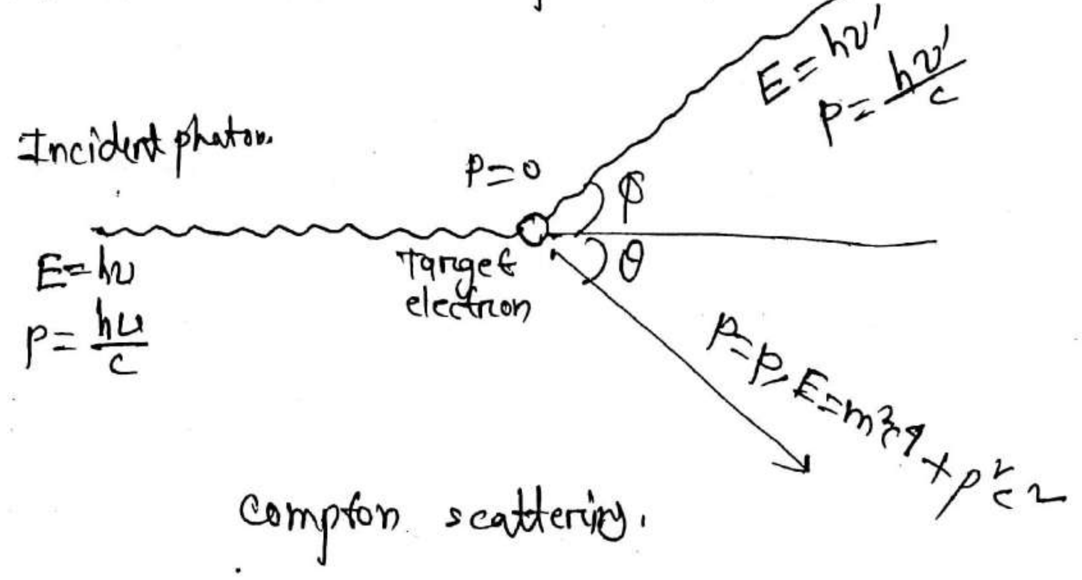
Ans. to the ques. No — 02

b

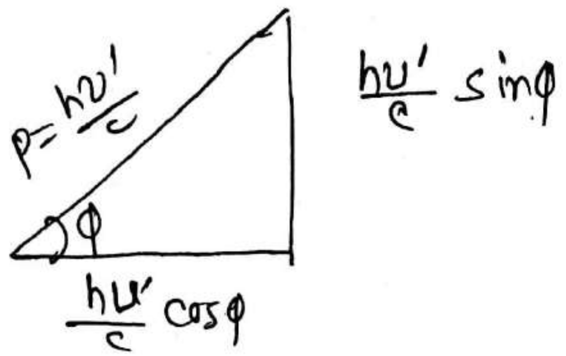
Compton effect:

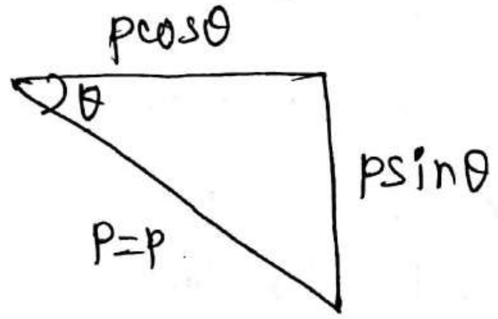
The scattering of a photon by an electron is called the compton effect. Energy and momentum are conserved in such an event and as a result the scattered photon has less energy (larger wave length) than the incident photon.

A phenomenon in which a collision betn a photon and a particle results in increase in the KE of the particle and corresponding increase in the wavelength of the photon called compton effect.



compton scattering.





vector diagram.

If the initial photon has the frequency ν associated with it, the scattered photon has, the linear frequency ν'.

where,

loss in photon energy = gain electron energy

$$\Rightarrow h\nu - h\nu' = KE \quad \text{——— (1)}$$

Again the momentum of particle.

$$E = pc$$

$$\Rightarrow p = \frac{E}{c} = \frac{h\nu}{c} \quad \text{——— (11)}$$

The initial momentum of photon $= \frac{h\nu}{c}$

The scattered " " " $= \frac{h\nu'}{c}$

The initial " of electron $= 0$

The scattered " " " $= p$

In the orginal photon direction.

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\phi + p\cos\theta$$

$$\Rightarrow pc\cos\theta = h\nu - h\nu'\cos\phi \quad \text{(iii)}$$

In the perpendiculars to the direction,

$$0 + 0 = \frac{h\nu'}{c}\sin\phi - p\sin\theta$$

$$\Rightarrow pc\sin\theta = h\nu'\sin\phi \quad \text{(iv)}$$

(iii)^2 + (iv)^2,

$$p^2c^2(\cos^2\theta + \sin^2\theta) = (h\nu - h\nu'\cos\phi)^2 + (h\nu'\sin\phi)^2$$

$$\Rightarrow p^2c^2 = h^2\nu^2 - 2h^2\nu\nu'\cos\phi + h^2\nu'^2\cos^2\phi + h^2\nu'^2\sin^2\phi$$

$$\Rightarrow p^2c^2 = (h\nu)^2 - 2(h\nu)(h\nu')\cos\phi + (h\nu')^2 \quad \text{(v)}$$

The initial energy of the particle.

$$E = KE + mc^2 \quad \text{——— (vi)}$$

$$E = \sqrt{m^2c^4 + p^2c^2} \quad \text{——— (vii)}$$

From (vi) and (vii)

$$KE + mc^2 = \sqrt{m^2c^4 + p^2c^2}$$

$$\Rightarrow (KE)^2 + 2 \cdot KE \cdot mc^2 + m^2c^4 = m^2c^4 + p^2c^2$$

$$\Rightarrow p^2c^2 = (KE)^2 + 2mc^2 \cdot KE$$

$$\Rightarrow p^2c^2 = (h\nu - h\nu')^2 + 2mc^2(h\nu - h\nu')$$

$$\Rightarrow p^2c^2 = (h\nu)^2 - 2h\nu \cdot h\nu' + (h\nu')^2 + 2mc^2(h\nu - h\nu') \quad \text{——— (viii)}$$

From (v) and (viii)

$$(h\nu)^2 - 2h\nu \cdot h\nu' \cdot \cos\phi + (h\nu')^2 = (h\nu')^2 - 2(h\nu)(h\nu') + (h\nu)^2 + 2mc^2(h\nu - h\nu')$$

$$\Rightarrow 2mc^2(h\nu - h\nu') = -2h\nu . h\nu' . \cos\phi + 2h\nu . h\nu'$$

$$= 2h\nu . h\nu'(1-\cos\phi) \quad \text{--- (ix)}$$

dividing. (ix) by $2h^2c^2$

$$\frac{m}{h^2}(h\nu - h\nu') = \frac{\nu . \nu'}{c^2}(1-\cos\phi)$$

$$\Rightarrow \frac{mc}{h}\left(\frac{\nu}{c} - \frac{\nu'}{c}\right) = \frac{\nu}{c} . \frac{\nu'}{c}(1-\cos\phi)$$

$$\Rightarrow \frac{mc}{h}\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = \frac{1}{\lambda} . \frac{1}{\lambda'}(1-\cos\phi)$$

$$\Rightarrow \frac{mc}{h} . \frac{\lambda' - \lambda}{\lambda\lambda'} = \frac{1}{\lambda\lambda'}(1-\cos\phi)$$

$$\Rightarrow \frac{mc}{h}(\lambda' - \lambda) = (1-\cos\phi)$$

$$\Rightarrow \lambda' - \lambda = \frac{h}{mc}(1-\cos\phi)$$

so, the compton shift $(\lambda' - \lambda)$ is independent on the wavelength of the incident photon.

(2)

c

$$\lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\phi)$$

$$\Rightarrow 10.5\times10^{-12} - 10\times10^{-12} = \frac{6.624\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}(1-\cos\phi)$$

$$\Rightarrow \phi = 37.445^\circ.$$

$$p = \frac{h}{\lambda'}$$

$$= \frac{6.624\times10^{-34}}{10.5\times10^{-12}}$$

$$= 6.308\times10^{-23}$$

$$\left[\begin{array}{l} p = \frac{E}{c} \\ = \frac{h\nu'}{c} \\ = \frac{h}{c}\times\frac{c}{\lambda'} \end{array}\right.$$

Answer to the Question No: 4/a

The common defects ~~are~~ in the image produced by a single lens are called aberrations. Abberations are of two types

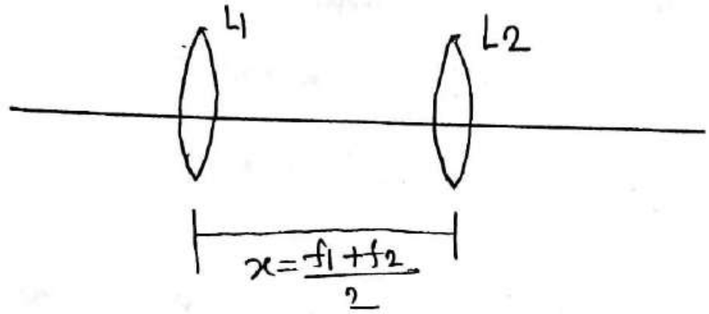
1) Monochromatic abberations

2) Chromatic abberations.

Monochromatic abberation: Monochromatic ~~abbero~~ aberration is also called as spherical aberration. The failure of paraxial and marginal rays to form a common image for pointed object in front of a lens on the principal axis is called spherical aberration.

2) chromatic aberration:
When a beam of white light passes through a lens, the failure of different colors of light to form a common is called chromatic aberration. It is caused due to variation of refractive index of the lens with the color.

Condition for achromatism of two thin lenses of same material placed at a distance apart:



Let,

f_1 and f_2 be the focal lengths of two lenses placed coaxial and separated by a distance. The mean focal length F of the combination is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2} \quad \text{.....(i)}$$

Differentiating the above equation, we obtain

$$-\frac{\partial F}{F^2} = -\frac{\partial f_1}{f_1^2} - \frac{\partial f_2}{f_2^2} - \frac{x}{f_2}\left(-\frac{\partial f_1}{f_1^2}\right) - \frac{x}{f_1}\left(-\frac{\partial f_2}{f_2^2}\right) \quad \text{---(ii)}$$

It follows from $-\frac{\partial f}{f^2} = \frac{\omega}{f}$ that

$$\left(-\frac{\partial f_1}{f_1^2} = \frac{\omega_1}{f_1}\right) \text{ and } \left(-\frac{\partial f_2}{f_2^2} = \frac{\omega_2}{f_2}\right)$$

where ω_1 and ω_2 are the dispersive powers of the first and second lense respectively.

We get,

$$-\frac{\partial F}{F^2} = \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} - \frac{x}{f_2}\left(\frac{\omega_1}{f_1}\right) - \frac{x}{f_1}\left(\frac{\omega_2}{f_2}\right)$$

or, $$-\frac{\partial F}{F^2} = \frac{\omega_1 f_2 + \omega_2 f_1}{f_1 f_2} - \frac{x(\omega_1+\omega_2)}{f_1 f_2} \quad \ldots \text{(iii)}$$

For an achromatic combination, F should be the same for violet and red colours. That is change in F must be zero. Thus,

$$\partial F = 0$$

$$\frac{\omega_1 f_2 + \omega_2 f_1}{f_1 f_2} - \frac{x(\omega_1+\omega_2)}{f_1 f_2} = 0$$

$$(\omega_1 f_2 + \omega_2 f_1) - x(\omega_1 + \omega_2) = 0$$

$$\Rightarrow x = \frac{\omega_1 f_2 + \omega_2 f_1}{\omega_1 + \omega_2} \quad \ldots \text{(iv)}$$

When the two lenses are of same material,

$$\omega_1 = \omega_2 .$$

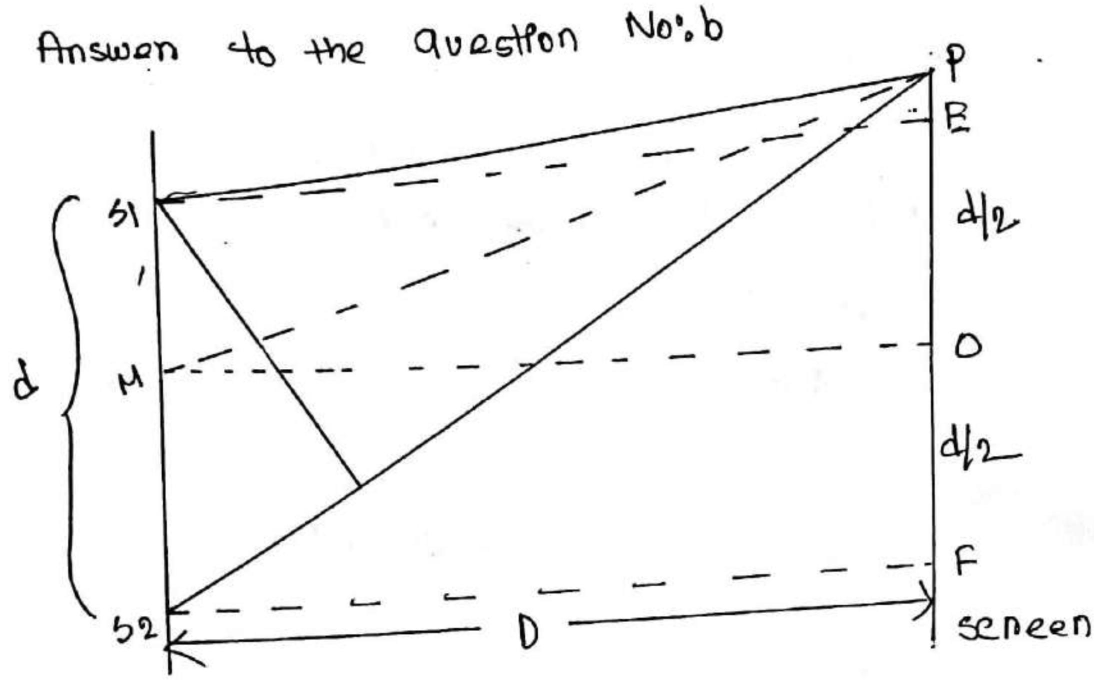
The equ (iv) becomes

$$x = \frac{f_1 + f_2}{2}$$

Thus the condition for achromatism of two thin co-axial lenses made of same material and separated by a distance is that the distance between the two lenses must be equal to the mean focal length of the two lenses.

b) Why, in Young's double slit experiment, the slit-slit separation should be small and the slit-screen separation large?

Answer to the question No: b



The fringe separation, $\beta = \frac{\lambda D}{d}$ . . . (i)

where,

D is the slit-screen separation

d is the slit-slit separation

From the equation (1),

The width of the fringe is directly propotional to the distance of the screen from the two slits, $\beta \propto D$. If the sources are ~~far apart~~ close enough to the screen, the fringe width will be very small and fringes are not seen properly.

The width of the fringe is inversely propotional to the distance between the two slits.

$\beta \propto \frac{1}{d}$.

So, the closer ~~th~~ are the slits, the wider will be the fringes. and they will be seen separately.

2

Answer to the question No: 4/c

14

Given that,

$$(r_m)_{air} = 60\ mm = 0.6\ cm$$

$$(r_m)_{liq} = 50\ mm = 0.5\ cm$$

$$\therefore \mu = \frac{(r_m)_{air}}{(r_m)_{liq}}$$

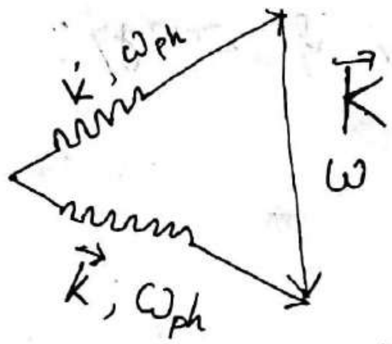
$$= \frac{0.6}{0.5}$$

$$= 1.2$$

So, the refractive index of the liquid 1.2.

Question-6

(a)



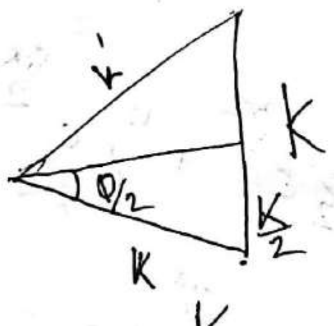
$$\hbar\omega'_{ph} + \hbar\omega = \hbar\omega_{ph}$$

$$\omega'_{ph} \simeq \omega_{ph}$$

$$k' \simeq k$$

We know, $\omega_{ph} = k\frac{c}{\mu}$

$$k = \frac{\omega_{ph} \times \mu}{c}$$



$$\sin\frac{\phi}{2} = \frac{\frac{K}{2}}{k}$$

$$K = 2k\sin\frac{\phi}{2}$$

$$\frac{\omega}{v_s} = 2k\sin\frac{\phi}{2}$$

$$\omega = 2k v_s \sin\frac{\phi}{2} = \frac{2\omega_{ph}\mu v_s}{c}\sin\frac{\phi}{2}$$

(b) $\varepsilon_n = (n+\frac{1}{2})\hbar\omega_0$

Maxwell Boltzman distribution of energy:

$$\bar{\varepsilon} = \frac{\sum_{n=0}^{\infty} \varepsilon_n e^{\frac{-\varepsilon_n}{k_B T}}}{\sum_{n=0}^{\infty} e^{\frac{-\varepsilon_n}{k_B T}}}$$

$$= \frac{\sum_{n=0}^{\infty} (n+\frac{1}{2})\hbar\omega_0\, e^{\frac{-(n+\frac{1}{2})\hbar\omega_0}{k_B T}}}{\sum_{n=0}^{\infty} e^{\frac{-(n+\frac{1}{2})\hbar\omega_0}{k_B T}}} \qquad \because \frac{-\hbar\omega_0}{k_B T} = x$$

$$= \frac{\hbar\omega_0 \sum_{n=0}^{\infty} (n+\frac{1}{2}) e^{x(n+\frac{1}{2})}}{\sum_{n=0}^{\infty} e^{x(n+\frac{1}{2})}}$$

$$= \frac{\hbar\omega_0 \left(\frac{1}{2}e^{\frac{x}{2}} + \frac{3}{2}e^{\frac{3x}{2}} + \frac{5}{2}e^{\frac{5x}{2}} + \cdots\right)}{e^{\frac{x}{2}} + e^{\frac{3x}{2}} + e^{\frac{5x}{2}} + \cdots}$$

$$= \hbar\omega_0 \left[\frac{d}{dx} \ln\left(e^{\frac{x}{2}} + e^{\frac{3x}{2}} + e^{\frac{5x}{2}} + \cdots\right)\right]$$

$$= \hbar\omega_0 \left[\frac{d}{dx} \ln e^{\frac{1}{2}x}\left(1 + e^{x} + e^{2x} + \cdots\right)\right]$$

$$= \hbar\omega_0 \left[\frac{d}{dx} \ln e^{\frac{x}{2}} + \frac{d}{dx}\ln(1-e^{x})\right]$$

$$= \hbar\omega_0 \left(\frac{1}{2} - \frac{\frac{-e^{x}/e^{x}}{(1-e^{x})}}{e^{x}}\right)$$

$$= \hbar\omega_0 \left(\frac{1}{2} + \frac{1}{e^{-x} - 1}\right)$$

$$= \hbar\omega_0 \left(\frac{1}{2} + \frac{1}{e^{\frac{\hbar\omega_0}{k_B T}} - 1}\right)$$

$$= \frac{1}{2}\hbar\omega_0 + \frac{\hbar\omega_0}{e^{\frac{\hbar\omega_0}{k_B T}} - 1}$$

$$E = 3N\bar{\varepsilon}$$

$$= \frac{3}{2}N\hbar\omega_0 + \frac{3N\hbar\omega_0}{e^{\frac{\hbar\omega_0}{k_B T}} - 1}$$

$$c_v = \frac{dE}{dT} = 3N\hbar\omega_0 \left[\frac{e^{\frac{\hbar\omega_0}{k_B - 1}} \times 0 - 1 \cdot \frac{d}{dt}\left(e^{\frac{\hbar\omega_0}{k_B T}} - 1\right)}{\left(e^{\frac{\hbar\omega_0}{k_B T}} - 1\right)^2} \right.$$

$$= 3N\frac{(\hbar\omega_0)^2}{k_B T^2} \frac{e^{\frac{\hbar\omega_0}{k_B T}}}{\left(e^{\frac{\hbar\omega_0}{k_B T}} - 1\right)^2}$$

$$= 3Nk_B \frac{\hbar^2\omega_0^2}{k_B^2 T^2} \frac{e^{\frac{\hbar\omega_0}{k_B T}}}{\left(e^{\frac{\hbar\omega_0}{k_B T}} - 1\right)^2}$$

$$c_v = 3Nk_B\left(\frac{\theta_E}{T}\right)^2 \frac{e^{\frac{\theta_E}{T}}}{\left(e^{\frac{\theta_E}{T}} - 1\right)^2}$$

(2

(18)

(c) Attempted and failed

Ans: TRY it yourself!

(8)(a) What are temporal and spatial coherence? Which properties of the LASER they lead to?

Temporal Coherence:
It's a measure of the average correlation between the value of a wave and itself delayed by T, at any pair of times. Temporal coherence tells us how monochromatic a source is.

Spatial Coherence:
The spatial coherence is the phase relationship between the radiation field at different points in space. In some system such as a water waves or optics, wave-like states com extend over one or two dimensions. Spatial coherence describes the ability for two points in space in the extent of a wave to interfere when averaged time is over.

(b) Explain the terms (i) Induced absorption (ii) Spontaneous emission and (iii) stimulated emission. Describe the working principle of Ruby LASER with suitable diagram.

Ans: Try it yourself

⇒

(c) A LASER beam has a power of 50 mW. It has an aperture of 5.1×10^{-3} m and it emits light of wave lengths 7200 Å. The beam is focused with a lens of focal length 0.1 m. Calculate the area and the intensity of the image.

Ans: Try it yourself.

43

~~54~~ 65

KHULNA UNIVERSITY OF ENGINEERING & TECHNOLOGY
B.Sc. Engineering 1st Year 1st Term Examination, 2017
Department of Computer Science and Engineering
PHY 1107
Physics

TIME: 3 hours

N.B. i) Answer **ANY THREE** questions from each section in separate scripts. FULL MARKS: 210

ii) Figures in the right margin indicate full marks.

iii) Assume reasonable data if any missing.

SECTION A
(Answer **ANY THREE** questions from this section in Script A)

1. a) Show that the spectral distribution derived by Plank is $n(\nu)d\nu = \frac{8\pi h\nu^3}{c^3}\frac{1}{\exp(h\nu/kt - 1)}d\nu$ and which reduce to the Rayleigh-Jeans law, $n(\nu)d\nu = \frac{8\pi\nu^2 kt}{c^3}d\nu$ in the long wavelength limit. (15)

b) In what sense wave theory is inadequate to explain photoelectric effect? (10)

c) In the spectral distribution of blackbody radiation, the wavelength λ_{max} at which the intensity reaches its maximum value decreases as the temperature is increased, in inverse proportional to the temperature: $\boxed{\lambda_{max}\alpha\frac{1}{T}}$. That is called the Wein's displacement law. The proportional constant is experimentally determined to be $\lambda_{max}T = 2.898\times10^{-3}m.k$. (10)

math

i) At what wavelength does a room temperature $(T = 20°C)$ object emit the maximum thermal radiation?

ii) To what temperature must we heat it until its peak thermal radiation is in the red region of the spectrum?

2. a) By deriving the expression for the Compton shift, show that the scattered photon always have a larger wavelength than the incident photon. (15)

b) X-rays with a wavelength $0.0045\,nm$ are used in a scattering experiment. If the X-rays are scattered through an angle of $160°$, what is the wavelength of the scattered photon? (10)

c) Discuss vector atom model. Write down the names of all quantum number associated with vector atom model. (10)

3. a) Explain physical significance of wave function. Deduce the time-independent Schrödinger wave equation. (12)

b) What is understood by stationary state and probability current density? Show that the probability density p and probability current density $\overline{j}$, satisfy the equation of continuity $\frac{\delta p}{dt} + \nabla.\overline{j} = 0$. (13)

c) A particle of mass "m" in the infinite square well (of width "a") starts out in the left half of the well, and is (at $t = 0$) equally likely to be found at any point in the region. What is the initial wave function, $\psi(x,0)$? (10)

4. a) Obtain the condition for achromatism of two thin lenses of the same material placed at a distance apart. (12)

b) Two monochromatic waves emanating from two coherent sources have the displacements represented by (13)

$$y_1 = a\cos\omega t$$
$$y_2 = a\cos(\omega t + \phi)$$

Where ϕ is the phase difference between two waves. Show that the resultant intensity at a point due to their superposition is given by $I = 4I_0\cos^2\phi/2$, where $I_0 = a^2$. Hence, obtain the conditions for constructive interference.

$\lambda = 6000 \times 10^{-8}$, $\mu = 1.4$

c) Light of wavelength 6000 Å falls normally on a thin wedge film of refractive index 1.4 forming fringes that are 2 *mm* apart. $\boxed{\text{Find the angle of the wedge.}}$

SECTION B

(Answer ANY THREE questions from this section in Script B)

5. a) Discuss seven crystal systems by giving one example of each and describe the various types of Bravais lattices in case of three dimension with the help of neat and clear diagrams. (15)

b) What is Miller indices? Discuss in brief the procedure for finding Miller indices. (10)

(33) c) In a unit cell of simple cubic structure, find the angle between the normal to pair of plane whose Miller indices are (i) [110] & [101] and (ii) [121] & [222]. (10)

6. a) What is reciprocal lattice? How is reciprocal lattice constructed? List its important properties. (12)

b) Explain the concept of phonon. Show that the dispersion relation for the lattice waves in a monoatomic linear lattice of mass "*m*" spacing "*a*" and the nearest neighbor interaction "*f*" is $w = \sqrt{\frac{4f}{m}}\left|\sin\left(\frac{ka}{2}\right)\right|$. Where *w* is the angular frequency and *k* is the wave vector. (13)

4th c) Compare the frequencies of sound waves of wavelength $\lambda = 10^{-7}$ cm for (i) homogeneous line, (ii) acoustic waves on a linear lattice containing two identical atoms per primitive cell of inter–atomic spacing 2.46 Å and (iii) light waves of the same wavelength, given that $v_0 = 10^5$ cm/sec. (10)

7. a) What are assumptions of Einstein's theory of specific heat of solid? Derive relation for lattice heat capacity following Einstein model. (1

b) Show that the ratio between thermal and electrical conductivity is proportional to absolute temperature.

c) Show that average kinetic energy of a free electron at 0k is $\frac{3}{5}E_f$. where E_f is Fermi energy and average speed is $\frac{3}{4}v_f$. where v_f is the velocity at Fermi surface.

8. a) Give the brief outlines of the form of input energy of a LASER. Give some characteristics properties of a LASER light.

b) What is population inversion? How can it be achieved? Explain, with neat diagram, the working of a He-Ne LASER.

c) The coherence length for the real cadmium line of wavelength 6.55×10^{-7} *m* is 30.5 *cm*. Calculate (i) the number of oscillations corresponding to the coherence length and (ii) the coherence time.

4th

Answer question no 1

a) Show that the spectral distribution derived by Plank is $n(\nu)\, d\nu = \frac{8\pi h\nu^3}{c^3} \frac{1}{\exp(\frac{h\nu}{kt} - 1)} d\nu$ and which reduce to the Rayleigh-Jeans Law, $n(\nu)d\nu = \frac{8\pi \nu^2 kt}{c^3} d\nu$ in the long wavelength limit.

Solve:

The quantisation of electromagnetic radiation means that the energy of a particular mode of frequency ν cannot have any arbitrary value but only those energies which are multiples of $h\nu$, in other words the energy of the mode is $E(\nu) = nh\nu$, where we associate n photons with this move. ~~Consider~~ Considering all the modes (and photons) to be in thermal equilibrium at temperature T. In order to establish equilibrium, there must be ways of exchanging energy between the modes. and this can occur through interections with any particles or ~~ge~~ oscillators.

Now, we use the Boltzman distribution ~~the~~ to determine the expected occupancy of the modes. the probablity that a single mode has energy $E_n = nh\nu$ is given by the Boltzman factor

$$p(n) = \frac{\exp(-E_n/KT)}{\sum_{n=0}^{\infty} \exp(-E_n/KT)}$$

Where the denominator ensures that the total probability is unity, the usual normalisation procedure. In the language of photons, this is the probability that the state contains n photons of frequency ν.

the mean energy of the mode of frequency ν is therefore

$$\overline{E}_r = \sum_{n=0}^{\infty} E_n \, p(n) = \frac{\sum_{n=0}^{\infty} E_n \exp(-E_n/KT)}{\sum_{n=0}^{\infty} \exp(-E_n/KT)}$$

$$= \frac{\sum_{n=0}^{\infty} nh\nu \exp(-nh\nu/KT)}{\sum^{\infty} \exp(-nh\nu/KT)}$$

$$\bar{E}_\nu = h\nu \frac{\sum_{n=0}^{\infty} n x^n}{\sum_{n=0}^{\infty} x^n} = h\nu \frac{(x+2x^2+3x^3+\cdots)}{(1+x+x^2+\cdots)}$$

$$= h\nu x \left(\frac{1+2x+3x^2+\cdots}{1+x+x^2+\cdots}\right)$$

Hence the mean energy of the mode is

$$\bar{E} = \frac{h\nu x}{1-x} = \frac{h\nu}{x^{-1}-1} = \frac{h\nu}{e^{h\nu/KT}-1}$$

This is the result we have been seeking. To find the classical limit, we allow the energy quanta $h\nu$ to tend to zero. Expanding $e^{h\nu/KT}-1$ for small values of $h\nu/KT$,

$$e^{h\nu/KT}-1 = 1 + \frac{h\nu}{KT} + \frac{1}{2!}\left(\frac{h\nu}{KT}\right)^2 + \cdots - 1$$

Thus, for small values of $h\nu/KT$,

$$e^{h\nu/KT} - 1 = \frac{h\nu}{KT}$$

and so, $\bar{E} = \dfrac{h\nu}{e^{h\nu/KT}-1} = \dfrac{h\nu}{e/KT} = KT$

Thus, if we take the classical limit, we recover ex-actly the expression for the average energy of a harmonic oscillator in thermal equilibrium, $\bar{E} = KT$.

Furthermore, wave theory could not explain the linear relationship between the change in frequency and change in stopping voltage.

By considering light to be made of photons of energy $h\nu$; Einstein could explain both the minimum frequency required and extra energy imparted being linearly related to the change in frequency. Also increasing the intensity of incident light meant more photons were incident on the metal surface - and thus liberated more electrons without changing their kinetic energy.

(c)(i) Given that

$$\lambda_{max} T = 2.898 \times 10^{-3} \text{ m.K}$$

and the temperature $T = 20^\circ C = 298$ K

$$\therefore \lambda_{max} = \frac{2.898 \times 10^{-3} \text{ m.K.}}{298 \text{ K}}$$

$$= 1.003 \times 10^{-5} \text{ m}$$

(ii)

The red region of the spectrum 680 nm

So, we must heat to the temperature of $T = \frac{2.898 \times 10^{-3}}{\lambda_{max}}$ K

$$= \frac{2.898 \times 10^{-3}}{680}$$

So, the tempeature is 4395.59 K

$$= 4395.59$$

PHY 1107

1st Year, 1st Term 2017

Section A

Ans. to the Q No - 2

a) By deriving the expression for the compton shift, show that the scattered photon always have a larger wavelength than the incident photon.

Ans:

If the initial photon has the frequency ν associated with it the scattred photon has the lowe frequency.

Then,

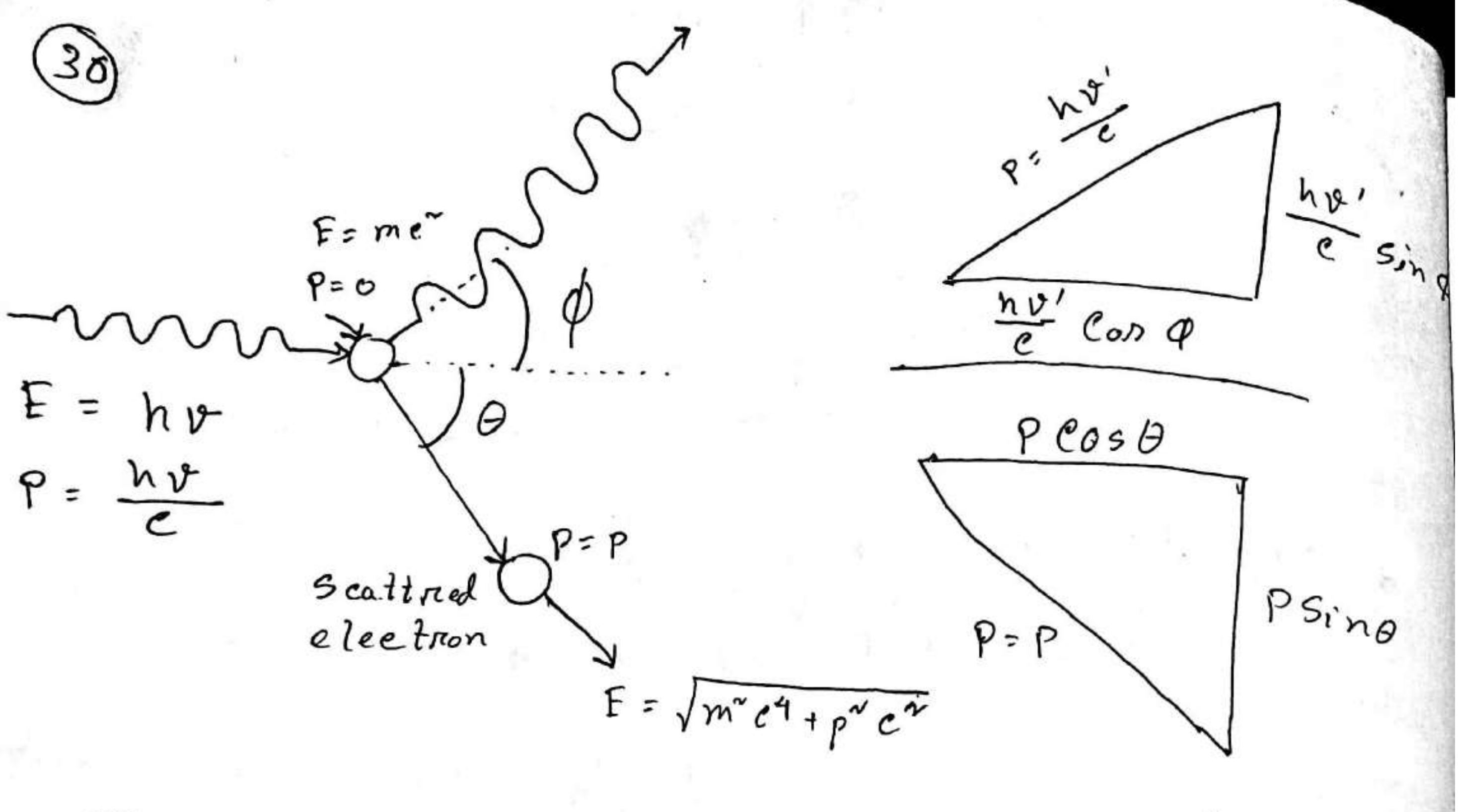
Less the photo energy = gain the electron energy

$$h\nu - h\nu' = KE \quad \text{---} \; (i)$$

Again momentum of partical,

$$E = pc$$

$$\therefore \; p = \frac{E}{c} = \frac{h\nu}{c} \quad \text{----} \; (ii)$$



The initial momentum of photon $= \frac{h\nu}{c}$

The final momentum of photon $= p$

~~The~~ The original photon direction,

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\phi + \cancel{p\cos\phi}\ p\cos\theta$$

$$pc\cos\theta = h\nu - h\nu'\cos\phi \quad \text{---- (iii)}$$

In the per-pendicular direction,

$$0 + 0 = \frac{h\nu}{c}\sin\phi - p\sin\theta$$

$$\therefore pc\sin\theta = h\nu\sin\phi \quad \text{.... (iv)}$$

$(iii)^2 + (iv)^2 \longrightarrow$

$$p^2c^2 = (h\nu - h\nu'\cos\phi)^2 + (h\nu'\sin\phi)^2$$

$$\Rightarrow p^2c^2 = (h\nu)^2 - 2h\nu \cdot h\nu' \cos\varphi + (h\nu')^2 \cos^2\varphi + (h\nu')^2 \sin^2\theta$$

$$\therefore p^2c^2 = (h\nu)^2 - 2(h\nu)(h\nu')\cos\varphi + (h\nu')^2 \quad \text{.....} (v)$$

The total energy of particale,

$$E = KE + mc^2 \quad \text{....} (vi)$$

intence of momentum,

$$E = \sqrt{m^2c^4 + p^2c^2} \quad \text{....} (vii)$$

From equ (vi) and (vii)

$$KE + mc^2 = \sqrt{m^2c^4 + p^2c^2}$$

$$\Rightarrow p^2c^2 = KE^2 + 2mc^2 KE$$

$$\Rightarrow p^2c^2 = (h\nu - h\nu')^2 + 2mc^2(h\nu - h\nu')$$

[using 1]

$$\Rightarrow p^2c^2 = (h\nu)^2 - 2(h\nu)(h\nu) + (h\nu')^2 + 2mc^2(h\nu - h\nu')$$

$$\text{....} (viii)$$

From (v) and (viii) $\rightarrow$

$$(h\nu)^2 - 2(h\nu)(h\nu')\cos\varphi + h\nu' = (h\nu)^2 - 2(h\nu)(h\nu') + (h\nu')^2 + 2mc^2(h\nu - h\nu')$$

(32)

$\Rightarrow 2mc^2(h\nu - h\nu') = -2(h\nu)(h\nu')\cos\varphi + 2(h\nu)(h\nu')$

$\Rightarrow \cancel{2}mc^2(h\nu - h\nu') = (h\nu)(h\nu')(1-\cos\varphi)$ .... (1

dividing (1x) by $h^2c^2 \longrightarrow$

$$\frac{m}{h^2}(h\nu - h\nu') = \frac{\cancel{2}h^2\nu\nu'}{\cancel{2}h^2c^2}(1-\cos\varphi)$$

$$\Rightarrow \frac{mc}{h}\left(\frac{\nu}{c} - \frac{\nu'}{c}\right) = \frac{\nu}{c}\cdot\frac{\nu'}{c}(1-\cos\varphi)$$

Since, $c = \nu\lambda \quad \therefore \frac{\nu}{c} = \frac{1}{\lambda}$ $\quad \lambda$ = W.L. of incident photon

$c = \nu'\lambda' \quad \therefore \frac{\nu'}{c} = \frac{1}{\lambda'}$ $\quad \lambda'$ = W.L. of scatt photon

So,

$$\frac{mc}{h}\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = \frac{1}{\lambda}\cdot\frac{1}{\lambda'}(1-\cos\varphi)$$

$$\boxed{\lambda' - \lambda = \frac{h}{mc}(1-\cos\varphi)}$$

$\frac{h}{mc} = \lambda_c = 2.426$ pm

the ~~highest~~ ~~value~~ range of $\cos\varphi$ is -1 to $+1$

So the scattred photon always have a large wave length than the incident photon

[~~Pro~~ showed]

b) X-rays with a wave length 0.0045 nm are used in a scattering experiment. If the x-rays are scattred through an angle of 160°, what is the wave length of the scattred photon?

Ans:

We know,

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\varphi)$$

$$\lambda' - 4.5 \times 10^{-12} = 2.426 \times 10^{-12}(1 - \cos(160))$$

$$\therefore \lambda' = 9.2057 \times 10^{12} \text{ m}$$

$$= 9.2057 \text{ pm}$$

where,

$\lambda = 0.0045 \text{ nm}$

$= 4.5 \times 10^{-12} \text{ m}$

$\varphi = 160°$

$\frac{h}{mc} = 2.426 \times 10^{-12}$

34

c) Discuss ~~it~~ vector atom model. Write down the name of all quantum number associated with vector atom model.

Ans: The two distined features of the vector atom model are –

i) The conception of speitial quantisat
ii) The spinning electron hypothesis.

The name of quantum number associated with the vector atom model are –

i) The principle quantum number (

It can take only integer value ~~Ex~~. excluding zero (0).

i.e.: $n = 1, 2, 3, 4 \ldots$

ii) The orbital quantum number (l):

It may take only any values between 0 to $(n-1)$

If $n = 4$, l can take the four value

0, 1, 2, 3,

By convention,

an electron for which $l = 0$ is called s orbital

$l = 1$ " " p orbital

$l = 2$ " " d "

$l = 3$ " " f "

iii) Spin quantum number (s):

The spin q.n has only one magnitude $\frac{1}{2}$

IV) Total angula momentum (J):

If represent the total angular momentum of the electron which is the sum of the orbital momentum and spin angular momentum.

The vector $\vec{j}$ is defined by eqation:-

$$\vec{J} = \vec{l} + \vec{s}$$

(36)

v) Magnatic quantum number (m):

To explain the splitting of spectral lin
in a magnatic field, ~~there~~ three more
magnetic quamtum number are
introduced.

~~Ques. 1i~~ Question Include

Ans. to the ques. 3 (a)

physical significance of wave function:

li (i) The wave function 'ψ' has no physical meaning. It is a complex quantity representing the variation of a matter wave.

(ii) The wave function $\psi(r,t)$ describes the position of particle with respect of time.

(iii) It can be considered as 'PROBABILITY' AMPLITUDE' since it used to find the location of the particle.

(iv) The square of the wave function gives the - PROBABILITY DENSITY of the particle which is represented by the wave function itself.

(v) More the value of probabolity density, more likely to find the particles of that region.

Schrodiner equation:

As we know, Total energy = K.E + P.E.

$$E = \frac{1}{2}mv^2 + U$$

$$= \frac{p^2}{2m} + U \quad \text{--- (1)}$$

$$K.E = \frac{1}{2}mv^2$$

$$P.E = U$$

where, $p = mv$

$$\frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{m^2v^2}{m}\right)$$

$$= \frac{p^2}{2m}$$

$$\psi = e^{i(kx - \omega t)}$$

As it is, time dependent,

$$\frac{d\psi}{dx} = \frac{d\left(e^{i(kx-\omega t)}\right)}{dx}$$

$$= ik\, e^{i(kx-\omega t)}$$

$$= iK\psi \quad (\because \psi = e^{i(Kx - \omega t)})$$

$$\frac{d^2\psi}{dx^2} = i^2 K^2 \psi$$

$$= -K^2\psi \quad (i^2 = -1)$$

As we know that

$$K = \frac{p}{\hbar}$$

So, $\dfrac{d^2\psi}{dx^2} = \dfrac{-p^2}{\hbar^2}\psi$

$$\frac{d^2\psi}{dx^2} = -\frac{p^2}{h^2}4\pi^2\psi$$

Now, we have,

$$p^2\psi = -\frac{h^2}{4\pi^2}\frac{d^2\psi}{dx^2} \quad \text{--- (11)}$$

We know that

$$\hbar = \frac{h}{2\pi}$$

Now,

$$E = \frac{p^2}{2m} + U \quad (\text{From eq (1)})$$

$$E\psi = \frac{p^2\psi}{2m} + U\psi$$

$$= -\frac{h^2}{8\pi^2 m}\frac{d^2\psi}{dx^2} + U\psi$$

$$\boxed{\therefore\ E\psi - U\psi = \frac{-h^2}{8\pi^2 m}\frac{d^2\psi}{dx^2}}$$

Time independent equation

b) ~~The time derivative of~~ the integral of ~~the~~ position

stationary state is a quantum state with all observables independent of time.

The probability current is a mathematical quantity describing the flow of probability per unit time per unit area.

The time derivative of the integral of the position probability density $P(\vec{r},t)$ over a finite volume τ gives,

$$\frac{d}{dt}\int_{\tau} P(\vec{r},t)\, d\tau = \int_{\tau} \frac{\delta}{\delta t}(\psi^* \psi)\, d\tau$$

$$= \int_{\tau} \left(\psi^* \frac{\delta \psi}{\delta t} + \frac{\delta \psi^*}{\delta t}\psi\right) d\tau \quad \text{——} (1)$$

But, we have,

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi \quad \text{——} (2)$$

Using (2) and (3) in eq^n (1) we have,

$$\frac{d}{dt}\int_{\tau} P(\vec{r},t)\, d\tau = \frac{i\hbar}{2m}\int_{\tau} \psi^*(\nabla^2\psi) - (\nabla^2\psi^*)\psi\, d\tau$$

$$= \frac{i\hbar}{2m}\int_{\tau} \vec{\nabla}\cdot\left[\psi^* \vec{\nabla}\psi - (\vec{\nabla}\psi^*)\psi\right] d\tau \quad \text{——} (4)$$

From Schrodinger's time dependent eqⁿ we have,

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi$$

or $$i\hbar \frac{\partial \psi}{\partial t} = \frac{i^2\hbar^2}{2m} \nabla^2 \psi + V\psi$$

or $$\frac{\partial \psi}{\partial t} = \frac{i\hbar}{2m} \nabla^2 \psi + V\psi \quad \text{--- (2)}$$

Also, $$-i\hbar \frac{\partial \psi^*}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi^* + V\psi^*$$

or $$\frac{\partial \psi^*}{\partial t} = -\frac{i\hbar}{2m} \nabla^2 \psi^* + V\psi^* \quad \text{--- (3)}$$

Using (2) and (3) in (1) we have,

$$\frac{d}{dt}\int_\tau \rho(\vec{r},t)\, d\tau = \int_\tau \left[\psi^*\left(\frac{i\hbar}{2m}\nabla^2\psi + V\psi\right) + \left(-\frac{i\hbar}{2m}\nabla^2\psi^* + V\psi^*\right)\psi\right] d\tau$$

$$= \int_\tau \left[\psi^*\left(\frac{i\hbar}{2m}\nabla^2\psi + V\psi\right) - \left(\frac{i\hbar}{2m}\nabla^2\psi^* - V\psi^*\right)\psi\right] d\tau$$

$$= \int_\tau \left[\frac{i\hbar}{2m}\psi^*\left(\nabla^2\psi\right) + V\psi^*\psi - \frac{i\hbar}{2m}\left(\nabla^2\psi^*\right)\psi - V\psi^*\psi\right] d\tau$$

$$= \frac{i\hbar}{2m}\int_\tau \left[\psi^*\left(\nabla^2\psi\right) - \left(\nabla^2\psi^*\right)\psi\right] d\tau$$

$$= \frac{i\hbar}{2m}\int_\tau \vec{\nabla}\left[\psi^*\vec{\nabla}\psi - \left(\vec{\nabla}\psi^*\right)\psi\right] d\tau \quad \text{--- (4a)}$$

or $$\frac{d}{dt}\int_\tau \rho(\vec{r},t)\, d\tau = \frac{i\hbar}{2m}\oint\left[\psi^*\vec{\nabla}\psi - \left(\vec{\nabla}\psi^*\right)\psi\right] \quad \text{--- (4)}$$

Here, S is the surface bonding the volume τ, and the subscript 'n' indicating the component of the vector (along the outward normal) to the surface element ds.

Let, us define a vector $\vec{j}_p(\vec{r},t) = -\frac{i\hbar}{2m}\left[\psi^*\vec{\nabla}\psi - (\vec{\nabla}\psi^*)\psi\right]$ — ⑤

from ④,

$$\frac{d}{dt}\int \rho(\vec{r},t)\, d\tau = -\int_\tau \vec{\nabla}\cdot\vec{j}\, d\tau$$

or $$\int \frac{\partial \rho(\vec{r},t)}{\partial t}\, d\tau + \int_\tau \vec{\nabla}\cdot\vec{j}\, d\tau = 0$$

or $$\int_\tau \left(\frac{\partial \rho(\vec{r},t)}{\partial t} + \vec{\nabla}\cdot\vec{j}_p(\vec{r},t)\right) d\tau = 0 \quad \text{— ⑥}$$

Since eq^n ⑥ is valid for any volume τ we must have,

$$\frac{\partial \rho(\vec{r},t)}{\partial t} + \vec{\nabla}\cdot\vec{j}_p(\vec{r},t) = 0$$

or $$\vec{\nabla}\cdot\vec{j}_p(\vec{r},t) + \frac{\partial \rho(\vec{r},t)}{\partial t} = 0 \quad \text{— ⑦}$$

Eq^n ⑦ is similar to the continuity eq^n in electricity.

$$\vec{\nabla}\cdot\vec{j} + \frac{\partial \rho}{\partial t} = 0 \quad \text{— ⑧}$$

42

Comparing eq^n 7 and 8 we can interpret the vector $\vec{J}_p(\vec{r},t)$ as the probability current density and eq^n 7 as the continuity eq^n.

(showed)

B.Sc. Engineering 1st Year 1st Term Examination, 2017

PHY-1107.

Section-A

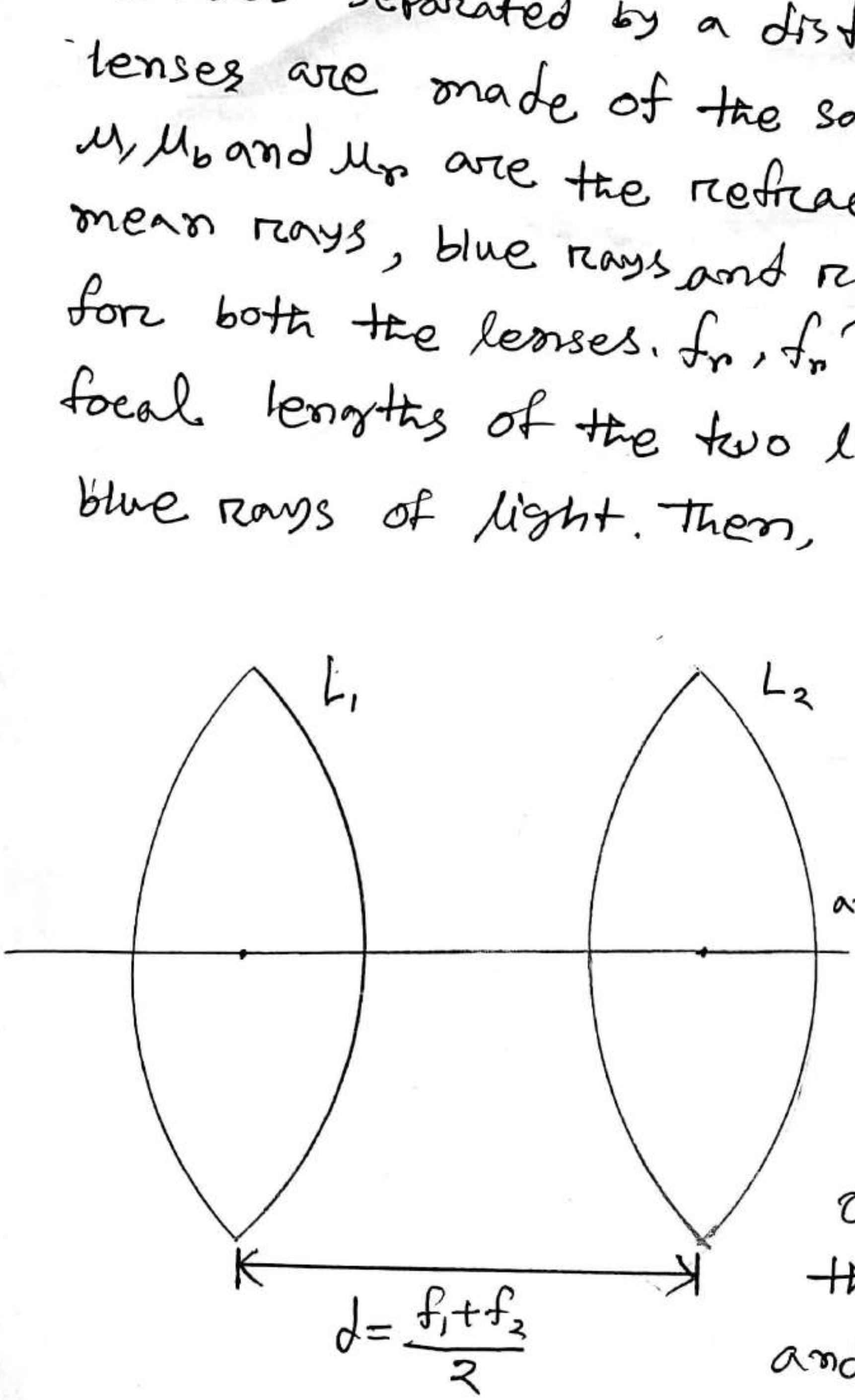
⇒ Q. 4. a) Obtain the condition for achromatism of two thin lenses of the same material placed at a distance apart.

⇒ Ans: Let f_1 and f_2 be the focal lengths of two lenses separated by a distance d. The two lenses are made of the same material and μ, μ_b and μ_r are the refractive indices for the mean rays, blue rays and red rays respectively for both the lenses. f_r, f_r' and f_b, f_b' are the focal lengths of the two lenses for red and blue rays of light. Then,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$\frac{1}{F_r} = \frac{1}{f_r} + \frac{1}{f_r'} - \frac{d}{f_r f_r'}$$

and $$\frac{1}{F_b} = \frac{1}{f_b} + \frac{1}{f_b'} - \frac{d}{f_b f_b'}.$$



where F, F_r and F_b are the focal lengths of the combination for the mean rays, red rays and blue rays.

(44)

But $\frac{1}{f_r} = \frac{(\mu_r-1)}{(\mu-1)f_1}$, $\frac{1}{f_r'} = \frac{(\mu_r-1)}{(\mu-1)f_2}$

$$\frac{1}{f_b} = \frac{(\mu_b-1)}{(\mu-1)f_1}, \quad \frac{1}{f_b'} = \frac{(\mu_b-1)}{(\mu-1)f_2}$$

$$\frac{1}{F_r} = \frac{(\mu_r-1)}{(\mu-1)f_1} + \frac{(\mu_r-1)}{(\mu-1)f_2} - \frac{(\mu_r-1)^2}{(\mu-1)^2}\cdot\frac{d}{f_1f_2}$$

and $\frac{1}{F_b} = \frac{(\mu_b-1)}{(\mu-1)f_1} + \frac{(\mu_b-1)}{(\mu-1)f_2} - \frac{(\mu_b-1)^2}{(\mu-1)^2}\cdot\frac{d}{f_1f_2}$

For the combination to be achromatic,

$$F_r = F_b$$

$$\Rightarrow \frac{(\mu_r-1)}{(\mu-1)f_1} + \frac{(\mu_r-1)}{(\mu-1)f_2} - \frac{d(\mu_r-1)^2}{(\mu-1)^2f_1f_2} = \frac{(\mu_b-1)}{(\mu-1)f_1} + \frac{(\mu_b-1)}{(\mu-1)f_3} - \frac{d(\mu_b-1)^2}{(\mu-1)^2f_1f_2}$$

$$\Rightarrow \frac{(\mu_r-1)}{(\mu-1)}\left(\frac{1}{f_1}+\frac{1}{f_2}\right) - \frac{(\mu_r-1)^2 d}{(\mu-1)^2f_1f_2}$$
$$= \frac{(\mu_b-1)}{(\mu-1)}\left(\frac{1}{f_2}+\frac{1}{f_1}\right) - \frac{(\mu_b-1)^2}{(\mu-1)^2}\cdot\frac{d}{f_1f_2}$$

or $\frac{(\mu_b-\mu_r)}{\mu-1}\left(\frac{1}{f_1}+\frac{1}{f_2}\right) = \frac{d}{(\mu-1)^2f_1f_2}\left[(\mu_b-1)^2-(\mu_r-1)^2\right]$

p. t. o

$$\frac{(\mu_b-\mu_r)}{(\mu-1)}\left(\frac{1}{f_1}+\frac{1}{f_2}\right)=\frac{d}{(\mu-1)^2 f_1 f_2}\left[(\mu_b-1)^2-(\mu_r-1)^2\right]$$

$$=\frac{d}{(\mu-1)^2 f_1 f_2}\left[(\mu_b-\mu_r)(\mu_b+\mu_r-2)\right]$$

$$=\frac{(\mu_b-\mu_r)d}{(\mu-1) f_1 f_2}(2\mu-2)\left[\begin{array}{l}\text{Taking}\\ \mu_b+\mu_r=2\mu\end{array}\right]$$

$$=\frac{(\mu_b-\mu_r)d}{(\mu-1)^2 f_1 f_2}\cdot 2(\mu-1)$$

$$\therefore \frac{(\mu_b-\mu_r)}{(\mu-1)}\left(\frac{1}{f_1}+\frac{1}{f_2}\right)=\frac{2d(\mu_b-\mu_r)}{(\mu-1) f_1 f_2}$$

or $$\frac{1}{f_1}+\frac{1}{f_2}=\frac{2d}{f_1 f_2}$$

$$\Rightarrow \frac{f_1+f_2}{f_1 f_2}=\frac{2d}{f_1 f_2}$$

$$\therefore d=\frac{f_1+f_2}{2}$$

Thus, the condition for achromatism of two thin co-axial lenses (of the same material) separated by a distance is that the distance between the two lenses must be equal to the mean focal ~~lenses~~ length of the two lenses.

(46)

⇒ Que: 4.b) Two monochromatic waves emanating from two ~~waves~~ Coherent sources have the displacements represented by

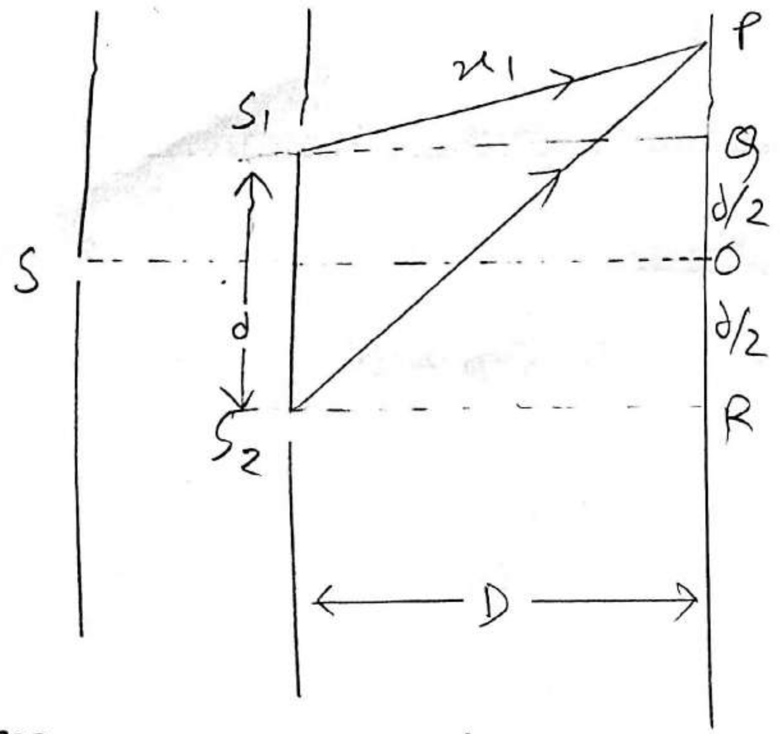
$$y_1 = a\cos\omega t$$

$$\cancel{y_2 = a\cos(\omega t+\varphi)} \quad y_2 = a\cos(\omega t+\varphi)$$

where φ is the phase difference between two waves. Show that the resultant intensity at a point due to their superposition is given by $I = 4I_0\cos^2\frac{\varphi}{2}$, where $I_0 = a^2$, Hence, obtain the conditions for constructive interference.

⇒ **Ans:** Consider a monochromatic ~~source of light S emitting waves of wavelength of λ and two narrow pilot pinholes A and B. A and B~~ are ~~equidistance~~ plane wave illuminating a narrow slit S.

Suppose that this Primary wave is falls on two parallel narrow closed spaced slits S_1 and S_2. That means when a monochromatic light falls into the slit S, it will constitute two coherent secondary sources. This secondary sources emit secondary wave. Interferance will occur where the waves overlap at point P. when they are overlap in same phase Bright fringe is observed and if they are superimposed in different phase dark fringe is observed.



given, $y_1 = a\cos\omega t$

$y_2 = a\cos(\omega t + \varphi)$

so, due to the intensences the resultant were eqn:

$$y = y_1 + y_2$$

$$y = a\cos\omega t + a\cos(\omega t + \varphi)$$

$$= a\cos\omega t + a\cos\omega t\cos\varphi - a\sin\omega t\cdot\sin\varphi$$

$$\therefore y = a\cos\omega t(1+\cos\varphi) - a\sin\omega t\cdot\sin\varphi \quad (1)$$

let, $a(1+\cos\varphi) = R\cos\theta$ —— (ii)

$a\sin\varphi = R\sin\theta$ —— (iii)

$\cancel{y = R\cos\omega t - R\sin\omega t}$.

$y = R\cos\omega t\cos\theta - R\sin\omega t\sin\theta$

$y = R\cos(\omega t + \theta)$ —— (iv)

squaring (ii) and (iii).

$\cancel{a^2(1+\cos\delta)^2 + a^2\sin^2\delta = A^2\cos^2\theta + A^2\sin^2\theta}$

$\cancel{\Rightarrow A^2(\cos^2\theta + \sin^2\theta)}$

$a^2(1+\cos\varphi)^2 + a^2\sin^2\varphi = R^2(\sin^2\theta + \cos^2\theta)$

$\Rightarrow R^2 = a^2 + 2a^2\cos\varphi + a^2\cos^2\varphi + a^2\sin^2\varphi$

$= 2a^2 + 2a^2\cos\varphi$

$= 2a^2(1+\cos^2\frac{\varphi}{2})$

$\therefore R^2 = 4a^2\cos^2\frac{\varphi}{2}$ —— (v)

The intensity at a point is given by the square of the amplitude.

$I = R^2$. & $I_0 = a^2$

$\therefore I = 4a^2\cos^2\frac{\varphi}{2} = 4I_0\cos^2\frac{\varphi}{2}$ —— (vi)

This is the resultant Intensity distribution due to the interference at any point.

For constructive Interference:

For constructive interference / maximum intensity for equation (vi) it is clearly seen that the intensity will be maximum if $\cos^2\frac{\varphi}{2}=1$

when, $\varphi = 0, \pm 2\pi, \pm 4\pi \pm \ldots\ldots + n(\pm 2\pi)$

then $\cos^2\frac{\varphi}{2}=1$, when, $n=0,1,2\ldots\ldots$

so, $I_{max} = 4a^2$

so, the intensity is maximum when the phase difference is a whole number multiple of 2π. In this case the interference is called constructive interference.

5.a) Discuss seven crystal system by giving by one example of each and describe the various types of Bravais lattices incase of three dimension with the helf of neat and clean diagram.

Ans. The seven crystal system are

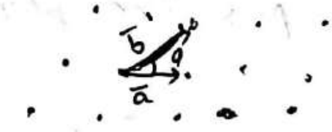
1. Cubic structure : NaCl
2. Tetragonal structure : $NiSO_4$
3. Hexagonal structure : SiO_2
4. Trigonal structure : $CaSO_4$
5. Orthogonal structure : KNO_3
   or
   Orthorhombic structure
6. Monoclinic structure : ~~Ca~~ $CuSO_4.2H_2O$
7. Triclinic structure : $CuSO_4$, $K_2Cr_2O_7$

There are 5 types of Brevais lattice depending upon the relative values: ~~of~~

1) oblique lattice:

(i) $\bar{a} \neq \bar{b}$ and $\varphi \neq 90$

(ii) φ is arbitrary



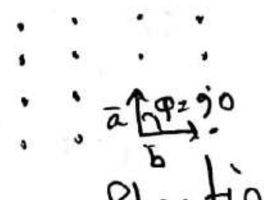
(iii) Symmetries: fold rotation

(iv) Convenient unit cell: Parallelogram

(v) Special lattice type: oblique

vi) Invariant under rotation : $\frac{2\pi}{3}$, $\frac{2\pi}{4}$, and $\frac{2\pi}{6}$

2) Square lattice



(i) $\bar{a} = \bar{b}$, $\varphi = 90$

(ii) Symmetries: 4 fold, rotation, reflection

(iii) Inverient index a reflection of : $\frac{2\pi}{4}$

(iv) Conventional unit cell: Square

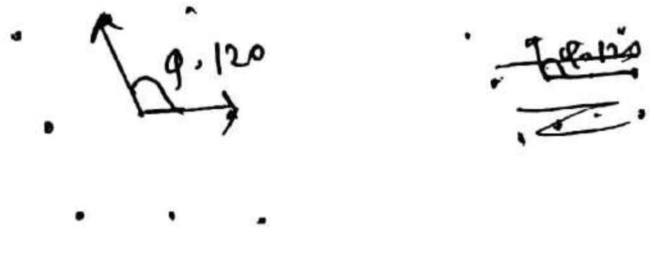
3) Hexagonal lattice:

(i) $\bar{a} = \bar{b}$, $\varphi = 120$

(ii) Symmetries : 6 fold, rotation, reflection

(iii) Invariant under: mirror reflection

(iv) Conventional unit cell: 60 Rhombus



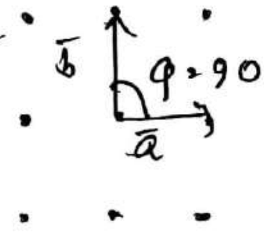
4. Rectangular lattice:

(i) $\bar{a} \neq b$, $\varphi = 90$

(ii) Symmetries: 2 fold, rotation, reflection

(iii) Invariant under: mirror reflection

(iv) Conventional unit cell: rectangle



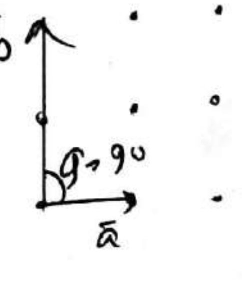
5. Centered Rectingular lattice

(i) $\bar{a} \neq b$ but $\bar{a} < b$, $\varphi = 90$

(ii) Symmetries: 2 fold, rotation, reflection

(iii) Invariant under: Inversion operation

(iv) Convenient unit cell: Rectangle



b) What is Millar indices? Discuss in brief the procedure for miller indices

→ ~~Orti~~- Miller indices represents the orientation of an atomic plane in a crystal lattice Orientation of planes or faces in a crystal may also be described in terms of their intercept on the three axis is called millar indices or the orientation of a plane by the recipocal of its numerical parameters rather then by its linear paramiter and when appropriately converted to whole number are called miller indices.

The procedure for miller indices:

① Find the intercepts of the plane on the crystal axis $\bar{a}, \bar{b}, \bar{c}$ in terms of lattice constant.

② Reciprocal of his number and then reduces to the smallest three integers having the same ratio

~~Defo~~ Denote (h, k, l)

③ These three integers (h, k, l) are called miller indices

Miller indices of cubic crystal plans:

① when a plane is parallel to one of the co-ordinate axis at infinity

miller index $= \frac{1}{\infty} = 0$

(ii) when the intercept of a plane is on the negative (-ve) part of any axis, the miller index is distinguished by a (-ve) signal put directly over it.

c) In a unit cell of simple cubic storvere, find the angle between the normal to pair plane whose Miller indices are

① [110] & [101]

(ii) [121] [2 22]

① $\cos\theta = \dfrac{1+0+0}{\sqrt{(1+1+0)(1+0+1}}$

$\Rightarrow \quad \theta = \cos^{-1}\left(\dfrac{1}{\sqrt{2.2}}\right)$

$\theta = 60^\circ$

(ii) $\cos\theta = \dfrac{2+4+2}{\sqrt{(1+4+1)(4+4+4}}$

$\theta = \cos^{-1}\left(\dfrac{8}{\sqrt{6\times 12}}\right)$

$= 19.47$

7. a) The assumptions of ~~Ein~~ Einstein's theory of ~~&~~ specific heat of solid are:

1. The atoms are identical independent simple Harmonic Oscillator.
2. All atoms vibrate in identical environment and have the same natural frequency.
3. Every solid is composed of atoms. The atoms are at rest at 0 K. Thermal energy = 0 J.
4. The vibration of each atom in the solid may resolve into components along three $\perp$ axes so that atom is represented by 3 Harmonic Oscillator. Contains N-atoms is considered to be equivalent to 3N H.O.
5. The energy spectrum of atomic oscillator is not continuous but has discrete values as given by quantum theory.

$$E = nh\nu$$

$$n = 0, 1, 2, 3, \ldots$$

(57)

6. The oscillators form an ~~apparently~~ assembly of system which are distinguishable due to their location at separate and distinct lattice sites in the crystal.

Relation for lattice heat capacity following Einstein model

Relative probabilities of the atom oscillator

$E_n = nh\nu$

Boltzmann's distribution of energy function:

$n = 0, \quad n = \infty$

$$\sum_{n=0}^{\infty} e^{-\frac{E_n}{kT}}$$

$$\bar{E} = \frac{\sum_{n=0}^{\infty} E_n e^{-E_n/kT}}{\sum_{n=0}^{\infty} e^{-E_n/kT}}$$

or $$\bar{E} = \frac{\sum_{n=0}^{\infty} nh\nu e^{-\frac{nh\nu}{kT}}}{\sum_{n=0}^{\infty} e^{-\frac{nh\nu}{kT}}}$$

$$= \frac{h\nu\left[0 + e^{-h\nu/kT} + 2e^{-2h\nu/kT} + \cdots\right]}{1 + e^{-h\nu/kT} + e^{-2h\nu/kT} + \cdots}$$

Let, $-\frac{h\nu}{kT} = x$

or $$\bar{E} = \frac{h\nu[0 + e^{x} + 2e^{2x} + \cdots + \infty]}{1 + e^{x} + e^{2x} + \cdots + \infty}$$

or $$\bar{E} = h\nu \frac{d}{dx} \ln[1 + e^{x} + e^{2x} + \cdots]$$

or $$\bar{E} = h\nu \frac{d}{dx} \ln\left(\frac{1}{1-e^{x}}\right)$$

$$= -h\nu \frac{d}{dx} \ln(1-e^{x})$$

$$= \frac{h\nu e^{x}}{1-e^{x}} = \frac{h\nu}{e^{-x}-1}$$

$$\bar{E} = \frac{h\nu}{e^{h\nu/kT}-1}$$

According to wave mechanics the ground state energy at a harmonic oscillator is not zero.

$$E_n = nh\nu + \frac{1}{2}h\nu$$

$$= \left(n+\frac{1}{2}\right)h\nu$$

(59) $\bar{E} = h\nu \left[\frac{1}{2} + \frac{1}{e^{h\nu/kT} - 1} \right]$

$\frac{1}{2} h\nu$ is Temp. independent

$\bar{E}$ at $T = 0$ and not $\bar{E} = kT$

The energy of each atom $= \frac{3 h\nu}{e^{h\nu/kT} - 1}$

Total internal energy of a k mole of a solid, $U = N\bar{E}$

$$= \frac{3N h\nu}{e^{h\nu/kT} - 1}$$

Atomic heat capacity at constant volume,

$$C_V = \frac{dU}{dT}$$

$$= 3NK \left(\frac{h\nu}{kT}\right)^2 \cdot \frac{e^{h\nu/kT}}{\left(e^{h\nu/kT} - 1\right)^2}$$

$$\therefore C_V = 3R \left(\frac{\theta_e}{T}\right)^2 \cdot \frac{e^{\theta_e/T}}{\left(e^{\theta_e/T} - 1\right)^2}$$

b) Electrical conductivity:

we know, time taken betn two successive collision

$$\tau = \frac{\lambda}{u}$$ $\lambda \rightarrow$ mean free path, $u \rightarrow$ velocity along an direction le u

If the applied field on the electron on charge (-e) be E,

$\therefore$ Eqn of motion along x-direction,

$$m\frac{d^2x}{dt^2} = (-e)E$$

$$\text{or}\quad \frac{d^2x}{dt^2} = \left(-\frac{e}{m}\right)E$$

$$\text{or}\quad \int\frac{dx}{dt} = \int -\frac{e}{m}E\,dt$$

$$\text{or,}\quad \frac{dx}{dt} = \left(-\frac{e}{m}\right)Et + c, \quad \text{At } t=0,\ \frac{dx}{dt}=0,\ c=0$$

$$\text{or}\quad \frac{dx}{dt} = \left(-\frac{e}{m}\right)Et = v$$

Average velocity between two collision $= \frac{1}{\tau}$

$$v = -\frac{eE\tau}{2m}$$

59 61 If i is the current density and n be the no. of electron per unit volume.

$$i = (-e)\, n \vec{v}$$

$$= (-en)\left(-\frac{eE\tau}{2m}\right)$$

$$= \frac{ne^2 E\tau}{2m}$$

$$= \left(\frac{ne^2 E}{2m} \times \frac{\lambda}{u}\right)$$

$$= \frac{1}{2}\, \frac{ne^2 E \lambda u}{mv^2}$$

We know,

$$\frac{1}{2} mv^2 = \frac{3}{2} kT$$

or $mv^2 = 3kT$

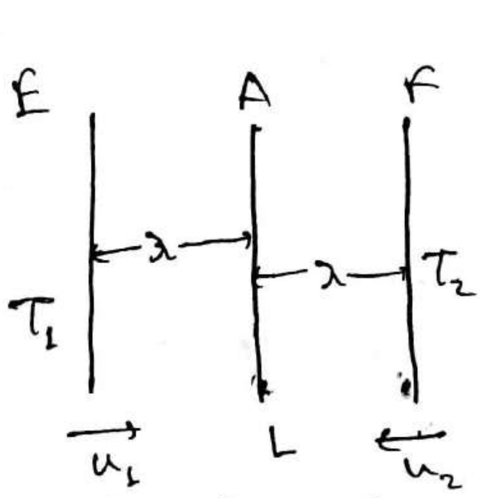
$$\therefore\ i = \frac{ne^2 E \lambda u}{6kT}$$

or $i \propto E$

or $i = \sigma E$

$$\therefore\ \boxed{\sigma = \frac{ne^2 \lambda u}{6kT}} \rightarrow \text{electrical conductivity}$$

Thermal contuctivity:



On the basis of kinetic theory, two temp. are equal.

$$T_1 = T_2$$

There is no transfer of energy.

There is transfer of energy ($T_1 > T_2$) from E to F.

$$E \to F$$

The no. of electrons per unit area per unit time $= \frac{1}{6} n u$

Each electron has energy $= \frac{1}{2} m u_1^2$

$\therefore$ Total energy transferred from E to F

$$= \frac{1}{6} n u \cdot \frac{1}{2} m u_1^2$$

$$= \frac{1}{6} n u \cdot \frac{3}{2} k T_1 \qquad \left[\because \frac{1}{2} m u_1^2 = \frac{3}{2} k T_1\right]$$

$$= \frac{1}{4} n u k T_1$$

Similarly net energy tranferred from $E \to F$ per unit area per unit time $= \frac{1}{4} n u k (T_1 - T_2)$

(63) If K be the thermal conductivity.

Transfer of energy per unit time $\propto \frac{T_2 - T_2}{2\lambda}$

From law of conservation of energy $= K \frac{T_1 - T_2}{2\lambda}$

or $K \frac{T_1 - T_2}{2\lambda} = \frac{1}{4} n u k (T_1 - T_2)$

or $K = \frac{1}{4} n u k \cdot 2\lambda = \frac{1}{2} \lambda n u k$

$\therefore \boxed{K = \frac{1}{2} \lambda u n k} \rightarrow$ thermal conductivity

Wiedemann - Franz law:

The ratio of thermal and electrical conductivity,

$$\frac{K}{\sigma} = \frac{\frac{1}{2} \lambda n u k}{\frac{n e^2 \lambda u}{6 k T}} = 3 \left(\frac{k}{e}\right)^2 T$$

$$\therefore \frac{K}{\sigma} \propto T$$

(showed)

c) Average kinetic energy at 0K :

At 0K all the electrons have energy less than E_f.

$$F(E) = \frac{1}{e^{(E-E_f)/kT}+1} = 1$$

$$D(E) = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}} E^{\frac{1}{2}}$$

Now, average K.E. per electron,

$$\langle E_0 \rangle = \frac{1}{N}\int_0^{E_f} E\,D(E)\,f(E)\,dE$$

$$= \frac{1}{N}\int_0^{E_f} E\,D(E)\,dE \qquad \Big|\because f(E) = 1$$

$$= \frac{1}{N}\left(\frac{V}{2\pi^2}\right)\left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}}\int_0^{E_f} E^{3/2}\,dE$$

$$= \frac{3}{5}E_f$$

(65) Average speed:

The amplitude of the wave vector is not the fermi surface.

$$K_f = \left(\frac{3\pi^2 N}{V}\right)^{\frac{1}{3}}$$

v_f is the velocity of the electron atom of the fermi surface (level)

$$mv_f = \hbar K_f$$

or $V_f = \frac{\hbar}{m} K_f$

or $V_f = \frac{\hbar}{m}\left(\frac{3\pi^2 N}{V}\right)^{\frac{1}{3}}$

or, $V_f^3 = \left(\frac{\hbar}{m}\right)^3 \cdot \frac{3\pi^2 N}{V}$

or $N = \frac{3V}{3\pi^2}\left(\frac{m}{\hbar}\right)^3 V_f^3$

Hence, the number of states having velocity betn v and $v+dv$

$$dN = \frac{V}{3\pi^2}\left(\frac{m}{\hbar}\right)^3 3v\,dv$$

Now, average speed at 0 K

$$\langle V \rangle = \frac{1}{N}\int_0^{V_f} V\,dN$$

$$= \frac{3}{4} V_f$$

8.

a) some characteristics properties of a laser light:

=> Monochromaticity:

Laser light has four unique characteristics that differentiate it from ordinary light: these are:

=> coherence => Directionality => Monochromatic => High intensity

coherence:

In laser, eletron transition occurs artificially. In other words, in laser, electron transition occurs in specific time. All the photons emitted in laser have the same energy, frequency, or wave length. Hence, the light waves of laser light have single wavelength or color. There, the wave lengths of the laser light are in phase in space and time. Thus, light genarated by laser is highly coherent

Directionality:

Laser ~~light~~ emits light only in one direction. This is called directionality of laser light. A laser beam can travel to long distance without spreading because of extremely narrow width.

Like ⇒ if an ordinary light travels a distance of 2km, it spreads to about 2km in diameter. If a laser light travels a distance of 2km it spreads to a diameter less than 2cm.

Monochromatic:

Monochromatic ~~me~~ light means a light containing a single color or wavelength. In laser, all emitted photons have the same energy, frequency, or wavelength. Hence, the light waves ~~hav~~ of laser have single wavelength or color. Therefore, laser light covers a very narrow range of frequencies or wavelengths.

High intensity:

In laser, the light spreads in small region of space and in a small wavelength range. Hence laser light has greater intensity when compared to the ordinary light.

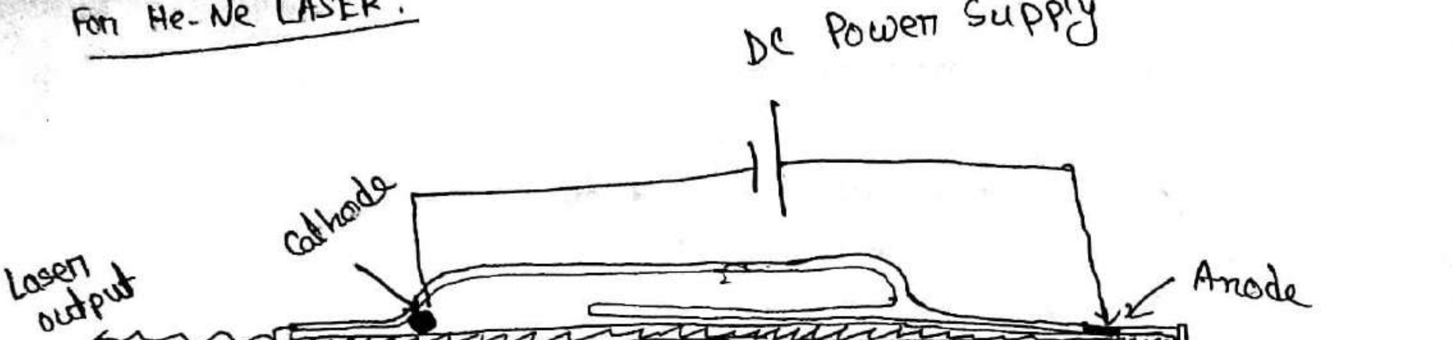
(1b) Population inversion:

A population inversion occurs while a system exists in state in which more members of the systems are in higher, exicited stated than in lower, unexcited enery states. It is called an "inversion" because in many familiar and commonly encountered physical systems, this is not possible. The concept is of fundamental importance in laser science because the production is necessary step in the workings of a standard laser.

Generally it cannot be achieved in our theoretical group of atoms with two energy-levels when they are in thermal equilibrium. In fact, any method by which the atoms are directly and continuously excited from the ground state to the excited state (such as optical absorption) will eventually reach equilibrium with de-exciting process of spontaneous and stimulated emission. At best, an equal population of the two states - $N_1 = N_2 = N/2$, can be achieved, resulting in optical transparency but no net optical gain.

(69)

For He-Ne LASER:



CSE - 1

73

KHULNA UNIVERSITY OF ENGINEERING & TECHNOLOGY
B.Sc. Engineering 1st Year 1st Term Examination, 2016
Department of Computer Science and Engineering
PHY 1107
Physics

TIME: 3 hours

FULL MARKS: 210

N.B. i) Answer **ANY THREE** questions from each section in separate scripts.
ii) Figures in the right margin indicate full marks.
iii) Assume reasonable data if any missing.

SECTION A

(Answer **ANY THREE** questions from this section in Script A)

1 a) Distinguish between photoelectric effect and Compton effect. Discuss their significance in the development of modern physics. (10)

b) Derive an expression for the Compton shift and give reason how it support the concept of photon. (15)

c) The longest wave length of light that will cause photo emission from sodium is approximately 540 nm. (10)

i) Find the work function of sodium.

ii) Find the maximum kinetic energy for photo electrons emitted when light of wavelength 400 nm strikes a sodium plate.

2. a) According to Bohr model under what condition an electron can radiate electromagnetic energy? Mention the quantum numbers associated with vector atom model. (12)

b) What is wave packet? What do you understand by phase velocity and group velocity of the matter waves? Derive a relation between them. (13)

c) What is the de-Broglie wavelength of Ping-Pong ball of mass 2.0g after it is slammed across the table with speed 5m/s? (10)

3. a) The one-dimensional time-independent Schrödinger equation is (25)

$$\left(-\frac{\hbar^2}{2m}\right)\frac{d^2\psi(x)}{dx^2}+V(x)\Psi(x)=E\,\Psi(x)$$

i) Give the meanings of the symbols in this equation.

ii) A particle of mass "m" is contained in a one-dimensional box of width "a". The potential energy V(x) is infinite at the walls of the box ($x = 0$ and $x = a$) and zero in between ($0 < x < a$). Solve the Schrödinger equation for this particle and hence show that the normalized solution have the form $\Psi_n(x)=\sqrt{2/a}\,\sin\left(\frac{n\pi x}{a}\right)$, with energy

$$E_n=\frac{h^2n^2}{8ma^2}$$, where "n" is an integer.

iii) Sketch the wave functions and the probability density distribution for the cases $n = 1, 2$ and 3.

b) Find $\langle P\rangle$ and $\langle P^2\rangle$ for the ground state wave function of the infinite square well. (10)

4. a) What is spherical aberration? How is it minimized when two thin lenses are placed at a distance from each other? (10)

b) Obtain the condition for maximum and minimum intensity of light in Young's double slit experiment. Find the average intensity of the interference pattern and show that it is exactly that which would exist in the absence of interference. (15)

c) In a Newton's ring's experiment the radius of curvature of the lens is 5 m and its diameter is 20 mm. How many bright rings are produced in the reflected rays? The wavelength of light used is 589 nm. (10)

SECTION B

(Answer **ANY THREE** questions from this section in Script B)

5. a) In practice how many crystal systems are possible? Give the names starting the relationship between crystallography axes and the angle between them. (15)

   b) State the properties of a reciprocal lattice. How is a reciprocal lattice constructed from a direct lattice? (10)

   c) In a unit cell of simple cubic structure, find the angle between the normal to pair of planes whose Miller indices are (i) [211] & [110] and (ii) [111] & [312]. (10)

6. a) What is the difference between photons and phonons? Explain 'Normal' process and 'Umklapp' process. (10)

   b) What are the assumptions of Debye model for the lattice specific heat? Calculate the lattice specific heat according to the Debye theory. (15)

   c) Calculate the maximum phonon frequency generated by scattering of visible light of wavelength $\lambda = 4000\ \text{Å}$. Given that velocity of sound in medium is 5×10^5 cm/s and refractive index is 1.5. (10)

7. a) What are static and transport properties in the case of free electron model? Obtain an expression for the electrical conductivity of a metal on the basis of free electron theory. Hence prove Ohm's law. (12)

   b) Show that the density of states of free electron is given by $D(E) = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}} E^{\frac{1}{2}}$. Use this expression to discuss number of filled states between electronic energy levels. (13)

   c) Aluminium metal crystallizes in *fcc* structure. If each contributes single electron as free electron and the lattice constant 'a' is $4.1\ \text{Å}$. Calculate treating conduction electrons as free electron Fermi gas (i) Fermi energy and Fermi vector, (ii) Total kinetic energy of free electron gas per unit volume at $0K$ [$\hbar = 1.054\times10^{-27}$ erg-sec, Electron rest mass $= 9.11\times10^{-28}$ gm]. (10)

8. a) Give brief outlines of generation of coherent radiation. Discuss the characteristics of a LASER light. (12)

   b) Describe briefly the principle, construction and working of a ruby LASER. (13)

   c) The coherence length for sodium light is 2.84×10^{-2} m. the wavelength of sodium light is $5890\ \text{Å}$. Calculate (i) the number of oscillation corresponding to the coherence length and (ii) the coherence time. (10)

a) Question: Distinguish between photoeletric effect nd compton effect. Discuss their ~~et~~ significance n the development of modern physics.

nswer: The photoelectric effect and compton fect are two different types of interactions tween light and matter. The photoelectric effect an effect where weakly bound (e^-) within metals re ejected from the material when electromagnetic adiation interacts with those (e^-). A collision tween a photon and a particle results in an increase n the kinetic energy of the particle and a orresponding increase in the wavelength of the photon called compton effect. Photoelectric effect is a low ergy phenomenon. compton effect is a mid energy enomenon. In photoelectric effect photon delivers total energy to a single electron. In compton fect the photon transfers a part of its energy a single (e^-). ~~After~~ In photoelectric effect the oton disappears after the interaction. In compton ect after the interaction the wavelength of the scattered photon is higher than that of the incident photon. These are the main difference between them.

The photoelectric effect ~~was~~ is very important because it provided scientists with an alternative method of describing light. For centuries, researchers had thought of light as a form of energy that travels in waves. And that explanation works many phenomena.

The compton effect is ~~a~~ significant because it ~~demores~~ demonstrates that light can't be explained purely as a wave phenomenon. It is also important to prove the wave function of in matter in the momentum representation.

2(b) Question: Derive an expression for the compton Shift and give reason how it support the concept of photon.

Answer: compton scattering is the inelastic scattering of a photon and ~~off~~ of an (e^-). If the photon ~~les~~ loses energy, its wavelength will increase and this is called compton scattering. If the (e^-) has sufficient initial kinetic energy, the photon can gain energy, this is called inverse compton scattering. The compton shift (the change in the photon wavelength), we use the conversation of energy and momentum. After the scatter, the photon has energy and is travelling at an angle θ relative to the original direction.

compton shift support the concept of photon. It is important to understand the particle nature of photon. Particle nature is part of complete photon theory. compton

(73)

effect indecates the interaction of photon with matter. When a photon interacts with an elect there is an increase in the wavelength of pho thus indicating that energy gained by an (e^-). We can easily calculate the change in the wavelength of photon from the defination of photon found through experiments.

i) The longest wave length of light that will cause photo emission from sodium is approximately 540 nm.

ii) find the work function of sodium.

Answer: work function, $\phi = h\nu_0$

$$\Rightarrow \phi = \frac{h \cancel{\lambda_0} c}{\lambda_0} = \cancel{\frac{6 \times 62}{6.626 \times 10^{-34} \times 540 \times 10^{-9}}}$$

$$= 3.68 \times 10^{-19} \qquad \cancel{= 3.578 \times 10^{-40}}$$

) Find the max kinetic energy for photo (e⁻) emitted when light of wavelength 400 nm stricks a sodium plate.

Answer: $K_{max} = h\nu - \phi$

$$= h(\nu - \nu_0)$$

$$= hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

$$= hc\left(\frac{1}{400 \times 10^{-9}} - \frac{1}{540 \times 10^{-9}}\right)$$

$$= 1.288 \times 10^{-19} \text{ J}$$

2.

(a) According to Bohr model under what condition an electron can radiate electromagnetic energy? Mention the quantum numbers associated with vector atom model.

Ans:

The electron is only allowed to exist at certain energy levels according to the Bohr model, there are only a few possible energies of light which can be released when electrons fall from one energy level to another. As a result, the Bohr model explains why atomic spectra are discontinuous.

So, ~~I~~ an electron can radiate electromagnetic energy when it moves from one energy level to another.

Quantum number associated with vector atom model:

76

(i) Principle quontum number

(ii) Orbital quontum number

(iii) Spin quontum number.

(b) What is wave packet? What do you understand by phase velocity and group velocity of the matter waves? Derive a relation between them.

Ans:

Wave packet: A wave packet is a short "burst" or "envelope" of localized wave action that travels as a unit. A wave packet can be analyzed into, or can be synthesized from, an infinite set of component ~~sinusodi~~ sinusoidal waves of different wave-numbers, with phases and amplitudes such that they constructively only over a small region of space and destructively elsewhere.

Phase Velocity: The phase velocity of a wave is the rate at which the phase of the wave propagates in space.

phase velocity, $v_p = \frac{\omega}{k}$

Group velocity: The group velocity of a wave is the velocity which the overall shape of the waves propagates through space.

Group velocity, $v_g = \frac{d\omega}{dk}$

Relation between phase velocity and group velocity

phase velocity, $v_p = \frac{\omega}{k}$ ..... (1)

group velocity, $v_g = \frac{d\omega}{dk}$ ..... (2)

wave no, $k = \frac{2\pi}{\lambda}$

$$\Rightarrow \frac{dk}{d\lambda} = -\frac{2\pi}{\lambda^2} \text{ ..... (3)}$$

The angular frequency,

$$\omega = 2\pi\nu = 2\pi \frac{v_p}{\lambda}$$

$$\Rightarrow \frac{d\omega}{d\lambda} = -\frac{2\pi v_p}{\lambda^2} + \frac{2\pi}{\lambda}\frac{dv_p}{d\lambda}$$

$$\Rightarrow \frac{d\omega}{d\lambda} = -\frac{2\pi}{\lambda^2}\left[v_p - \lambda\frac{dv_p}{d\lambda}\right] \text{ ...... (4)}$$

(4) ÷ (3) ⇒

$$\frac{d\omega}{d\lambda} \times \frac{d\lambda}{dk} = \frac{-\frac{2\pi}{\lambda^2}\left[v_p - \lambda\frac{dv_p}{d\lambda}\right]}{-2\pi/\lambda^2}$$

$$\Rightarrow \frac{d\omega}{dk} = v_p - \lambda\frac{dv_p}{d\lambda}$$

$$\therefore \boxed{v_g = v_p - \lambda\frac{dv_p}{d\lambda}}$$

(c) What is the de-Broglie wavelength of Ping-Pong ball of mass 2.0g after it is slammed across the table with speed 5 m/s?

Ans:

We know,

here,
$m = 2\,g$
$= 2 \times 10^{-3}\,kg$

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.63 \times 10^{-34}}{2 \times 10^{-3} \times 5}\,m$$

$$= 6.63 \times 10^{-33}\,m$$

(Ans.)

3. (a) The one-dimensional time-independent Schrodinger equation is

$$\left(-\frac{h^2}{2m}\right)\frac{d^2\psi(x)}{dx^2}+V(x)\psi(x)=E\psi(x)$$

(i) Q. Give meaning of the symbols in this equation.

Solution

$$\left(-\frac{h^2}{2m}\right)\frac{d^2\psi(x)}{dx^2}+V(x)\psi(x)=E\psi(x)$$

h = Planck's Constant $=6.624\times10^{-34}$ J.s

m = mass of the particle

$\psi(x)$ = wave function

$V(x)$ = Potential Energy

E = System Energy.

(ii) Q. A particle of mass "m" is contained in a one-dimensional box of width "a". The potential energy $V(x)$ is infinite at the walls of the box ($x=0$ and $x=a$) and zero in between ($0<x<a$). Solve the Schrodinger equation for this particle and hence show that the normalized solution have the form

$\psi_{(n)}(x) = \sqrt{2/a}\ \sin\left(\frac{n\pi x}{a}\right)$, with energy $E_n = \frac{h^2 n^2}{8ma^2}$, where "n" is an integer.

Solution:

$$V(x) = \begin{cases} 0 & 0 < x < a \\ \infty & x = 0 \text{ and } x = a \end{cases}$$

~~[illegible]~~

Where potential is infinity, probability of the particle's existence is zero.

Probability of the availability of the particle from $x=0$ to $x=a$ is from 0 to 1.

Now, time independent Schrödinger equation is

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + 0 = E\psi(x)$$

$$\Rightarrow \frac{d^2\psi(x)}{dx^2} = -\frac{2mE}{\hbar^2}\psi(x)$$

$$\Rightarrow \frac{d^2\psi(x)}{dx^2} + K^2\psi(x) = 0 \qquad \left[K^2 = \frac{2mE}{\hbar^2}\right]$$

The solution of this equation is

$$\psi(x) = A\sin kx + B\cos kx$$
$$= e^{hx} + e^{-hx}$$

Now,

$$\psi(0) = 0$$
$$\Rightarrow A\sin(k.0) + B\cos(k.0) = 0$$
$$\Rightarrow A.0 + B.1 = 0$$
$$\therefore B = 0$$
$$\psi(x) = A\sin kx$$
$$\psi(a) = 0$$
$$\Rightarrow \psi(a) = A\sin ka = 0$$
$$\Rightarrow \sin ka = 0$$
$$\Rightarrow ka = n\pi$$
$$\Rightarrow k = \frac{n\pi}{a}$$

$$k^2 = \left(\frac{n\pi}{a}\right)^2 = \frac{2mE}{\hbar^2}$$
$$\Rightarrow E = \frac{\hbar^2 n^2 \pi^2}{2ma^2}$$

$$\psi(x) = A\sin\left(\frac{n\pi}{a}x\right)$$

Again

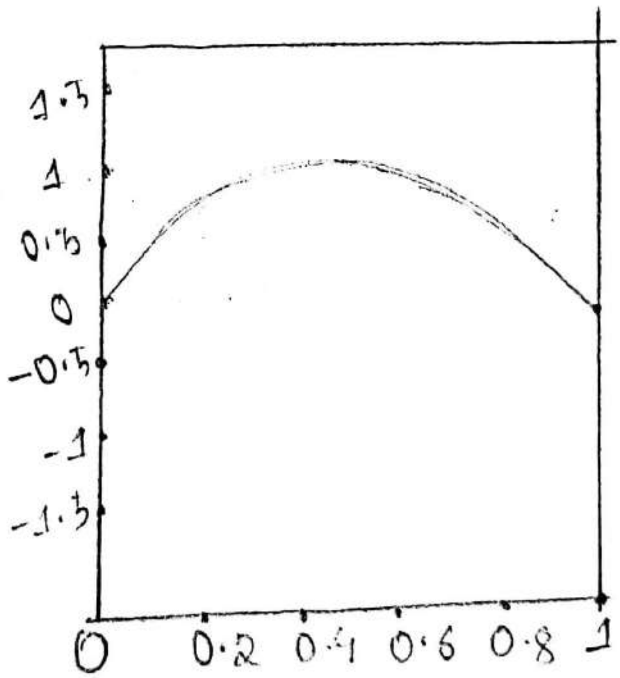
$$\int_0^a |\psi(x)|^2 dx = 1 \quad [\text{Condition}]$$
$$\Rightarrow A^2 \int_0^a \sin^2\left(\frac{n\pi}{a}x\right) = 1$$

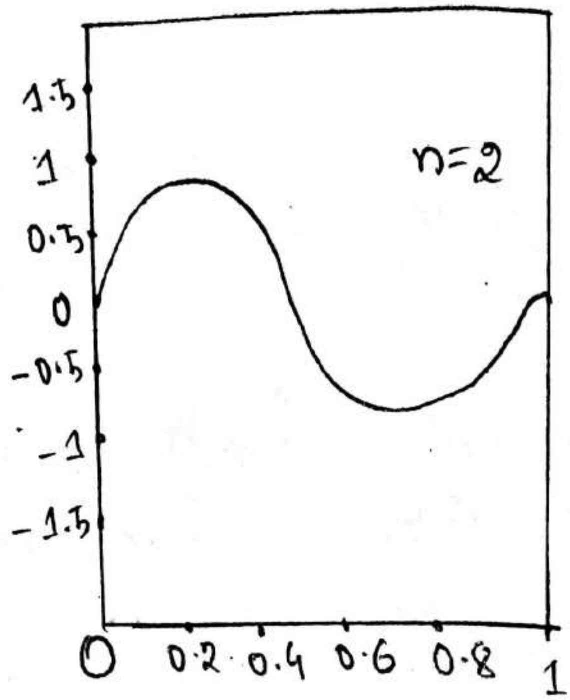
$\Rightarrow \ |A| = \sqrt{\frac{2}{a}}$

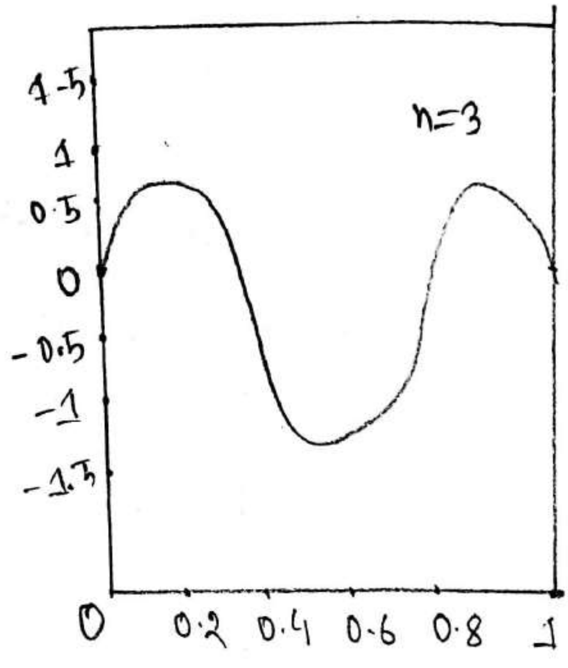
$\therefore \ \psi(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a} x\right)$

(iii) Sketch the wave function and the probability density distribution for the cases $n=1, 2$ and 3.

Solution




1.5
1
0.5
0
-0.5
-1
-1.5
n=2
0
0.2
0.4
0.6
0.8
1


1.5
1
0.5
0
-0.5
-1
-1.5
n=3
0
0.2
0.4
0.6
0.8
1

(b) Find $\langle p \rangle$ and $\langle p^2 \rangle$ for the ground state wave function of the infinite square well.

Solution: We can ignore the time dependence of Ψ, in which case we have,

$$\langle p \rangle = \int_0^L \left(\sqrt{\frac{2}{L}} \sin \frac{\pi x}{L}\right) \left(\frac{\hbar}{i} \cdot \frac{\partial}{\partial x}\right)\left(\sqrt{\frac{2}{L}} \sin \frac{\pi x}{L}\right) dx$$

$$= \frac{\hbar}{i} \cdot \frac{2}{L} \cdot \frac{\pi}{L} \int_0^L \sin \frac{\pi x}{L} \cos \frac{\pi x}{L} \, dx$$

$\therefore \langle p \rangle = 0$

The particle is likely to be moving in the $-x$ as in the $+x$ direction, so its average momentum is zero.

Similarly, since

$$\frac{\hbar}{i} \frac{\partial}{\partial x}\left(\frac{\hbar}{i} \frac{\partial}{\partial x}\right) \Psi = -\hbar^2 \frac{\partial^2 \Psi}{\partial x^2}$$

$$= -\hbar^3 \left(-\frac{\pi^2}{L^2} \sqrt{\frac{2}{L}} \sin \frac{\pi x}{L}\right)$$

$$= + \frac{\hbar^2 \pi^2}{L^2} \Psi$$

We have,

$$\langle p^2 \rangle = \frac{\hbar^2 \pi^2}{L^2} \int_0^L \Psi^* \Psi \, dx = \frac{\hbar^2 \pi^2}{L^2} \int_0^L \Psi^* \Psi \, dx$$

$$= \frac{\hbar^2 \pi^2}{L^2} \quad \text{(Ans.)}$$

Source: www.albany.edu/physics/ Phy 240-2016 / Homework 7-240.pdf

CSE - 1

66

KHULNA UNIVERSITY OF ENGINEERING & TECHNOLOGY
B.Sc. Engineering 1st Year 1st Term Examination, 2016
Department of Computer Science and Engineering
PHY 1107
Physics

TIME: 3 hours

FULL MARKS: 210

N.B. i) Answer **ANY THREE** questions from each section in separate scripts.
ii) Figures in the right margin indicate full marks.
iii) Assume reasonable data if any missing.

SECTION A

(Answer **ANY THREE** questions from this section in Script A)

1 a) Distinguish between photoelectric effect and Compton effect. Discuss their significance in the development of modern physics. (10)

b) Derive an expression for the Compton shift and give reason how it support the concept of photon. (15)

c) The longest wave length of light that will cause photo emission from sodium is approximately 540 nm. (10)

i) Find the work function of sodium.

ii) Find the maximum kinetic energy for photo electrons emitted when light of wavelength 400 nm strikes a sodium plate.

2. a) According to Bohr model under what condition an electron can radiate electromagnetic energy? Mention the quantum numbers associated with vector atom model. (12)

b) What is wave packet? What do you understand by phase velocity and group velocity of the matter waves? Derive a relation between them. (13)

c) What is the de-Broglie wavelength of Ping-Pong ball of mass 2.0g after it is slammed across the table with speed 5m/s? (10)

3. a) The one-dimensional time-independent Schrödinger equation is (25)

$$\left(-\frac{\hbar^2}{2m}\right)\frac{d^2\psi(x)}{dx^2}+V(x)\Psi(x)=E\,\Psi(x)$$

i) Give the meanings of the symbols in this equation.

ii) A particle of mass "m" is contained in a one-dimensional box of width "a". The potential energy V(x) is infinite at the walls of the box ($x = 0$ and $x = a$) and zero in between ($0 < x < a$). Solve the Schrödinger equation for this particle and hence show that the normalized solution have the form $\Psi_n(x)=\sqrt{2/a}\sin\left(\frac{n\pi x}{a}\right)$, with energy $E_n=\dfrac{h^2n^2}{8ma^2}$, where "$n$" is an integer.

iii) Sketch the wave functions and the probability density distribution for the cases $n = 1, 2$ and 3.

b) Find $\langle P\rangle$ and $\langle P^2\rangle$ for the ground state wave function of the infinite square well. (10)

4. a) What is spherical aberration? How is it minimized when two thin lenses are placed at a distance from each other? (10)

b) Obtain the condition for maximum and minimum intensity of light in Young's double slit experiment. Find the average intensity of the interference pattern and show that it is exactly that which would exist in the absence of interference. (15)

c) In a Newton's ring's experiment the radius of curvature of the lens is 5 m and its diameter is 20 mm. How many bright rings are produced in the reflected rays? The wavelength of light used is 589 nm. (10)

Page: 1 of 2

SECTION B

(Answer ANY THREE questions from this section in Script B)

5\. a) In practice how many crystal systems are possible? Give the names starting the relationship between crystallography axes and the angle between them. (15)

b) State the properties of a reciprocal lattice. How is a reciprocal lattice constructed from a direct lattice? (10)

c) In a unit cell of simple cubic structure, find the angle between the normal to pair of planes whose Miller indices are (i) [211] & [110] and (ii) [111] & [312]. (10)

6\. a) What is the difference between photons and phonons? Explain 'Normal' process and 'Umklapp' process. (10)

b) What are the assumptions of Debye model for the lattice specific heat? Calculate the lattice specific heat according to the Debye theory. (15)

c) Calculate the maximum phonon frequency generated by scattering of visible light of wavelength $\lambda = 4000\,\mathring{A}$. Given that velocity of sound in medium is 5×10^5 cm/s and refractive index is 1.5. (10)

7\. a) What are static and transport properties in the case of free electron model? Obtain an expression for the electrical conductivity of a metal on the basis of free electron theory. Hence prove Ohm's law. (12)

b) Show that the density of states of free electron is given by $D(E) = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}} E^{\frac{1}{2}}$. Use this expression to discuss number of filled states between electronic energy levels. (13)

c) Aluminium metal crystallizes in fcc structure. If each contributes single electron as free electron and the lattice constant 'a' is $4.1\,\mathring{A}$. Calculate treating conduction electrons as free electron Fermi gas (i) Fermi energy and Fermi vector, (ii) Total kinetic energy of free electron gas per unit volume at $0K$ [$\hbar = 1.054\times10^{-27}$ erg-sec, Electron rest mass $= 9.11\times10^{-28}$ gm]. (10)

8\. a) Give brief outlines of generation of coherent radiation. Discuss the characteristics of a LASER light. (12)

b) Describe briefly the principle, construction and working of a ruby LASER. (13)

c) The coherence length for sodium light is 2.84×10^{-2} m, the wavelength of sodium light is $5890\,\mathring{A}$. Calculate (i) the number of oscillation corresponding to the coherence length and (ii) the coherence time. (10)

a) What is spherical aberration? How is it minimized when two thin lenses are placed at a distance from each other?

→ Spherical Aberration is an optical problem that occurs when all incoming light rays end up focusing at different points after passing through a spherical surface.

It can be minimized:

1) by using stops, which reduces the effective len aperture The stop used can be such as to permit either the axial rays of light or the marginal rays of the light. However, as the amount of light passing through the lense is reduced, corresponding the image is less bright.

2) Plane convex lenses are used ~~to~~ in optical instruments so as to reduce the spherical aberration. When the curved surface of thin lens faces the incident or

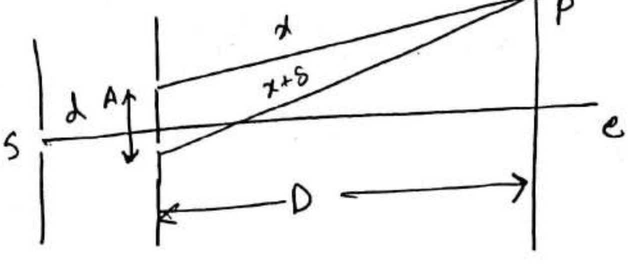
emergent light whichever is more parallel to the axis, the spherical aberration is minimum.

~~Test~~

By ~~meas~~ taking aforesaid measures, spherical aberration can be minimized.

b) Obtain the condition for minimum and maximum intensity of light in Young's double slit experiment. Find the average intensity of the interference pattern and show that it is exactly that which exist in the absence of interfence.

We Consider a monochromatic source of light emitting waves of wave length λ and two narrow pin whole A and B equidistance from S. A and B act as a coherrent sources separated by a distance d. Let a screen be placed at a distance D from the coherrent ~~distance~~ sources. Let a be the amplitude of the waves and the phase difference between the two waves reaching the point P, at any instant is δ.



If Y_1 and Y_2 are the displacements,

$$Y_1 = a \sin \omega t$$
$$Y_2 = a \sin (\omega t + \delta)$$

$$\therefore \; Y = Y_1 + Y_2$$
$$= a \sin \omega t + a \sin(\omega t + \delta)$$
$$= a \sin \omega t + a \sin \omega t \cos \delta + \cancel{a \cos \delta \sin} \; a \sin \delta \cos \omega t$$
$$= a \sin \omega t \, (1 + \cos \delta) + a \cos \omega t \sin \delta$$

Taking $a(1 + \cos \delta) = R \cos \theta \quad \dots \quad (1)$

and, $a \sin \delta = R \sin \theta \quad \dots \quad (2)$

$$Y = R \sin \omega t \cos \theta + R \cos \omega t \sin \theta$$
$$= R \sin (\omega t + \theta)$$

Which represents the equation of simple harmonic vibration of amplitude R.

Squaring eq^n (1) and (2) and adding we g[et]

$$R^2 \cos^2 \theta + R^2 \sin^2 \theta = a^2 (1 + \cos \delta)^2 + a^2 \sin^2 \delta$$
$$\Rightarrow R^2 = a^2 \sin^2 \delta + a^2 (1 + \cos^2 \delta + 2 \cos \delta)$$
$$\Rightarrow R^2 = a^2 \sin^2 \delta + a^2 + a^2 \cos^2 \delta + 2a^2 \cos \delta$$
$$= 2a^2 + 2a^2 \cos \delta \quad = 2a^2 (1 + \cos \delta)$$

$$R^2 = 2a^2 \times 2\cos^2 \frac{\delta}{2}$$
$$= 4a^2 \cos^2 \frac{\delta}{2}$$

The intensity at a point is given by the square of the amplitude. $I = R^2$

$\therefore I = \cancel{4R} \; 4a^2 \cos^2 \frac{\delta}{2}$ $\qquad I_0 = a^2$. then

MAXIMUM,

$$I = 4I_0 \cos^2 \frac{\delta}{2}$$

when, the phase difference $\delta = 0, 2\pi, 2\times 2\pi \ldots \ldots n\times 2\pi$. i.e. the even multiplication of π. ~~ph~~ path difference, $x = 0, \lambda, 2\lambda \ldots$

Then, $\cos^2 \frac{\delta}{2} = 1$

$$I = 4a^2$$

MINIMUM :

When, the phase difference $\delta = \pi, 3\pi \ldots$

$(2n+1)\pi$ or the path difference $x = \frac{\lambda}{2}, \frac{3\lambda}{2}$,

$\frac{5\lambda}{2} \ldots (2n+1)\frac{\lambda}{2}$, Then, $\cos^2\frac{\lambda}{2} = 0$. $I = 0$.

The average intensity of the interference pattern:

$$\int_0^{n\pi} \frac{1}{n} \times \left(4I_0 \cos^2\frac{\delta}{2}\right) d\delta$$

$$\Rightarrow \frac{4I_0}{2} \times \frac{1}{n} \int_0^{n\pi} 2\cos^2\left(\frac{\delta}{2}\right) d\delta$$

$$= \frac{2I_0}{n} \int_0^{n\pi} (\cos\delta - 1)\, d\delta$$

$$= \frac{2I_0}{n} \left[\int_0^{n\pi} \cos\delta\, d\delta - \int_0^{n\pi} d\delta\right]$$

$$\Rightarrow \frac{2I_0}{n} \left\{ [\sin\delta]_0^{n\pi} - [\delta]_0^{n\pi} \right\}$$

$$= \frac{2I_0}{n} \left\{ \sin(n\pi - 0) - (\overset{0-n\pi}{\cancel{-n\pi - 0}}) \right\}$$

$$\Rightarrow \frac{2I_0}{n} (0 + n\pi)$$

$$\Rightarrow \underline{\underline{2I_0 \times \pi}}$$

c) In a Newton's ring's experiment the radius of curvature of ~~then~~ lens is 5 m and its ~~distance~~ diameter is 20 nm. How many bright rings are produced in the reflected ~~wo~~ rays? The wave-length of light is 589 nm.

The curvature of the lense, $R = 5m$

The diameter of nth bright ring, $D = 20\times10^{-3} m$

$r_n = 10\times10^{-3} m$

wave length, $\lambda = 589\times10^{-9} m$

$$\frac{(r_n)^2}{R} = (2n-1)\frac{\lambda}{2}$$

$$\Rightarrow r_n{}^2 = \frac{2n-1}{2} \times \lambda R$$

$$\Rightarrow r_n = \sqrt{\frac{2n-1}{2} \times \lambda \times R}$$

$$\Rightarrow \frac{(10\times10^{-3})^2}{5} \times \frac{2}{\lambda} = 2n-1$$

$$\Rightarrow \left\{\left(\frac{(10\times10^{-3})^2}{5} \times \frac{2}{589\times10^{-9}}\right) + 1\right\}\frac{1}{2} = n$$

$$n = 34.46$$

$$\approx 35$$

Roll. 78

5.a) In practice how many crystal systems are possible? Give the names starting the relationship between crystallography axes and the angle between them.

⇒ In practice 7 crystal systems are possible. They are-

i) cubic system ($a=b=c; \alpha=\beta=\gamma=90^\circ$)

ii) Monoclinic " ($a\neq b\neq c; \alpha=\beta=90^\circ\neq\gamma$)

iii) Triclinic " ($a\neq b\neq c; \alpha\neq\beta\neq\gamma\neq 90^\circ$)

iv) Tetragonal " ($a=b\neq c; \alpha=\beta=\gamma=90^\circ$)

v) orthorhombic" ($a\neq b\neq c; \alpha=\beta=\gamma=90^\circ$)

vi) Rhombohedral/Trigonal system ($a=b=c; \alpha=\beta=\gamma\neq$

vii) Hexagonal system ($a=b\neq c; \alpha=\beta=90^\circ; \gamma=120^\circ$)

b) State the properties of reciprocal lattice. How is a reciprocal lattice constructed from a direct lattice?

⇒ Properties of reciprocal lattice-

i) The direct lattice is the reciprocal lattice to its own reciprocal lattice.

ii) Each point in a reciprocal lattice correspo to a particular set of ||-planes of the direct lattice.

iii) The distance of reciprocal lattice point from an orbitrary fixed origin is inversely proportional to the interplaner spaces of the corresponding ||-planes of the direct lattice.

iv) The unit cell of the reciprocal lattice is not necessarily a parallelepiped.

v) The volume of the unit cell of the reciprocal lattice is inversely proportional to the volume of the corresponding unit cell of the direct lattice.

A reciprocal lattice is constructed from a direct lattice in the following ways-

i) Take origin at some orbitrary point and draw normal to every part of ||-planes of the direct lattice.

ii) Take length of each normal equal to the reciprocal of the interplaner space for the corresponding set of planes.

c) In a unit cell of simple cubic structure, find the angle between the normal to pair planes whose Miller indices are i) [211] & [110] and ii) [111] & [312]

→ The directions of the two normals are [211], [110] and [111], [312] respectively.

i) $\cos\theta = \dfrac{2\times 1 + 1\times 1 + 1\times 0}{\sqrt{2^2+1^2+1^2}\ \sqrt{1^2+1^2+0^2}}$

$\therefore\ \theta = 30^\circ$

ii) $\cos\theta = \dfrac{1\times 3 + 1\times 1 + 1\times 2}{\sqrt{1^2+1^2+1^2}\ \sqrt{3^2+1^2+2^2}}$

$\theta = 22.2^\circ$

Que. NO. 6
Physics 1107

[Tasnia Haque Kheya
Roll: 1807071]

(a) What is the difference between Photons and Phonons? Explain 'normal' process and 'Umklapp' process.

(b) What are the assumptions of Debye model for the lattice specific heat? Calculate the lattice specific heat according to the Debye theory.

(c) Calculate the maximum phonon frequency generated by scattering of visible light of wavelength $\lambda = 4000\,A^\circ$. Given the velocity of sound medium is 5×10^5 cm/s and refractive is 1.5.

57

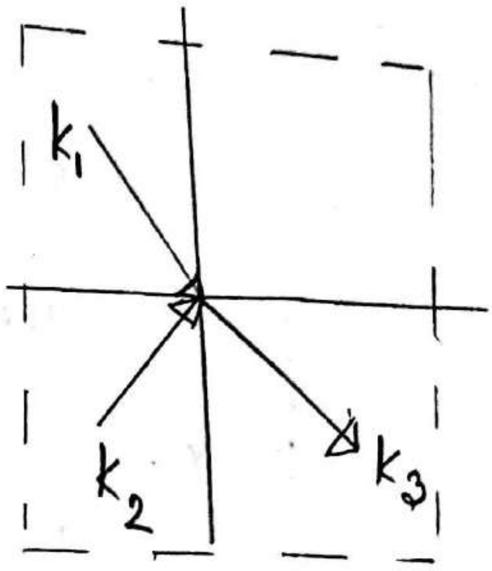
Answer to the que. no. (a)

The differances between photons and phonons are given below:-

Photon	Phonon
(i) Photons are particle of light.	(i) Phonons are particle of sound or heat.
(ii) Transition between differant energy levels of electron emits photons.	(ii) The vibration of atom and molecules produce phonons.
(iii) A photon is a packet of energy which is the base of quantam mechanics.	(iii) A phonon is a collec oscillation of several atoms.
(iv) A Photon is a form of energy.	(iv) Phonon is mode of oscillation that occurs in lattice structure.
(v) photon in free space have one type of dispension (for example- energy and momentum are always proportional).	(v) Phonons in crystallin solid typically show two types of dispension (for example- optical and acoustic).

Normal Process:

From the figure, figure-02 schematically shows the possible scattering process of two incoming Phonons with wave vectors (k-vectors) k_1 and k_2 creating one outgoing Phonon with a wave vector k_3. As long as the sum of k_1 and k_2 stay inside the Brillouin zone, k_3 is the sum of the former two thus conserving phonon momentum. This Process is called normal scattering or N-Process.



Brillouin zone

Fig 2: Normal Process

Umklapp Process :

In crystalline materials, Umklapp scattering ~~or~~ is a scattering process that results in a wave vector which falls outside the first Brill zone. Umklapp scattering is the dominant process for thermal resistivity at high tempe ture for low defect crystals. The thermal conductivity for an insulating crystal when the U-process are dominant has $1/T$ depende phonon-Phonon scattering are caused due to the Umklapp Process.

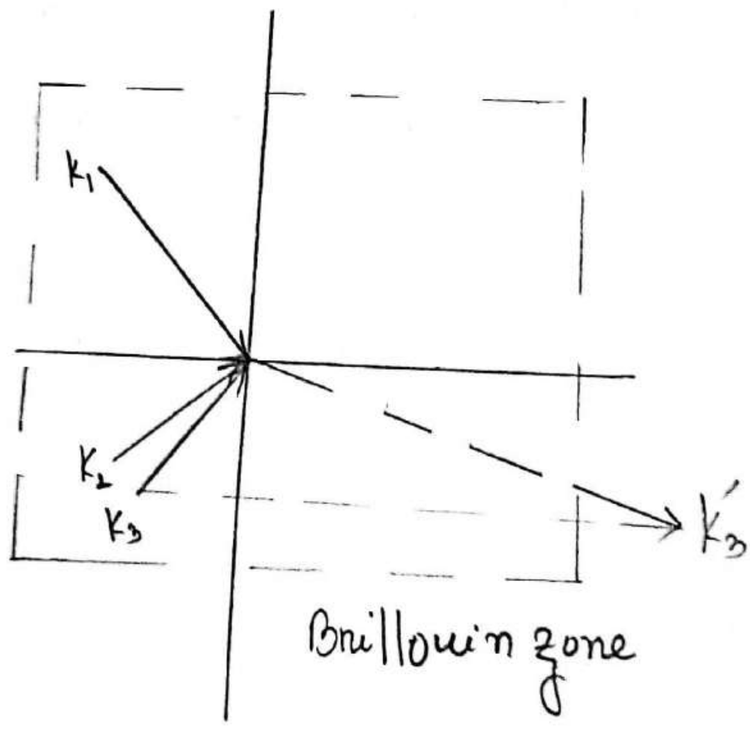


Fig 01: Umklapp process

Answer to the que. no. (b)

The assemptions of Debye Model of lattice specific heat is given below:-

(i) Any solid is capable of vibrating elastically in many differant nodes.

(ii) The frequency of vibration in one node is differant from that in another node.

(iii) The numbers of vibrations of solids are limited in number.

(iv) The density of vibrational modes of crystal solids make it as a continuous medium.

(v) The number of possible vibration of atoms of any ~~model~~ solid in the frequency range ν and

$$z(\nu)d\nu = \left(\frac{4\pi V}{\nu^3}\right)\nu^2 d\nu$$

$\uparrow$

~~z(ν)dν is the~~

$z(\nu)d\nu$ is the number of vibrational modes per unit frequency range is known as a density mode.

Calculation of lattice specific heat according to the Debye theory:-

The vibrational modes of a crystal as a whole.

$$V^2 \cdot \frac{d^2y}{dx^2} = \frac{d^2y}{dz^2}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{1}{V^2} \cdot \frac{d^2y}{dz^2}}$$

↳ This is known as simple harmonic equation.

Now,

$$\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz^2} = \frac{1}{C_s^2} \cdot \frac{d^2u}{dt^2} \quad \text{—(i)}$$

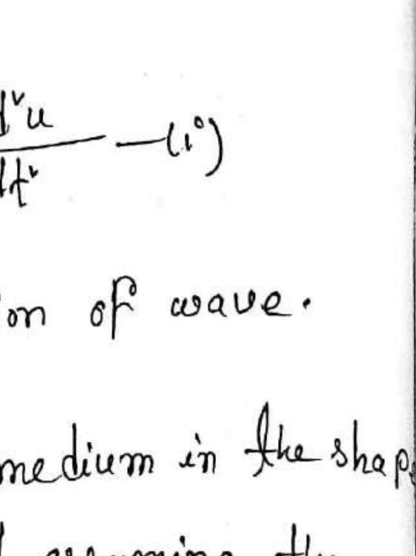
c_s is the velocity of propogation of wave.

According to a continuous medium in the shape of a cubic of edge h and assuming the forces the cubic at fixed.

$u(x, y, z, t) =$

$A \sin\left(\frac{n_x \pi}{L}\right) \sin\left(\frac{n_y \pi}{L}\right) \sin\left(\frac{n_z \pi}{L}\right) \times$

$\cos 2\pi \nu t$ —(ii)



solving eqn (i) and (ii) we get,

$$\frac{\pi^2}{L^2}(n_x^2+n_y^2+n_z^2)=\frac{4\pi^2\nu^2}{c_s^2}=\frac{4\pi^2}{\lambda^2}$$

$$\text{or, } \frac{\pi^2}{L^2}\times R^2=\frac{4\pi^2\nu^2}{c_s^2}$$

$$\text{or, } R^2=\frac{4\cancel{\pi^2}L^2\nu^2}{c_s^2}$$

$$\text{or, } R=\frac{2\cancel{\pi}L\nu}{c_s}$$

$$\text{so; } dR=\frac{2L}{c_s}\cancel{m}\,d(\nu) \quad \text{—(iv)}$$

Now,

the, no. of points in the unit cell between R and dR is,

$$=\frac{1}{8}\times(2\pi R)\,2R\,dR$$

$$=\frac{1}{8}\times 4\pi R^2\times dR$$

$$=\frac{1}{8}\times 4\pi\times\frac{4L^2\nu^2}{c_s^2}\times\frac{2L}{c_s}(d\nu)$$

$$=\frac{\pi}{2}\left(\frac{8V}{c_s^3}\nu^2\,d\nu\right)$$

$$=\frac{4\pi V}{c_s^2}\nu^2\,d\nu \quad \text{—(v)}$$

The no. of frequency range between ν and $\nu+d\nu$ is,

$$z(\nu)\,d\nu = \frac{4\pi V}{(c_s)^3} \times \nu^2 \cdot d\nu$$

In case of elastic waves,

longitudinal wave $\rightarrow c_l$

transverse wave $\rightarrow c_t$

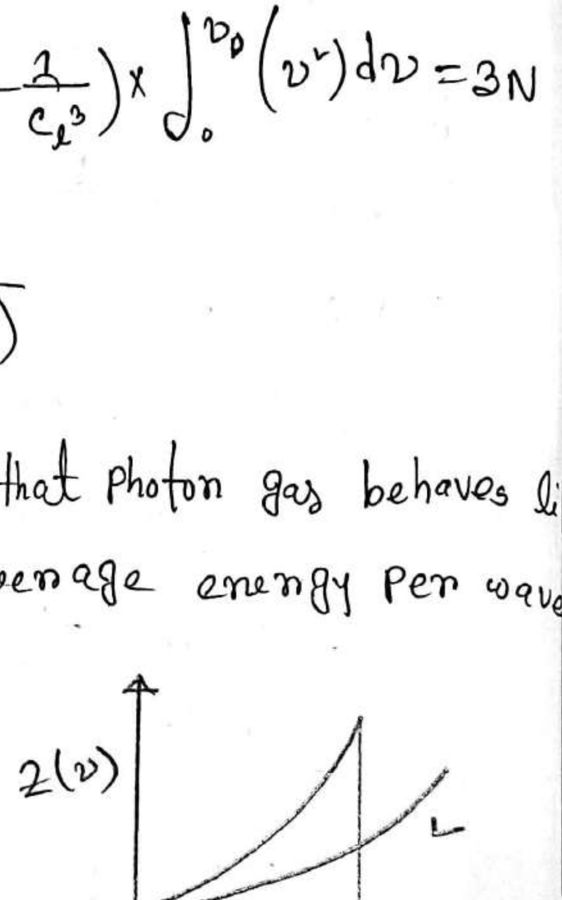
$$z(\nu)\,d\nu = 4\pi V \times \left(\frac{2}{c_t^3} + \frac{1}{c_l^3}\right) \times \frac{\nu^2}{1} \times d\nu$$

$$\int_0^{\nu_D} z(\nu)\,d\nu = 4\pi V \times \left(\frac{2}{c_t^3} + \frac{1}{c_l^3}\right) \times \int_0^{\nu_D} (\nu^2)\,d\nu = 3N$$

$$\therefore \nu_D^3 = \frac{9N}{4\pi V\left(\frac{2}{c_t^3} + \frac{1}{c_l^3}\right)}$$

Debyte further assumed that photon gas behaves li a plain gas and the average energy per wave is provides by,

$$E = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$$



Energy contained between in the waves with the frequencies lying between ν and $\nu+d\nu$ is,

$$= \langle E \rangle Z(\nu) d\nu$$

$$\therefore \nu = \frac{9N}{\nu_D^3} \int_0^{\nu_D} \frac{h\nu^3}{e^{h\nu/kt}-1}$$

$$So, V\theta = 4RT\left(\frac{T}{T_0}\right)^3 \cdot \int_0^{\frac{T_0}{T}} \frac{m^3 dx}{em-1}$$

For very very light temperature,

$T >> T_0 \rightarrow m$ is very small.

The range of the integral,

$$e^x \simeq 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\cdots$$

$$\simeq 1 + x$$

$$V\theta = 9RT\left(\frac{T}{T_0}\right)^3 \times \frac{1}{3}\left(\frac{T_0}{T}\right)^3$$

$$= 3RT$$

$$\therefore C_V = \left(\frac{\delta V}{\partial T}\right)_V = 3R \rightarrow \text{Dulong \& Petit's law.}$$

$$\therefore V = \frac{9RT}{1} \times \left(\frac{T}{T_0}\right)^3 \times \int_0^{\frac{T_0}{T}} \frac{x^3 dx}{e^x - 1}$$

Case-II

At very low temp; $T << T_0$

$$\frac{T_0}{T} \to \infty$$

$$V = \frac{9RT}{1}\left(\frac{T}{T_0}\right)^3 \int_0^\infty \frac{x^3\,dx}{e^x - 1}$$

$$V = 9RT \times \left(\frac{T}{T_0}\right)^3 \times \frac{\pi^4}{15}$$

$$= \frac{3}{5} \times \frac{R\pi^4}{T_0^{\,3}} \times T^4$$

$$C_v = \left(\frac{\partial V}{\partial T}\right)_v$$

$$= \frac{3}{5} \times \frac{R\pi^4}{T_0^3} \times 4T^3$$

$$= \frac{12}{5} \times \frac{R\pi^4}{T_0^3} \times T^3$$

So, $C_v \propto T^3$

So, the Debye's theorem for calculating specific + heat say that,

$$\boxed{C_v \propto T^3}$$

Ans. to the que. no.(c)

Given that,

the wave length of visible light, $\lambda = 4000\ A^{\circ}$

$= 4000 \times 10^{-10} m$,

The velocity of sound in medium is, $v = 5 \times 10^{5} cm/s$

$= 5 \times 10^{3} m/s$,

refractive index, $n = 1.5$,

Refractive index, $n = \dfrac{\text{Velocity of sound in zero medium}}{\text{Velocity of sound in medium}}$

$$n = \frac{V_0}{V};$$

The maximum frequency of phonon,

$$\nu = \frac{V_0}{\lambda}$$

$$= \frac{v \times n}{\lambda}$$

$$= \frac{5 \times 10^{3} \times 1.5}{4000 \times 10^{-10}}$$

$$= 1.875 \times 10^{10} Hz$$

Ans: 1.875×10^{10} Hz.

Ans to the Q. No. 7 Roll - 72 (107)

7. (a) What are the static and transport properties in the case of free electron model? Obtain an expression for the electrical conductivity of a ~~model~~ metal on the basis of free ~~e~~ electron theory. Hence prove the ohm's law?

Answer: Static properties are those properties which can be treated effectively by considering effectively by -- considering the energy levels by considering the energy levels only of the distribution of energy levels to which the electron belong.

And the static properties are:

(i) Electron emission properties.

(ii) Magnetic.

(iii) properties like: (i) heat capacity (ii) Contact potential

Transport properties which can treated by considering the detailed response of the electron to an external field.

Electrical conductivity of Metals:

$$\sigma = \frac{\Sigma}{AtE}$$

Here, σ = electrical conductivity

Σ = charge flow through a conductor.

A = Area of the conductor
t = time.
E = electric field.

Dc conductivity: A large influence of electron field is reflected in a large mean free time ~~ree~~/ relaxe time velocity

$$\tau = \frac{\lambda}{u} \rightarrow \text{mean free path}$$

If the applied field of the electron of charge

$$e^q n \text{ of motion, } m\frac{d^2x}{dt^2} = (-e)E$$

$$\Rightarrow \frac{d}{dt}\left(\frac{dx}{dt}\right) = \left(-\frac{e}{m}\right)E$$

$$\int d\frac{dx}{dt} = \int -\left(\frac{e}{m}\right)E\,dt$$

$$\frac{dx}{dt} = -\frac{e}{m}Et + C$$

At $t=0$, $\frac{dx}{dt} = 0$ $\quad C = 0$

$$\frac{dx}{dt} = -\left(\frac{e}{m}\right)Et = v$$

Average velocity between two collisions is

$$\bar{v} = \frac{1}{\tau}\int_0^{\tau} -\left(\frac{e}{m}\right)Et\,dt = -\frac{eE\tau}{2m}$$

If I is the current density and n is the no of electrons per unit volume.

$$i = n(-e)\bar{v} = \left(\frac{ne^2E}{2m}\right)\tau = \frac{1}{2}\left(\frac{ne^2E}{m}\right)\frac{\lambda}{u} = \frac{1}{2}\left(\frac{ne^2E\lambda u}{mu^2}\right)$$

Kinetic theory $\frac{1}{2}mu^2 = \frac{3kT}{2}$, k = Boltzmann constant

Dc electric conductivity, $\sigma = \frac{ne^2\lambda u}{6kT}$ $\quad i = \frac{ne^2E\lambda u}{6kT}$

Ohm's law, $i \propto E$ $\quad i = \frac{ne^2\lambda u}{6kT}E$

$i = \sigma E$ $\rightarrow$ (electrical conductivity)

b) Show that the density of states of free electron is given by $D(E) = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2}$. Use this expression to discuss number of filled states between electronic energy levels.

Answer: The no of orbital (energy state) per unit energy range is known as density of state.

$$D(E) = \frac{dN}{dE} = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2}$$

The density of filled electronic state at a particular temp. TK:

$$N(E) = D(E) f(E)$$

This is the fermi function which gives the probability that a particular quantum state at E is filled in accordance with fermi. Direct distribution law.

All absolute zero.

$f(E) = 1$, for $E < E_f$

All energy state for $E < E_f$ are occupied.

$f(E) = 0$ for $E > E_f$

All energy state for $E > E_f$ are vacant (empty)

No of filled state between energy E and E+dE at a Temp is given by -

$$N(E)\,dE = D(E) f(E)\, dE$$

$$= \frac{V}{2\pi^2}\left(\frac{2m}{h^2}\right)^{3/2} \frac{E^{1/2}}{e^{(E-E_f)/kT}+1}\, dE$$

<u>Average K.E at 0K :</u>

At 0K all the electrons have energy less than E_f

$$F(E) = \frac{1}{e^{(E-E_f)/kT}+1} = 1$$

$$D(E) = \frac{V}{2\pi^2}\left(\frac{2m}{h^2}\right)^{3/2} E^{1/2}$$

<u>Now ~~avg~~ average K.E per electron :</u>

$$\langle E_0 \rangle = \frac{1}{N}\int_0^{E_f} E D(E) f(E)\, dE \quad \Big| \because f(E) = 1$$

$$= \frac{1}{N}\int_0^{E_f} E D(E)\, dE$$

$$= \frac{1}{N}\left(\frac{V}{2\pi^2}\right)\left(\frac{2m}{h^2}\right)^{3/2}\int_0^{E_f} E^{3/2}\, dE$$

$$= \frac{3}{5} E_f$$

a) Aluminium metal crystallize in fcc structure. If each contributes single electron as free electron and the lattice constant a is 4.1 Å. Calculate treating conduction electrons as free electron fermi gas (i) Fermi energy and Fermi vector, (ii) total kinetic energy of free electron gas per unit volume at 0K [$\hbar = 1.054 \times 10^{-27}$ erg-sec], Electron rest mass = 9.11×10^{-28} gm

8. a. Give brief outlines of generation of coherent radiation. Discuss the characteristics of a LASER light.

b. Describe briefly the principle, construction and working of a ruby LASER.

c. The coherence length for sodium ~~length~~ light is 5890A°. Calculate (i) the number of oscillation corresponding to the coherence length and (ii) the coherence time.

Answer:

a. If the sum of energies of a signal, E_s and stimulated emission, E_e, is higher than the energy loss E_{loss} and energy E_l given up to the load,

$$E_s + E_e > E_{loss} + E_l$$

The quantum system enters the mode of self-excitation and begins to operate as a quantum oscillator (LASER), in which oscillation are also built up in the absence of an external signal under the action of random spontaneously emitted quant

The optical range within the spectrum interval extending from the ultraviolet to the sub-milimetric wave region: $\lambda = 0.1$ to $800\,\mu m$.

The energy levels between which optical transitions occur always have a finite width ΔE because the time of the settle life of electrons on these levels is finite, which, according to uncertainty relation, must cause broadening of the levels and their spreading into narrow bands. Consequently, the radiation emitted during optical transitions never happens to be trcitly monochromatic, and its frequencies cover a certain band $\Delta \nu$.

LASER excites oscillations only at resonant frequencies satisfying the condition,

$$L = n\left(\frac{\lambda}{2}\right)$$

The LASER emits the waves of one or less frequencies, the wavelength of which satisfy the resonances condition eqn and lie within the band $\Delta\nu$. The bandwith of each of these waves is a function of the Q-factor of an optical cavity and can be rather short (less than 100 Hz). The frequency stability depends on the stability of the dimension L of the cavity.

Characteristics Properties of a LASER Light:

i. Coherent: Different parts of the laser beam are related to each other in phase. The phase relationships are maintained over long enough time so that interference effects may be seen or recorded photographically. This coherence property is what makes holograms possible. The light is coherent with the waves all exactly in phase with one another.

ii. Monochromatic: The light is very nearly monochromatic. Laser light consis of essentially one wavelength, having its origin in stimulated emission from one set of atomic energy levels.

iii. Collimated: Because of bouncing back between mirrored ends of a laser cavity those paths which sustain amplification must pass between the mirrors many times and be very nearly perpendicular to the mirrors. As a result, laser beams are very narrow and do not spread very much.

iv. Intensity: The LASER light beam is extremely intense. A LASER beam diverge hardly at all.

b. A ruby laser is a solid-state laser that uses the synthetic ruby crystal as its laser medium. Ruby laser is the first successful laser developed by Maiman in 1960.

Ruby laser is one of the few solid-state lasers that produce visible light. It emits deep red light of wavelength 694.3 nm.

The ruby laser is used as a pulsed laser. After receiving a pumping flash from the flash tube, the laser light emerges for as long as the excited atoms persist in the ruby rod, which is typically about a milisecond.

The ruby laser consists of ruby cylindrical rod whose ends are optically flat and accurately parallel. ~~Oned~~ One end is fully silvered and the other is only partially silvered. The rod is surrounded by a glass tube. The glass tube is surrounded by a helical Xenon flash tube which acts as the optical pumping system.

Working of a ruby laser:

Consider a ruby laser medium consisting of three energy levels E_1, E_2, E_3 with N number of electrons.

We assume that the energy levels will be $E_1 < E_2 < E_3$. The energy level E_1 is known as ground state or lower energy state, the energy level E_2 is known as metastable state, and the energy level E_3 is known as pump state.

Let us assume that initially most of the electrons are in the lower energy state (E_1) and only a tiny number of electrons are in the excited states (E_2 and E_3).