CSE 1107 Solution

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... UNIVERSITY OF ENGINEERING & TECHNOLOGY
B.Sc. Engineering 1st Year 1st Term Examination, 2018
Department of Computer Science and Engineering
CSE 1107
Discrete Mathematics

TIME: 3 hours   FULL MARKS: 210

N.B. i) Answer **ANY THREE** questions from each section in separate scripts.
ii) Figures in the right margin indicate full marks.

SECTION A

(Answer **ANY THREE** questions from this section in Script A)

1. a) Define "Contrapositive" and "Inverse" propositions using example(s). (08)
   b) Find a proposition that is equivalent to $p \to q$ which uses the basic connectives. Hence prove its validity. (09)
   c) What are logical quantifiers? Prove the expression with example: $\neg\forall x Q(x) \Leftrightarrow \exists x \neg Q(x)$ (10)
   d) Suppose that we have an array of size 5×5. Are these statements equivalent? (08)

$$\forall \text{ row } x \, \exists \text{ column } y \; A(x, y) = 1$$
$$\exists \text{ row } x \, \forall \text{ column } y \; A(x, y) = 1$$

   Justify your answer.

2. a) What is induction method? Explain the idea of it. (10)
   b) Use induction method to find the orderings of all n-bit strings in such a way that two consecutive n-bit strings differed by only one bit. (10)
   c) Provide the recursive definition of the sequence $\{a_n\}$, $n = 1, 2, 3, \ldots$, if (10)

$$\text{(i) } a_n = 3n + 1, \text{ (ii) } a_n = 3^n$$

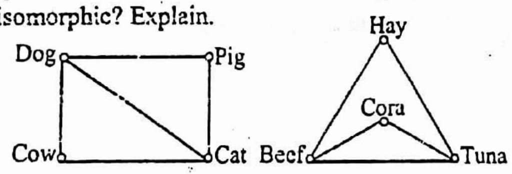
   d) Draw the recursive Fibonacci evaluation tree for $f(5)$, where (05)

$$f(0) = 0, f(1) = 1 \text{ and } f(n) = f(n-1) + f(n-2) \text{ for } n = 2, 3, 4, 5, \ldots$$

3. a) What is a graph? Let G = (V, E) be a graph with directed edges. Then prove that $\sum_{v \in V} \deg^-(v) = \sum_{v \in V} \deg^+(v) = |E|$. (10)
   b) Name the different methods to represent graphs. Draw the graphs for the following matrices: (13)

$$\text{(i) } A_G = \begin{bmatrix} 0 & 1 & 1 & 2 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 3 \\ 2 & 1 & 3 & 0 \end{bmatrix}; \quad \text{(ii) } A_G = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

   c) What are the necessary and sufficient conditions for two graphs to be isomorphic? Are the following two graphs isomorphic? Explain. (12)



4. a) What is linear congruential method. Use this method to find pseudo random numbers within the limit 5 to 20. (10)
   b) Let m be a positive integer. If $a \equiv b (\text{mod}\, m)$ and $c \equiv d (\text{mod}\, m)$, then prove that $ac \equiv bd (\text{mod}\, m)$. (08)
   c) Produce a secret message from the message "SEE YOU IN THE LOBBY" using the Caesar cipher method. (10)
   d) If a and b are positive integers. Then prove that $gcd(a, b) \times lcm(a,b) = a \times b$, where the symbols have their usual meaning. (07)

SECTION B

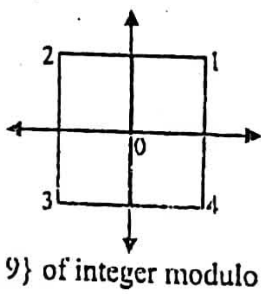
(Answer **ANY THREE** questions from this section in Script B)

5. a) What is a function? "A function must possesses one-to-one correspondence property if it wish to have an inverse."– justify the statement. (12)
   b) Describe the applications of *floor* function and *ceil* function with example. (13)
   c) Discuss the various representations of a sum. Mention the pros and cons of the representations. (10)

6. a) Show that the set of odd positive integers is a countable set. (10)
b) "Relations are a generalization of functions" – justify the statement with example. (08)
c) Let R be a reflexive relation. Is R^n (where $R^n = R^{n-1} \circ R$) necessarily reflexive? Give a reason for your answer. (10)
d) What do you mean by n-ary relation? Mention some applications of an n-ary relation with example. (07)

7. a) Let n be a positive integer and S be a set of strings. R_n is a relation on S such that $s \, R_n \, t$ if and only if $s = t$ or both s and t have at least n characters and first n characters of s and t are the same where $s, t \in S$. Is R_n an equivalence relation on S? Explain. (10)
b) During a month with 30 days, a baseball team plays at least one game a day, but no more than 45 games. Show that there must be a period of some consecutive days during which each team must play exactly 14 games. (10)
c) Define Algebraic System. Discuss the properties of the operations of an Algebraic System. (09)
d) Define Monoid. Prove that (N, ×) is a Monoid. (06)

8. a) Suppose that a valid codeword is n-digit number in decimal notation containing an even number 0s. let a_n denote the number of valid codewrds of length n. the sequence $\{a_n\}$ satisfies the recurrence relation $a_n = 7a_{n-1} + 2$ where $a_0 = 5$. Use generating function to find an explicit formula for a_n. (10)
b) Let S be the square in the plane R^2 mentioned in the figure below, with its center at the origin 0. The vertices of S are numbered counterclockwise from 1 to 4. For $\alpha = 0°, 90°, 180°$ and $270°$. Let $r(\alpha)$ be the symmetry obtained by rotating S about its center α degree and let $\gamma(\alpha)$ be the symmetry obtained by reflecting S about the y-axis and rotating S about its center α degree. Show the permutation group for S_4. (10)



c) Consider the ring $Z_{10} = \{0, 1, 2, \ldots, 9\}$ of integer modulo 10. (10)
i) Find the units of Z_{10}.
ii) Find -3, -8 and 3^{-1}.
iii) Let $f(x) = 2x^2 + 4x + 4$. Find the roots of $f(x)$ over Z_{10}.

d) Define injective function. If composite function gof is onto, does it follow that g is so? (05)

1.a. Let, a sentence, "If it rained last night, then the side walk is wet."

The ~~contrapositive~~ of the conditional statement is "If ~~it did~~ the sidewalk is not wet, then ~~the~~ it did not rain last night

The inverse of the conditional statement is "If it did not rain last night then the side walk is not wet.

b. $p \to q$ and $\neg p \vee q$ are logically equivalent.

Proof: Here we see a truth table.

p	q	$\neg p$	$p \to q$	$\neg p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

From this table we can see that $p \to q$ and $\neg p \vee q$ are logically same.

(Proved).

10. Logical quantifier is a way to state that a certain number of elements fulfill some criteria.

Example: $\neg \forall x\, Q(x) \Leftrightarrow \exists x \neg Q(x)$

To show that $\neg \forall x\, Q(x)$ and $\exists x Q(x)$ are logically ~~are logically~~ equivalent no matter what the propositional function $Q(x)$ is and what the domain is: Not that $\neg \forall x\, Q(x)$ is true if and only if $\forall x\, Q(x)$ is false. Next, not that $\forall x\, Q(x)$ is false iff. there is an element in domain for which $\neg Q(x)$ is true putting this steps together, we can conclude that $\neg \forall x\, Q(x)$ is true iff. $\exists x \neg Q(x)$ is true. It follows that $\neg \forall x\, Q(x)$ and $\exists x \neg Q(x)$ are logically equivalent so, $\neg \forall x\, Q(x) \Leftrightarrow \exists x \neg Q(x)$.

2 (a) . What is induction method? Explain the idea of it?

Mathematical induction: is a technique for proving results or establishing statements for natural numbers, This part illustrates the method through a variety of examples.

Definition: Mathematical Induction is a mathematical technique which is used to prove a statement, a formula or a theorom is true for every natural number.

The technique involves two steps to prove a statement, as stated below:

Step 1 (Base step) - It proves that a statement is true for the initial value.

Step 2 (Inductive step) - It proves that if the statement is true for the ~~nth~~ nth iteration (or number n), then it is also true for $(n+1)^{th}$ iteration (or number n+1)

How to do it:

Step 1 - Consider an initial value for which the statement is true. It is to be shown that the statement is true for $n =$ initial value.

Step 2 Assume the statement is true for any value of $n = k$. Then proove the statement is true for $n = k+1$. We actually break $n=k$ into two parts, one part is $n = k$ (which is already prooved) and try to proove the other part.

For example:

Proove that $3^n - 1$ is a multiple of 2 for $n = 1, 2 ----$

Solution: ~~For~~

Step 1: For $n = 1$, $3^1 - 1 = 3 - 1 = 2$ which is a multiple of 2.

Step 2 Let us assume $3^n - 1$ is true for $n = k$, Hence, $3^k - 1$ is true (It is an assumption)

We have to prove that $3^{k+1} - 1$ is also a multiple of 2 $3^{k+1} - 1 = 3 \times 3^k - 1$

$= (2 \times 3^k) + (3^k - 1)$

The first part $\boxed{(2 \times 3^k)}$ is certain to be a multiple of 2 and the second part $(3k - 1)$ is also true as our previous assumption.

Hence, $3^{k+1} - 1$ is a multiple of 2

Reference: https://www.tutorialspoint.com > discrete-mathematical-induction.htm.

(6)

S-32

~~S) Ans~~

~~S. What are logical quantifiers? Proove the~~

C. Provide the recursive definition of the sequence $\{a_n\}$, $n = 1, 2, 3 \ldots$, if

i) $a_n = 3n + 1$; (ii) $a_n = 3^n$

Ans. There are many possible correct answers. We will supply relatively simple ones.

i) $a_{n+1} = 3(n+1) + 1$

$= 3n + 3 + 1$

$= 3n + 4$

$= 3n + 1 + 3$

$= a_n + 3$ ~~$a_1 = 4$~~

~~and for even~~

for $n \geq 1$ and $a_1 = 4$

~~323, 306~~ ~~[illegible]~~

~~Ans~~)

(ii) $a_n = 3^n$

$a_{n+1} = 3^{n+1}$

$= 3^n \cdot 3$

$= 3a_n$

for $n \geq 1$ and $a_1 = 3$

(8)

d) Ques: Draw the recursive Fibonacci evaluation tree for $f(5)$, where

$$f(0) = 0, \quad f(1) = 1 \text{ and}$$

$$f(n) = f(n-1) + f(n-2) \text{ for } n = 2, 3, 4, 5 \ldots$$

Sol.n: Because the first part of the definition states that $f(0) = 0$ and $f(1) = 1$, it follows from the second part of the definition that,

$$f(2) = f(1) + f(0) = 1 + 0 = 1$$

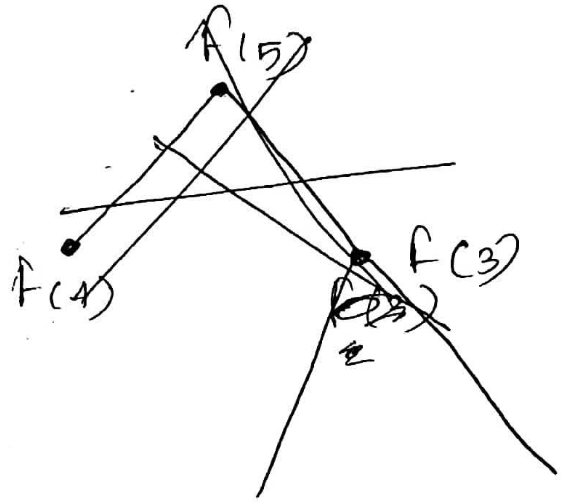
$$f(3) = f(2) + f(1) = 1 + 1 = 2$$

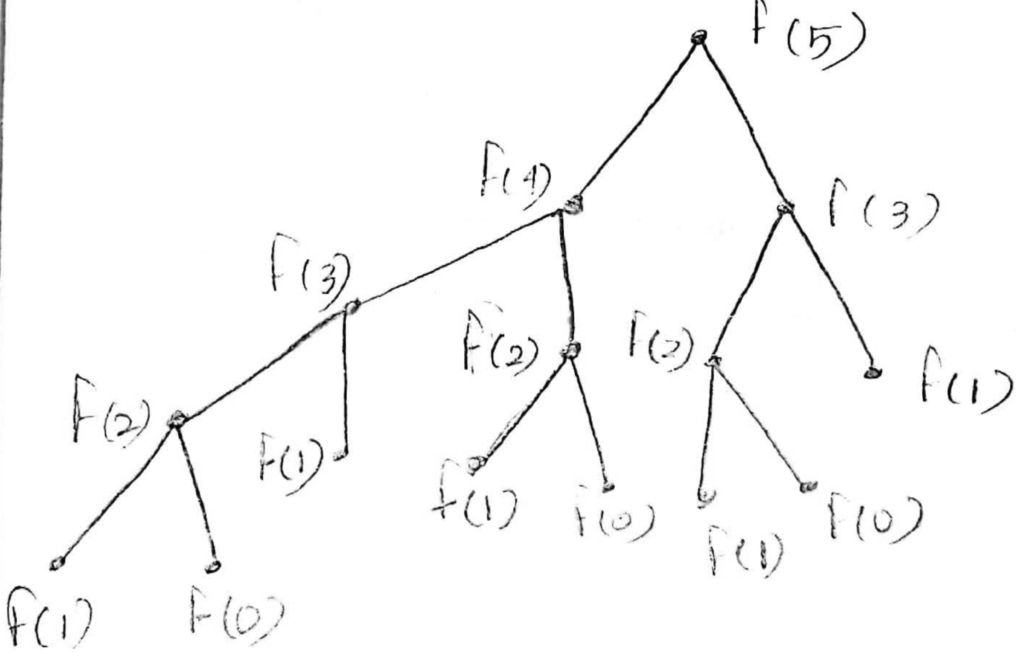
$$f(4) = f(3) + f(2) = 2 + 1 = 3$$

$$f(5) = f(4) + f(3) = 3 + 2 = 5$$

$$f(6) = f(5) + f(4) = 5 + 3 = 8$$

evaluating $f(5)$ recurssively : the tree will be :





Figure; Evaluating $f(5)$ Recursively

2)b) Use induction method to find the orderings of all n-bit strings in such a way that two consecutive n-bit strings differed by only one bit.

Sol^n: Let a_n denote the number of permissible bit strings of length n.

A bit string of length 1 cannot ~~be~~ have two consecutive ones. Since there are two ways to fill the digit, $a_1 = 2$

The only bit string of length 2 that has two consecutive ones is 11. Since there's are two ways to fill each of the two digits in the bit string, $a_2 = 2.2 - 1 = 3$

Any permissible bit string of length n+1 that ends in 0 can be formed by appending a 0 to the end of a permissible bits bit string n, of which there are a_n.

(11)

For a bit string of length $n+1$ to end in a 1, the entry in the nth position must be a zero. Thus, any permissible bit string of length $n-1$, of which there are a_{n-1}.

Thus, we have the recurrence relation

$$a_1 = 2$$

$$a_2 = 3.$$

$$a_{n+1} = a_n + a_{n-1}, \ n \geq 2$$

Or, you can write

$$a(n) = a(n-1) + a(n-2)$$

base case is $a(1)$

Solution no: 03

Solution no:03 (a)

~~A Key concepts~~ concept

Ques: a — What is a graph? Let $G=(V,E)$ be a graph with directed edges. Then prove that

$$\sum_{v \in V} deg^{-}(v) = \sum_{v \in V} deg^{+}(v) = |E|$$

Solution: A Key concept of the system is the graph (or edges or relationship), which directly relates data items in the store a collection of nodes of data and edges representing the relationships between the nodes.

A graph G consists of two things:

(i) A set $V = V(G)$ whose elements are called vertices, points or nodes of G.

(ii) A set $E = E(G)$ of unordered pairs of distincts verticks called edges of G.

We denote such a graph by $G(V, E)$ ~~whose elements are~~ which we want to emphasize the two parts of G.

(B) Name of the different method to represent graphs. Draw the graphs ~~of~~ for ~~the following~~ ~~structures~~ following matrices.

(i) $A_G = \begin{bmatrix} 0 & 1 & 1 & 2 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 3 \\ 2 & 1 & 3 & 0 \end{bmatrix}$

(ii) $A_G = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

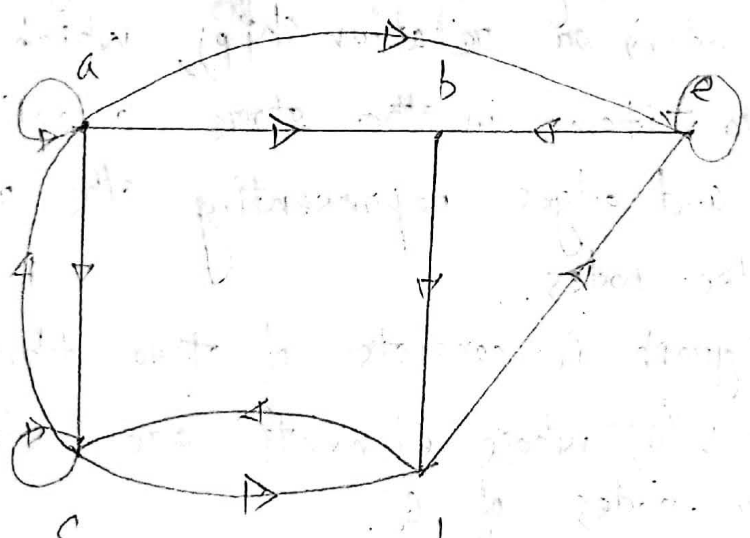
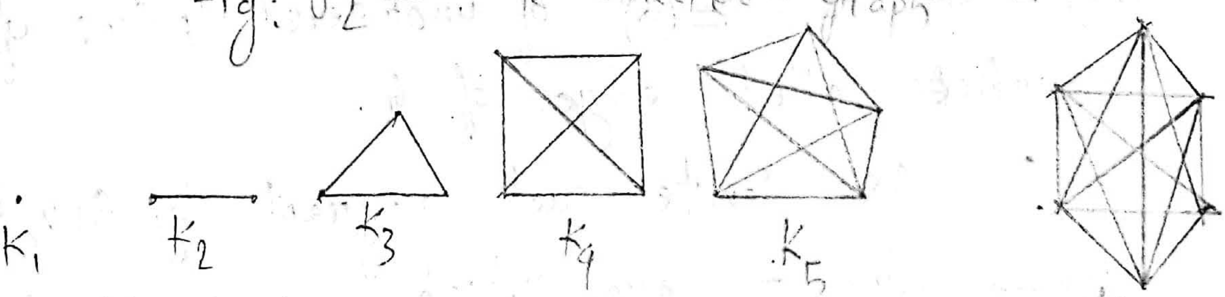


Fig: 0.2 — The directed graph



K_1 K_2 K_3 K_4 K_5 K_6

So, we can say that $\sum_{v \in V} deg^-(v) = \sum_{v \in V} deg^+(v) = |E|$. (Pro

③ Name the different methods to represent graphs. Draw the graphs for the following structures.

(i) $A_G = \begin{bmatrix} 0 & 1 & 1 & 2 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 3 \\ 2 & 1 & 3 & 6 \end{bmatrix}$

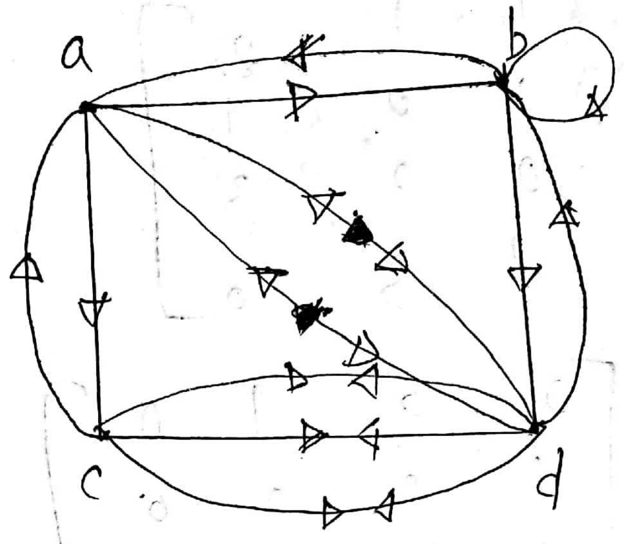
(ii) $A_G = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

Answer: There are 6 Kinds of graphs.

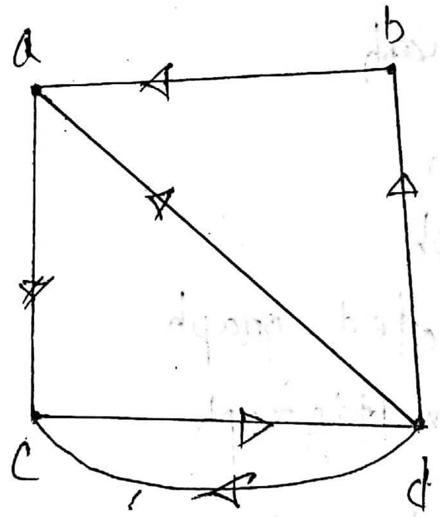
(i) Simple graph
(ii) Multigraph
(iii) Pseudograph
(iv) Simple directed graph
(v) Directed multigraph
(vi) Mixed graph

Q4 The graph for the following adjacency matrix will be:

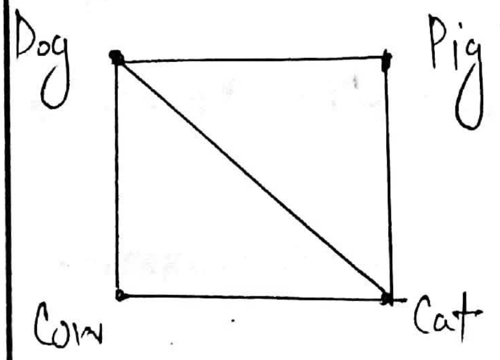
(i) A_G:

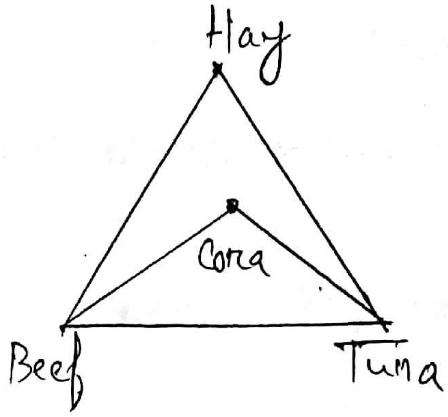


(ii) A_{G_2}:



No: 03(a): What are the necessary and sufficient conditions for two graphs to be isomorphic? Are the following two graphs isomorphic? Explain





Ans: In Graph theory, an isomorphism of graphs G and H is a bijection between the vertex sets of G and H

$$f: V(G) \to V(H).$$

If an isomorphism exists between two graphs, then the graphs are called isomorphic and denoted as $G \simeq H$. In the case, when the bijection is a mapping of a graph onto itself, i.e, when G and H are one and the same graph, the bijection is called an automorphism of G.

Question no: 4

a) What is a ~~to~~ linear congruential method? Use this method to find pseudo random numbers within the limit 5 to 20. (10)

b) Let m be a positive integer. If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then prove that $ac \equiv bd \pmod{m}$. (08)

c) Produce a secret message from the message "SEE YOU IN THE LOBBY" using the Ceaser cipher method. (10)

d) If a and b are positive integers. Then prove that $\gcd(a,b) \times \operatorname{lcm}(a,b) = a \times b$, where the symbols have their usual meaning. (07)

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Solution

a) Linear congruential method is a kind of procedure for generating pseudo-random numbers. In this method, four integers are used → the modulous m, multiplier a, increament c and seed x_0, with values ⇒ $2 \le a < m$, $0 \le c < m$ and $0 \le x_0 < m$.
To generate a sequence of pseudo-random numbers $\{x_n\}$, with $0 \le x_n < m$ for all n, by successively using the congruence:

$$x_{n+1} = (ax_n + c) \bmod m$$

Pseudo. ~~numbers~~ random numbers within the limit 5 to 20:

Here, maximum value $= 20$
minimum value $= 5$
∴ modulous, $m = 20 - 5 = 15$

Choosing a, c and x_0 according to the conditions
$a = 7$, $c = 9$ and $x_0 = 10$.

Now,

$x_1 = (7\times10+9) \bmod 15 = 4+5 = 9$

$x_2 = (7\times9+9) \bmod 15 = 12+5 = 17$

$x_3 = (7\times17+9) \bmod 15 = 8+5 = 13$

$x_4 = (7\times13+9) \bmod 15 = 10+5 = 15$

$x_5 = (7\times15+9) \bmod 15 = 9+5 = 14$

$x_6 = (7\times14+9) \bmod 15 = 2+5 = 7$

$x_7 = (7\times7+9) \bmod 15 = 13+5 = 18$

$x_8 = (7\times18+9) \bmod 15 = 0+5 = 5$

$x_9 = (7\times5+9) \bmod 15 = 14+5 = 19$

$x_{10} = (7\times19+9) \bmod 15 = 7+5 = 12$

$x_{11} = (7\times12+9) \bmod 15 = 3+5 = 8$

$x_{12} = (7\times8+9) \bmod 15 = 5+5 = 10$

Because, $x_{12} = x_0$ and because, each terms depends only on the previous term, this sequence is generated,

10, 9, 17, 13, 15, 14, 7, 18, 5, 19, 12, 8, 10, 9, 17, 13, 15, 14, 7, ...

This sequence contains 12 different numbers before repeating.

b) It is given that,

$$a \equiv b \pmod{m}$$
$$c \equiv d \pmod{m}$$

These imply that there are integers s and t with, $b = a + sm$ —(I) and $d = c + tm$ —(II)

Therefore, (I) × (II) ⇒

$$bd = (a+sm) \times (c+tm)$$
$$= ac + atm + scm + stm^2$$
$$\Rightarrow bd = ac + m(at + sc + stm)$$

Hence, $ac = bd \pmod{m}$

c) b

Given sentence ⇒

"SEE YOU IN THE LOBBY"

First replacing the letters in the message with corresponding numbers ⇒

18 4 4 24 14 20 8 13 19 7 4 11 14 1 1 24

Now, replacing each of these numbers p by $f(p) = (p+3) \bmod 26$. This gives ⇒

21 7 7 1 17 23 11 16 22 10 7 14 17 4 4 1

Now, translating these back to letters ⇒

"VHH BRX LQ WKH OREEB"

d)

~~Let,~~ ~~a = p~~

d) According to prime factorisation process,

Let, $a = p_1^{k_1} p_2^{k_2} \cdots\cdots p_n^{k_n}$

$b = p_1^{l_1} p_2^{l_2} \cdots\cdots p_n^{l_n}$

So, the lcm would be,

$$lcm(a,b) = p_1^{max(k_1,l_1)} \, p_2^{max(k_2,l_2)} \cdots\cdots p_n^{max(k_n, l}$$

and, the gcd would be,

$$gcd(a,b) = p_1^{min(k_1,l_1)} \, p_2^{min(k_2,l_2)} \cdots\cdots p_n^{min(k_n,l_n)}$$

From sum of maximum and minimum, for all $i \in \{1, 2, \ldots, n\}$

$$min(k_i, l_i) + max(k_i, l_i) = k_i + l_i$$

Hence, $gcd(a,b) \times lcm(a,b) = \cancel{p_1^{max(k_1,l_1)+min(k_2,l_2)}}$

$$= p_1^{k_1+l_1} \, p_2^{k_2+l_2} \cdots\cdots p_n^{k_n+l_n}$$

$$= (p_1^{k_1} p_2^{k_2} \cdots p_n^{k_n}) \times (p_1^{l_1} p_2^{l_2} \cdots p_n^{l_n})$$

$$= ab$$

$\therefore$ $gcd(a,b) \times lcm(a,b) = a \times b$ (Proved)

Ans. to the Ques. No-5

a) Let. A and B be non-empty sets. A function. f from A to B is an assignment of exactly one element of B to each element of A. If f is a function from A to B, we write : $f: A \to B$.

A function is called one to one correspondence if it is both one-to-one and onto.

A one to one correspondence is called invertible because we can define an inverse of that function. A function is not invertible if it is not a one-to-one correspondence, because the inverse of such a function doesn't exist.

If f is not one-to-one ~~correspondence~~, some element b in the co-domain is the image of more than one element in the domain. If f is not onto, for some element b in the co-domain no element a in the domain exists for which $f(a) = b$.

Consequently, if f is not a one-to-one correspondence, we cannot assign to each element b in the co-domain a unique element a in the domain such that $f(a) = b$, beacause for some b there ~~es~~ either more than one such a or no such a.

So, it is justified that, a function must possesses one-to-one correspondence property if it wish to have an inverse.

b) Floor function: The floor function assigns to the real number x the largest integer that is less than or equal to x. The value of the floor function at x is denoted by $\lfloor x \rfloor$.

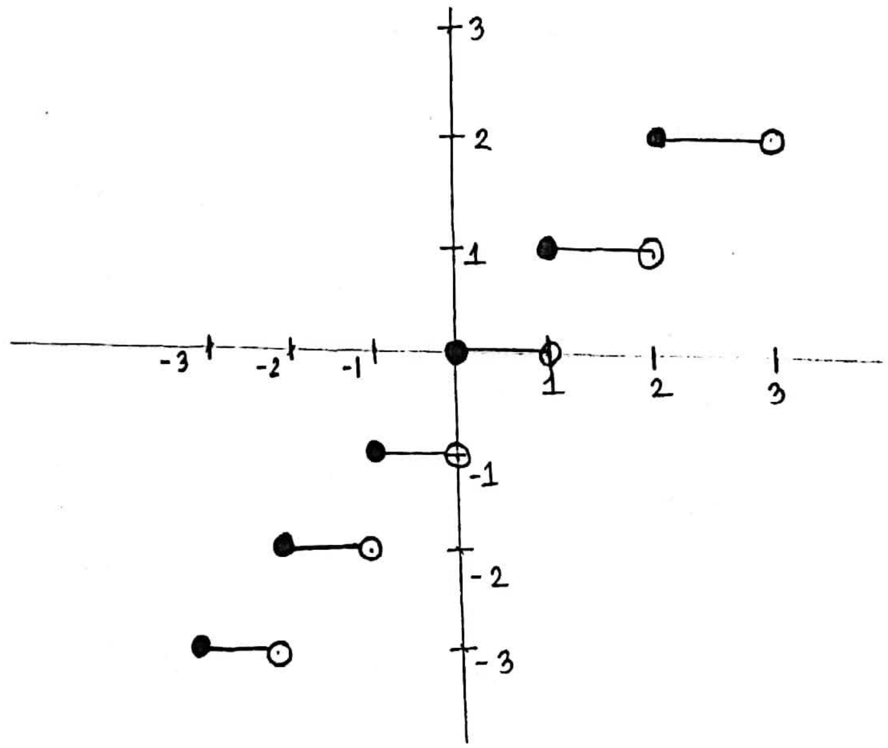

Fig: The Graph of floor function

The graph shows the floor function.

Ceil function : The ceiling function assigns to the real number x the smallest integer that is greater than or equal to x. The value of the ceiling function at x is denoted by $\lceil x \rceil$.

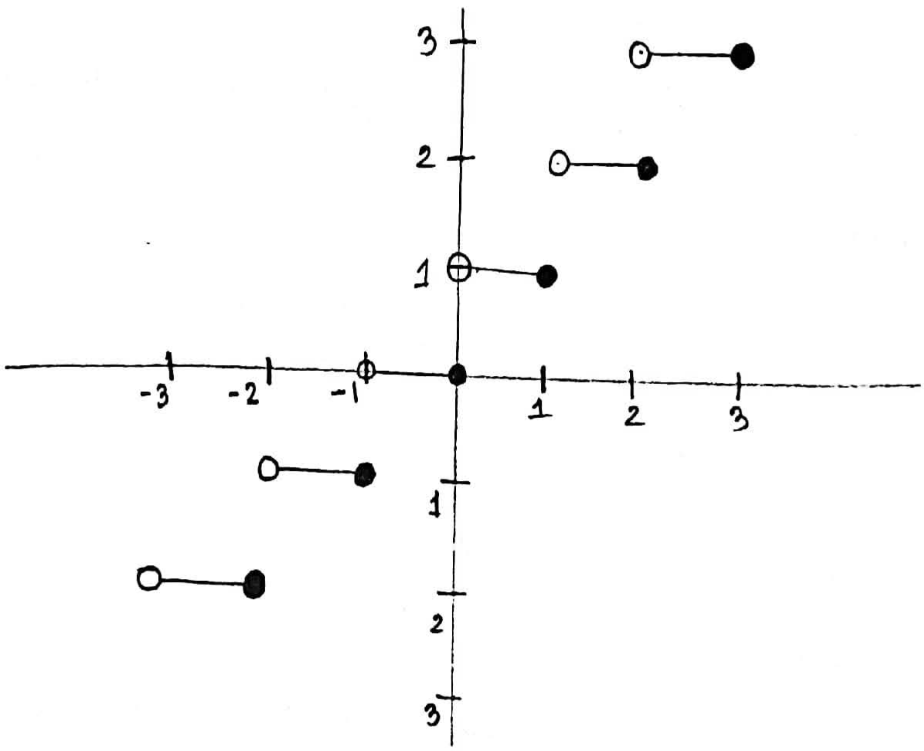

Fig: The graph of ceil function

The graph shows the ceil of some numbers.

The floor and ceil functions are useful in wide variety of applications, including those involving data storage and data transmission.

For example, when data is stored on a computer disk, it is reprented by as a string of bytes. Each byte is made up of 8 bits. For determining how many bytes needed for encoding 100 bits of data, the smallest integer that is atleast as large as the quotient when 100 is divided by 8, has to be determined.

So, $\lceil 100/8 \rceil = \lceil 12.5 \rceil = 13$ bytes are required.

This is how ceil function helps. In the same way, even floor function is needed for determining largest integer.

(c) There are different ways to introduce summation notation.

The notation used to express the sum of the terms $a_m, a_{m+1}, \dots, a_n$ from the sequence $\{a_n\}$. The notations used to represent $a_m + a_{m+1} + \dots + a_n$ are

$$\sum_{j=m}^{n} a_j, \quad \sum_{j=m}^{n} a_j \quad \text{or} \quad \sum_{m \leq j \leq n} a_j$$

Here, the variable j is called index of summation, and the choice of the letter j as the variable is arbitraty, that is, we could have used any other letter, such as i or k.

Or in notation,

$$\sum_{j=m}^{n} a_j = \sum_{i=m}^{n} a_i = \sum_{K=m}^{n} a_k$$

Here, the index of summation runs through all integers starting with its lower limit m and ending with its upper limit n. A large uppercase Greek letter sigma, Σ, is used to denote summation.

Now, when a and b are real numbers,

$$\sum_{j=1}^{m} (ax_j + by_j) = a\sum_{y=1}^{n} x_j + b\sum_{j=1}^{n} y_j \,;$$

where, $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_n$ are real numbers.

Having so many ways of representing sum, sometimes can get confusing. Cause all the forms might not be known by everybody. And again having so many ways of representing sum has advantages, as we can use any notation to show and specify the details of the sum.

Answer to the question no : 6 (28)

a) Show that the set of odd positive integer is a countable set.

Ans: let, O be the set of odd integer.

and, $f(x) = (2x+1)$ be a function from N to O.

Now,

$$f(0) = 1$$
$$f(1) = 3$$
$$f(2) = 5$$
$$f(3) = 7$$

and so on.

then, f is a bijection from N to O since f is both one-to-one and onto.

Again, $f(n) = f(m)$

$$\Rightarrow 2n+1 = 2m+1$$

$$\therefore n = m$$

so, $f(x)$ is injective.

Again, let, $t = 2k+1$ [~~k is~~ $k \in N$]

$$\therefore k = \frac{t-1}{2}$$

$$\therefore f(k) = 2\left(\frac{t-1}{2}\right) + 1$$
$$= t$$

So, $f(x)$ is surjective.

(29) Since, $f(x)$ is bijective, injective and surjective

So, $f(x)$ is ~~a~~ countable.

$\therefore$ The set of odd positive integers is a countable set

b) "Relations are a generalization of function" – justify the statement with example.

Ans: A relations is a set of inputs and outputs. on the other hand, a function is a Relation which each input has only one output.

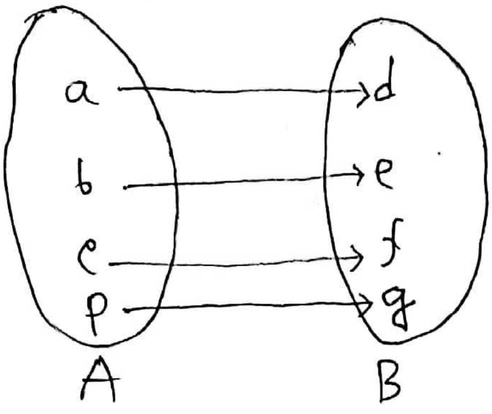

figure: 1

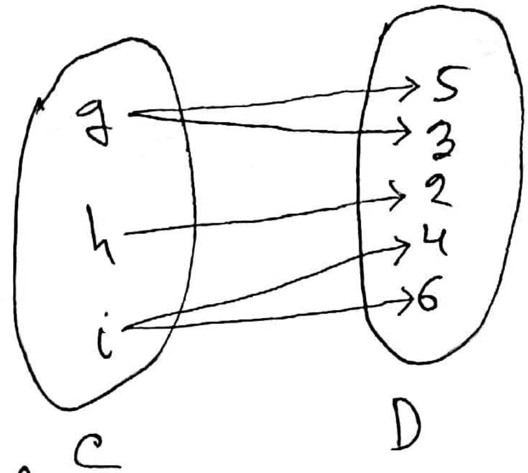

figure: 2

In figure: 1, it is both Relation and function.

and In figure: 2, it is a Relation. It is not a function.

So, we can say that, "Relations are a generalization of function".

(31)

b)

TRY IT YOURSELF

c) TRY it yourself

d) What do you mean by n-ary relation? mention some application of an n-ary relation with example

Ans:

An n-ary relation on sets $A_1, \ldots, A_n$ is a set of ordered n-tuples $\langle a_1, \ldots, a_n \rangle$ where a_i is an element of A_i for all i, $1 \leq i \leq n$. Thus an n-ary relation on sets $A_1, \ldots, A_n$ is a subset of cartesian product $A_1 \times \ldots \times A_n$.

Application of n-ary relation:

i) Relational database model has been developed for information processing.

example:

- a database consists of records, which are n-tuples made up of fields.
- The fields contains information such as:
  - Name
  - Student #
  - Major
  - Grade point average ~~of~~ of the student.

- The relational database model represent a database of records of n-ary relation.
- The relation is R (Student-Name, id-Number, Major, GPA)

(34) example of records

- (Smith, 3214, Mathematics, 3.9)

(Stevens, 1412, computer.science, 4.0)

(Rao, 6633, physics, 3.5)

(Adams, 1320, Biology, 3.0)

(Lee, 1030, computer science, 3.7)

Table

Student names	ID #	Major	GPA
Smith	3214	Mathematics	3.9
Stevens	1412	Computer science	4.0
Rao	6633	physics	3.5
Adams	1320	Biology	3.0
Lee	1030	Computer science	3.7

Ans. to the ques. no - 08

(a)

Let $G(x)$ be the generation function for the sequence $\{a_n\}$, that is,

$$G(x) = \sum_{n=0}^{\infty} a_n x^n .$$

First note that,

$$x\,G(x) = \sum_{n=0}^{\infty} a_n x^{n+1} = \sum_{n=0}^{\infty} a_{n-1} x^n .$$

Using the recurrence relation, we see that,

$$G(x) - 7x\,G(x) = \sum_{n=0}^{\infty} a_n x^n - 7\sum_{n=1}^{\infty} a_{n-1} x^n$$

$$= \sum_{n=0}^{\infty} a_n x^n - 7a_0 - 7\sum_{n=0}^{\infty} a_{n-1} x^n + 7a_0$$

$$= \sum_{n=0}^{\infty} a_n x^n - 7\left(a_0 + \sum_{n=1}^{\infty} a_{n-1} x^n\right) + 7\times 5 \quad [\because a_0 = 5]$$

$$= \sum_{n=0}^{\infty} a_n x^n - 7\sum_{n=0}^{\infty} a_{n-1} x^n + 35$$

$$= \sum_{n=0}^{\infty} (a_n - 7a_{n-1}) x^n + 35$$

$$= \sum_{n=0}^{\infty} 2x^n + 35 \qquad [\because a_n = 7a_{n-1} + 2]$$

$$= 2\sum_{n=0}^{\infty} x^n + 35$$

$$= 2 \times \frac{1}{1-x} + 35$$

$$= \frac{2}{1-x} + 35$$

$$= \frac{2+35-35x}{1-x}$$

$$= \frac{37-35x}{1-x}$$

$$\therefore (1-7x)G(x) = \frac{37-35x}{1-x}$$

$$\therefore G(x) = \frac{37-35x}{(1-x)(1-7x)}$$

Expanding the right hand side of the above equation into partial fractions gives,

$$G(x) = \frac{1}{3(x-1)} - \frac{112}{3(7x-1)}.$$

$$= \frac{1}{3}\left(\frac{1}{x-1} - \frac{112}{7x-1}\right)$$

$$= \frac{1}{3}\left(-\frac{1}{1-x} + \frac{112}{1-7x}\right)$$

$$= \frac{1}{3}\left(112 \times \frac{1}{1-7x} - \frac{1}{1-x}\right)$$

$$= \frac{1}{3}\left(112 \times \sum_{n=0}^{\infty} 7^n x^n - \sum_{n=0}^{\infty} x^n\right)$$

$$= \sum_{n=0}^{\infty} \frac{1}{3}(112 \times 7^n - 1)x^n$$

$$\therefore a_n = \frac{1}{3}(112 \times 7^n - 1)$$

(b)

Permutation group for S_4 :

$$\sigma_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

$$\sigma_2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 4 & 3 \end{pmatrix}$$

$$\sigma_3 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 2 & 3 \end{pmatrix}$$

$$\sigma_4 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 2 & 4 \end{pmatrix}$$

$$\sigma_5 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \end{pmatrix}$$

$$\sigma_6 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 3 & 2 \end{pmatrix}$$

$$\varphi_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 3 & 4 \end{pmatrix}$$

$$\varphi_2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$$

$$\varphi_3 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$$

$$\varphi_4 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix}$$

$$\varphi_5 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$$

$$\varphi_6 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 3 & 2 \end{pmatrix}$$

$$\theta_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix}$$

$$\theta_2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}$$

$$\theta_3 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 1 & 3 \end{pmatrix}$$

$$\theta_4 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix}$$

$$\theta_5 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$$

$$\theta_6 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix}$$

vi)

$$\lambda_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}.$$

$$\lambda_2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}$$

$$\lambda_3 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 3 & 1 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 4 & 1 \end{pmatrix}$$

$$\lambda_5 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}$$

$$\lambda_6 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}$$

Here, the total number of permutations for S_4 are $= 4! = 24$.

(c)

(i) $\gcd(10, 1) = 1$

$\gcd(10, 2) = 2 \neq 1$

$\gcd(10, 3) = 1$

$\gcd(10, 4) = 2 \neq 1$

$\gcd(10, 5) = 5 \neq 1$.

$\gcd(10, 6) = 2 \neq 1$

$\gcd(10, 7) = 1$

$\gcd(10, 8) = 2 \neq 1$

$\gcd(10, 9) = 1$

$\therefore$ Units of Z_{10}, $U(10) = \{1, 3, 7, 9\}$.

(ii) For integer modulo 10,

$[Z_{10}, +]$	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	0
2	2	3	4	5	6	7	8	9	0	1
3	3	4	5	6	7	8	9	0	1	2
4	4	5	6	7	8	9	0	1	2	3
5	5	6	7	8	9	0	1	2	3	4
6	6	7	8	9	0	1	2	3	4	5
7	7	8	9	0	1	2	3	4	5	6
8	8	9	0	1	2	3	4	5	6	7
9	9	0	1	2	3	4	5	6	7	8

From above, we can see,

$$-3 = 7$$

and, $-8 = 2$.

Again, for integer modulo 10,

$[Z_{10}, X]$	1	3	7	9
1	1	3	7	9
3	3	9	1	7
7	7	1	9	3
9	9	7	3	1

From above, we can see,

$$3^{-1} = 7.$$

(iii) $f(x) = 2x^2 + 4x + 4$.

$\mathbb{Z}_{10} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

The roots of $f(x)$ over $\mathbb{Z}_{10}$ will be the value of x from set $\mathbb{Z}_{10}$ if,

$f(x) = 2x^2 + 4x + 4 \equiv 0 \pmod{10}$.

Now,

$f(0) = 2\times0^2 + 4\times0 + 4 = 4 \not\equiv 0 \pmod{10}$

$f(1) = 2\times1^2 + 4\times1 + 4 = 10 \equiv 0 \pmod{10}$

$f(2) = 2\times2^2 + 4\times2 + 4 = 20 \equiv 0 \pmod{10}$

$f(3) = 2\times3^2 + 4\times3 + 4 = 34 \not\equiv 0 \pmod{10}$

$f(4) = 2\times4^2 + 4\times4 + 4 = 52 \not\equiv 0 \pmod{10}$

$f(5) = 2\times5^2 + 4\times5 + 4 = 74 \not\equiv 0 \pmod{10}$

$f(6) = 2\times6^2 + 4\times6 + 4 = 100 \equiv 0 \pmod{10}$

$f(7) = 2\times7^2 + 4\times7 + 4 = 130 \equiv 0 \pmod{10}$

$f(8) = 2\times8^2 + 4\times8 + 4 = 164 \not\equiv 0 \pmod{10}$

$f(9) = 2\times9^2 + 4\times9 + 4 = 202 \not\equiv 0 \pmod{10}$

$\therefore$ 1, 2, 6, 7 are the roots of $f(x)$ over $\mathbb{Z}_{10}$.

(d) <u>Injective function:</u> A function is said to be injective if and only if $f(a) = f(b)$ implies that $a = b$ for all a and b in the domain of f.

<u>If composite function $g \circ f$ is onto, does it follow that g is so:</u>

Let, $f : A \to B$ and $g : B \to C$

$g \circ f : A \to C$ is onto.

Take any $y \in C$.

Since, $g \circ f$ is onto,

$$\exists a \in A \left((g \circ f)(a) = y \right)$$

$$\therefore \exists a \in A \left(g(f(a)) = y \right).$$

Let, $b = f(a) \in B$

Then, $g(b) = g(f(a)) = y$.

But, we proved the above discussion for any $y \in C$.
Therefore g is onto

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KHULNA UNIVERSITY OF ENGINEERING & TECHNOLOGY

B.Sc. Engineering 1st Year 1st Term Examination, 2017

Department of Computer Science and Engineering

CSE 1107

Discrete Mathematics

TIME: 3 hours FULL MARKS: 210

N.B. i) Answer ANY THREE questions from each section in separate scripts.
ii) Figures in the right margin indicate full marks.

SECTION A

(Answer ANY THREE questions from this section in Script A)

1. a) What is contradiction? Determine whether the proposition $[(p\vee q)\wedge(p\to r)\wedge(q\to r)]\to r$ is a tautology or contradiction. (10)

   b) What are logical quantifiers? Provide the physical example(s) for the following relations: (10)

   i) $\neg\exists x Q(x)\Leftrightarrow\forall x\neg Q(x)$

   ii) $\neg\forall x P(x)\Leftrightarrow\exists x\neg P(x)$

   c) Express the following statements in the logical arguments using quantifiers, connectives and corresponding predicates: (10)
   "All lions are fierce", "Some lions do not drink coffee", "Some fierce creatures do not drink coffee".

   d) Prove the validity of mathematical induction. (05)

2. a) What do you mean by recursion? Formulate a recursive solution to the "Tower of Hanoi" problem. (10)

   b) Using mathematical induction, prove that the sum of the first n odd positive integer is n^2. (10)

   c) Elaborate the rules of inference for propositional logic. (10)

   d) Provide the examples of "Converse" and "Contrapositive" statements. (05)

3. a) Let n and d be positive integers. How many positive integers not exceeding n are divisible by d? (08)

   b) If a and b are positive integers, then prove that $gcd(a, b)\times lcm(a, b) = a\times b$, where the symbols have their usual meaning. (09)

   c) Using Caesar cipher method construct a secret message for "MEET YOU IN THE CLASSROOM". (10)

   d) There is a number which is divided by 3, the remainder is 2; when divided by 5, the remainder is 3; and when divided by 7, the remainder is 2. What is the number? (08)

4. a) What is a tree? Prove that a tree with n vertices has n-1 edges. (08)

   b) What is a graph? Let $G=(V,E)$ be a graph with directed edges. Then prove that $\sum_{v\in V}\deg^-(v)=\sum_{v\in V}\deg^+(v)=|E|$. (09)

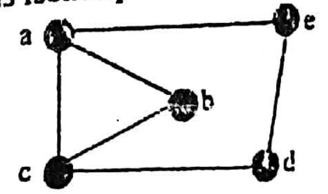
   c) List the different methods to represent the graphs. Draw the graphs for the following adjacency matrices: (10)

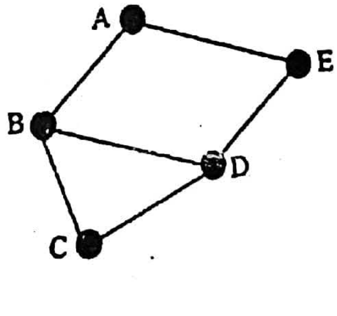
i) $\begin{array}{c c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$, ii) $\begin{array}{c c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$.

a b

c d

   d) What are the necessary and sufficient conditions for two graphs to be isomorphic? Are the following two graphs isomorphic? (08)





SECTION II

(Answer ANY THREE questions from this section in Script II)

5. a) Justify the statement – "Every function is a relation but not every relation is a function". (10)

b) Explain graph of function with example. How can it be used to distinguish various types of function? (12)

c) Define equivalence relation. Consider a relation $R = \{(a,b) \in R \times R \mid a-b \text{ is an integer}\}$. Is R an equivalence relation? (12)

6. a) How many relations are there on a set with n elements? (04)

b) Let $A = \{1, 2, 3, 4\}$ and R is a relation on A such that $R = \{(a,b) \mid a \text{ divides } b\}$, show the different representations of R and check whether it is symmetric, anti-symmetric and transitive or not. (14)

c) What is the composite of the relations R and S where R is the relation from $\{1, 2, 3\}$ to $\{1, 2, 3, 4\}$ with $R = \{(1, 1), (1, 4), (2, 3), (3, 1), (3, 4)\}$ and S is the relation from $\{1, 2, 3, 4\}$ to $\{0, 1, 2\}$ with $S = \{(1, 0), (2, 0), (3, 1), (3, 2), (4, 1)\}$? (08)

d) Construct a Hasse Diagram for the poset $(\{1, 2, 3, 4, 6, 8, 12\}, |)$. Is the poset a lattice? (09)

7. a) Answer the following questions concerning the poset $(\{2, 4, 6, 9, 12, 18, 27, 36, 48, 60, 72\}, |)$. (10)

i) Find the maximal elements.
ii) Find the minimal elements.
iii) Find all upper bounds of $\{2, 9\}$.
iv) Find all lower bounds of $\{60, 72\}$.

b) Define sequence. Let the sum of the geometric series $a, ar, ar^2, \ldots, ar^k$ is given by $S = \sum_{i=0}^{k} ar^i$, where $a, r \in R$. Reduce the closed form of S.

c) State the following principles with example
i) Pigeonhole principle.
ii) Inclusion-Exclusion Principle.

d) Suppose that the number of bacteria in a colony triples every hour.
i) Set up a recurrence relation for the number of bacteria after n hours have elapsed.
ii) If 100 bacteria are used to begin a new colony, how many bacteria will be in the colony in 10 hours?

8. a) Use generating functions to determine the number of different ways 15 identical stuffed animals can be given to six children so that each child receives at least one but no more than three stuffed animals.

b) Find an explicit formula for the Fibonacci numbers.

c) Define algebraic system. Discuss the properties of operations in an algebraic system.

d) What are the conditions that an algebraic system to be a ring? Give example.

CSE-2K17

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Answer to the Question no: 1

1(a) A contradiction is a proposition which is always false.

$[(p \vee q) \wedge (p \to r) \wedge (q \to r)$

$= ((p \vee q) \wedge (\neg p \vee r) \wedge (\neg q \vee r))$

$= \{(p \vee q \wedge \neg p) \vee (p \vee q \wedge r)\} \wedge (\neg q \vee r)$

$= \{(p \vee q \vee \neg p) \wedge (\neg q \vee r)\} \vee \{(p \vee q \wedge r) \wedge (\neg q \vee r)\}$

$= \{r\} \vee \{r\}$

$= r$

This is a tautology; not a contradiction

1(b). Quantifiers express the meaning of words all and some. The following example:

(i) $\neg \exists x\, Q(x) \longleftrightarrow \forall x\, \neg Q(x)$ where $Q(x)$ is x has taken a course in Java

That means,

There doesn't exist a student who has taken a course in Java if and only if all students haven't taken a course in Java

(ii) $\neg \forall x\, P(x) \longleftrightarrow \exists x\, \neg P(x)$ where $P(x)$ is x likes football.

(17)

Not all students like football if and only if there exists a student who doesn't like a football.

1(c)

"All lions are fierce"

" Some lions do not drink coffee"

" Some fierce creature do not drink coffee"

Let $P(x)$, $Q(x)$, $R(x)$ be the statement x is a lion, x is fierce, x drinks coffee.

Assumping the domain consists of all creatures express the statements in the argument using quantifiers and ~~P(x)~~

$$\forall x \, (P(x) \longrightarrow Q(x))$$

$$\exists x \, (P(x) \land \neg R(x))$$

$$\exists x \, (Q(x) \land \neg R(x))$$

Answer to the question no-2

(a) What do you mean by recursion? Formulate a recursive solution to the "Tower of Hanoi" problem.

Ans: The process in which a function calls itself directly or indirectly is called recursion. Recursion occurs when a thing is defined in terms of itself or of its type.

Solution to the "Tower of Hanoi" problem:

n = number of total disks

T_n = the minimum number of moves that will transfer n disks from one peg to another.

Here,

$n=1$ - we have 1 disk and 1 move, i.e. $T_1 = 1$

$n=2$ - we have 2 disks and 3 moves: top (smaller) disk from peg 1 to peg 2, remaining (larger) disk from peg 1 to peg 3, the disk from peg 2 (smaller) on the top of the disk (larger) on peg 3 and hence $T_2 = 3$

$n=3$ - (i) transfer top 2 disks as in previous case for $n=2$, we use T_2 moves.

(ii) move remaining (largest) disk to empty peg - we use 1 move.

(iii) bring the 2 disks to the top of the largest disk as in previous case for $n=2$, we use T_2 moves;

together we have: $T_2 + T_2 + 1 = 3+3+1 = 7$ moves

For n, (i) In order to move the bottom disk, we need to move all the $n-1$ disks above it to a empty peg first.

(ii) Then we can move the bottom disk to the remaining empty peg, and

(iii) move the $n-1$ smaller disks back on top of it

Together we have: $T_{n-1} + T_{n-1} + 1 = 2T_{n-1} + 1$ moves.

$$T_n = \begin{cases} 0, & \text{if } n=0; \\ 2T_{n-1}+1 & \text{if } n>0. \end{cases}$$

(b) Using mathematical induction, prove that the sum of the first n odd positive integer is n^2.

Ans:

We have to prove, $1+3+5+7+\cdots+(2n-1)=n^2$ for all $n \in N$.

Basis:

If $n=1$, $(2n-1)=n^2$

$\Rightarrow (2\cdot1-1)=1^2$

$\Rightarrow (2-1)=1^2$

$\therefore 1=1$ ✓ True

Induction:

Assume $n=k$ is true for some $k \in N$

$1+3+5+7+\cdots+(2k-1)=k^2$ for some $k \in N$. —(i)

If $n=k+1$, show $n=k+1$ is also true.

~~$1+3+5+7+\cdots+(2(k+1)-1)=(k+1)^2$~~

$$\underbrace{1+3+5+7+\cdots+(2k-1)}+(2(k+1)-1)=(k+1)^2$$

$k^2+(2k+2-1)=(k+1)^2$ [from (i)]

$\Rightarrow k^2+2k+1=(k+1)^2$

$\therefore (k+1)^2=(k+1)^2$ ✓ True

So, since the statement is true for $n=1$ and truth for $n=k$ implied that $n=k+1$ is also true, then it follows by induction that it is true for all $n \in N$.

(c) Elaborate the rules of inference for propositional logic.

Ans:

Modus Ponens:

$p \to q$
p
$\therefore q$

corresponding tautology:
$(p \land (p \to q)) \to q$

Here, as p implies q and p is asserted to be true, therefore q must be true.

Modus Tollens:

$p \to q$
$\neg q$
$\therefore \neg p$

corresponding tautology:
$(\neg q \land (p \to q)) \to \neg p$

This is a valid argument form and it asserts that the inference from p implies q to the negation of q implies the negation of p is valid.

Hypothetical Syllogism:

$p \to q$
$q \to r$
$\therefore p \to r$

corresponding tautology:
$((p \to q) \land (q \to r)) \to (p \to r)$

Here, if p implies q and q implies r, then, p must imply r.

Disjunctive Syllogism:

$p \lor q$
$\neg p$
$\therefore q$

corresponding tautology
$(\neg p \land (p \lor q)) \to q$

Here, if p is true or q is true and p is false, then q is true.

Addition:

p
$\therefore p \lor q$

corresponding tautology
$p \to (p \lor q)$

It is the inference that if p is true, the p or q must be true.

Simplification:

$$\begin{array}{l} p \wedge q \\ \hline \therefore q \end{array}$$

corresponding tautology:

$(p \wedge q) \rightarrow p$

It says here that, if the conjunction p and q is true, then p is true and q is true.

Conjunction:

$$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}$$

corresponding tautology:

$((p) \wedge (q)) \rightarrow (p \wedge q)$

If the proposition p is true, and proposition q is true, then the logical conjunction of the two proposition p and q is true.

Resolution:

$$\begin{array}{l} \neg p \vee r \\ p \vee q \\ \hline \therefore q \vee r \end{array}$$

corresponding tautology:

$((\neg p \vee r) \wedge (p \vee q)) \rightarrow (q \vee r)$

Suppose p is ~~false~~ true. In order for the premise $\neg p \vee r$ to be true, r must be true. Again, suppose p is false. In order for the premise $p \vee q$ to be true, q must be true. Therefore regardless of falsehood or veracity of p, if both premises hold, then the conclusion $q \vee r$ is true.

(d) Provide the examples of "Converse" and "Contrapositive" statements. (52)

Ans:

Example: "It raining is a sufficient condition for my not going to town"

The converse of this statement:

"If I do not go to town, then it is raining."

The contrapositive of this statement:

"If I go to town, then it is not raining."

3) ⓐ Let n and d pe be positive integers. How many positive integers not exceeding n are divisible by d?

Solve:-
The positive ~~number~~ integer numbers divisible by d ~~are~~ all the integers of the form dk. where k is a positive integers. Hence The number of positive integers divisible by d that ~~do~~ not exceeding 'n' equals to n ~~or less~~ ~~then~~ the number of ~~k~~. integer k.

$$\text{Here } 0 < dk \leq n$$

$$0 < k \leq n/d$$

$$\boxed{\therefore k = \lfloor n/d \rfloor}$$

k is the number of positive integers divisible by d not exceeding 'n'.

3 (b) If a and b are positive integers, then prove that $\gcd(a,b) \times \text{lcm}(a,b) = a \times b$, where the symbols have their usual meaning.

Ans:

Let $c = \gcd(a,b)$

then $a = c x$ for some x, and $b = c y$

therefore $\text{lcm}(a,b) = c x y$ (since x and y are coprime)

$\text{lcm}(a,b) \times \gcd(a,b) = c \times c \times x y = (c \times x) \times (c \times y)$

$= a \times b$

$\therefore \text{lcm}(a,b) \times \gcd(a,b) = a \times b$

[Proved]

3(c) Using caesar cipher method construct a secret message for "MEET YOU IN THE CLASSROOM". (55)

Answer: First replace the letters in the message with numbers. This produces

12 4 4 19 24 14 20 8 13 19 7 4 2 11 0 18 18 17 14 14 12

Now replace each of these numbers p by $f(p) = (p+3) \bmod 26$. This gives

15 7 7 22 1 17 23 11 16 22 10 7 5 14 3 21 21 20 17 17 15

Translating this back to letters produces the encrypted message: "PHHW BRX LQ WKH FODVVURRP".

To recover the original message from a secret message encrypted by the caesar cipher, the function f^{-1} the inverse of f, is used. Note that the function f^{-1} sends and integer p from $\{0, 1, 2, \ldots 25\}$ to $f^{-1}(p) = (p-3) \bmod 26$. In other words to find the original message each letter is shifted back letters in the alphabet with the first three letters sent to the last three letters of the alphabet, the process of determining the original message from the encrypted message is called ~~alphabet~~ decryption.

There are various ways to generalize the caesar cipher. For example insted of shifting each letter by 3. we can shift each letter by k so that

$f(p) = (p+k)$ ~~mod~~ $\bmod 26$

Such a cipher is called a shift cipher. Note that decryption can be carried out using

$f^{-1}(p) = (p-k) \bmod 26$

· Obviously Caesar's method and shift ciphers do not provide a high level of security. There are various ways to enhance this method. One approach that slightly enhances the security is to use a function of the form

$$f(p) = (ap+b) \bmod 26$$

where a and b are integers chosen such that f is a bijection. This provides a number of possible encryption system.

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3(d) Ques: There is a number which is divided by 3 the reminder is 2; when divided by 5, the remainder is 3; when divided by 7, the remainder is 2, what is the number ?

Answer:

let, $m = 3 \cdot 5 \cdot 7 = 105$

$M_1 = m/3 = 35$, $M_2 = m/5 = 21$ and $M_3 = m/7 = 15$

We see that 2 is an inverse of $M_1 = 35$ modulo 3. because $35 \equiv 2 \pmod 3$; 1 is an inverse of $M_2 = 21$ Modulo 5, because $21 \equiv 1 \pmod 5$; and 1 is an inverse of $M_3 = 15 \pmod 7$ because $15 \equiv 1 \pmod 7$; The solutions to this system are those x such that

$$x \equiv a_1 M_1 y_1 + a_2 M_2 y_2 + a_3 M_3 y_3$$
$$= 2 \cdot 35 \cdot 2 + 3 \cdot 21 \cdot 1 + 2 \cdot 15 \cdot 1$$
$$= 233$$
$$= 23 \pmod{105}$$

It follows that 23 is the smallest positive integer number that is a simultaneous solution. We conclude that 23 is that number.

Ans. to the que. no. ⇒ 4

(a) A tree is a connected undirected graph with no simple circuits.

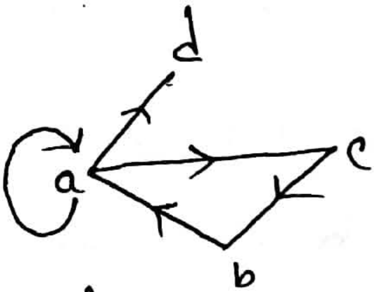
A tree with n vertices has n-1 edges.
The prove is given below:
Basic Step: When n=1, a tree with n=1 vertex has no edges. It follows that the theorem is true for n=1.
Inductive step: The induction hypothesis states that every tree with k vertices has k-1 edges, where k is a positive integer. Suppose that a tree T has k+1 vertices and that is a leaf of T (which must exist because the tree is finite), and let w be the parent of. Removing from T the vertex and the edge connecting w to produces a tree T' with k vertices, because the resulting graph is still connected and has no simple circuits. By the ~~edge~~ induction hypothesis, T' has k-1 edges. It follows that T has k edges because it has

one more edge than T, the edge connecting and w. This completes the induction step.

The number of vertices in a full m-ary tree with a specified number of internal vertices is determined, as

(b) A Graph $G = (V, E)$ consists of V, a non-empty sets of vertices (or nodes) and E, a set of edges. Each edge has either one or two vertices associated with it, called its endpoints. An edge is said to connect its end points.

Let a grap be,



$$2e = \sum \deg(v) + \sum \deg(v)$$

where, $v \in V_1$ and $V \in V_2$

$\deg^+(a) = 4$, $\deg^-(a) = 1$,

(c) Different methods to represent the graph is –

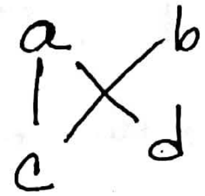
(I) Adjacency lists.

(II) Adjacency matrix

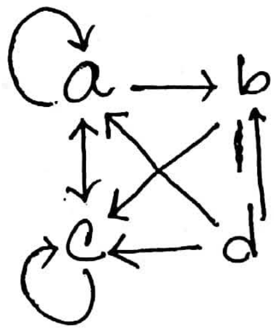
(III) Incident matrix.

Graphs are given below for the following adjacency matrices:

(i)
$$\begin{array}{c|c} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array} \Rightarrow$$



(ii)
$$\begin{array}{c|c} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array} \Rightarrow$$



5) a) Justify the statement "Every function is a relation but not every ~~f~~ relation is function"

Solve:- //

Function:-

Let 'A' and 'B' be non empty sets, A function 'f' from 'A' to 'B' is an assignment of exactly one element of 'B' to each element of 'A'

Relation:-

Let 'A' and 'B' be sets. A binary relation from 'A' to 'B' is a subset of $A \times B$.

Now, suppose, $A = \{1, 2, 3\}$

$B = \{1, 4, 9\}$

$A \times B = \{(1,1), (1,4), (1,9), (2,1), (2,4), (2,9)$
$(3,1), (3,4), (3,9)\}$

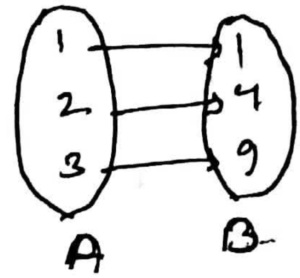
$R_1 = \{(1,1), (2,4), (3,9)\}$

$R_2 = \{(1,1), (1,4), (2,4), (3,9)\}$

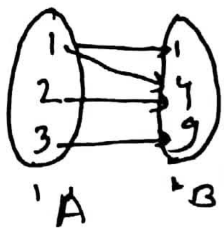
Here $R_1 \in A \times B$ and $R_2 \in A \times B$. So, ~~Bo~~ both are relation.

But here only R_1 is a function. Because

In R_1 we can see that every element of 'A' has only one relation with elements of 'B'. That's wy R_1 is a function and a Relation at the same time.



But in R_2 we see that '1' has relation with two element of B '1' and '4' so it is not a function. But as R_2 is a subset of $A \times B$, so it is a Relation.



So, we can say function doesn't allow more than one relation of any element of set 'A' with elements of set 'B'. ~~More over,~~ Moreover, every function is subset of $(A \times B)$. So, every function is relation. But every relation dosen't fulfil conditions of function.

So the statement is true.

5) b) Explain graph of function with example. How can it be used to distinguish various types of function?

Solve:-

~~Function of graph~~:-

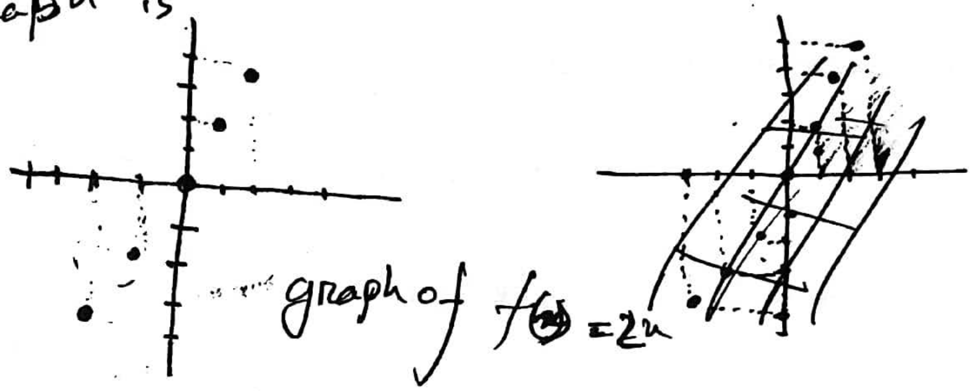
Graph of function:-

Let f be a function from the set 'A' to 'B'. The graph of the function 'f' is the set of ordered pairs

$$\{(a,b) \mid a \in A \text{ and } f(a) = b\}$$

From the defination, the graph of a function ~~is~~ f' from A to B is the subset of $A \times B$ containing the ordered pair with the second entry equal to the element of 'B' ~~assighe~~ assigned by 'f' to the first entry.

Suppose $f(x) = 2x$ is a function where ~~xx is~~ (x, y) belongs to integer number. The graph of 'f' is the set of $(x, 2x)$ when x is an integer number. And the graph is



5) c) Define equivalance relation. Considers a relation $R = \{(a,b) \in R \times R \mid a-b$ is an integer$\}$. Is R a equivalence relation?

Solve: Equivalence relation:-

A relation on a set A is called an equivalence relation if it is reflective, symmetric and transitive at the same time.

A relation R on a set A,

is reflexive if $\forall a \in A, (a,a) \in R$

is symmetric if $\forall (a,b) \in A, (a,b) \in R$ then $(b,a) \in R$

is transitive if $\forall (a,b) \in A, (a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$

If a Relation 'R' has all those properties we can call 'R' an equivalence relation

Now, $R = \{(a,b) \in R \times R \mid a-b$ is an integer$\}$.

R is a binary relation such that $(a-b)$ is an integer.

Reflexive :-

For all real numbers a, $(a-a)=0$ and '0' is an integer. which mean aRa for all real numbers a. Hence R is a reflexive relation.

Symmetric :-

Now suppose ~~a relat~~ aRb

Then $(a-b)$ is an integer and so $(b-a)$ is also an integer. Hence bRa.

It proves, R is a symmetric relation.

Transitive :-

If aRb and bRc then $(a-b)$ and $(b-c)$ are integers, Now $(a-c)=(a-b)+(b-c)$ is also an integer number.

So aRc. It proves that R is transitive.

Consequently, R is an equivalence relation.

Answer to the Question No.06

(a) There are 2^{n^2} relations on a set with n elements.

(b) $A = \{1, 2, 3, 4\}$ Relations

$R = \{ (a,b) \mid a \text{ divides } b \}$

Symmetric:

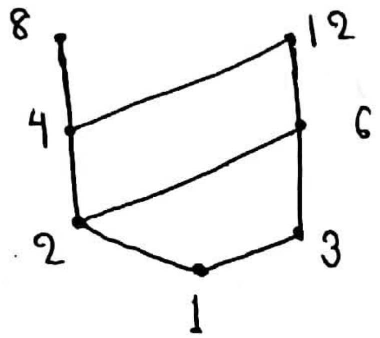
~~When~~ If (a,b) exists then (b, a) will also exist.

(c) $R = \{(1,1), (1,4), (2,3), (3,1), (3,4)\}$

$S = \{(1,0), (2,0), (3,1), (3,2), (4,1)\}$

Composite of R and $S = \{(1,0), (1,1), (2,1), (2,2), (3,0), (3,1)\}$

(d) Hasse Diagram for the poset $(\{1,2,3,4,6,8,12\}, 1)$



In the hasse diagram,

~~The upper bounds of (8, 12) are (8, 12)~~

~~The lower boun~~

The pair (8, 12) does not have a least upper bound as 8 and 12 both are at the same stage of the hasse diagram.

∴ The poset is not a lattice.

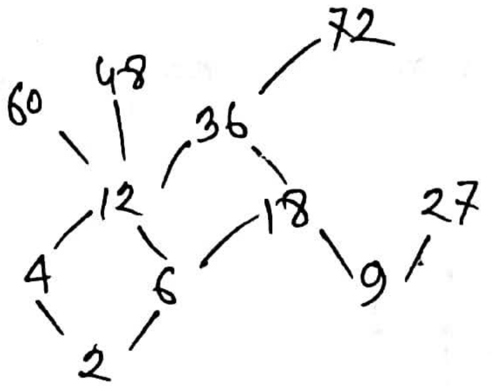
7. a) Answer the following questions concerning the poset $(\{2,4,6,9,12,18,27,36,48,60,72\}, \mid)$

i) Find the maximal elements

ii) Find the minimal elements

iii) Find all upper bounds of $\{2,9\}$

iv) Find all lower bounds of $\{60,72\}$



i) the maximal elements — 27, 48, 60, 72

ii) the minimal elements – 2, 9

iii) upper bounds of $\{2,9\}$ — 18, 36, 72

iv) lower bounds of $\{60,72\}$ – 2, 4, 6, 12

8.(a)

Generating function $= (x+x^2+x^3)^8 = G(x)$

Need the co-efficient of x^{15}

$G(x) = [x+x^2+x^3]^6$

$= x^6 (1+x+x^2)^6$

$= x^6 \left(\frac{1-x^3}{1-x}\right)^6$

$= x^6 (1-x^3)^6 (1-x)^{-6}$

$= x^6 \left(1-6x^3+\frac{6.5}{2}x^6-\frac{6.5.4}{6}x^9+\frac{6.5.4.3}{24}x^{12}-\dots\right)$

$\left(1+6x+\frac{6.7x^2}{2}+\frac{6.7.8}{6}x^3+\frac{6.7.8.9}{24}x^4+\dots\right)$

co-efficient of $x^{15} = (-6).\frac{6.7.8.9.10.11}{720} + 1.\frac{\frac{6.7.8.9.10.11.12.13.14}{}}{9!}$

$+\frac{6.5}{2}.\frac{6.7.8}{6} - \frac{6.5.4}{6}$

$= -2772 + 2002 + 840 - 20$

$= 50$

$\therefore$ there are 50 possible ways.

(b) Recall that the sequence of Fibonacci numbers satisfies the recurrence $f_n = f_{n-1} + f_{n-2}$ and also satisfies the initial conditions $f_0 = 0$ and $f_1 = 1$, the roots of the characteristic equation $r^2 - r - 1 = 0$ are $r_1 = (1+\sqrt{5})/2$ and $r_2 = (1-\sqrt{5})/2$, therefore, from theorem 1 it follows that the fibonacci numbers are given by

$$f_n = \alpha_1\left(\frac{1+\sqrt{5}}{2}\right)^n + \alpha_2\left(\frac{1-\sqrt{5}}{2}\right)^n$$

for some constants α_1 and α_2. The initial conditions $f_0 = 0$ and $f_1 = 1$ can be used to find these constants.

we have

$$f_0 = \alpha_1 + \alpha_2 = 0,$$

$$f_1 = \alpha_1\left(\frac{1+\sqrt{5}}{2}\right) + \alpha_2\left(\frac{1-\sqrt{5}}{2}\right) = 1.$$

The solution to ~~these~~ these simultaneous equations for α_1 and α_2 is

$$\alpha_1 = 1/\sqrt{5}, \quad \alpha_2 = -1/\sqrt{5}$$

consequently the fibonacci numbers are given by

$$f_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$$

~~Theorem~~

(e) Algebraic System:

An Algebraic system is a nonempty set S in which at least one or more operations $Q_1, \ldots, Q_K$ $(K \geq 1)$, are defined. We denoted by $[S; Q_1, \ldots, Q_K]$.

Properties of Operation:

1. Associative law: Let $\#$ be a binary operation on a set S. $a \# (b \# c) = (a \# b) \# c$ for $\forall a, b, c \in S$.

2. Commutative law: Let $\#$ be a binary operation on a set S. $a \# b = b \# a$ for $\forall a, b, \in S$.

3. Identity ~~law~~ element: Let $\#$ be a binary operation on a set S. An element e of S is an identity element if $a \# e = e \# a = a$ for all $a \in S$.

4. Inverse element: Let $\#$ be a binary operation on set S with Identity element e. Let $a \in S$

then b is an inverse of a if $a \# b = b \# a = e$

Cancelation Law: An operation $\#$ on a set S is said to satisfy left cancelation law if $a \# b = a \# c$ implies $b = c$ and right cancelation law if $b \# a = c \# a$ implies $b = c$.

Distributive Law: Let $\#$ and $\bullet$ be two binary operations on non empty S. For $\forall a, b, c \in S$,

$$a \bullet (b \# c) = (a \bullet b) \# (a \bullet c)$$

$$(b \# c) \bullet a = (b \bullet a) \# (c \bullet a)$$

8.(d) what are the conditions that an algebraic system to be a ring? Give example

The conditions that an algebraic system to be a ring are given below :

☐ R is called a commutative ring if $ab = ba$ for every $a, b \in R$

☐ R is called a ring with an identity element 1 if $a \cdot 1 = 1 \cdot a = a$ for every $a \in R$,

☐ R is called a unit with an identity element 1 if $a \cdot a^{-1} = a^{-1} \cdot a = 1$ for every $a \in R$.

CSE-(1) 48

KHULNA UNIVERSITY OF ENGINEERING & TECHNOLOGY

B.Sc. Engineering 1st Year 1st Term Examination, 2016

Department of Computer Science and Engineering

CSE 1107

Discrete Mathematics

TIME: 3 hours FULL MARKS: 210

N.B. i) Answer **ANY THREE** questions from each section in separate scripts.
ii) Figures in the right margin indicate full marks.

SECTION A

(Answer **ANY THREE** questions from this section in Script A)

1. a) Define "Contrapositive", "Converse" and "Inverse" propositions using example(s). (09)
b) Find a proposition that is equivalent to $p \to q$, but that uses only the connectives $\neg$, $\wedge$, and $\vee$. Hence, prove its validity. (08)
c) What are logical quantifiers? Give the physical example(s) for the following relations: (10)
(i) $\neg\exists x P(x) \Leftrightarrow \forall x \neg P(x)$ (ii) $\neg\forall x Q(x) \Leftrightarrow \exists x \neg Q(x)$
d) Suppose that we have an array of size 6×6. Are these statements equivalent? (08)
$\forall$ row x $\exists$ column y $A(x,y) = 1$
$\exists$ row x $\forall$ column y $A(x,y) = 1$
Justify your answer.

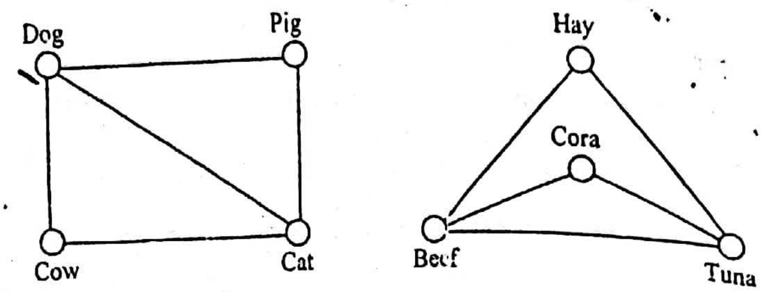
2. a) What do you mean by "Rules of Inference" in mathematical reasoning? List the rules of inferences for mathematical reasoning. (10)
b) Express the following statements in the logical arguments using quantifiers, connectives and corresponding predicates. (10)
"All lions are fierce"
"Some lions do not drink coffee"
"Some fierce creatures do not drink coffee".
c) What is linear congruential method? Use this method to find pseudo random numbers within the limit 0 to 12. (10)
d) Let a be an integer and d a positive integer. Then prove that there are unique integers q and r, with $0 \le r \le d$, such that $a = dq + r$. (05)

3. a) Explain the idea of induction method using example. (10)
b) Use induction method to find an ordering of all the n-bit strings in such a way that two consecutive n-bit strings differed by only one bit. (10)
c) Provide a recursive definition of the sequence $\{a_n\}$, $n = 1, 2, 3, \ldots..$, if (i) $a_n = 3n + 1$ (ii) $a_n = 2^n$. (10)
d) The Fibonacci numbers $f_0, f_1, f_2, \ldots..$ are defined by the equations $f(0) = 0$, $f(1) = 1$ and $f(n) = f(n-1) + f(n-2)$ for $n = 2, 3, 4, \ldots..$ Draw the recursive Fibonacci evaluation tree for $f(5)$. (05)

4. a) What is a graph? Let G = (V, E) be a graph with directed edges. Then prove that $\sum_{v \in V} \deg^-(v) = \sum_{v \in V} \deg^+(v) = |E|$. (10)
b) In a round-robin tournament, each team plays against each other team exactly once. Suppose that Alpha, Beta, Gamma and Theta teams are registered for the tournament. Represent the result of the tournament (which team beats which other team) using a proper graph. Hence, find the in-degrees and out-degrees of the teams. (08)
c) Name the different methods to represent graphs. Draw the graph for the following adjacency matrices: (10)

(i) $A_G = \begin{bmatrix} 0 & 1 & 1 & 2 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 3 \\ 2 & 1 & 3 & 0 \end{bmatrix}$ (ii) $A_G = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

d) What are the necessary and sufficient conditions for two graphs to be isomorphic? Are the following two graphs isomorphic? Explain. (07)



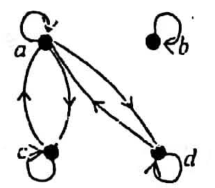
SECTION B

(Answer ANY THREE questions from this section in Script B)

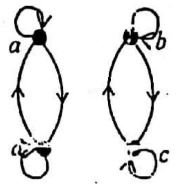
5. a) What is a tree? Prove that a tree with n vertices has $n-1$ edges? (08)
   b) What is power set? Prove De-Morgan's law by membership table. (05)
   c) Let "f" is a function from Z^+ to R such that $f(x)=\lfloor\sqrt{x}\rfloor$. What is the domain and co-domain of "f"? Determine whether "f" is injective or surjective. Explain. (10)
   d) Prove that if x is real number then $\lfloor 2x\rfloor=\lfloor x\rfloor+\lfloor x+\frac{1}{2}\rfloor$. (12)

6. a) Let $A=\{1, 2, 3, 4\}$ and R is a relation on A such that $R=\{(a, b) \mid a \text{ divides } b\}$, show the different representations of R. (10)
   b) What is equivalence relation? Determine whether the relation with the directed graphs shown as follows is an equivalence relation. Explain. (12)

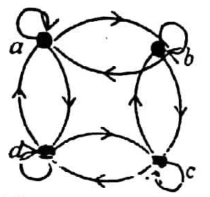
(i)



(ii)



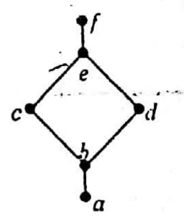
(iii)



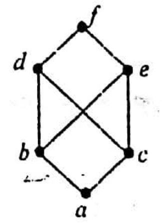
   c) Using proper steps draw a Hasse diagram of the poset $(\{2, 4, 5, 10, 12, 20, 25\}, \mid)$ and find the followings: (13)
      i) Maximal elements ii) Minimal elements
      iii) Greatest and least element iv) Upper and lower bound of $\{4, 10\}$

7. a) Define Lattice. Determine whether the posets with the following Hasse diagrams are Lattices. Explain. (10)

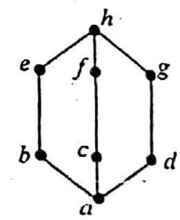
(i)



(ii)



(iii)



   b) Define sequence. Sum of the geometric series $a, ar, ar^2, \ldots, ar^k$ is given by $S=\sum_{i=0}^{k} ar^i$, where (12) $a, r \in R$. Reduce the closed form of S.
   c) Define product rule. How many bit strings of length eight either start with a 1 or end with the two bits 00? (08)
   d) Draw the tree diagram for finding the number of bit strings of length 4 that do not have two consecutive 0's. (05)

8. a) What is algebraic system? What are the conditions needed for an algebraic system to be a group? Give example. (09)
   b) Suppose that a person deposits \$10,000 in a savings account at a bank yielding 12% per year with interest compounded annually. How much will be in the account after 40 years? (08)
   c) Find the solution of the recurrence relation $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$, with the initial conditions $a_0=2$, $a_1=5$ and $a_2=15$. (10)
   d) Using Generating function determine the number of different ways eight identical balloons can be distributed among three distinct children if each child receives at least two balloons and no more than four balloons. (08)

Ans. No. 1

(a) Contrapositive: The contrapositive of $p \to q$ is $\neg q \to \neg p$.

Example: Consider a statement–

"Raining is a sufficient condition for my not going to town".

The contrapositive of the statement is –

"If I go to town, then it is not raining".

Converse: The converse of $p \to q$ is $q \to p$.

Example: The converse of the same statement is–

"If I do not go to town, then it is raining".

Inverse: The inverse of $p \to q$ is $\neg p \to \neg q$.

Example: The inverse of the same statement is–

"If it is not raining, then I will go to town".

(b) Two propositions are equivalent if they always have the same truth table.

$\neg(\neg q \wedge (p \vee q))$
~~$\neg p \vee q$~~ is a proposition that is equivalent to $p \rightarrow q$.

Let's prove it's validity by showing the truth table -

p	q	$\neg q$	$p \vee q$	$\neg q \wedge (p \vee q)$	$p \rightarrow q$	$\neg(\neg q \wedge (p \vee q))$
T	T	F	T	F	T	T
T	F	T	T	T	F	F
F	T	F	T	F	T	T
F	F	T	F	F	T	T

Same truth value

As, both of them have the same truth value always, so we can say that $\neg(\neg q \wedge (p \vee q))$ is a proposition that is equivalent to $p \rightarrow q$. (Proved)

(c) Logical quantifiers are applied to values in a given domain U that express the meaning of the words "all" and "some".

① $\neg \exists x P(x) \leftrightarrow \forall x \neg P(x)$

Consider - (i) the domain is students in this class
(ii) $P(x)$ is "x has taken a course in Java".

if we consider $\exists x P(x)$ firstly that denotes - "There is a student in this class who has taken a course in Java".
Then, negating the statement: "It is not the case that there is a student in this class who has taken a course in Java".
It will be expressed as $\neg \exists x P(x)$.

This implies that: "No student in this class has taken Java" or "Every student in this class has not taken a course in Java".

So, we can express it as $\forall x \neg P(x)$.

2. $\neg \exists x P(x) \leftrightarrow \forall x \neg P(x)$ and the example is - "It is not the case that there is a student in this class who has taken a course in Java" if and only if every student in this class has not taken a course in Java".

(iii) $\neg \forall x Q(x) \leftrightarrow \exists x \neg Q(x)$

consider - (i) the domain is students in this class

(ii) $Q(x)$ is "x has taken a course in Python".

if we consider $\forall x Q(x)$ firstly that denotes - "Every student in this class has taken a course in Python".

Then, $\neg \forall x Q(x)$ denotes that "It is not the case that every student in this class has taken a course in Python" (negating the statement)

This implies that ; "There is a student in this class who has not taken a course in Python"

we can express it as $\exists x \neg Q(x)$.

$\therefore \neg \forall x Q(x) \leftrightarrow \exists x \neg Q(x)$ and the example is - "It is not the case that every student in this class has taken a course in Python if and only if there is a student in this class who has not taken a course in Python".

(d) $\rightarrow$ Not Solved.

Ans. to the Q. NO-02

(a)

Rules of ~~Inter~~ference:

Proofs in mathematics are valid arguments that establish the truth of mathematical statements. By an argument, we mean a sequence of statements that end with a conclusion. By valid, we mean that the conclusion, or final statement of the argument, must follow from the truth of the preceding statements or premises, of the argument. That is, an argument is valid if and only if it is impossible for all the premises to be true and the conclusion to be false. To deduce new statements from statements we already have, we use rules of inference which are templates for constructing valid arguments. Rules of inference are our basic tools for establishing the truth of statements.

The rules of inference:

1) Modus Ponens:

$p \to q$
p
___
$\therefore q$

corresponding Tautology:

$(p \land (p \to q)) \to q$

2) Modus Tollens:

corresponding Tautology:

$p \to q$
$\neg q$
___
$\therefore \neg p$

$(\neg q \land (p \to q)) \to \neg p$

3) Hypothetical Syllogism:

$p \to q$
$q \to r$
___
$\therefore p \to r$

corresponding Tautology

$((p \to q) \land (q \to r)) \to (p \to r)$

4) Disjunctive Syllogism:

$p \vee q$

$\neg p$

$\therefore q$

corresponding Tautology

$(\neg p \wedge (p \vee q)) \to q$

5) Addition:

corresponding Tautology

p

$\therefore p \vee q$

$p \to (p \vee q)$

6) Simplification:

corresponding tautology

$p \wedge q$

$\therefore q$

$(p \wedge q) \to p$

7) Conjunction:

p

q

$\therefore p \wedge q$

corresponding tautology

$((p) \wedge (q)) \to (p \wedge q)$

8) Resolution:

$\neg p \vee r$

$p \vee q$

$\therefore q \vee r$

corresponding tautology

$((\neg p \vee r) \wedge (p \vee q)) \to (q \vee r)$

(b)

Let $P(x)$, $Q(x)$ and $R(x)$ be the statements "x is a lion," "x is fierce," and "x drinks coffee," respectively.

We can express the statements as:

$\forall x \, (P(x) \rightarrow Q(x))$

$\exists x \, (P(x) \land \neg R(x))$

$\exists x \, (Q(x) \land \neg R(x))$

(c)

Linear congruential method:

A linear congruential generator is an algorithm that yields a sequence of pseudorandomized numbers calculated with a discontinous piecewise linear equation. The method represents one of the oldest and best-known pseudorandom number generator algorithms. The theory behind them is relatively easy to understand, and they are easily implemonted and fast, especially on computer hardware which can provide modulo arithmetic by storage-bit truncation.

3) a) Mathematical induction can be used to prove statements that assert that P(n) is true for all possible integer n, where P(n) is a propositional function. To prove that P(n) is true for all positive integer n, where P(n) is a propositional function, we complete two steps:

Basic step: We verify that P(1) is true.

inductive step: We show that the conditional statement $P(k) \to P(k+1)$ is true for all positive integers k.

To prove it, we ~~assume~~ show that P(1) is true. Then we know that P(2) is true, because P(1) implies P(2). Further, we know that P(3) is true, because P(2) implies P(3), Continuing along these lines, we see that P(n) is true for every positive integer n.

b) recursion is a principle closely related to mathematical induction.

(i) $a_n = 3n+1$. $\therefore a_0 = 1$; $a_1 = 4$; $a_2 = 7$, $a_3 = 10$.
$a_4 = 13$.

The sequence can be defined recursively,

$a_0 = 1$,
$a_n = a_{n-1} + 3$ where $n = 1, 2, 3 \ldots$

$a_{n+1} = a_n + 3$.

(ii) $a_n = 2^n$; $a_0 = 1$; $a_1 = 2$, $a_2 = 4$, $a_3 = 8$, $a_4 = 16 \ldots$

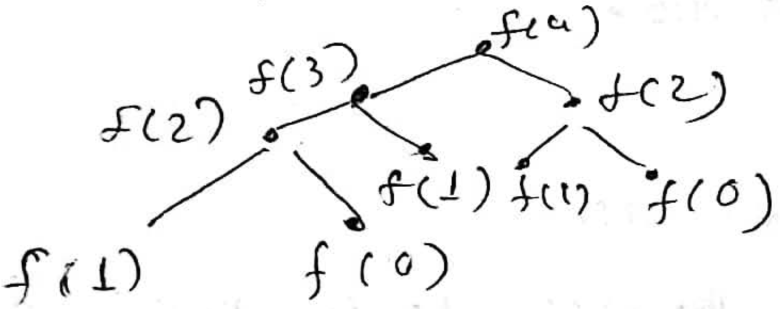
$\therefore a_0 = 1$; $a_{n+1} = 2a_n$.

(1) $f(0) = 0,\ f(1) = 1;\quad f(n) = f(n-1) + f(n-2)$

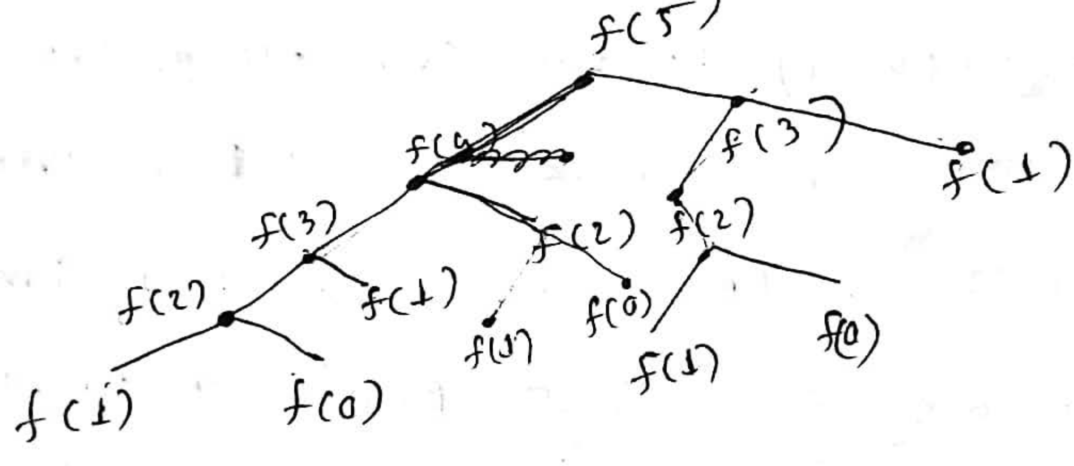
$f(2) = f(1) + f(0) = 1 + 0 = 1$

$f(3) = f(2) + f(1) = 1 + 1 = 2$

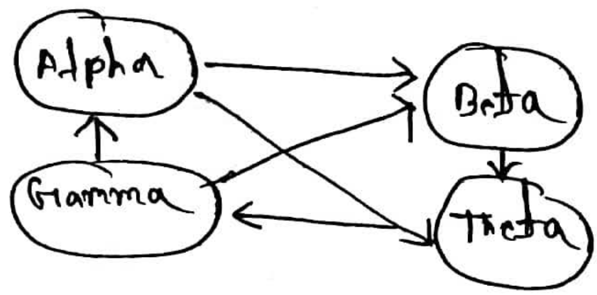
$f(4) = f(3) + f(2) = 2 + 1 = 3$ ....



(b) tree for f(5)



Ans to the Q No-01 (b)



Thin in the graph of the tournament and each team playn with each other according to the graph.

Ans to the Q NO - 01(c)

methods of representing graphs are Edge list, Adjacency matrices, adjacency lists.

Edge lists: One simple way to represent a graph is just a list, an array of |E| edges, which we call an edge list. To represent an edge, we just have an array of two vertex numbers or, an array of objects containing the vertex numbers of the vertices that the edges are incident on.

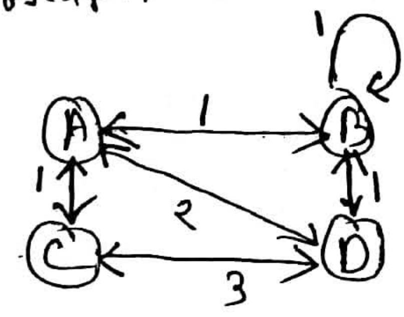
Adjacency matrices:

For a graph with N vertices, an adjacency matrix is a N × N matrix of 0s and 1s where where the entry in row i and column j is 1 if and only if the edge (i,j) is in the graph. If you want to indicate an edge weight, put in the row i, column j entry and reserve a special value (perhaps null) to indicate an absent edge.

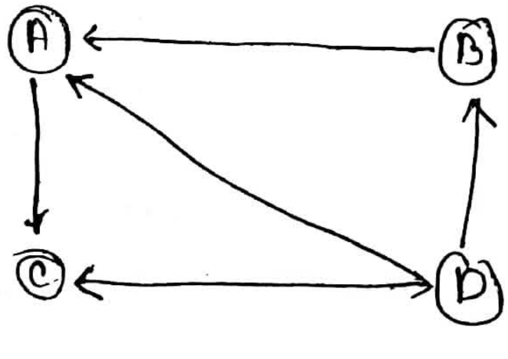
Adjency list:

Representing a graph with adjency lists combines adjancy matrices with edge lists. For each vertex i, store an array of the vertices adjacent to it. We typically have an array of |V| adjaceency lists one adjaceency list per vertex. ~~vertex~~

Graph of the first matrice:



Graph of the 2nd matrice:

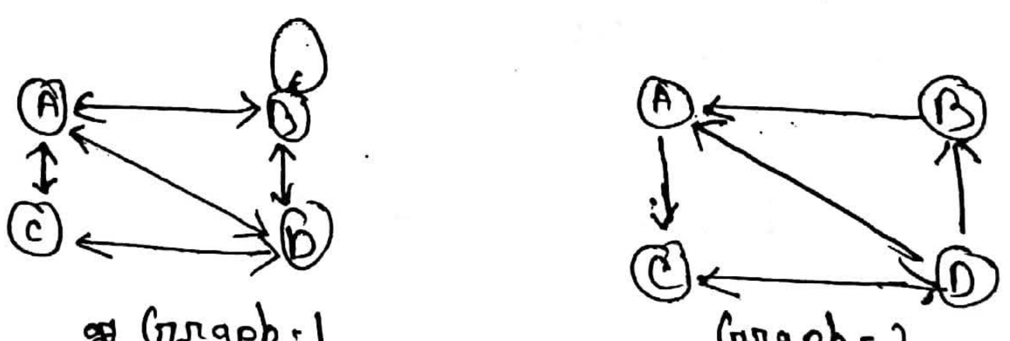


Ans to the Q No - 4(d)

The condition for the two graphs to be isomorphic —

1. They must have same number of vertices
2. Same number of edges.
3. Same degrees of corresponding vertices.
4. The same number of connected components.
5. Same number of loops.
6. Same number of parallel edges.

The graphs were



Graph-1 Graph-2

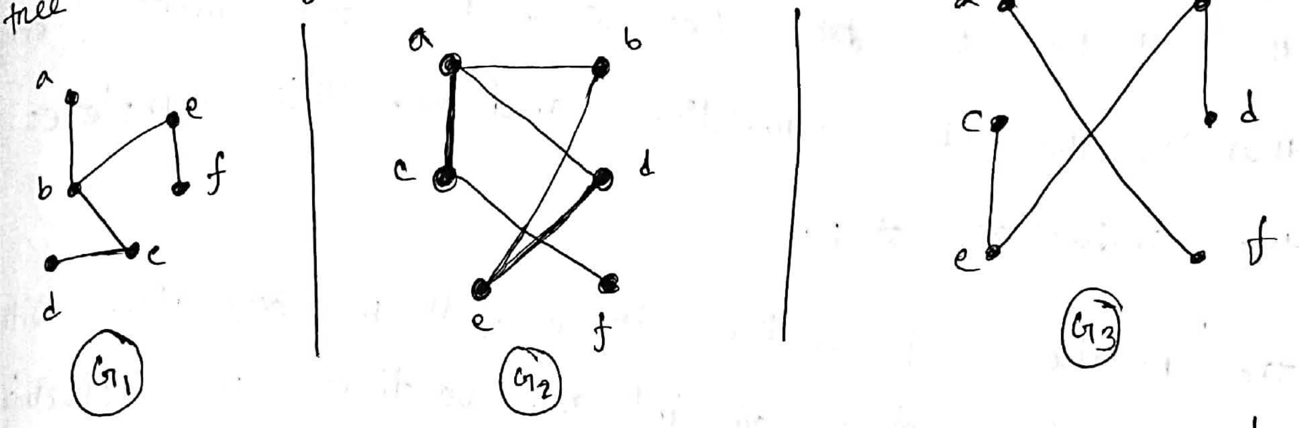
The graphs aren't isomorphic. Though they have the same vertices same edges but the B has a loop which the graph 2 doesn't. That's why the graphs aren't isomorphic.

Discrete Mathematics

CSE - 1107

Year - 2016

5 a) A tree is a connected undirect graph with no simple circuit. Trees can be used to model procedure carried out using a sequence of dicision. An undirect graph is a tree if and only if there is a unique simple path between any two of its vertices. Examples of trees and graphs that are not tree are given below:-



G_1 = Tree, $G_2 \neq$ Tree, as a d e b a is a circuit.

$G_3 \neq$ Tree, as it is not properly connected.

Proof: A tree with n vertices has n-1 edges.

→ Here mathematical induction to prove this theorem.

Note that for all the trees here we can choose a

root and consider the tree rooted.

When $n=1$, a tree with $n=1$ vertex has no edges. It

follows that the theorem is true for $n=1$.

The induction hypotesis states that every tree with k vertices has k-1 edges. where k is a positive integer. Suppose that a tree T has k+1 vertices and that is a leaf of T (which must exist because the tree is finite) and let w be the parent of. Removing from T the vertex and the edge connecting w to produce a tree T' with K vertices, because the resulting graph is still connected and has no simple circuit. By the induction hypothesis, T' has k-1 edges. It follows that T has K edges because it has one more edges than T, the edge conneting and w. This completes the induction step.

The number of vertices in a full m-ary tree with a specified number of internal vertices is determined.

5(b) Let S be a set. Then the power set of S will be the set of all subsets of S. It is denoted by P(S). If $S = \{0,1,2\}$ $\therefore P(S) = \{\{0\},\{1\},\{2\},\{0,1\},\{1,2\},\{0,2\},\{0,$

We know De' Morgan's Law is :-

1. $\overline{A \cup B} = \bar{A} \cap \bar{B}$. 2. $\overline{A \cap B} = \bar{A} \cup \bar{B}$.

The Law has been proved by membership table below:-

A	B	$\bar{A}$	$\bar{B}$	$\overline{A\cup B}$	$\bar{A}\cap\bar{B}$	$\overline{A\cap B}$	$\bar{A}\cup\bar{B}$
0	0	1	1	1	1	1	1
0	1	1	0	0	0	1	1
1	0	0	1	0	0	1	1
1	1	0	0	0	0	0	0

From the above membership table, we find,

1. $\overline{A\cup B} = \bar{A}\cap\bar{B}$. 2. $\overline{A\cap B} = \bar{A}\cup\bar{B}$.

That is what De Morgan's Law is.

5(c) The given function is $f: \mathbb{Z}^+ \to \mathbb{R}$, $f(x) = |\sqrt{x}|$.

Here for the function Domain is $\mathbb{Z}^+$ and co-domain is $\mathbb{R}$.

f is injective. As $\forall x, x \in \mathbb{Z}^+$ we get only one image of each x.

But the range of f is $\mathbb{Z}^+$. And $\mathbb{Z}^+ \subset \mathbb{R}$. So f is not surjective. f would be surjective if and only if $|\mathbb{Z}^+| = |\mathbb{R}|$. But it is not possible.

So, $f: \mathbb{Z}^+ \to \mathbb{R}$ is only injective but not surjective.

5(d) Given x is a real number.

So, let $x = n + \epsilon$, where n is a positive integer and $0 \leq \epsilon \leq 1$. There are two cases to consider, depending on whether ϵ is less than or greater than or equal to $\frac{1}{2}$. (According to the proof).

firstly, consider $0 \leq \epsilon < \frac{1}{2}$.

So, $2x = 2n + 2\epsilon$. $\therefore$ $\lfloor 2x \rfloor = 2n$, [as $0 \leq 2\epsilon \leq 1$.]

similarly, $x + \frac{1}{2} = n + (\frac{1}{2} + \epsilon)$ $\therefore$ $\lfloor x + \frac{1}{2} \rfloor = n$. (as $0 < \frac{1}{2} + \epsilon < 1$).

So, $\lfloor x \rfloor + \lfloor x + \frac{1}{2} \rfloor = n + n = 2n$.

Next, we consider when $\frac{1}{2} \leq \epsilon < 1$.

In this case, $2x = 2n + 2\epsilon = (2n+1) + (2\epsilon - 1)$.

$\therefore$ $\lfloor 2x \rfloor = 2n+1$ (as $0 \leq 2\epsilon - 1 < 1$).

Again, $\lfloor x + \frac{1}{2} \rfloor = \lfloor n + 1 + (\epsilon - \frac{1}{2}) \rfloor$ as $(0 \leq \epsilon \frac{1}{2} < 1)$.

$\therefore$ $\lfloor x + \frac{1}{2} \rfloor = n + 1$.

Consequently, $\lfloor 2x \rfloor = 2n + 1$.

And $\lfloor x \rfloor + \lfloor x + \frac{1}{2} \rfloor = 2n + 1$.

$\therefore$ $\lfloor 2x \rfloor = \lfloor x \rfloor + \lfloor x + \frac{1}{2} \rfloor$.

This satisfies the proof.

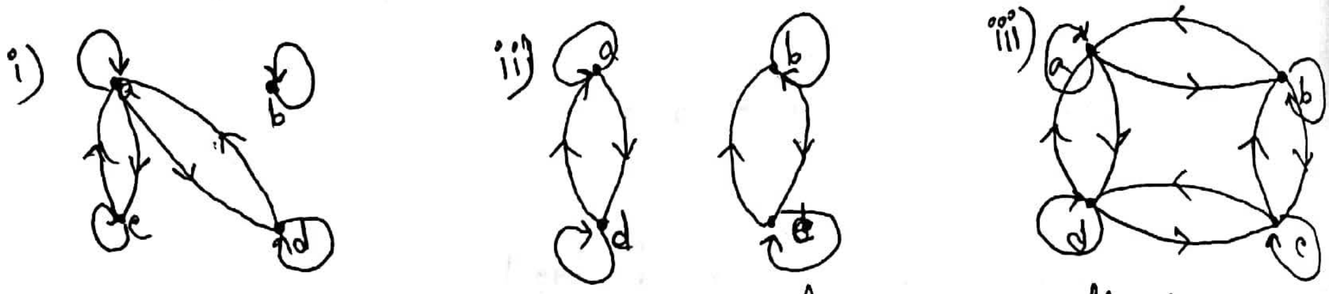
6.(a) Let $A = \{1, 2, 3, 4\}$ and R is a relation on A such that $R = \{(a, b) \mid a \text{ devides } b\}$, show the different representation of R.

Here, $A = \{1, 2, 3, 4\}$

$R = \{(a, b) \mid a \text{ devided } b\}$

$\therefore R = \{(1,1), (2,2), (3,3), (4,4), (1,2), (1,4), (1,3), (2,4)\}$ Ans.

6) What is equivalance relation? Determine whether the relation with the directed graphs shows is an equvalance relation. Explain.



Solution: A relation on set A is called an equivalence relation if it is reflexive, symmetric and transitive.

- A relation will be reflexive if every element of ~~of~~ the relation is related to itself.
- A relation will be symmetric if a is Related to b then b is related to a for all (a, b)
- A relation will be transitive if a is related to b and b is related to c for all (a, b), the a is related to c

(b, c), (a, c).

i) The relation is =

~~{(1, 1), (1, 2)}~~ {(a, a), (b, b), (c, c), (d, d), (a, c), (c, a), (a, d), (d, a)}

The relation is not equivalence relation because for the pair (a,a), (b,b), (c,c), (d,d) the relation is reflexive. Moreover for the pairs (a,c), (c,a) (a,d), (d,a) the relations is symmetric. But the relation is not transitive ~~Be~~ because as there (a,c) and (a,d) so there should be a pair (c,d) for being transitive. But such pair does not remain in the relation. So the relation is not equivalence.

ii) The relation is $= \{(a,a), (b,b), (c,c), (d,d), (a,d), (d,a), (b,c), (c,a)\}$

For the pairs (a,a), (b,b), (c,c), (d,d) the relation is transitive. For pairs (a,d), (d,a), (b,c), (c,d) the relation is symmetric. The relation is also transitive.

So, the relation is an equivalence relation.

iii) The relation is =
{(a, a), (b, b), (c, c), (d, d), (a, b), (b, a), (b, c), (c, b),
(c, d), (d, c), (d, a), (a, d)}

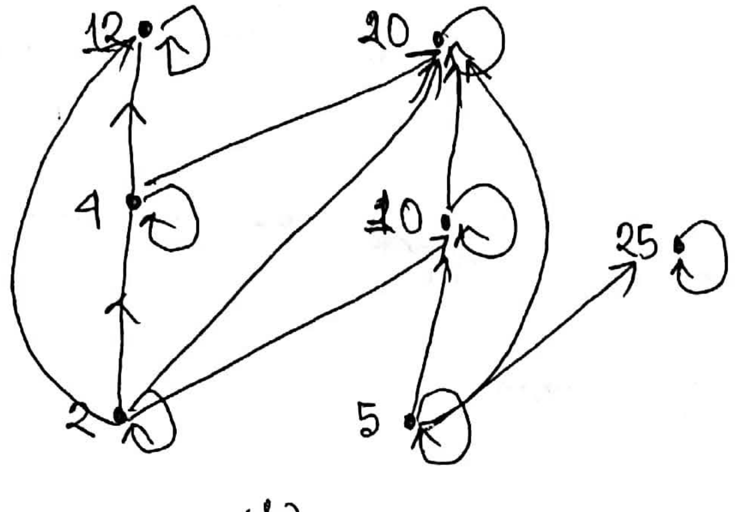
The relation is reflexive for the pairs (a, a), (b, b), (c, c), (d, d). The relation is symmetric for the pairs (a, b), (b, a), (b, c), (c, b), (c, d), (d, c), (d, a), (a, d). The relation is not transitive. As there is (a, b) and (b, c) pairs, so there should be (a, c) pairs for being transitive. but such pair does not remain in the relation. So, the relation is not transitive.

The relation is not an equivalence relation.

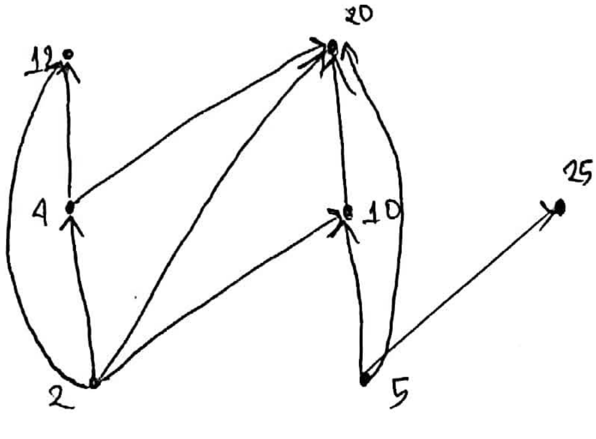
c) Using proper steps draw a Hasse diagram of the poset ({2, 4, 5, 10, 12, 20, 25}, |) and find the following.

i) Maximal elements
ii) Minimal elements
iii) Greatest and least element
iv) Upper and lower bound of (4, 10)

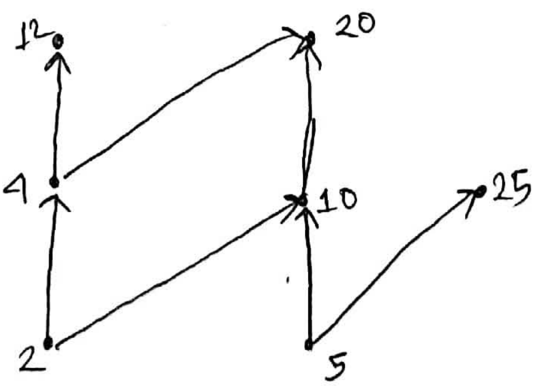
The relation is = {(2, 2), (2,4), (2, 10), (2, 12), (2, 20), (4, 4), (4, 12), (4, 20), (5, 5), (5, 10), (5, 20), (5, 25), (10, 10), (10, 20), (12, 12), (20, 20), (25, 25)}



(i)



(ii)



(iii)

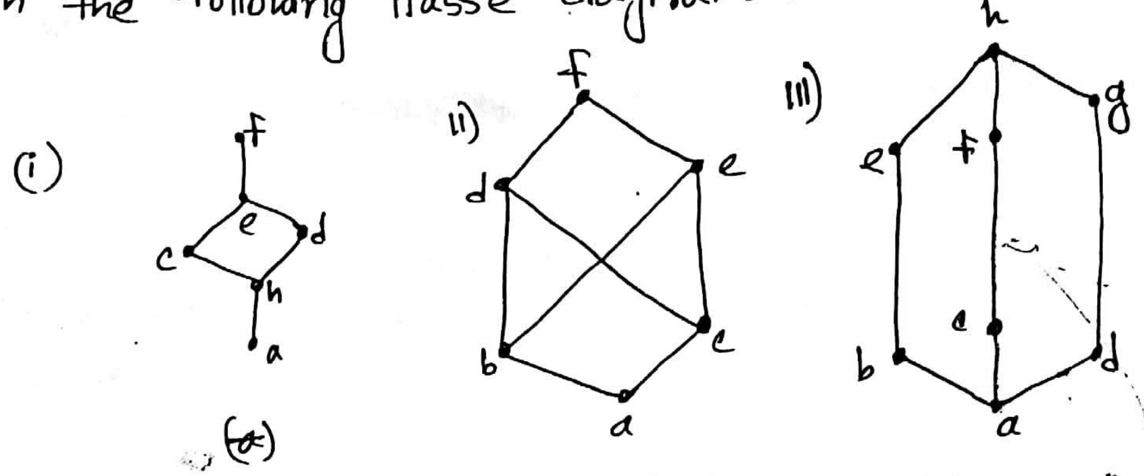
i) For the poset the maximal elements are 12, 20 and 25.

ii) For the poset the minimal elements are 2, 5.

iii) There is neither least element nor great element.

iv) For {4, 10} upper bounds are 12, 4, 20 lower bouds are 2, 10, 5.

7. a) Define Lattice. Determine whether the po-sets with the following Hasse diagrams are Lattices.



Lattice: A partially ordered set in which every pair of elements has both a least upper bound and a greatest lower bound is called lattice.

The posets represented by the Hasse diagrams in (i) and (iii) are both lattices because in each poset every pair of elements has both a least upper bound and a greatest lower bound, as On the other hand, the poset with the Hasse diagram shown in (ii) is not a lattice, because the elements b and c has no least upper bound. To see this, note that each of the elements d, e and f is an upper bound. but none of these three elements precedes the other two with respect to the ordering of this poset.

b) Define sequence. Sum of the geometric series $a, ar, ar^2, \ldots, ar^k$ is given by $S = \sum_{i=0}^{k} ar^i$; where $a, r \in \mathbb{R}$. Reduce the closed form of S.

Sequence: TRY IT YOURSELF

c) Define product rule. How many bit strings of length eight either start with a 1 or end with the two bits 00?

Product rule: If two events are not mutually exclusive, then product rule is applied.

Theorem: suppose a procedure can be accomplished with two disjoint sub-tasks. If there are:

→ n_1 ways of doing the first task and

→ n_2 ways of doing the second task.

then there are $n_1 \cdot n_2$ ways of doing the overall procedure.

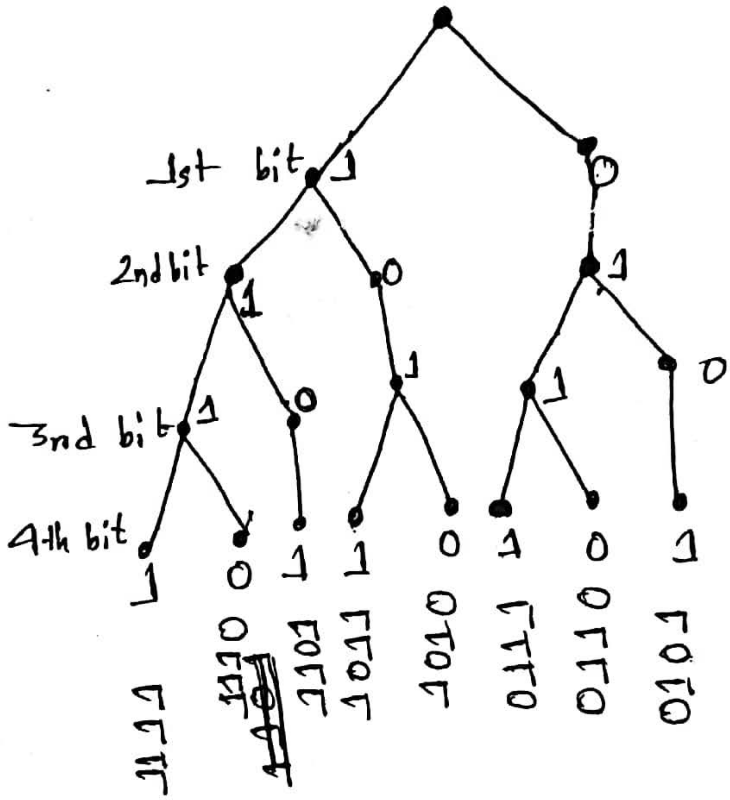
We can construct a bit string of length eight tat begins wit a 1 in $2^7 = 128$ ways.

Similarly we can ~~constuct~~ construct a bit strings of length eight ending with the two bits 00, in $2^6 = 64$ ways.

Some of the ways to construct a bit string of length eight starting with a 1 are the same as the ways to construct a bit string of length eight that ends with the two bits 00, in $2^5 = 32$ ways.

So, the solution is $128 + 64 - 32 = 160$

d) Draw the tree diagram for finding the number of bit strings of length 4 that do not have two consecutive 0's.



Ans to.8

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(b)

Principle Amount (P) = 10,000

Interest rate (R) = 12%.

Time (t) = 40 years

Number of interest compounded (n) = 1.

Total Balance (x) = ?

$$x = P\left(1+\frac{R}{N}\right)^{nt}$$

$$= 10000\left(1+\frac{.12}{1}\right) \times 1 \times 40$$

$$= 448000$$

(Ans)

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c

$a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$

Characteristic equation: $r^3 - 6r^2 + 11r - 6 = 0$

$r = 1, 3, 2$; $r_1 = 1$, $r_2 = 3$, $r_3 = 2$.

General equation: $a_n = \alpha_1 1^n + \alpha_2 3^n + \alpha_3 2^n$

For,

$a_0 = 2$,

$a_0 = \alpha_1 1^0 + \alpha_2 3^0 + \alpha_3 2^0$

$2 = \alpha_1 + \alpha_2 + \alpha_3$ — (i)

$a_1 = 5$,

$a_1 = \alpha_1 1^1 + \alpha_2 3^1 + \alpha_3 2^1$

$5 = \alpha_1 + 3\alpha_2 + 2\alpha_3$ — (ii)

$a_2 = 15$

$a_2 = \alpha_1 1^2 + \alpha_2 3^2 + \alpha_3 2^2$

$15 = \alpha_1 + 9\alpha_2 + 4\alpha_3$ — (iii)

~~α~~ Solving. (i), (ii) and (iii)

$\alpha_1 = 1$, $\alpha_2 = 2$, $\alpha_3 = -1$

$\therefore$ the equation is,

$$a_n = 1^n + 2 \cdot 3^n - 2^n$$

(Ans)

(d)

No idea about generating function.

(a)

An algebric system is defined as a set along with operations on elements of the set.

An algebric system $(G, *)$ is said to be a group if the following

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conditions are satisfied.

1) $*$ is a closed operation.

2) $*$ is an associative operation.

3) There is ~~a~~ an identity in G.

4) Every element in G has inverse in G.

example for,

1. For all $x, y \in G$.

   $x * y \in G$.

2. For all $x, y, z \in G$.

   $x * (y * z) = (x * y) * z$

3. There exists an element $e \in G$ such that for any $a \in G$; $x * e = x = e * x$

4. For every $x \in G$, there exists an element denoted by $a^{-1} \in G$ such that

   $x^{-1} * x = x * x^{-1} = e$.