$\frac{d}{dx}(K)$; এখানে identity ব্যবহৃত variable

↓

dependent variable $K = f(x)$

K must be function of x.

→ Independent function

$f(x) = \underbrace{ax^2 + bx + c}_{eqn}$

$ax^2 + bx + c$ numbers of

$x = \alpha$, $x = \beta$ $x = 0$;

$\beta = x = -b/a$.

$\alpha + \beta = -b/a$.

Independent variable operator এ থাকবে। dependent কে operate করবে।

$d(\tan x) = \sec^2 x\, dx$

$\frac{d}{dx}\tan x = \sec^2 x$.

(i) $a < x < b$

(ii) $a \leq x \leq b$ ⟶ $[a, b]$ - closed set

$a < x < b$ → open set.

Lt $x \to 1$ → যেইখানে ২টা মানগুলো যদি সমান হয়; তাহলে limit value exists.

Lt $x \to -1$

✓ A function $f(x)$ is defined as follows;

$f(x) = 1 + 2x$ when $-\frac{1}{2} \leq x < 0$

$= 1 - 2x$ when $0 \leq x < \frac{1}{2}$

$= -1 + 2x$ when $x > \frac{1}{2}$

Q) Does $\underset{x \to 0}{Lt}$ এর জন্য $f(x)$ কী exist করে?

when $x < 0$; $f(x) = 1 + 2x$. $= \underset{x \to 0}{Lt} f(x)$ $\underset{x \to 0}{Lt}(1 + 2x)$

L.H.S

$= 1 + 2 \cdot 0 = 1$.

When; $x>0$;

$f(x) = 1-2x$

Since; L.H.L = R.H.L = 1; so that the given function $f(x)$ exist at $x = 0$;

R.H.S.

$= \lim_{x \to 0^+} f(x)$

$= \lim_{x \to 0^+} 1-2x$

$= 1-2\cdot 0$

$= 1.$

When $x < \frac{1}{2}$, $f(x) =$

$\lim_{x \to \frac{1}{2}^-} (1-2x)$ $\quad$ $\lim_{x \to \frac{1}{2}^+} (-1+2x)$

$= 1-2\times\frac{1}{2} = 0$; $\quad$ $= -1+2\times\frac{1}{2}$

$= -1+1 = 0$;

$\therefore f(x)$ exists at $x = \frac{1}{2}$.

limit $(a+b) =$ limit a + limit b.

Limit, variable, constant, function, absolute '?' value

Limit এর problems; এটা solve করতে হবে।

Limit:

A limit is the value that a function approaches as the input approaches some value.

Variable;

A symbolic name associated with a value and whose associated value may be changed.

Constant;

constant is a non-varying value.

Function;

Function is a relation between sets that associates to every element of a first set exactly one element of the second set.

9th February.

$\boxed{=}$ $x \neq 0 \Rightarrow x > 0$ or $x < 0$

$\boxed{>}$ $x = 0$

$\boxed{<}$

Q A function $f(x)$ is defined as follows;

$f(x) = x \sin(\frac{1}{x})$ when; $x \neq 0$

$= 4$ when; $x = 0$

Does $\underset{x \to 0}{Lt} f(x)$ exist?

$f(x) = -3x+3$ for $x < 3$

$= -x+3$ for $0 \leq x < 1$

$= x+1$ for $1 \leq x < 2$

$= 3x-3$ for $x \geq 2$

Relation (x) with $\rightarrow 0, 1, 2$

Show that, limit exists ~~or~~ at any point of x

Does limit exist at any point of x?

$f(x) = |x| + |x-1| + |x-2|$

$x < 0$

$(a-b)^2 \geq 0$

$f(x) = -|x| - |x-1| - |x-2|$

$= -x - x + 1 - x + 2$

$= -3x + 3$

$f(x) = -3x+3$, when $x < 0$;

$(a-b)^2 \geq 0$

Q A function $f(x)$ is defined as follows;

$f(x) = \frac{|x-4|}{x-4}$ when; $x \neq 4$

$= 0$ when; $x = 4$

Does $Lt_{x \to 4} f(x)$ exist?

$f(x)$
continuous for $x=0$;
provided
$\lim_{x\to a} f(x)$ exist.
(i) Finite
(ii) is Equal to $f(a)$

$\lim_{x=a} f(x) = f(a)$
$\lim_{x\to a^+} f(x) =$
$\lim_{x\to a^-} f(x)$

Define limit and continuity;
limit exist করবে ও functional value সমান হবে।

$f(x)$ at $x:a$

$$\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = f(a)$$

OR

$$\lim_{h\to 0} f(a+h) = \lim_{h\to 0} f(a-h) = f(a)$$

A function $f(x)$ is defined as follows:

$$f(x) = 3+2x;\ \text{for } -3/2 \le x < 0$$
$$= 3-2x;\ \text{for } 0 \le x < 3/2.$$
$$= -3-2x;\ \text{for } x \geq 3/2$$

Is the function continuous at $x=0$ & $x=3/2$.

(i) odd (ii) Even

$x=0$; এর জন্য

L.H.L. $\lim_{x\to 0^+} f(x) = 3-2x$
$= 3$

R.H.L $\lim_{x\to 0^-} f(x) = 3+2x$
$= 3+2\times 0$
$= 3$

now, $f(0) = 3-2\cdot 0 = 3$
f.v; $\therefore f(x)$ is continuous for $x=0$;

Since;
$\therefore$ L.H.L = R.H.L. & function value $= 3$ (finite)

Even
L.H.
R.H

A

8

even

$L.H.L = \underset{x\to 3/2^-}{Lt} f(x) = \lim_{x\to 3/2^-} (3-2x) = 3 - 2\times\frac{3}{2} = 3-3 = 0;$

$R.H.L = \underset{x\to 3/2^+}{Lt} f(x) = \lim_{x\to 3/2^+} (-3x-2x) = -3-2.3/2 = -3-3 = -6$

Since, $L.H.L \neq R.H.L$.

functional value $= f(v) = -3-2.3/2 = -3-3 = -6.$

$\therefore L.H.L = R.H.L.$

B) A function defined as follows:

$f(x) = \frac{x^2-4}{x-2}$ for $x \neq 2$

$= 3$ for $x = 2$

$x \neq 2$

$x > 2 \quad x < 2$

Does con: at $x = 2$.

$f(x) = \frac{|x-3|}{x-3}$, when $x \neq 3$

$= 0$, when $x = 3$

; L.H.L

$\underset{x\to 3^+}{Lt} \frac{|x-3|}{x-3} =$

$L.H.L. \underset{x\to 3^-}{Lt} \frac{\cancel{-x+3}\ -(x-3)}{(x-3)} = -1.$

$R.H.L. \underset{x\to 3^+}{Lt} \frac{(x-3)}{(x-3)} = 1.$

Here, $L.H.L \neq R.H.L$.

So, Limit does not exist. So, the function is not continuous.

B) $x^2 \cos \frac{1}{x}$

for $x \neq 0$

$x < 0$ | $x > 0$

$x \to 0^+$

যেহেতু; L.H.L. = R.

$\cos(-x) = \cos x$

L.H.L

$\sin(-x) = -\sin x$

$\sin x = \sin x$

R.H.L.

$\frac{1}{0} = \infty$

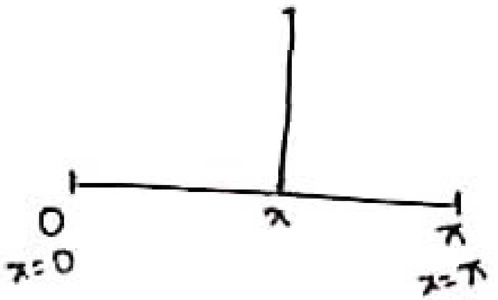
L.H.Lt = R.H.Lt = f.v

is $f(x)$ continuous

Find the value of a and b. such that the function

$$f(x) = x + a\sqrt{2}\sin x, \text{ for } 0 \le x \le \pi/4$$
$$= 2x\cot x + b, \text{ for } \pi/4 \le x \le \pi/2.$$
$$= a\cos 2x - b\sin x, \text{ for } \pi/2 < x \le \pi$$

the function $f(x)$ will be continuous for $0 \le x \le \pi$.



Differentiability:

Def^n: A function $f(x)$ is said to be differentiable at $x=a$, if $\underset{h\to 0}{Lt} \frac{f(a+h)-f(a)}{h}$,

$\underset{h\to 0}{Lt} \frac{f(a-h)-f(a)}{-h}$ exist and are equal.

$f(x)$; $f'(x)$

If $f(x) = \sin x$.

$f(x+h) = \sin(x+h)$

$$f'(x) = \underset{h\to 0}{Lt} \frac{f(x+h)-f(x)}{h}$$

$$\underset{h\to a}{Lt} = \frac{f(x+h)-f(x)}{h}$$

$= f(a)$; exist.

⊡ $f(x)$ differentiable;

$f'(a) = \underset{h\to 0}{Lt} \frac{f(a+h) - f(a)}{h}$ exists;

$\underset{h\to 0}{Limit}[f(a+h) - f(a)] = \underset{h\to 0}{Lt} \frac{f(a+h) - f(a)}{h} \times h.$

$f'(a) \underset{h\to 0}{Lt} h.$

$= f'(a) \times 0$

$= 0$

$\underset{h\to 0}{Lt} f(a+h) - f(a) = 0;$

$= \underset{h\to 0}{Lt} f(a+h) = f(a)$

$f(x)$ cont. at $x = a$;

⊡ Consider the function $f(x) = |x|$ } Test the continuity; & differentiability.

we can write;

$f(x) = x$ for $x > 0$

$= -x$ for $x < 0$

$= 0$ for $x = 0$;

$\underset{x\to 0^+}{Lt} f(x) = \underset{x\to 0^+}{Lt} = x \cdot \underset{x\to 0^-}{Lt} (-x) = 0;$

Now;

$L f'(0) = \underset{h\to 0^-}{Lt} \frac{f(0-h) - f(0)}{-h}$

$= \underset{h\to 0}{Lt} \frac{h-0}{-h}$

$= -1$

$R f'(0) = \underset{h\to 0}{Lt} \frac{f(0+h) - f(0)}{h}.$

$= \underset{h\to 0}{Lt} \frac{h-0}{h} = 1$

$\underset{h\to 0}{Lt} f(a+h) = f(a).$

⊡ Set is a proper subset of a domain.

A function $f(x)$ is defined as follows;

$f(x)=1$; for $x<0$

$= 1+\sin x \quad 0 \le x \le \pi/2$

$= 2+(x-\pi/2)^2$ for $x \geq \pi/2$

Test $f'(x)$ exist at $x=\pi/2$

$f(x)$.

$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ $\qquad \lim_{h\to \pi/2^-} f\ (1+\cancel{\sin}\ \pi/2 - \pi/2$

$\lim_{h\to 0} \frac{f(x-h)-f(x)}{-h}$

$= \underline{\underline{L.H.L}}$ $\quad \therefore \lim_{h\to 0} \frac{f(\pi/2-h)-f(\pi/2)}{-h}$

$x \geq \pi/2$

$x > \pi/2$

$x=\pi/2$

$= \frac{1+\sin(\pi/2+h)-[2+(\pi/2-\pi/2)^2]}{-h}$

$= \frac{\cancel{1+\cos h}\ 1+\cos h-2}{-h}$

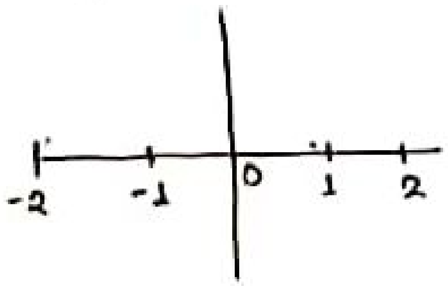
$= \frac{-1+\cos h}{-h}$.

$\lim_{h\to 0} = \frac{1-\cos h}{h}$ [using L. hospital].

$= 0$;

L.H.L = R.H.L = F.V.

A function $f(x)$ is defined in the interval $[0,2]$.



$\boxed{m \cdot 2}$ $[0, 1)$

$f(x) = x$; when $0 \leq x \leq 1$

$= 2x-1$ when $1 \leq x \leq 2$;

at; $x = 1$, test the continuity;

L.H.L:

$\lim_{x \to 1^-} f(x) = x$.

$= 1$

R.H.L; $\lim_{x \to 1^+} f(x) = 2x-1$

$= 2(1)-1 = (2-1) = 1$;

$\therefore$ L.H.L = R.H.L.

F.V = $f(x) = f(1) = 1$

As, L.H.L = R.H.L. exists and the functional value is 1., so the function is continuous.

$f' =$ exist.

$f(x) = x$; $0 \leq x < \frac{1}{2}$.

$= 1$; $x = \frac{1}{2}$

$= 1-x$; $\frac{1}{2} < x < 1$

at; $x = \frac{1}{2}$; test the continuity;

L.H.L; $= \lim_{x \to \frac{1}{2}^-} f(x) = x = \frac{1}{2}$

L.H.L = R.H.L.

$\lim_{x \to \frac{1}{2}^+} f(x) = (1-x)$

$= 1 - \frac{1}{2} = \frac{1}{2}$.

f.v = $f(x) \cdot 1$; f(v) not same.

(b) A function $f(x)$ is defined as follows;

$$f(x) = x^2 \sin(\tfrac{1}{x}) \text{ for; } x \neq 0;$$
$$= 0 \qquad \text{for } x = 0;$$

Show that

$f(x) = \cancel{x^2 \sin\frac{1}{x}}$ ~~fo~~ is differentiable at $x=0$, but its derivative is not continuous, at $x=0$;

$$f(x) = x^2 \sin\frac{1}{x}.$$

$$f'(x) = 2x \sin\frac{1}{x} - \cos\frac{1}{x}$$

$$F(x) = 2x \sin\frac{1}{x} - \cos\frac{1}{x}$$