Lecture 1
27.01.2020

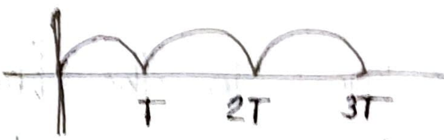
Alternating Current (AC) :

AC is a periodic current whose average value over a period is zero.

square wave

Periodic Current (PC) :

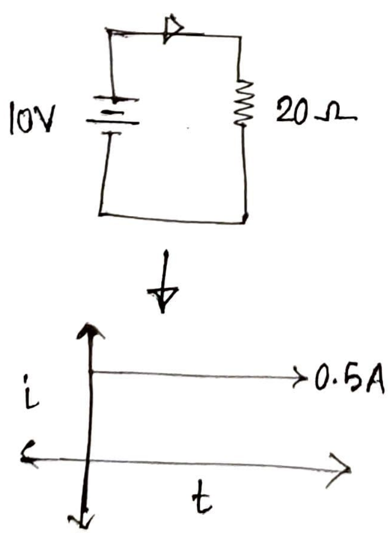
An oscillating current which reccurs at a certain time.



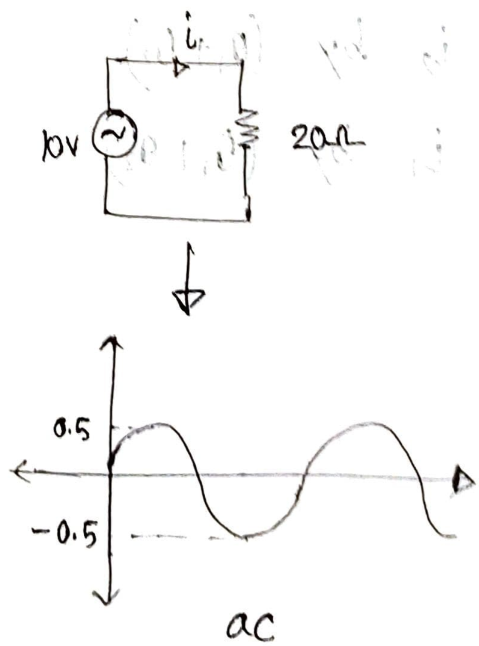
pulsating dc

Oscillating Current :

Alternately increases and decreases at definite law.



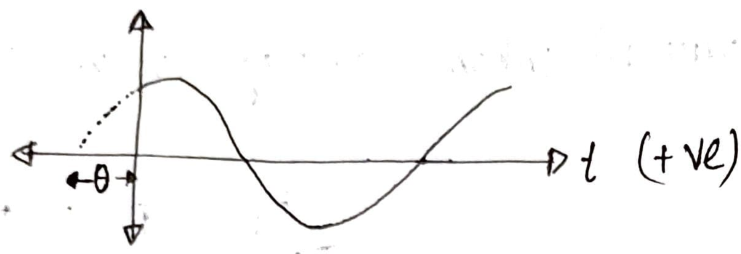
pure dc



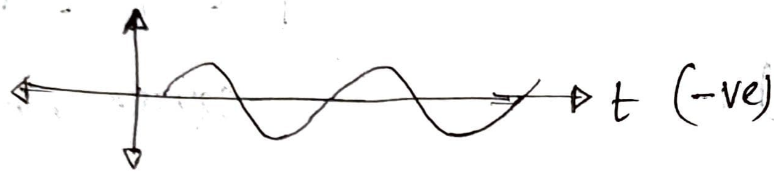
ac

→ AC can be represented by sine waves.

→ Smallest recurring values → T (period)



$i_1 = I_m \sin(\omega t + \theta_1)$



$i_2 = I_m \sin(\omega t - \theta_2)$

phase difference $= \theta_1 - (-\theta_2) = \theta_1 + \theta_2$

Phase is the fractional part of the period through which time or associated time angle has advanced from an arbitery reference.

i_1 leads i_2 by $(\theta_1 + \theta_2)$.

i_2 lags i_1 by $(\theta_1 + \theta_2)$

Lecture 2
03.02.2020

$\theta_1 - \theta_2 = +ve$ i_1 leading, i_2 laging

$\theta_1 - \theta_2 = -ve$ i_1 laging, i_2 leading

Impedance function or Impedance :

Ratio of V_{max} to I_{max}

Angle of difference between v and i

→ impedance $Z\angle Angle = \frac{V_{max}}{I_{max}} \angle \theta_v - \theta_i$

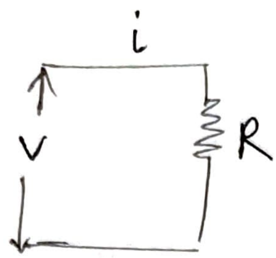
example : $v = 150 \sin(\omega t + 75°)$

$i = 10 \sin(\omega t + 30°)$

impedance $Z\angle Angle = \frac{150}{10} \angle (75-30)°$

$= 15 \angle 45°$

→ parameters : R (resistance), L (inductance), C (capacitance), f (frequency).



The R branch : Let $v = V_{max} \sin \omega t$ is applied to an R branch. we know, $v = iR \Rightarrow i = v/R$

$$\Rightarrow i = \frac{V_m \sin \omega t}{R}$$

$$= I_m \sin \omega t$$

$$\Rightarrow \frac{V_m}{I_m} = R$$

impedance $Z_R = R\angle 0°$ ★★★

example : $V = 100 \sin(\omega t + 30°)$

$i = 20 \sin(\omega t + 30°)$

$Z = 5\angle 0° \rightarrow$ R Branch

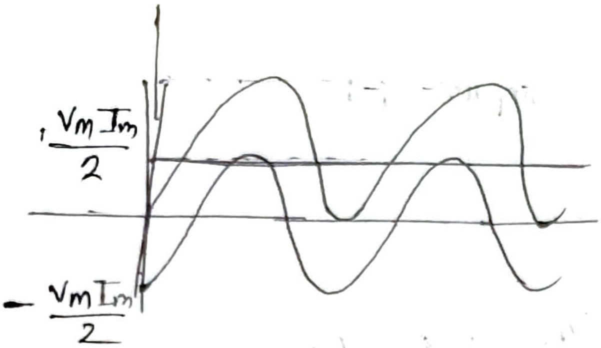
$\therefore R = 5$

→ $P = vi = V_m I_m \sin^2 \omega t$

$$= \frac{V_m I_m}{2}(1 - \cos 2\omega t)$$

$$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t$$

∴ In Resistant Branch → power will always be lossed



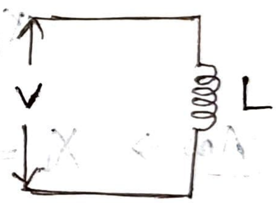
$$P_{av} = \frac{V_m I_m}{2}$$

Lecture 3
05.02.2020

$Z = 5\angle 0^\circ$

যেহেতু, $\theta_v - \theta_i = 0^\circ$. সুতরাং, impedance ⇒ resistance

The L Branch :



$$V = L\frac{di}{dt} = V_m \sin\omega t$$

$$\Rightarrow \frac{di}{dt} = \frac{V_m}{L}\sin\omega t$$

$$\Rightarrow i = \frac{-V_m}{L\omega}\cos\omega t + c_1 \text{ (on integration)}$$

c_1 → transient current

At steady state value, $C_1 = 0$

$$\therefore i = -\frac{V_m}{L\omega}\cos\omega t = \frac{V_m}{\omega L}\sin(\omega t - 90^\circ)$$

$i = I_m \sin \omega t$ - এর সাথে compare করে,

$$I_m = \frac{V_m}{\omega L}$$

$\therefore$ impedance, $Z_L = \frac{V_m}{I_m} \angle \theta_v - \theta_i$

$= \omega L \angle 0 - (-90°)$

$= \omega L \angle 90°$ ★★★

inductance reactance, $X_L = \omega L$

Here, L is in "henry"

$X_L = \omega L$ is in "ohm (Ω)".

example: $f = 60 Hz$,

$L = 10 \times 10^{-3} H$

$X_L = ?$

Ans ⇒ $X_L = \omega L$

$= 2\pi f . L$

$= (2\pi \times 60 \times 10 \times 10^{-3}) \Omega$

$=$

$V_m = 100 V$, $v = ?$ $i = ?$

Ans ⇒ $V = V_m \sin \omega t$

$= 100 \sin(2\pi \times 60 t)$

$i = \frac{V_m}{\omega L} \sin(\omega t - 90) = \frac{100}{2\pi \times 60 \times 10 \times 10^{-3}} \sin(2 \times 60 \times \pi t - 90)$

(Ans)

$P = vi$

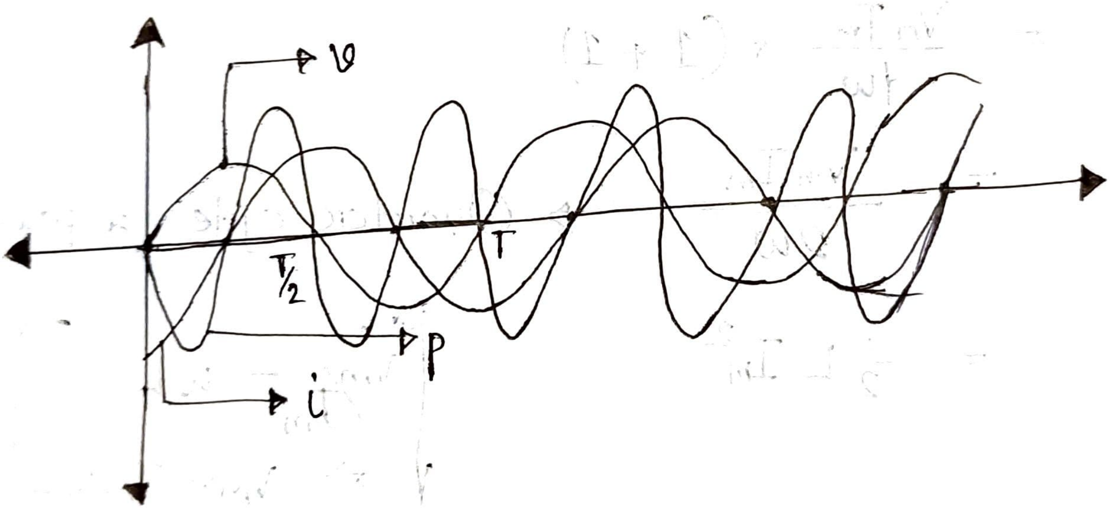
$= (V_m \sin \omega t) \cdot \{I_m \sin(\omega t - 90°)\}$

$= -\frac{V_m I_m}{2} \sin 2\omega t$

$P_{avg} = 0$

Here, v and i waves are single frequency variation but P double frequency variation.

There is no loss in pure inductance.



Time period → half of v or i wave.

10.02.2020

$$P = \int_{T/4}^{T/2} -\frac{V_m I_m}{2} \sin 2\omega t \, dt$$

$$= \left[-\frac{V_m I_m}{2} (-\cos 2\omega t) \cdot \frac{1}{2\omega} \right]_{T/4}^{T/2}$$

$$= \frac{V_m I_m}{4\omega} \left\{ \cos 2 \cdot \frac{2\pi}{T} \cdot \frac{T}{2} - \cos 2 \cdot \frac{2\pi}{T} \cdot \frac{T}{4} \right\}$$

$$= \frac{V_m I_m}{4\omega} \left\{ \cos 2\pi - \cos \pi \right\}$$

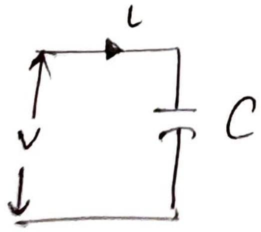
$$= \frac{V_m I_m}{4\omega} \times (1 + 1)$$

$$= \frac{V_m I_m}{2\omega} \longrightarrow \text{Quarter cycle} \rightarrow \text{power}$$

$$= \frac{1}{2} L I_m^2$$

$$\left[\frac{V_m}{I_m} = \omega L \;\Rightarrow\; V_m = I_m \omega L \right]$$

~~Capact~~ Capacitance :



$$v = \frac{q}{C} = V_m \sin\omega t$$

$$\Rightarrow \cancel{C} = \cancel{\frac{q}{V_m}} \quad q = V_m C \sin\omega t$$

$$i = \frac{dq}{dt} = V_m \omega C \cos\omega t$$

$$= \frac{V_m}{\frac{1}{\omega C}} \sin(\omega t + 90°)$$

$i = I_m \sin(\omega t + 90^\circ)$

$$\therefore \quad I_m = \frac{V_m}{\frac{1}{\omega C}}$$

$$\Rightarrow \quad \frac{V_m}{I_m} = \frac{1}{\omega C}$$

impedence, $Z_c = \left(\frac{1}{\omega C}\right) \angle -90^\circ$ ★★★

capacitive reactance $X_c = \frac{1}{\omega C}$

Here, C is in Farade (F)

$\frac{1}{\omega C}$ is in ohm (Ω)

example : $f = 25\,Hz$, $C = 15\,\mu F$, $X_c = ?$, $V_m = 200V$

$$X_c = \frac{1}{\omega C}$$

$$= \frac{1}{2\pi f \times C} = \frac{1}{2 \times 3.1416 \times 25 \times 15 \times 10^{-6}} = 424.41\,\Omega$$

(Ans)

$$i = \frac{V_m}{\frac{1}{\omega C}} \sin(\omega t + 90^\circ)$$

$$= 0.471 \sin(157.08t + 90^\circ) \quad \text{(Ans)}$$

$$P = vi = V_m \sin \omega t \cdot I_m \sin(\omega t + 90^\circ)$$

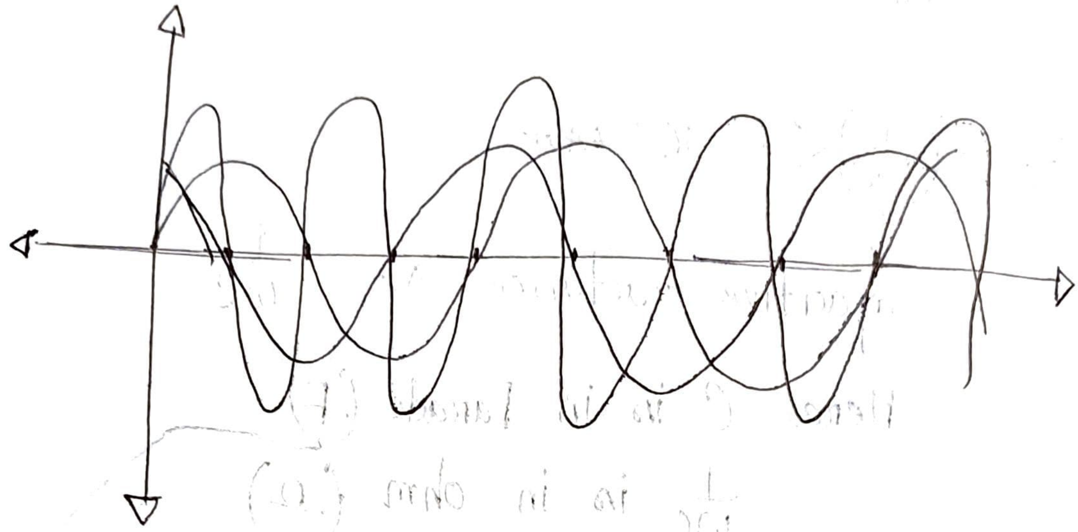
$$= 200 \times 0.471 \sin(157.08t) \cdot \sin(157.08t + 90^\circ)$$

$$= 94.2$$

$$P = vi$$

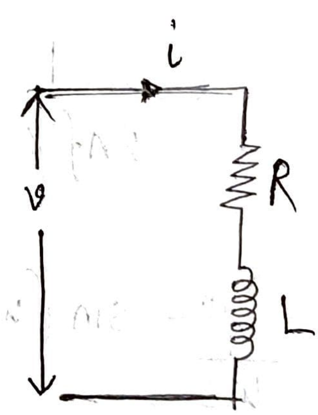
$$= V_m \sin \omega t \cdot I_m \sin(\omega t + 90^\circ)$$

$$= \frac{V_m I_m}{2} \sin(2\omega t).$$



$$W_C = \int_0^{T/4} \frac{V_m I_m}{2} \sin 2\omega t \, dt$$

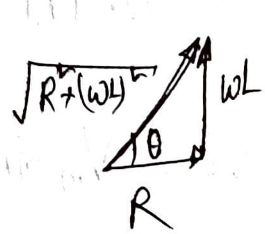
R-L Branch:



voltage drop, $v = Ri + L \cdot \frac{di}{dt}$

Let, $i = I_m \sin \omega t$

$\Rightarrow v = R I_m \sin \omega t + \omega L I_m \cos \omega t$



$$\Rightarrow \frac{v}{\sqrt{R^2+(\omega L)^2}} = \frac{R}{\sqrt{R^2+(\omega L)^2}} I_m \sin\omega t + \frac{\omega L}{\sqrt{R^2+(\omega L)^2}} \cdot I_m \cdot \cos\omega t$$

$$\Rightarrow \frac{v}{\sqrt{R^2+(\omega L)^2}} = I_m \cos\theta . \sin\omega t + I_m \sin\theta . \cos\omega t$$

$$\Rightarrow \frac{v}{\sqrt{R^2+(\omega L)^2}} = I_m \sin(\omega t+\theta)$$

$$\Rightarrow v = I_m \cdot \sqrt{R^2+(\omega L)^2} \sin(\omega t+\theta)$$

$$\Rightarrow v = V_m \sin(\omega t+\theta)$$

$$\therefore \frac{V_m}{I_m} = \sqrt{R^2+(\omega L)^2}$$

$$\Rightarrow \frac{V_m}{I_m} = \sqrt{R^2+(\omega L)^2} \qquad \tan\theta = \frac{\omega L}{R} \Rightarrow \theta = \tan^{-1}\frac{\omega L}{R}$$

impedence of R-L Branch $= \sqrt{R^2+(\omega L)^2} \angle \theta$

$= \sqrt{R^2+(\omega L)^2} \angle \tan^{-1}\frac{\omega L}{R}$

Voltage leading,
Current lagging,

ex: $R = 20\Omega$

$L = 0.056$ Henry

$f = 60 Hz$

$$Z_{RL} = \sqrt{R^2 + (\omega L)^2} < \tan^{-1}\frac{\omega L}{R}$$

$$= 29.08 < 46.55^\circ$$

$v = 200 \sin 377t,$

$i = I_m \sin \omega t$ $\quad V_m = 200$

$= 6.88 \sin(376.99t - 46.5)$ $\quad \frac{V_m}{I_m} = \sqrt{R^2 + (\omega L)^2}$

$\Rightarrow I_m = 6.88$

$$i = 6.88 \sin(376.99t - 46.5)$$

$P = vi$

$$= \frac{V_m I_m}{2} 2\sin^2\omega t \cdot \cos\theta + \frac{V_m I_m}{2} \cdot 2\sin\omega t \cdot \cos\omega t \cdot \sin\theta$$

$$= \frac{V_m I_m}{2} \cos\theta - \frac{V_m I_m}{2} \cos\theta \cdot \cos 2\omega t + \frac{V_m I_m}{2} \sin\theta \cdot \sin 2\omega t$$

$$= \frac{V_m I_m}{2} \cos\theta - \frac{V_m I_m}{2} \cos(2\omega t + \theta)$$

$$P_{avg} = \frac{V_m I_m}{2} \cos\theta \quad \text{(Real Value Power)}$$

~~Power~~ Reactive power $= \frac{V_m I_m}{2} \sin\theta$

Power factor $= \cos\theta$.

Reactive factor $= \sin\theta$

R-L CKT

$$Z_{R-L} = \sqrt{R^2 + (\omega L)^2} \angle \tan^{-1}\frac{\omega L}{R}$$

$v = V_m \sin(\omega t + \theta)$, $i = I_m \sin\omega t$

$$P = vi = \frac{I_m V_m}{2}\cos\theta - \frac{V_m I_m}{2}\cos\theta \cdot \cos 2\omega t + \frac{V_m I_m}{2}\sin\theta \cdot \sin 2\omega t$$

$\frac{V_m I_m}{2}\cos\theta - \frac{V_m I_m}{2}\cos\theta \cdot \cos 2\omega t \rightarrow$ Instaneous Real Power (always +ve)

$\frac{V_m I_m}{2}\cos\theta \rightarrow$ Real Power ; Power factor (p.f.) $= \cos\theta$

$\frac{V_m I_m}{2}\sin\theta \cdot \sin 2\omega t \rightarrow$ Instantaneous Reactive Power

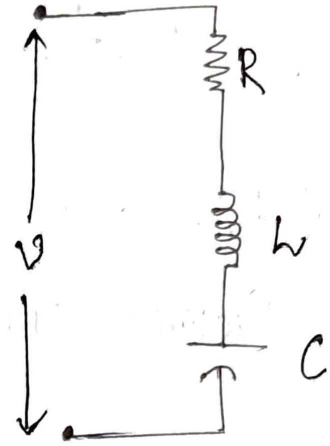
$P_{av.} = \frac{V_m I_m}{2}\cos\theta \rightarrow$ measured by a wattmeter.

$\frac{V_m I_m}{2}\sin\theta \rightarrow$ Reactive power ; Reactive factor (r.f) $= \sin\theta$

$\downarrow$

measured by varmeter

R-L-C Branch :



$$v = Ri + L\frac{di}{dt} + \frac{q}{C}$$

Let, $i = I_m \sin\omega t$ and

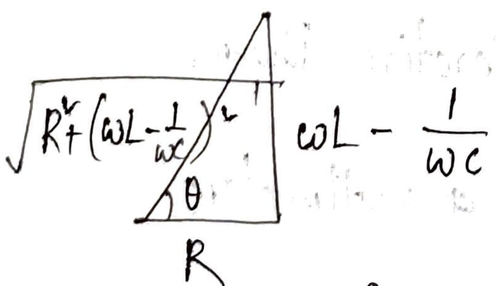
$$\frac{q}{C} = \frac{\int i\,dt}{C} = \frac{-I_m \cos\omega t}{\omega C}$$

$$\therefore v = RI_m \sin\omega t + \omega L I_m \cos\omega t - \frac{1}{\omega C} I_m \cos\omega t$$

$$= I_m R \sin\omega t + I_m\left(\omega L - \frac{1}{\omega C}\right)\cos\omega t$$

dividing both the sides by $\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

$$\frac{v}{\sqrt{R^2+\left(\omega L - \frac{1}{\omega C}\right)^2}} = I_m \cdot \frac{R}{\sqrt{R^2+\left(\omega L - \frac{1}{\omega C}\right)^2}} \sin\omega t + I_m \cdot \frac{\left(\omega L - \frac{1}{\omega C}\right)}{\sqrt{R^2+\left(\omega L - \frac{1}{\omega C}\right)^2}}$$



Let $\cos\theta = \frac{R}{\sqrt{R^2+\left(\omega L - \frac{1}{\omega C}\right)^2}}$

then $\sin\theta = \frac{\omega L - \frac{1}{\omega C}}{\sqrt{R^2+\left(\omega L - \frac{1}{\omega C}\right)^2}}$

$$\Rightarrow v = I_m \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \sin(\omega t + \theta)$$

$$= V_m \sin(\omega t + \theta)$$

impedence $Z_{RLC} = \frac{V_m}{I_m} \angle \theta$

$$= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \angle \tan^{-1} \frac{\omega L - \frac{1}{\omega C}}{R}$$

e.g: $R = 20\Omega$ $L = 0.056$ henry $C = 50\mu F$

$V = 200 \sin 377t$, $Z_{RLC} = ?$ $i = ?$

$\Rightarrow$ $V = V_m \sin \omega t$

$\therefore \omega = 377$

$$\therefore Z_{RLC} = \sqrt{(20)^2 + \left(377 \times 0.056 - \frac{1}{377 \times 50 \times 10^{-6}}\right)^2} \angle \tan^{-1} \frac{377 \times 0.056 - \frac{1}{377 \times 50 \times 10^{-6}}}{20}$$

$$= \cancel{37.68 \angle}$$

$$= \cancel{33.467 \angle 88.03^\circ} \quad 33.467 \angle -72.61^\circ$$

$$i = \frac{200}{33.5} \sin(377t + 72.61^\circ)$$

12. $v = 100 \sin(\omega t - 30)$ and $i = 10 \sin(\omega t - 60)$

$\underbrace{i \text{ and } v}_{\text{phase difference}} = ?$ lead = ?

$v =$

~~$V_m = 100$~~

Ans: phase difference $= -60 - (-30)$

$= -30$

v is leading.

13. $v = 100 \cos(\omega t - 30)$ & $i = -10 \sin(\omega t - 60°)$

phase difference v & i = ? lag ?

$v = 100 \sin(\pi/2 + \omega t - 30)$

$= 100 \sin(\omega t + 60)$

$\frac{V}{\sqrt{R^2 + (}}$

$i = 10 \sin(180 + \omega t - 60°)$

$= 10 \sin(\omega t + 120)$

$\sqrt{R^2 + (\omega}$

v & i $= \overset{+60}{\cancel{120}} - (+120) = \cancel{60}° \; -60$

Lo ∴ i lagging.

th

18. $v = 150 \cos 314t$

$R = 30\,\Omega$

a) $i = ?$

b) $f_v = ?$ $f_i = ?$

c) $P = ?$

d) $f_p = ?$

A: $v = 150 \sin(90 + 314t)$

$\omega = 314$

$\Rightarrow 2\pi f = 314$

$\Rightarrow \boxed{f_v = 50 = f_i}$

$\frac{\cancel{I_m}}{V_m} = R \qquad \frac{V_m}{I_m} = R$

$\Rightarrow I_m = \frac{150}{30} A = 5A$

$\boxed{i = 5 \cos(314t)}$

$\boxed{\therefore f_p = 100}$

$P = vi$

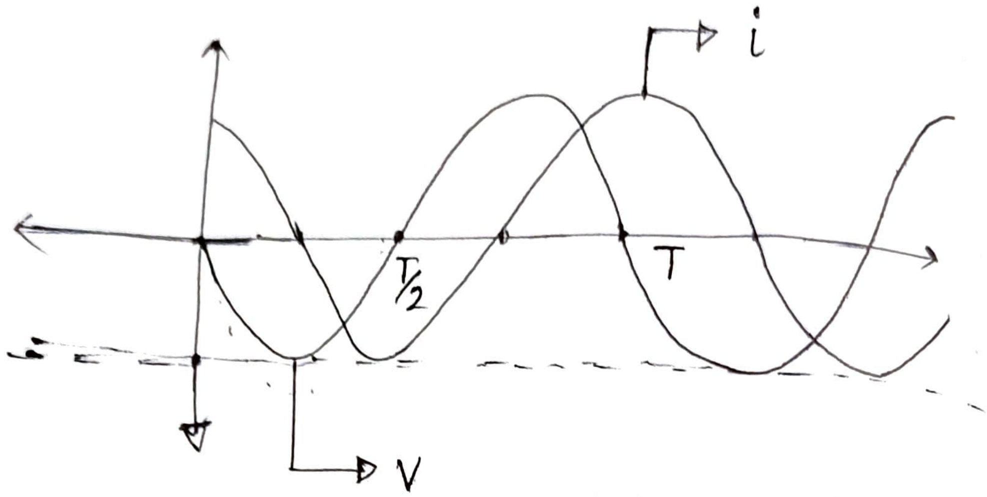
$= 150 \times 5 \times \cos^2(314t)$

$= \boxed{750 \cos^2(314t)} = 375(1 + \cos 628t)$

$= 375 + 375 \cos 628t$

22. $v = -150 \sin 377t$

$i = 10 \cos 377t$



$V = 150 \sin(180 + 377t)$

$i = 100 \sin(90 + 377t)$

phase difference $= \theta_v - \theta_i = 180^\circ - 90^\circ$

$= 90^\circ$

$\therefore$ V leading and i lagging.

Nature : Inductance.

$I_m = \dfrac{V_m}{\omega L}$

$\Rightarrow L = \dfrac{V_m}{I_m \omega} = 0.0397$ henry

29. $R = 10\,\Omega$, $L = 0.05$ henry

$f = 25$ cycles , $V_m = 150\,V$

a) $Z_{R-L} = ?$

b) $v = 75$ at $t = 0$, voltage equation = ?

c) $i = ?$ d) $P = ?$

a) $Z_{R-L} = \sqrt{R^2 + (\omega L)^2} \angle \tan^{-1} \frac{\omega L}{R}$

$= \sqrt{10^2 + (2\pi \times 25 \times 0.05)^2} \angle \tan^{-1} \frac{2\pi \times 25 \times 0.05}{10}$

$= 12.7 \angle 38.14^\circ$

b) $v = V_m \sin(\omega t + \theta_v)$

$= 150 \sin (157t + 30)$

$75 = 150 \sin\theta$

$\Rightarrow \theta = 30^\circ$

c) $i = I_m \sin(\omega t + \theta_i)$

$\frac{V_m}{I_m} = \sqrt{R^2 + (\omega L)^2}$

$\Rightarrow I_m = 11.811$

$\theta_v - \theta_i = 38.14 \Rightarrow \theta_i = -8.14$ $\therefore$ $i = 11.811\,(157t - 8.14)$