



UNIVERSITY OF GHANA

DEPARTMENT OF STATISTICS & ACTUARIAL SCIENCE

SCHOOL OF PHYSICAL & MATHEMATICAL SCIENCES

FIRST SEMESTER, 2023/2024 ACADEMIC YEAR

B.A/B.Sc. STATISTICS

STAT 111: INTRODUCTION TO STATISTICS AND PROBABILITY 1

EXERCISE 3 ON LECTURES 5 AND 6

1.
 - (a) Describe the sample space and all 16 events for a trial in which two coins are thrown and each shows either a head or a tail.
 - (b) A fair coin is tossed, and a fair die is thrown. Write down sample spaces for
 - (i) the toss of the coin;
 - (ii) the throw of the die;
 - (iii) the combination of these experiments.
 - (c) Let A be the event that a head is tossed, and B be the event that an odd number is thrown. Directly from the sample space, calculate $P(A \cap B)$ and $P(A \cup B)$.
 - (d) A bag contains fifteen balls distinguishable only by their colours; ten are blue and five are red. I reach into the bag with both hands and pull out two balls (one with each hand) and record their colours.
 - (i) What is the random phenomenon?
 - (ii) What is the sample space?
 - (iii) Express the event that the ball in my left hand is red as a subset of the sample space.

2. (a) Eight chairs are numbered 1 to 8. Two women and 3 men wish to occupy one chair each. First the women choose the chairs from amongst the chairs 1 to 4 and then men select from the remaining chairs. Find the total number of possible arrangements.
- (b) If the letters of the word RACHIT are arranged in all possible ways as listed in dictionary. Then what is the rank of the word RACHIT?
- (c) A candidate is required to answer 7 questions out of 12 questions, which are divided into two groups, each containing 6 questions. He is not permitted to attempt more than 5 questions from either group. Find the number of different ways of doing questions.
3. (a) We wish to select 6 persons from 8, but if the person A is chosen, then B must be chosen. In how many ways can selections be made?
- (b) How many committee of five persons with a chairperson can be selected from 12 persons.
- (c) How many automobile license plates can be made if each plate contains two different letters followed by three different digits?
4. (a) A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected from the lot.
- (b) Find the number of permutations of n distinct things taken r together, in which 3 particular things must occur together.
5. (a) Find the number of different words that can be formed from the letters of the word 'TRIANGLE' so that no vowels are together.
- (b) Find the number of positive integers greater than 6000 and less than 7000 which are divisible by 5, provided that no digit is to be repeated.
- (c) There are 10 persons named P1, P2, P3, ... P10. Out of 10 persons, 5 persons are to be arranged in a line such that in each arrangement P1 must occur whereas P4 and P5 do not occur. Find the number of such possible arrangements.

6. (a) There are 10 lamps in a hall. Each one of them can be switched on independently. Find the number of ways in which the hall can be illuminated.
- (b) A box contains two white, three black and four red balls. In how many ways can three balls be drawn from the box, if at least one black ball is to be included in the draw.
- (c) If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$, then find rC_2 .
- (d) Find the number of integers greater than 7000 that can be formed with the digits 3, 5, 7, 8 and 9 where no digits are repeated.
7. (a) In a certain city, all telephone numbers have six digits, the first two digits always being 41 or 42 or 46 or 62 or 64. How many telephone numbers have all six digits distinct?
- (b) 18 mice were placed in two experimental groups and one control group, with all groups equally large. In how many ways can the mice be placed into three groups?
- (c) A bag contains six white marbles and five red marbles. Find the number of ways in which four marbles can be drawn from the bag if:
- they can be of any colour
 - two must be white and two red and
 - they must all be of the same colour.
8. (a) In how many ways can a football team of 11 players be selected from 16 players? How many of them will:
- include 2 particular players?
 - exclude 2 particular players?
- (b) A sports team of 11 students is to be constituted, choosing at least 5 from Class XI and at least 5 from Class XII. If there are 20 students in each of these classes, in how many ways can the team be constituted?
- (c) A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has
- no girls
 - at least one boy and one girl
 - at least three girls.

- 9. (a)** A box contains 10 red marbles, 20 blue marbles and 30 green marbles. 5 marbles are drawn from the box, what is the probability that:
- (i) all will be blue?
 - (ii) at least one will be green?
- (b)** 4 cards are drawn from a well-shuffled deck of 52 cards. What is the probability of obtaining 3 diamonds and one spade?
- 10. (a)** A die has two faces each with number '1', three faces each with number '2' and one face with number '3'. If this die is rolled once, determine
- (i) $P(2)$ (ii) $P(1 \text{ or } 3)$ (iii) $P(\text{not } 3)$
- (b)** In a certain lottery 10,000 tickets are sold and ten equal prizes are awarded. What is the probability of not getting a prize if you buy:
- (i) one ticket (ii) two tickets (iii) 10 tickets.
- 11. (a)** Out of 100 students, two sections of 40 and 60 are formed. If you and your friend are among the 100 students, what is the probability that
- (i) you both enter the same section?
 - (ii) you both enter the different sections?
- (b)** Three letters are dictated to three persons and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that at least one letter is in its proper envelope.
- 12.** A and B are two events such that
 $P(A) = 0.54$, $P(B) = 0.69$ and $P(A \cap B) = 0.35$.
 Find
- (i) $P(A \cup B)$
 - (ii) $P(A' \cap B')$
 - (iii) $P(A \cap B')$
 - (iv) $P(B \cap A')$

- 13.** From the employees of a company, 5 persons are selected to represent them in the managing committee of the company. Particulars of five persons are as follows:

S. No.	Name	Sex	Age in years
1.	Miriam	F	30
2.	Inkoom	M	33
3.	Deborah	F	46
4.	Nellisa	F	28
5.	Nedrick	M	41

A person is selected at random from this group to act as a spokesperson.

What is the probability that the spokesperson will:

- (i) be either male or over 35 years?
 - (ii) a female and over 35 years?
 - (iii) neither be female nor below 35 years?
- 14.** (a) If 4-digit numbers greater than 5,000 are randomly formed from the digits 0, 1, 3, 5, and 7, what is the probability of forming a number divisible by 5 when,
- (i) the digits are repeated?
 - (ii) the repetition of digits is not allowed?
- (b) The number lock of a suitcase has 4 wheels, each labelled with ten digits i.e., from 0 to 9. The lock opens with a sequence of four digits with no repeats. What is the probability of a person getting the right sequence to open the suitcase?
- 15.** (a) If two fair dice are rolled together, which of the following is a more likely event:
- (i) Event A = Getting a total of 7 or more?
 - (ii) Event B = Getting a total of 7 or less?
- (b) In a group of 12 international referees, there are three from Africa, four from Asia and five from Europe. To officiate at a tournament, three referees are chosen at random from the group. Find the probability that:
- (i) A referee is chosen from each Continent.

- (ii) Two referees are chosen from Asia.
 - (iii) All the three referees are chosen from the same Continent.
- (c)
- (i) In how many different ways can the letters of the word “MISSISSIPPI” be arranged so that all the letter I always come together?
 - (ii) There are 8 men and 10 women and you need to form a committee of 5 men and 5 women. In how many ways can the committee be formed?
16. (a) If 8 members of a tennis club are classified as A players, 6 are classified as B players and 10 are classified as C players, in how many different ways can 2 players from each group be chosen to represent the club.
- (b) The rate of inflation in four successive years in a country was 7 percent, 11 percent, 15 percent and 19 percent. Using the harmonic mean, find the average rate of inflation per year.
- (c) A sample survey conducted in a city shows that the probabilities are 0.87, 0.36 and 0.29 that a family randomly chosen will own a colour T.V set, a black-and-white T.V. set, or both, respectively. What is the probability that such a family will own at least one of the two kinds of set?
- (d) A survey asked people how often they exceed speed limits. The data are then categorized into the following contingency table of counts showing the relationship between age group and response.

Age (in years)	Possible Exceed Limit		
	Always	Not always	Total
Under 30	100	100	200
Over 30	40	160	200
Total	140	260	400

Compute the Phi (ϕ) contingency coefficient between age and possible speed limit and comment appropriately.

17. M&M sweets are of varying colours and the different colours occur in different proportions. The table below gives the probability that a randomly chosen M&M has each colour, but the value for tan candies is missing.

Colour	Brown	Red	Yellow	Green	Orange	Tan
Probability	0.3	0.2	0.2	0.1	0.1	?

- (a) What value must the missing probability be?
- (b) You draw an M&M at random from a packet. What is the probability of each of the following events?
- (i) You get a brown one or a red one.
 - (ii) You don't get a yellow one.
 - (iii) You don't get either an orange one or a tan one.
 - (iv) You get one that is brown or red or yellow or green or orange or tan.
18. (a) You play draughts against an opponent who is your equal. Which of the following is more likely:
- (i) winning three games out of four or winning five out of eight;
 - (ii) winning at least three out of four or at least five out of eight?
- (b) Count the number of distinct ways of putting 3 balls into 4 boxes of the following events described when:
- MB = {all boxes and balls are distinguishable};
- BE = {the boxes are different but the balls are identical};
- FD = the balls are identical, the boxes are different but hold at most a single ball
- (c) See if you can do the counting when there are m balls and n boxes
19. I have in my pocket ten coins. Nine of them are ordinary coins with equal chances of coming up head and tail when tossed and the tenth has two heads.
- (a) If I take one of the coins at random from my pocket, what is the probability that it is the coin with two heads ?

- (b) If I toss the coin and it comes up heads, what is the probability that it is the coin with two heads?
 - (c) If I toss the coin one further time and it comes up tails, what is the probability that it is one of the nine ordinary coins?
20. A certain person considers that he can drink and drive: usually he believes he has a negligible chance of being involved in an accident, whereas he believes that if he drinks two pints of beer, his chance of being involved in an accident on the way home is only one in five hundred. Assuming that he drives home from the same pub every night, having drunk two pints of beer, what is the chance that he is involved in at least one accident in one year? Are there any assumptions that you make in answering the question?