UNIT 6: DEDUCTION VS INDUCTION

Introduction

For general purposes and widest application, it is best to think of human reasoning as a process with two aspects. One aspect concerns the form or pattern of our sequences of thought, independent of whatever it is our thoughts are about. Thoughts that are formed into sentences have a structure defined by a small set of fixed operations that comprise different types of sentences, which are organised in packages called arguments. The validity of an argument is determined by how the shapes of its constituent individual thoughts fit together. A valid argument, also called a demonstration, or a proof, indicates a conclusion of a certain form will be true if its supporting premises are true. The principles or rules governing the form of thoughts fitting together are called DEDUCTIVE.

The other equally important aspect of our declarative thoughts concerns how they reflect the world around us. We call this side of reasoning INDUCTIVE.

Each aspect requires different principles and techniques to evaluate whether the reasoning has been executed well or poorly. All actualreasoning that humans do (practical, aesthetic, spiritual, philosophical, moral, and scientific) except for veryspecialised situations is both inductive and deductive.

Reasoning is always about problem solving. Disciplines that construct knowledge—philosophy and many liberal arts and social studies and physical sciences—begin with a problem or question that prompts a quest for solution or a satisfying answer, using both deductive and inductive patterns of thinking.

From a cognitive, psychological point of view (concerned with how we actually think and survive) the two cannot be isolated from each other. Yet they can be distinguished, like all interdependent things must be distinguished, in order for their reciprocal dependence to be seen clearly.

In this Unit we will learn to tell the difference between deductive and inductive reasoning, even though they can rarely be separated altogether as meaningful categories in isolation from each other (think of the biological categories male and female, or the physics of night and day). We need to distinguish induction from deduction because we need to evaluate each type of reasoning using different principles of dependability and credibility.

This unit will cover the following topics:

Section 1: How to contrast and how not to contrast deductiveand inductive arguments

Section 2: Some simple deductive argument patterns (four valid syllogisms)

Section 3: Three syllogistic fallacies

Section 4: The notion of a valid argument as distinct from a true statement

Section 5: Some famous samples of deductive proof

Objectives

Upon completion of this unit you should be able to

- Appreciate what is wrong with a standard way of contrasting induction and deduction
- understand that deduction is topic neutral, while induction is dependent upon subject matter
- Identify four types of deductive syllogism
- Discriminate these four valid syllogisms from fallacies that parody them
- Enjoy two examples of applying deductive proof to justify a commonplace arithmetic truth and a familiar arithmetic rule

Section 1 How to Contrast and how not to Contrast Deductive and Inductive Arguments

Introduction

This course is in part a researchcourse. This means you should learn to use the library as a source of information about the topics covered in the syllabus. Any library can be used to practice, because there will be at least one logic textbook in the library that you have access to.

You can equally well use the internet to do research for this course, because there are lots of websites devoted to information about introductory practical reasoning, and exercises for 'informal' logic courses online. Use a search engine and type in topics on the syllabus, and see what comes up.

It may be easier sometimes to go to the City Campus library or the Balme Library, (either the circulation section and the SRL (Student Reference Library)Reading Room on the second floor and use the logic section to browse the book indexes for the key terms in the syllabus, and see what you find. At the end of this module you will find a list of useful books all available in the Balme, upstairs on reserve and downstairs in circulation, the department library, and in the Hall libraries too.

But you must always be careful when you rely on research materials; sometimes what you find in elementary publications or on the internet is not altogether true. A critical thinker is always prepared to cross check and improve upon or update or deepen what is presented at an elementary level in publications, what is circulated in the public domain, and what most people take for granted.

Elementary level treatments and textbooks are by nature incomplete. You will find for instance that the Oxford Learner's Dictionary leaves out many words that you need for your studies in university. This module is also incomplete; for instance it leaves out almost all the important contributions of elementary formal symbolic logic.

Objectives

- define 'argument' in a technical sense
- recognise counterexamples to a standard way of contrasting induction and deduction
- distinguish reference class from attribute class of a statement
- distinguish particular from general statements based on the reference class of the statement
- understand that the quantity of things or the type of things referred to by the premises and the conclusion have nothing at all to do with whether or not the conclusion follows deductively from premises

What is an Argument?

In the technical sense we use for this course and module, an argument is a set of statements with premises and one conclusion. This was the sense of the term introduced in Unit 3.

Arguments in this sense need not involve altercation or heated conflict or differences of opinion. Deductive arguments are used to demonstrate that a given conclusion must be true provided linestarting points that led to it logically are all true. Such a deductive conclusion is sometimes called a theorem.

Arguments are also used to explain events or processes observed in nature.

Arguments are used to defend a value judgment. They are used to defend the application of a rule.

They are used to correct our deepest intuitions and convictions.

Rational argument can help us progress beyond our fallible sense of what must be true, which getwarped by the limitations of our experiences in the world.

In this Unit we will study the structure or anatomy of some deductive arguments. And then we will look at some famous ways that deductive arguments have been used to accomplish some of the things listed above.

Activity 1.1

Identifying the conclusion and the premises of an argument

Return to Unit 3 Activity 4.1.For each passage that you identified as an argument, discern which statement expresses the conclusion.

All the recommended logic textbooks at the introductory level are likely to contain some brief description of the difference between induction and deduction, and they will give you examples. You should do some research to see whether this is actually happening in the textbooks or online in elementary logic web pages: look at more than one of the textbooks available to you and compare treatments as your activity assigned along with this section.

Sometimes textbooks are misleading. Some textbooks and elementary presentations of deduction vs. induction claim that the difference between induction and deduction is dependent upon the premises and the conclusion being particular or general.

What is the difference between particular and general statements that appear in almost every argument?

To understand this mistaken way of drawing the contrast we need to first appreciate the difference between particular and general statements.

Identifying the reference class and the attribute class of a statement

This contrast between particular and general statements depends upon the reference class of the statement (also the grammatical subject of the statement, in many cases). This is to be distinguished from the attribute class of the statement.

For example, in the statement:

(i) *Joshua and Kofi are reading in the library,*

The reference class is 'Joshua and Kofi', while the attribute class (assigning them an action or property) is 'reading in the library'.

The reference class in (1) is finite because you can count the number of individuals in this reference class—there are two people referred to by the sentence. So we say that the statement
(1) is particular.

In the statement,

(2) *All freshmen are reading in the library when they are not in lecture.*

The reference class here is 'all freshmen' and that is a universal or infinite number of individuals, so (2) is called a general statement. In fact we call it a *universal generalisation* because it refers implicitly to each thing in the universe, and says that if this thing is a freshman, then it is reading in the library when it is not in lecture. The attribute class is 'reading in the library when not in lecture'.

Activity 1.2

Recognising what makes a statement particular

In the following list of statements (1-8), identify the reference class and the attribute class. Then explain why each of these is a particular statement. Later when we study inductive arguments in Unit 7 we will return to this exercise and we will be referring to one kind of particular statements as evidence and as verifiable statements.

1. This student is reading Philosophy.
2. That man is the Dean of Arts.
3. The water in this person's bucket is finished.
4. That table is green.
5. This stone is not a real diamond.
6. None of the students in this class have registered.
7. The fourth and seventh samples of the meat consignment were infected with the lethal bacteria.
8. All the voters interviewed said they would prefer a recount of theballots

Activity 1.3

Recognising what makes a statement general

In the following list of statements (9-18), identify the reference class and the attribute class. Then explain why each of these reference classes is not finite but is rather infinite (the number of individualshas no definite limit). Thus each of these is a general statement. Later in Unit 7 when we study inductive arguments, we will be referring to one type of general statement as an hypothesis and as confirmable.

9. All metals expand when heated.
10. Planets move in elliptical orbits around the Sun.
11. Every heavy smoker has a carbon film on his lung tissue.
12. Green tables are costly these days.
13. Meat from England is infected with e-coli bacteria.
14. No student registers unless forced.
15. All 100 level students must take UGRC 150.

16. 80% of all retailed stones are not diamonds.

17. Few Ghanaians are allergic to pineapple.

18. 75% of those who contract the human equivalent of mad cow disease die from eating the infected beef.

A mistaken way of distinguishing induction from deduction1

Let us return now to one very common but misleading way to contrast deduction and induction. Some textbooks tell you that:

(A) An inductive argument moves from particular premises to a general conclusion;

Whereas:

(B) A deductive argument always moves from general premises to a particular conclusion.

This is sometimes true but not always, so it fails to provide a good definition of the difference between induction and deduction.

In fact some valid deductive arguments, just like inductive arguments, move from general premises to a general conch or from particular premises to a particular conclusion:

Example 1. All reptiles are cold blooded: and no cold blooded creature is a mammal; so no reptile is a mammal.

Example 2. Amy is older than Patience; and Patience is older than Ama. So Amy is older than Ama.

Both example 1 and 2 are deductive arguments, for reasons we have yet to discuss. Notice that the two premises in example (1) are general. And so too the conclusion is general.So we call this a counterexample of claim B, which says to be a deductive argument the conclusion should be particular while the premises are general. Similarly, in example (2) the premises and the conclusion are all particular. So this is another counterexample to the definition that is a standard given at the elementary level for deductive arguments. Many such counterexamples can be created. This means the definition is faulty.

You have yet to be given a good definition of what makes an argument deductive. We will come to that by providing you an ostensive definition (giving you examples) in the next section of this Unit.

The correction of this misleading characterisation of the contrast between induction and deduction is made in other textbooks for instance by Merrilee Salmon Introduction to Logic and Critical Thinking (1983) p. 55. Used with permission of the author and publisher for classroom distribution and for inclusion as chapter 21 in volume II of History and Philosophy of Science for African Undergraduates (2003) ed. H, Lauer Ibadan: Hope Publishers.

Meanwhile, we can find easily inductive arguments that move from general premises to a particular conclusion just as deductive arguments do

Example 3 95% of the students m Annex B have contactedheadache, fever and painful joints in the last month. 80% patients presenting headache, fever and painful joints at the clinic were tested for malaria parasites and were diagnosed with malaria. So the doctor conduced thatYaa, who is residing in Annex B and has a headache and fever and painful joints, has malaria.

And some inductive arguments often move from general premises to a general conclusion:

Example 4.95% of the students interviewed preferred free access to a computer services instead of kitchen facilities. So we conclude that students coming to Legon will welcome the installation of computers instead of cooking pantries in the new residence blocks.

Activity 1.4

Analysing the quantity of reference class in premises and conclusion

Look at example 3, which is an inductive argument. We will discuss why it is an inductive argument in Unit 7. For now, identify the following features of the argument:

a) What is the conclusion?

b) What is the reference class of the conclusion?

c) Is the conclusion particular or general?

d) What is the reference class of the premises?

e) Are the premises particular or general?

f) Is this a counterexample to the definition of an inductive argument given in A above?

g) Go through the same exercise following steps (a-f) for Example.

So what is the correct way to describe the difference between deductive reasoning and inductive reasoning? The answer is this:

In a valid deductive argument, the conditions that ensure the truth of the promises require that the conclusion will also be true. That is, it would be a contradiction to assert the premises and deny the conclusion in a valid deductive argument. In a valid deductive argument the truth of the premises proves or guarantees or requires or contains the truth of the conclusion.

Now these are all metaphoric terms (guaranteeing, requiring, containing) and we have yet to get a clear picture of what makes an argument deductively valid, in this course we will provide adefinition by ostension, giving examples of valid arguments. Thisis merely introductory and will leave you dissatisfied. But it will suffice for the purpose of contrasting deductive arguments with inductive arguments, which is all we can accomplish in this overview course.

But in a good inductive argument, the premises provide a good reason to believe the conclusion will be true. The premises do not prove the conclusion to be true. The premises rather confirmthe likelihood of the conclusion being true. That means there will be no logical contradiction between the premises being true and finding the conclusion to be false.

This contrast has to be spelled out in more depth by making clear what inductive confirmation amounts to, and what deductive validity involves. This will be the focus of the rest of this Unit and the next Unit 7.

What is the correct way to contrast deductive and inductive arguments?

To study deductive arguments we pay no attention to the subject matter or content of the statements, we just look at the logical connectives within and between the statements. When we study inductive arguments, we have to look at the content of the statements to examine the quantity and the sorts of things described in the premises and the conclusion. This is why we separate the study of deduction and induction, even though in real life and thinking they complement each other.

Although you may not have thought about forms of argument before, the term ‘form’ in the sense used here is not unfamiliar to you. You probably have been concerned with structures in many real-life situations. Most secondary-school students are required to take a civics course in which various forms of government are discussed.

For example, in Ghana and Nigeria, the governments are now constitutional republics. Both would be characterised today as democratic, whereas in Zimbabwe today the government functions as an ‘authoritarian’ democracy. The former apartheid South Africa was a totalitarian state; in 1990 its form changed to a constitutionaldemocracy. In the United States, the democratic constitutional form of federal government is composed of executive, judicial, and legislative branches, whose respective powers are also outlined in a constitution.

The republic form of all these governments stresses their independence from any foreign power. But both Nigeria and Ghana, for example, follow in part the legislative and judicial structure of the British form of government which is a constitutional monarchy, in that both Nigeria and Ghana have a Parliament whose members are elected as well as a judicial branch whose members are appointed. However, the British monarchy features a crowned King or Queen, who—except in cases of abdication— holds his or her office for life. In contrast, the chieftaincy system in Ghana functions parallel to the state; the chiefs’ military strengths are nominal, and ostensibly, disloyalty to the Asantehene (Asante King) is no longer legally punishable by military force. Yet it does not follow that chiefs in Ghana are not defended by force, nor that the form of governance called chieftaincy is outmoded and merely ornamental. Another contrast is that the head of government in Britain is the Prime Minister, but unlike the President of Ghana or

Nigeria or South Africa, the British Prime Minister is not the head of state. T he powers and responsibilities of the various branches of the British government are defined in various acts of Parliament, traditions and customs. In Ghana, all the powers and functions of branches of government are outlined in the constitution while the allocation of a chief's powers is sustained by oral tradition, customary knowledge and the system of common law, which function parallel to the state judiciary and legislative systems, not unlike the way the traditional health care system functions parallel to the western hospital system. Ghana practises a unitary system of government in contrast with the federal system in the United States, whereby judicial and legislative powers and functions are balanced between two territorial levels, the central government and fifty-one subsidiary state governments. In Russia, the former Union of Soviet Socialist Republics comprised a very different form than the current Federation of Russia, and prior to that, the Czar of Russia was the head of a monarchy, which no longer exists even vestigially. In some ways, monarchies are recounised by political scientists generally as more stable than the other forms of government mentioned above.

When we study forms of government in this way, we learn something about these various forms of government in a rather abstract way, without taking particular notice of the individuals who fill the positions determined by the form of government at any given time. We learn about their parts and how these parts are (or should be) related to one another. Similarly, when we study forms of arguments, we will learn something about their parts and how they relate to one another. To discuss how the parts of a governmentshould function is to analyse it normatively. In contrast, when we give a descriptive account of that government we focus on how it actually does function under the circumstances of a certain period of history, by a particular group of individuals.

The distinction between normative and descriptive analysis carries over to arguments: when we discuss the general forms of argument independently of their content, we engage in a normative discussion of the structure of scientific reasoning. If we reconstruct the steps in reasoning that led someone to a certain discovery, then we are engaged in a descriptive or a psychological account of human inference.

On the standard or 'received' view of scientific theorising, the context of justification is distinguished from the context of discovery. This distinction is highlighted again in Part 11 where it is stressed that the techniques used to justify causal hypotheses should not be misconstrued as a general prescription for the discovery of causes.

In a valid deductive argument, the conditions that ensure the truth of the premises have nothing to do at all with the quantity of things referred to in the premises and the conclusion. Deductive reasoning is not reliable because of the content or subject matter of the statements involved; it is rather the form of the statements or thoughts that the comprise which determines whether or not the premises, if true, can guarantee the conclusion will also be true. That is, it would be a contradiction to assert the premises and deny the conclusion in a valid deductive argument. But the necessity that true premises will result in a true conclusion is not a result of what or how many things the premises and the conclusion refer to.

Deductive validity is preserved in our thoughts because it is all that prevents us from contradicting ourselves. This is what is involved when we say that in a valid deductive argument the premises prove the conclusion.

But in a good inductive argument, the premises provide a good reason to believe the conclusion will be true. The premises do not provethe conclusion to be true. The premises rather confirm the likelihood of the conclusion being true.That means there will be no logical contradiction between the premises being true and finding the conclusion to be false.

This way of understanding the contrast between proving a conclusion deductively, and confirming a conclusion inductively, is troublesome because it begs the question: What is a contradiction, such that to avoid committing one we are compelled to embrace the conclusion of a valid argument?

The best way to explain this will be by example. We will study such examples for the rest of this Unit.

Example of a deductive argument

> There is a big football match today and Kwame always watches the matches whenever he can. But this is the only day Kwame is able to study in the library. So Kwame will either be watching the match or he must be in the library. But he is definitely not in the library; his mates saw him leave there two hours ago. So you will find him watching the match.

In the passage above, we learn first that for any day when football matches are played, if Kwame can watch then he will. We then learn from the premises a disjunction: either Kwame is in the library, or he can watch the match. And he is not in the library. So the conclusion is, by disjunctive syllogism, that he is watching the match.

Example of an inductive argument

> 75% of the football matches played over the last year were attended by Kwame. Normally if he is not in the library when a match is played then he will watch the match. Today the library is closed, and there is a big chance. So Kwame is probably watching the match.

In this argument, we learn that based on his behaviour over the last year, there is a 3 out of 4 chance that if a big match is being played Kwame will be watching it. He tends to be there if he is not in the library. But we don't know this as an absolute regularity with no exception. So if the premises are true then it may not be true that Kwame is watching the match, but in the absence of further information these premises give us good reason to believe that the conclusion is true.

Activity 1.5

Contrasting deductive arguments with inductive arguments

Look at the passages in this list carefully, and see if you can discern by reading their contents which premises provide a guarantee that if they are true then the conclusion will necessarily be true—these are deductive; contrasted with passages where the premises might be true while the conclusion is discovered to be false and there would be no contradiction in such a discovery, since the premises give you good reason to believe the conclusion is likely to be true, but no guarantee that it must be true—these are inductive.

1. In a sample of 20 patients who ate meat infected with the lethal bacterium, and who contractedthe human equivalent of mad cow disease, 15 of the patients died of eating the infected meat. The study shows that 75% of people contracting the human equivalent of mad cow disease will die from eating infected meat.

2. Eighty of the one hundred rats fed on burnt meat developed brain tumours. So charcoal-burnt meat is carcinogenic in rats.

3. Every time I date a Commonwealth Hall boy I wind up spending a lot of time waiting for him to keep his appointment and then getting disappointed. So I am not going to date this Commonwealth Hall boy who has asked me out. Because he will just stand me up and I will get hurt.

4. All *clacks* are *quacks,* and all *quacks* have *smirkishsublaveens*in their anterior *postuly.* So all *clacks* have *smirkishsublaveens*in their anterior *postuly.*

5. When the accident occurred there were no witnesses. Without witnesses you cannot claim insurance. Without insurance you have to pay for the repairs yourself. So I will have to pay for therepairs myself.

6. If I want a mobile phone, then I have to use my school loan when it comes. But if I use the school loan up on the phone, Iwill not be able to return my mother's money for the tuition next year. I cannot afford to pay tuition next year unless my mother helps with the tuition. But I have to pay tuition next year. So I cannot buy a phone.

7. The politicians want votes. You can see they are corrupt. So don't trust any politician.

8. In my first and second years I only dated Commonwealth Hall boys. Every time I spent a lot of time waiting for the guy to keep his appointment and then I would be disappointed. So I am not going to date this Commonwealth Hall boy who has asked me out. Because he will just stand me up and I will get hurt.

Summary of Section

We have covered a good deal of vocabulary that we will need for the next Unit on inductive arguments as well as putting us in the right frame of mind to study deductive argument patterns in the rest of this Unit. We learned:

- A common way of contrasting deduction and induction is false; because it requires judging the size of the reference class of the premises and the conclusion of the argument, making this a central feature of being inductive rather than deductive.
- A statement is particular if its reference class is finite; a statement is general if its reference class is infinite.
- The difference between deduction and induction has nothing to do with the subject matter described by the statements' reference and attribute classes at all.
- We will see what deduction involves later in this Unit by example, and we will see what induction involves in the next Unit.
- Deductive arguments, when valid, guarantee their conclusions being true if their premises are true.
- If you do not accept the conclusion of a valid pattern ofreasoning deductively then you will be contradicting yourself.
- Inductive arguments, when good evidence is provided, give a reliable reason it believe the conclusion is likely to be true; but this is no guarantee of the contusion's being true. So you could embrace the premises as true and the conclusion false, and this would not result in your contradicting yourself. It would result in people wondering what you might know that is keeping you from depending upon the evidence cited in the premises which seems to suggest that the conclusion stated is likely to be true.

Section 2: Some Simple Deductive Argument Patterns (four Valid Syllogisms)

Introduction

In a valid deductive argument, the conditions or interpretation that determine the premises will be true, ensure that the conclusion will be true as well. This is what it means to say that in a valid deductive argument the truth of the premises prove the truth of the conclusion.

Objectives

- to recognisethe hidden conditional pattern 'if _____ then ______ in universal affirmations and universal negations
- to recognise 4 different syllogistic patterns

- to interpret passages that contain these 4 syllogistic patterns

A note on universal affirmations (generalizations) and Universal negotiations Consider the universal generalization.

All As are Bs, example: all cats are mammals.

This should be understood as a conditional: If (x is A) then (x is B) In this example, if x is a cat then x is a mammal.

In this sentence, (x is A) is called the antecedent and (x is B) is called the consequent.

In section 1unit 5, it was suggested that we think of this as a prediction, a conditional statement of this form: If x is A then x is B. We learned that ‘x is A’ counts as the antecedent placeholder of any ‘if (______) then (_____)’ or conditional form of sentence, and ‘x is B’ is called the consequent.

So to continue our example:

All cats are mammals. My pet is a cat; so my pet is a mammal.

Take a moment here to consider also the universal negation pattern:

No As are Bs (that is, every A is not B) This means ‘if (x is an A) then (x is not a B)’

In our example it would read No cats are mammals that is, if x is a cat then x is not a mammal.

Now you are ready to study four types of syllogism: A syllogism is a simple pattern of relations that always hold between two or three classes - we can call them A, B, C, or P,Q, R:it doesn’t matter. These letters simply represent the reference classes and the attribute classes described by two premises and one conclusion. There might be two or three such classes.

Consider this syllogism:

Premise 1: all As are Bs.

Premise 2: This individual is an A

Conclusion: Therefore this individual is a B.

Suppose now we interpret A as the class of all birds.

And B is the class of all flying creature.

Then the interpretation that makes the premises true will ensure the conclusion is also true. So for example:

All birds are flying creatures, and this individual is a bird therefore this individual must be a flying creature.

This is one of the four syllogism patterns we will study in this course. It is called MODUS PONENS (also called Affirming the antecedent)

Modus ponens (Affirming the antecedent) always has this form:

Example:

All As are Bs, and this individual is an A, so this individual is a B.

All my friends are Ga. This lady is my friend, so this lady is a Ga.

A second type of syllogism pattern is called MODUS TOLLENS (also called denying the consequent)

Modus tollens (denying the consequent) always has this form:

Example:

All As are Bs and this individual is not a B, so this individual is not an A.

All my friends are Ga. This lady is not a Ga, so this lady is not my friend.

A third category of syllogistic pattern is called HYPOTHETICAL SYLLOGISM (this is because every line is a hypothesis—if___then) Hypothetical syllogism always has this form:

Example:

All As are Bs., and All Bs are Cs. So all As are Cs.

All these freshmen are tall. All tall people are good at racing. So all these freshmen are good at racing.

The final category of syllogism we will study in this course (there are 25 or so) is called DISJUNCTIVE SYLLOGISM (this is because one premise is a disjunction—either __ or)

Disjunctive syllogism always has this form:

Example:

Either A is true or B is true,but A is false.So B is true.

Either I will live in Volta or at home; but I cannot get a room in Volta; so I will live at home.

Teaser

Looking at examples of syllogistic arguments with universal negations

Here are examples of the four syllogistic patterns taking the negative universal.See if you can turn each universal negation into an if _____ then _____ statement to observe the syllogistic form.See if you can complete the syllogism yourself.

Modus ponens

None of the students in this class needs to fear deductive logic.

Kofi is a student in this class. So . . .

Modus tollens

No Legon Hall gentleman has ever felt comfortable looking like a fool. That student dancing around naked doesn't feel uncomfortable looking like a fool.So . . .

Hypothetical syllogism

All of the goats in that truck are for sale now

Nothing which is for sale now is headed for Tema. So . . .

Disjunctive syllogism

Either none of my friends from my home town are in my courses or none of them have started attending lectures. But I know one of my friends has been attending lectures since the beginning of the semester very regularly. So . . .

Answers:

Modus ponens conclusion—Kofi doesn't need to fear deductivelogic.

Modus tollens conclusion—That student can't be a Legon Hallgentleman.

Hypothetical syllogism conclusion.—None of the goats in that truckare headed for Tema.

Disjunctive syllogism conclusion—None of my friends from myhome town are in my courses.

Activity 2.1

Symbolizing arguments to reveal valid patterns of thought

Identify the type of syllogism that each passage contains by following these 3 steps:

1. Substitute letters for the reference and attribute class in the conclusion: then find the reference classes and attribute classes of the premises, using the same names for crucial classes as and when possible;

2. Rewrite or symbolise the argument as a pattern of symbolised class relations;3.

Identify which syllogistic pattern you are looking *at*

Example:

1. Every herbivore is easy to breed. All goats are herbivores. Therefore all goats are easy to breed.

Working:

Step 1: The reference class of the conclusion is Goats = G. The attribute class is 'easy to breed': = B. In the first premise G is the reference class and the attribute class is Herbivores = H. That is all the relevant classes in the argument.

Step 2: All Hs are Bs. All Gs are Hs. Therefore all Gs are Bs.

Step 3: This is a hypothetical syllogism. The two premises can be in any order; the pattern of thinking is the same.

Now try the rest. Answers are at the end of the book.

2. This creature is a herbivore. All herbivores are easy to breed. So this creature is easy to breed.

3. All goats are herbivores. The creature in the shed eats meat. So the creature in the shed cannot be a goat.

4. No goats require vaccinations. Creatures that require no vaccinations are not expensive to breed. So goats are not expensive to breed.

Activity 2.2

Recognising deviations from the deductive ideal

For each of the following passages in italics, determine which of these four syllogism patterns (also called rules of inference) is characterised by the passage.

If the passage does not represent any of the patterns you have been introduced to, then it may be a fallacy. We will discuss kinds of fallacy in the next section.

The argument may be valid and represent a syllogistic pattern, but the conclusion is false. This means that the argument is not sound. Section 4 we will deal with the difference between arguments being valid and conclusions and premises being true.

1. All the wild mushroom varieties in this forest are poisonous. If a vegetable is poisonous then it could kill a small child. So all the wild mushroom varieties in this forest could kill small children.
2. Anyone who is registered for this course should use the Course Companion Reader and also do research from textbooks. Kofi is registered for this course, so Kofi should use the Course Reader and also do research from textbooks.
3. If you try hard then you can get an A in that course. Joseph tried hard, so he must have gotten an A.
4. All graduates who want to become lawyers are very smart. And all those who are very smart are dishonest. So any graduate who wants to become a lawyer must be dishonest.
5. All graduates are virtuous; all pastors are virtuous; so any pastormust be a graduate.

Activity 2.3

Recognising syllogistic arguments

Each of these passages requires a premise to be a valid syllogism. Since it is missing a premise we call it an enthymeme (ENTH-ehmeem).

Choose which of the following would provide the premise to make the passage into a valid syllogism, following these steps (a, b, c).

a. Try each of the options A, B, C as a premise and decide which sentence if any helps to complete the passage in italics as a valid syllogism.

b. Use your own choice of letters to capture the reference and attribute classes in the italicized passage and your chosen premise, and determine from the resulting pattern which type of syllogism your choice has helped complete.

c. if none of the options will create a valid syllogism, then mark the passage as invalid. Can you find premises of your own that willcreate a valid argument linking the given premises with the conclusion?

i. All *cats are mammals. So all cats need oxygen,*

A. Oxygen is given off by trees in the process of photosynthesis.

B. All cats have fur.

C. All mammals need oxygen.

ii. All the people sitting in this room are taking the Mature Students Examination. So Kofi cannot be sitting in this room.

A. Kofi is not taking the Mature Student exam.

B. All the Mature Students candidates are required to take an examination.

C. Kofi is a Mature Student candidate taking the exam.

iii. All mammals have at least two legs (unless they aredeformed or amputees). Therefore all mammals have fur or feathers or hair.

A. Two legged creatures are able to run but not as fast as four legged creatures.

B. Some mammals have fur.

C. All creatures with at least two legs have fur or feathers or hair.

iv. The occupation of the Gaza Strip will not end unless the military withdraws. So the terrorism will continue in Gaza.

A. The terrorism will continue in Gaza until the occupation is over.

B. The military is going to pull out.

C. The purpose of withdrawal of settlers in the Gaza Strip is to promote peace.

Summary of Section

- Universal affirmations (All As are Bs) are disguisedconditionals.
- Universal negations (No As are Bs) are also disguisedconditionals.
- A conditional has two components, the antecedent and the consequent.
- The syllogisms we discussed here are very common rules of inference: modus tollens, modus ponens, disjunctive syllogism, hypothetical syllogism.

- A valid argument may not have true premises, the conclusion then may be true or false.
- An argument with a missing or suppressed premise is called an enthymeme.

Section 3 Three Syllogistic Fallacies

Introduction

Careful attention to details is required otherwise we will be confused by passages that look like syllogisms but aren't really. These similar but faulty patterns are called fallacies.

Objectives

Recognise the difference between valid syllogisms and fallacies that are parodying the real thing.

Syllogistic Fallacies

—they look like the real thing but aren't really.

False hypothetical syllogism

All As are Cs.All Bs are Cs.So; All As are Bs.

This conclusion does not follow from these premises because the class C can include very many other individuals besides members of A and B. In any case, being a member of C, according to the premises, does not entail you are a member of BOTH A and of B.

Example 1: All the women in this room are enrolled in UGRC150. All the men in this room are enrolled in UGRC150. So All the women in this room are men.

Examples 2: Every lawyer needs to be intelligent. Every doctor needs to be intelligent. So every lawyer needs to be a doctor.

Fallacy of affirming the consequent

All As are Bs.This x is a B.Therefore this x is an A.

Again the conclusion cannot follow the premises because not everything which is B must be an A. Bs include other individuals besides As.

Examples: 1All the women in this room are enrolled in UGRC150.That student is enrolled in UGRC 150. So that student must be a woman in this room.

Example 2: Every cat is a mammal.My lecturer is a mammal. So my lecturer is a cat.

Fallacy of negating the antecedent.

All As are Bs.This x is not an A.So this x cannot be a B.

Again because the class of B can contain many more kinds of individuals other than As, the fact that something is not an A doesn't mean it cannot be a B.

Examples 1: All the women in this room are enrolled in UGRC150.That student is not a woman. So that student cannot be enrolled in UGRC 150.

Example 2: Every cat is a mammal. My lecturer is not a cat. So my lecturer is not a mammal.

Activity 3.1—Thinking drill

Demonstrate fallacies of syllogistic form

Choose your own classes A. B, C. Write three passages in English using your classes as the reference or attribute classes so that you have three demonstrations of the three types of fallacies labelled in this section.

Example: A = Vandals, B =Vikings C =Legon students

Writing an illustration of false hypothetical syllogism:

All Vandals are Legon students, and all Vikings are Legon students; so all Vandals are Vikings.

Activity 3.2

Identify fallacies of syllogistic form

For each of the following, symbolise the passage as a syllogistic fallacy and identify it by name.

1. The Vandals' student leaders are always making trouble. This student leader is not a Vandal. So surely he will not make any trouble.

2. Anytime he goes to town he passes by my mother's house first. Today he passed by my mother's house. So definitely he is going to town.

3. On last year's examination we did not see this question. If a question appears on the examination this year, then it follows that it did not appear on the examination last year. So this particular question will by all means come on the examination this year.

4. I am looking for a job. Anytime I have no luck with my job hunting, then I visit my girlfriend. This morning I have to visit my girlfriend. So I can expect that later today I will have no luck when I job hunt.

5. All his friends are from Nigeria. All the Muslims in this room are from Nigeria. So all the Muslims in this room must be his friends.

6. All the forensic medical experts I know make over 5,000 Ghana cedis a month. All the lawyers I know make over 5,000 Ghana cedis a month. So all the forensic medical experts I know are lawyers.

Summary of section

Syllogism fallacies bear close resemblances to the syllogistic patterns which are valid. Often a fallacy is committed when the antecedent of a conditional is confused with the consequent, and then a parody of the inference drawn from conditional forms can occur.

So, when you negate the antecedent of a conditional rather than the consequent, then you commit a fallacy. And if you assert the consequent of a conditional instead of the antecedent, then again a fallacy is committed. If your conclusion reflects the antecedents or only the consequents of two hypothetical conditionals, then again you have committed a fallacy. This is because when you change the pattern of the thinking, the premise being true will no longer guarantee that the conclusion will be true.

Take a simple example:You can see it is valid to say that

All Vandals are men and my classmate is a Vandal, so my classmate is a man. If it is true that all Vandals are men (it is true now, it might not always be) then this conclusion must also be true. So affirming the antecedent is a valid pattern of thinking. But if you affirm (assert) the consequent and say that my classmate is a man, then it would not follow that my classmate is a Vandal.Lots of men are not Vandals, even if all Vandals are men. So affirming the consequent commits a logical fallacy.

Section 4: The Notion of a Valid Argument as Distinct from a True Statement

Introduction

One of the important distinctions we need to make is the difference between truth and validity. A valid argument may not result in a true conclusion. Example:

(1) All elephants can fly; my lecturer is an elephant. So my lecture can fly.

The problem in (1) above is that the premises were false, so the conclusion need not be true just because it follows the premises using a valid rule of inference. In the case above we have a modus ponens syllogism which is valid, but it is unsound. It is not sound because it does not have true premises.

In another peculiar situation, a conclusion might accidentally be true even though it generates from false premises. So the validity of the argument again is not sufficient to provide a good logical reason to believe the conclusion, even if it is true. Example:

(2) All corrupt politicians are over fifteen years old. Any creature over fifteen years old is dishonest to some degree. So all corrupt politicians are dishonest to some degree.

In (2) above, the conclusion may be convincing, that is, the conclusion might well be true. But clearly the premises cannot possibly give a good reason for believing the conclusion is true; the conclusion is believable despite the ludicrous premises, not as a logical consequence of those premises, even though it is a logical consequence of those premises—a valid one at that. This is a hypothetical syllogism. But it is unsound. Again, the premises are false so the argument is not sound, even though the conclusion may happen to be true.

Activity 4,1 — Thinking drill

Create your own syllogisms to reflect valid but unsound arguments.

Choose any interpretation for the classes A, B, C. Then fit these reference classes and attributes of your own choosing into each of the valid syllogistic patterns.

Example: Let A = x is an Australian rhinoceros

E = x is a member of an endangered species

U = x is a member of a union

All Australian rhinoceri are unionised.All unions are endangered species.So all Australian rhinoceri are endangered species.

Here the conclusion may well be true, but the premises give no reason to believe that it is, since the second premise is absurd, unless the sentence is understood as a metaphor, indicating that nowadays it is difficult to organise labour under neo-liberal economics policies (which is true). But in this sense the second premise has nothing to do with either the first premise of the conclusion, and would render the passage a fallacy of equivocation, best interpreted as a joke.

Write a passage in completely grammatical English that houses these syllogistic patterns using your chosen classes. Does your interpretation result in a valid argument but does not result in a true conclusion? Or is the conclusion true but the premises turn out to be false under your interpretation, so that the premises do not supply a sound reason to support the conclusion?

SECTION 5: FOUR FAMOUS EXAMPLES OF DEDUCTIVE PROOF IN ACTION

Introduction

Strict deduction can be used for many purposes. One is to provide the proof for truths we take to be self evident. For instance we all ‘know’ that 2+1 = 3. We learned to accept this in primary school. We believe it because we were well motivated to accept what we are told at that formative age; to reject anything so basic in that context so early in life would have had severely negative consequences for our future and our acceptance in society.

But the question arises inside the critical thinker, I know why I have always believed this statement of arithmetic—it would be crazy not to believe it. But what justifies this statement? Why is it crazy not to believe it?

The answer is: a deductive proof exists to show that 2+1 =3 must be true. This formula follows necessarily from the definition of the counting numbers that place 1, 2, 3 and many more within our grasp as a tool for organizing experiences around us and well beyond our experience, for that matter. So if you accept that definition, you are stuck with the belief that 2+1=3.

If you want to reject the counting number definition we ail use, then go ahead and give us a different theory of how to count. And in your alternative system, if it does as good a job as the one we already have, then indeed 2+1 does not have to equal 3.But if we accept the standard meaning of 1, 2. 3 then this arithmetic sentence must be true. We provide the proof as Number 1 in this section. It's easy.

Another use of deduction is to justify rules. You know you cannot divide by zero. You know the motivation for obedience to this rule: early in life the consequences of not dividing by zero might be the shame of a poor mark on a test, maybe even a beating in school, discouraged parents or guardians, ridicule of class mates

But WHY should we never divide by zero? What is the justification for this strict rule in arithmetic?

The answer is also a deductive proof: we provide it in section 2. It's also easy. The style of this proof varies from the one to show why 2+1=3. This style of proof is called reduction ad absurdum or reducing to absurdity or indirect proof. It carries the message: 'Believe P because if you don't your beliefs will degenerate into complete absurdity.' The indirect method of proof is relied uponto prove a statement P is true by showing that if you try to prove the statement P is false, then step by step by relying on rules of inference you will wind up deriving such ridiculous further beliefs that you are forced to concede that P was never a good idea to reject in the first place. By being forced to admit P cannot be false, you are compelled to accept that P must be true.

Actually, this style of proof has come in for some serious, if radical and eccentric, controversy. If it makes you uncomfortable or wary, you are on good philosophical grounds for refusing to accept it. But most of mathematics depends upon this style of proof, so it is useful to realise how it works and get the feel for why it is supposed to be convincing. We cannot delve into the deep logical reasons why some 'intuitionistic' and 'constructivist' philosophers of mathematics argue that most of classical mathematics is based on very shaky logical ground since it depends upon indirect proofs such as this one. We give this lovely proof as our No. 2 in this section.

Deduction really is a very powerful tool, so effective it can vanquish dogma, no matter how entrenched. But you must be careful. You can deduce your way right into jail or the gallows. Galileo Galilei almost did. Through deduction he was able to show that standing doctrine of 1500 years acceptance in Europe were actually rubbish. He got sent to jail for exposing the contradiction within Aristotle's ancient explanation of the motion of all bodies near the earth in free fall. He was charged with treason and heresy; because he was right, and it disturbed the political powers that prevailed. His indirect proof of gravitation is legend; we look at it briefly as number 3.

In this final section of Unit 6 we will consider yet another use of deductive proof: to explore the unknown terrains of reality beyond our senses. We hold very deep, spiritual convictions about how the world must be, and yet we may be wrong. Deductive reasoning can lead us to realise our firmest convictions are totally wrong, and our reasoning mind can help us correct our deepest intuitions. Through deduction our mind can grow and learn about reality where no experience can take us.

Objectives

We look here at some classic examples of deductive reasoning in living action, which made tremendous impacts in the history of pure science. These are tasty preserved treats of reasoning that are part of your intellectual heritage to savour and enjoy for their beauty and gratifying elegance, and because they are fun.

- A direct proof of why necessarily 2+1=3.
- An indirect proof of why you should never divide by zero.
- Galileo's famous proof that refuted Aristotle's theory of free fall by reducing it to absurdity.
- Pythagorean's indirect proof of the existence of irrational numbers.

No. 1

Here is a 'direct' proof that 2+1 = 3. We begin by laying down some definitions as our starting assumptions about what the symbols 1, 2, 3 mean. This means we accept 4 of the 5 Axioms of Arithmetic (spelled out by Giuseppe Peano, published 1889):

Axioms of counting (natural) numbers.

1. 1 is a natural number.
2. For every natural number n, S9n) is a natural number. 'S(n)' reads as the 'successor of n'. S(n)=n+1.
3. For every natural number n, there is no number that precedes 1. 1 is the first natural number.

That is, for every number n, S(n) 1, (1 is not the successor of any other number. It's the first natural number)

4. For any two natural numbers m, n if S(m)=S(n) then m=n. That is to say, every natural number is the successor of one and only one number. Or: every counting number n has one and only one successor.

If one accepts these four features of the definition of natural number then one must accept as well that 2+1=3. Like this:

Proof that 2+1=3

1. 1 is in the set N of natural numbers (by Axiom 1)

2. So 1 + 1 = S(1) is in the set of natural numbers, (by Axiom 2)

3. A handy notation for S(1) is 2.

4. So 2 is in the set of natural numbers. (from 2, 3 above) 5. But then S(2) is in the set of natural numbers, (by Axiom 2)

6. That is, 2+1 = S(2) is in the set of natural numbers.

7. A handy notation for S(2) is 3.

8 So 2+1 = 3 which must be in the set of natural numbers.

Deductive proofs are used to demonstrate why obvious truths aretrue, as we just did.

Deductive proof is also used to justify rules and regulations.

The next proof you will read is convincing provided you accept the definition of counting numbers proposed above in Peano's axioms. In that scheme, if 1+1 =2, then 1+1 is not equals to1. In other words, 2 is not equals to 1.This follows from Axiom 4. In other words, if you accept a starting assumption which leads you to conclude that 2=1, then that is absurd; and so you have demonstrated that there must be something wrong with the starting assumption that led you, step by step, to deductively draw such an absurd conclusion. Indeed you will have shown (by indirect proof) that the starting assumption was false.

No. 2

An indirect proof of why you should never divide by zero.

1. Suppose you *can* divide by zero. (Assume this for reductio ad absurdum)

2 Let a = b assume a, b are two whole quantities whereby

3. a2 = ab by multiplying both sides of equation in (2) by whole quantity a.

4 $a2-b2 = ab—b2$ by subtracting both sides of (3) by positive whole b2

5. $(a-b)(a+b) =.b(a-b)$ by factoring both sides of (4).

6. $(a+b) = b$ by dividing through by (a-b).* Note from 1, $a-b = 0$

7. $2b = b$ by substituting equals for equals, from line 2

8. $2 = 1$ by dividing through by whole number b.

But this is absurd! 2 cannot equal 1!

9. So line 1 must be false. You CANNOT divide by zero.

Detail of 5-6:

If $a-b=0$Then $(a-b)(a+b) = b(a-b)$

$(0)(a+b) = b(0)$

$0 = 0$

But since we have agreed to allow (a-b) to figure into our algebraic operations. Then if we divide through by (a-b) we get Then $(a-b)(a+b)=b(a-b)$

$(a-b)(a+b) =b(a-b)$

$(a-b)(a-b)$ So

$1(a+b) = 1(b)$

$b+b = b$

No. 3

Galileo's refutation of Aristotle's Theory of gravitation using inductive and deductive methods

The premier example of a scientist utilising both of the approaches described above to support his theories was Galileo (1564-1642) of Galilei in Italy. Galileo is celebrated as the first and perhaps the most famous of modern scientists who withstood ridicule, accusation and lifethreatening condemnation for contradicting received official dogma. His living practice inspired both Descartes' and Bacon's written theories about the methods of the New Science.

Among Galileo's most revolutionary precedents (and worst offences in the eyes of the Inquisition) was to present in a court of law evidence derived from a strange, new gadget— designed apparently through sleight of hand to dupe the most dignified authorities—which he called a telescope.

He had used the telescope and discovered moons revolving around the planet Jupiter, contrary to the received doctrine concerning the pristine nature of celestial spheres, each of which was supposed to move in perfectly uniform symmetry over the stationary earth with no peculiar irregularities. Today, astronomy could hot proceed without telescopes, but at the time Galileo introduced the first copy of its contemporary prototype into public discourse, the contraption was ruled as an outrageous insult and its architect was deemed a dangerous trickster guilty of heresy in Europe.

Galileo employed the two complementary styles of reasoning (advocated by Descartes and by Bacon) to refute key principles of Aristotle's doctrines. This was the point that so outraged the authorities: Aristotle's doctrines were established by fiat for centuries as truth, the only truth. In the particular case dramatized here, Galileo was attacking Aristotle's theory of araviiatlon. Aristotle had lived 1,500 years before Galilee his doctrines were accepted by the Vatican. No one ever questioned them.

Very briefly, Aristotle said: each and every existing thing has a distinct essence, comprised of a characteristic combination of the four basic elements of substance: earth, air, fire, andwater. Each entity tends towards its natural place strictly in accord with its essence. Things with a predominantly earthy essence such as rocks or lead cannonballs fail toward their natural place of rest faster than objects whose essences are less earthy. Things with very little earth element and a predominance of air (e.g. bird feathers, flower petals, lamb's fleece, clouds) are things that float easily in the air which is their natural resting place; they likewise fall to earth with great reluctance because moving toward the earth is contrary to their essence.

Galileo argued deductively against the Aristotelian account of gravitation by demonstrating that Aristotle's theory yields a contradiction. Consider a thought experiment involving a 10-kg cannonball called A, and a 2 kg cannonball called B.

According to Aristotle's view, A will fall to earth faster than B because its essence contains more earth element. Now suppose A and B are tied together; call the resulting system C. By virtue of B's airier, lighter essence, it resists falling. So B retards A's fall when they are tied together in system C. Hence A falls faster than C. S. Drake (1976) p. 264.

But the cumulative density of C is 10 kg plus 2 kg. So C which is 12 kg will fall faster to earth than A which is only 10 kg.

Therefore if you embrace Aristotle's principles of motion and trace their logical consequences, you are obliged to deduce that A falls faster than C and also that C falls faster than A. But this is a contradiction. To say that C falls both faster and slower than A is an absurd conclusion; it cannot be true. Yet it follows by valid rules of deductive inference from Aristotle's assumptions about the motion of bodies. So there must be something false in Aristotle's assumptions about the motion of bodies.

This is Galileo's *reductio ad absurdum* of Aristotle's theory of motion. It is a *deductive* form of argument: the truth of the premises guarantees that the conclusion is true (the conclusion here being that Aristotle's theory contains some false precept).

Galileo started by accepting Aristotle's starting principles and then moved step by step to consider the logical consequences of these principles until he discovered a contradiction. He thus demonstrated by pure inference that embracing Aristotie'sassumptions about gravitation must be mistaken since jointly they lead one to conclude a blatant absurdity. No further considerations can detract from this validly deduced conclusion that something must be wrong with Aristotie's assumptions since they have led to a contradiction. Either the set of principles thus reduced to absurdity needs to be revised, or it needs to be rejected altogether if a better theory can be offered which is free of inconsistency.

No. 4

Deductive proof that **there** must exist irrational numbers **(Indirect** *proof—reductio ad absurdum)*

2,500 years ago the Pythagoreans subjected their own metaphysical theory and spiritual doctrine of numbers to deductive scrutiny, and thus discovered a mistake in their understanding of the cosmos, whose essence they took to be rational numbers.

We next give the oldest and most famous example of indirect proof in mathematics, the Pythagorean indirect proof of the existence of irrational numbers.

Historical note: The historical evidence of the use of *reductio ad absurdum* as a method of deductive proof goes back well over two thousand years, to the earliest records of Hellenic mathematics in the 6th century BC. Pythagoras, as well as other famous pre-socratic philosophers constituting the pinnacle of ancient Western abstract thought, had journeyed to Africa (Egypt) as students and studied with Africans (Egyptian priests) in order to develop their highly coveted and widely celebrated results in abstract mathematics—notably geometry. Thus the roots of Western mathematical abstraction and logical reasoning are chronologically traced to ancient African culture. This is neither controversial nor contested fact, and well documented.

To appreciate the significance and power of deductive reasoning, you need some background and context in which this discovery that not all numbers can be rational was made.

Historical and conceptual context of the Pythagorean proof of the existence of irrational numbers. What remains important about the ancient Greek practice of pure speculative reasoning inmathematics and cosmology is not the content of their speculations but rather the methodsby which they went about achieving and justifying their insights.

To say a scientist's method is 'pure' means its results are axiomatisable; it does not in general mean that the individual scientist harbours virtuous intentions or that he pursues his work in any particularly laudable manner—although science for the ancient Greeks was indeed an extension of fervent religious devotion, chastity, righteous living, and a pious dedication to universal harmony.

The ancient Greeks who engaged in cosmological speculation were doing what we now call metaphysics and meditation—pure contemplative reflection of the logical consequences of basic starting principles accepted as self-evident. Rival theories of cosmology, for instance, were judged by their relative coherence, simplicity, proportional beauty and elegance. Hypotheses were inspired not from the results of empirical experimentation, as in Bacon's later notion of science, but rather from contemplative meditation upon the pure basic geometrical forms—now referred to as the Platonic solids: the perfect sphere, cube, tetrahedron, hexagon, octagon, dodecahedron and so on. These foci of contemplation wholly absorbed the mind in the perfect symmetry and ideal balance presumed to reflect the ultimate orderliness and harmony of the Divine 'kosmos' (meaning 'creation' in Greek). From the ancient Greek's and modern scientist's point of view, creation is so mysterious and awesome that nothing is impossible in principle, not even the possibility that one's strongest convictions may be proven false.

A leading teacher of this methodology was Pythagoras (572-490 BC). He was the head of an esoteric order, a religious brotherhood that lasted for centuries until it was banned in 415 AD. The Pythagoreans were priests who regarded the cosmos as governed by a divine mathematical lawfulness. Man's reason reflected in numbers—or ratio—was presumed to be a microcosmic mirror of the divine order. To worship the divine it was necessary to cleanse the mind of preoccupations with worldly distractions. By refining and stilling oneself through ascetic rules for diet and daily discipline, together with meditating on pure forms and the essence of rational numbers, one brings the mind to a state of serenity and receptivity to the divine order, which is in essence mathematical. The foundations of reality for the Pythagoreans were rational, proportions, and rational numbers expressed these proportions. Thus rational numbers were regarded as the basis of all things.

Justice, reason, opportunity are all modifications of rational number. Contemplation of rational numbers, i.e. the study of mathematics, was the ancient Greek technique for knowing God.

To appreciate the cognitive significance of deductive methods in mathematics even in its very earliest use and interpretation, it is important to realise the depth of this conviction with which the Pythagoreans believed that 'number' *means* what we now still define as the 'rational numbers'. Their conviction about the structure underlying the observable world was on a par with contemporary convictions about the existence of gravitation and a monotheistic God. It was just such a deep intuition that the Pythagoreans recanted by the light shed through followingsystematic deduction. They used indirect proof.

They so trusted in the method of deductive reason that when they recognised a contradiction follows from assuming that all numbers are rational, they renounced and corrected their deepest conviction about the very foundation of the cosmos, and countenanced the existence of numbers that are not rational. This is strong faith indeed. It distinguishes faith in reason disciplined by scientific method from fundamentalist faith in a dogmatic creed.

In conclusion, here is the Pythagoreans' proof of the existence of irrational numbers (6th century BC). It has the form of a reductio ad absurdum proof. To prove that 2 is irrational (using reductio ad absurdurn)

Line

0. Definition:

A rational numberis a ratio m/n where m, n are integers {1, 2, 3, . . ,9} which are reduced to lowest terms. This means that m, n have no common factor other than 1.This definition is central to the main crux of the proof.

1. Assume that 2 is rational. So assume the definition at line (0) holds of a/2.

2. 2 = m/n by definition of 'rational1.

3.2 = m2/n2 by squaring ; both sides of the equation.

4. 2n2 = m2 by multiplying through by n2 in equation on line (3).

So m is even, since its square (m x m) in equation on line (4) is divisible by 2. So:

6. m = 2p where p is some integer less than m. So:

7. M2 = 4p2 by squaring both sides of the equation

8. 2n2 = 4p2 by substituting into step 7 from iine (4;

9. n2 = 2p2 by dividing through the equation by 2 on line (8).

10. So n is even, since (n x n) is equal to some whole number times 2, so (n x n) is divisible by 2.

11 -But by steps 5 and 10, m/n is not reduced to lowest terms (m, n share 2 as a common factor). And from step 1, m/n *is* reduced to lowest terms.

12. So 2 is both NOT rational and rational, by the *definition* assumed at line (0) plus step 11; this is a contradiction; it cannot be true!

13. Thus the assumption at step 1 must be false, because the falsehood (line 12) was derived by valid rules of inference from step 1. It is *not* the case that 2 can be rational, contrary to assumption 1. We conclude that there exists at least one number that is not rational, i.e. rational numbers must exist.

Summary of Section

Deductive proofs have played an essential role in the development of our human legacy of knowledge about the world that we can never see but are obligated to understand to the best of our ability. Our understanding is constantly open to improvement through revision and augmentation using basic patterns and rules for creating deductive proofs.

Assignment 1

Recognising arguments containing syllogistic structures.

a) Some of these passages in the list below are not arguments; some are. Determine which are arguments.

b) Some of the arguments below are simply representations of a single syllogistic argument pattern. If so, can you identify it? Use alphabet letters to identify the key reference and attribute classes, and illustrate the syllogistic pattern.

c) Other arguments go beyond one simple syllogistic form, and so they contain one or more of these patterns and they contain statements that may not directly count in any pattern. For those you can recognise, spell out the type of syllogisrn(s) that each reveals

1 Charles is a conceited snob.*He* is a penny-pincher, he is aggressive, and he is a bully, in addition, he steals other people's belongings and he doesn't work.

2. If Paul leaves the door open again I will complain to the house management. If he leaves the door open he is letting inmosquitoes every night. If mosquitoes get in we can catch malaria, and these days if you want to cure malaria you have to go to the hospital. We want to avoid going to the hospital, so the door must be kept closed as much as possible.

3. All the candidates in my District are interested in helping private schools settle in the area. But as a school teacher I know this will destroy the public school system. And the district needs a good public school system, since education is the only way to end the cycle of poverty. I want to do whatever I can to preventthe public school system from being destroyed.

So if the candidates change their platform and offer some resistance to the influx of private education entrepreneurs then I will vote for them.

4. If Kofi comes early to take over my post, then I will leave here and go to the farm before the sun goes down.

5. We need to do the weeding more than once a month. If you only weed once a month, then the weeds grow. The weeds take most of the nutrients because they grow faster than the tomato plants. Whenever the weeds get too much the tomatoes don't get nutrition from the soil. If they don't get nutrition the tomatoes will die. We need the tomatoes to live so we can take them to market. So we have to move the weeds out more than once a month.

6. Goats who eat wet grass always get sick. Since you allow your goats to graze after the sun goes down instead of bringing them in early, your goats will get sick.

7. The internet communication is down again, so I cannot read any of my mail.

8. Everyone who was invited to Georgina's party must come. She went to a great deal of trouble making all the decorations, cooking, making a new dress and looking very pretty, and she is looking forward to receiving gifts. So everyone should make every effort to attend.

9. Today is very hot and sunny. I am climbing the hill. Anytime you reach the top of a hill and the sun is bright and shining, the sun's powerful rays bounce off the surface of the pavement sharply, at the same angle that it refracts off the surface of water. You get an optical illusion or a mirage, and you think you are looking at water. So as I approach the top of the hill I will think I see a puddle.

10. It is raining and the sun is shining. A rainbow is the result of light beingdispersed into distinct wave length frequencies by refraction off the surface of raindrops. This occurs when sun hits rain drops of sufficient size. So we will probably see a rainbow.

11. Stars look as if they are twinkling, but they actually emit a great steady light. But because they are very far away, the light coming from stars is disturbed and interrupted by objects and particles in the earth's atmosphere which may periodically block the light on its great distance to reach our eyes.

12. Now that the meaning of the word 'planet' has changed, there is no consensus anymore about how many planets compose our Solar System.

13. Very few young people are actually interested in the problem of global climate change, according to a recent poll.

14. You cannot pass the test early on Monday morning if you do not study. You only have today left and tomorrow. But today is an important football match and you are the only member of the team who is not injured and who is trained as a goal keeper, so you cannot study today. Therefore you will have to study very hard tomorrow.

15. You and your friends are nothing but a bunch of thieves. You just take people's property without their knowing anything about it, and you don't ever give it back. That's what thieves do: they take other people's property. You're a bad sort. I am going to tell my friends not to mix with you.

16. None of these candidates running for office can be trusted; I won't vote for any of them. Everybody knows that power corrupts, and that all politicians are corrupt even before they get elected. They are corrupt by nature otherwise they would never become politicians.

17. Some of the people from my senior secondary school are politically concerned and active. And given that every politically concerned and active person is very intelligent, it shows that some of my former classmates are very intelligent.

18. Anywhere you go these days in the cities you find unemployed young people vandalising and begging on the street. Young urban dwellers who take recreational drugs tend to vandalise and to beg for money to buy more drugs. But if young people had access lo credit then they could start up their ownbusinesses and be gainfully employed. Youth who have created their own jobs for themselves become responsible and don't have time to involve in drug abuse. So if there were credit schemes for young adults, the street begging and vandalism would decrease.

19. There has been a sudden increase in criminals, gamblers and drug sellers on the campus. This is probably because SRC recently built a new Gaming Centre for students to congregate, which is open to the general public. The SRC wants the general public so they can make some money from the Gaming Centre. But a UGRC 150: Critical Thinking and Practical Reasoning Unit6: Deduction and induction Centre for Distance Education, 36 University of Ghana, Legon place where people relax, eat, drink and play games also tends to attract drug dealers, gamblers and criminal elements of society.

20. I really can't stand people who spend all their time playing computer games. They are just idle and ignoring their responsibilities. They are boring too. They don't care about anybody except the fake people in the computer.

21. What's the point of even talking to you? You don't pay any attention to anyone who disagrees with you.

Assignment 2

Some of the passages below are arguments.

(A). Identify the passages which are *not* clearly premises leading to a conclusion.

(B). Recognize the difference between arguments that areDEDUCTIVE and INDUCTIVE.

(C). Which of the inductive arguments do not seem to be well substantiated by the premises?

1. If you want to get an A it is necessary to study two hours at least for each lecture hour. Kofi is not able to study for each lecture period. He just goes to class and collects notes, but does not do any other work. So Kofi will not get an A in the course.

2. All goals are mammals. This creature that he keeps in the yard is a goat. Therefore he is keeping a mammal in the yard.

3- All the goats in my neighbours yard have contracted stomach flu. They have been eating scraps from the kitchen. So anytime a goat feeds on kitchen scraps it is likely to contract stomach flu.

4 All the people who bought diesel fuel pickup trucks over the last decade and imported them into Ghana up through 2009 were able to avoid the customs and excise tax on importing an automobile—none of them paid custom duty. Jackson was forced to pay the custom charge on the vehicle that he imported in July 2010. So it appears that he did not buyand import a dieselfuel pick-up truck.

5. Of the 15 Vandals who took part in this demonstration, 12 were caught by the authorities and four were punished with suspension for two years, while others were expelled from school. So Legon students should be aware that they are notentitled to demonstrate because they likely will be punished by the authorities or rusticated and lose their place in university.

6. If he wins this lottery then you know he is a lucky man. Since he is known to be a very lucky man, so it follows that he must win this lottery.

7. To say a number is even means it is divisible by 2 without -remainder. 7,008,956,784 is even. So it is divisible by two without remainder.

8. The Volta Hall girls are conformist; they are generally willing to do what they see most of the other Volta Hall girls doing. 85% of Volta Hall girls so far have registered to take part in this church function. Mary is a Volta Hall resident. So Marywill register to take part in this church function.

9. There is a big football match today and Kwame always watches the matches whenever he can. But this is the only day Kwame is able to do his laundry. So Kwame will either be watching the match or he must be doing his laundry. He is not doing his laundry; so you will find him watching the match.

10.If Guinness is willing to sponsor a Hall Week for millions of cedis, why doesn't someone convince them to help the students to buy a big bus? What about asking Guinness, or Coca Cola, or Lever Brothers, *or* any of the regular sponsors to help improve the water situation on campus?The transport problem and the water problem really worry students year after year. Can't we ever plan for the future of the University? All students only thinkabout their own personal welfare, the rising costs of their 'user fees', and finding their personal avenues to success. Does anything matter to BA students besides grabbing a quick and easy degree while conducting a promising social life?

11.Protozoa are microscopic life forms that lived under water . The first five rocks from the hills of Obuasi studied show evidence of fossilized protozoan life. The sixth and seventh rocks have no such fossils, but the eighth, ninth and tenths rocks studied from Obuasi also had fossils of protozoan life. When these fossils are carbon tested, they are found to be four million years old. So the rocks in Obuasi must have been under water at least four million years ago. 12. 98% of the Pro-Vice Chancellors in African universities are male, and 95% of the Directors of institutes of African Studies attracting meritorious awards at universities within and outside of Africa are also male. So at the University of Ghana, Legon, Professor AkuaDplphyne (a former Pro-Vice Chancellor) and Professor YaaOdotei (a current Director of the Institute of African Studies) are both males.

13. Eight of the ten rats fed charcoal-burnt meat developed brain tumours. So 80% of all rats fed on charcoal-burnt meat develop brain tumours.

14. In a sample of 20 patients who ate meat infected with the lethal bacterium, and who contracted the human equivalent of mad cow disease, 15 of the patients died of eating the infected meat.75% of people contracting the human equivalent of mad cow disease will die of eating infected meat.

15. Anyone who is registered for this course should use the Course Companion Reader and also do research in textbooks Kofi is registered for this course, so Kofi should use the Course Reader and also do research in textbooks.

16. You can get an A in that course only if you try hard. Joseph tried *very* hard, so he must have gotten an A.

Unit summary

In this unit you learned:

- How to contrast induction and deduction
- How to locate the reference class and the attribute class of a statement to distinguish particular from general statements.
- How to recognise four types of deductive syllogism
- How to differentiate some valid syllogisms for the fallacies that pretend to copy them
- Some examples of how deductive reasoning is used to expand and correct our understanding of the world