Espérance mathématique (ou moyenne) de X

$$E(X) = \int_{-\infty}^{+\infty} x f(x)\,dx = \underbrace{\int_{-\infty}^{5} \frac{1}{2} x e^{x-5}\,dx}_{=\frac{1}{2} I_1} + \underbrace{\int_{5}^{+\infty} \frac{1}{2} x e^{-x+5}\,dx}_{=\frac{1}{2} I_2} = \frac{1}{2} I_1 + \frac{1}{2} I_2$$

$$I_2 = \int_{5}^{+\infty} x e^{-x+5}\,dx$$

Par parties

$$\begin{cases} u = x \Rightarrow du = dx \\ dv = e^{-x+5}\,dx \Rightarrow v = -e^{-x+5} \end{cases}$$

alors $I_2 = [uv]_5^{+\infty} - \int_5^{+\infty} v\,du = [x e^{-x+5}]_5^{+\infty} - \int_5^{+\infty} -e^{-x+5}\,dx =$

$= +5 + [-e^{-x+5}]_5^{+\infty} = +5 + (+1) = 6$.

$I_1 = \int_{-\infty}^{5} x e^{x-5}\,dx$ par parties $\begin{cases} u = x \Rightarrow du = dx \\ dv = e^{x-5}\,dx \Rightarrow v = e^{x-5} \end{cases}$

alors $I_1 = [uv]_{-\infty}^{5} - \int_{-\infty}^{5} v\,du = [x e^{x-5}]_{-\infty}^{5} - \int_{-\infty}^{5} e^{x-5}\,dx =$

$= 5 - [e^{x-5}]_{-\infty}^{5} = 5 - [1] = 4$

$E(X) = \frac{1}{2} I_1 + \frac{1}{2} I_2 = 5$

Pour calculer la variance de X nous calculons d'abord le moment d'ordre 2 : $E(X^2) := \int_{-\infty}^{+\infty} x^2 f(x)\,dx = \underbrace{\int_{-\infty}^{5} \frac{1}{2} x^2 e^{x-5}\,dx}_{=\frac{1}{2} J_1} + \underbrace{\int_{5}^{+\infty} \frac{1}{2} x^2 e^{-x+5}\,dx}_{=\frac{1}{2} J_2}$

$E(X^2) = \frac{1}{2} J_1 + \frac{1}{2} J_2$

$J_1 = \int_{-\infty}^{5} x^2 e^{x-5}\,dx$ $\begin{cases} u = x^2 \Rightarrow du = 2x\,dx \\ dv = e^{x-5}\,dx \Rightarrow v = e^{x-5} \end{cases}$

$J_1 = [uv]_{-\infty}^{5} - \int_{-\infty}^{5} v\,du = [x^2 e^{x-5}]_{-\infty}^{5} - 2\int_{-\infty}^{5} x e^{x-5}\,dx =$

$J_1 = 25 - 2 I_1 = 25 - 2 \cdot 4 = 17$

$J_2 = \int_{5}^{+\infty} x^2 e^{-x+5}\,dx$ $\begin{cases} u = x^2 \Rightarrow du = 2x\,dx \\ dv = e^{-x+5}\,dx \Rightarrow v = -e^{-x+5} \end{cases}$

$J_2 = [uv]_5^{+\infty} - \int_5^{+\infty} v\,du = [-x^2 e^{-x+5}]_5^{+\infty} - \int_5^{+\infty} -2x e^{-x+5}\,dx$

$= 25 + 2 I_2 = 25 + 2 \cdot 6 = 37$

Au final $E(X^2) = \frac{1}{2} J_1 + \frac{1}{2} J_2 = \frac{17+37}{2} = \frac{54}{2} = 27$

Et $Var(X) = E(X^2) - [E(X)]^2 = 27 - 5^2 = 2$