| TD3 (suite) |

Ex4 : X : la durée d'un appel téléphonique (min)

$$X \sim \mathcal{E}xp(\lambda) \; ; \; \lambda = \frac{1}{10} \Rightarrow X \sim \mathcal{E}xp\left(\frac{1}{10}\right)$$

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & \text{si } x \geq 0 \\ 0 & \text{si } x < 0 \end{cases}$$

$$\boxed{f_X(x) = \begin{cases} \frac{1}{10} e^{-x/10} & \text{si } x \geq 0 \\ 0 & \text{si } x < 0 \end{cases}}$$

① ? $P(X \leq 5)$ ② ? $P(5 \leq X \leq 10)$

Méth 1 :

① $P(X \leq 5) = \int_0^5 f_X(x)\,dx = \int_0^5 \frac{1}{10} e^{-x/10}\,dx = \frac{1}{10}\int_0^5 e^{-x/10}\,dx$

$= \frac{1}{10}\left[\frac{1}{-1/10} e^{-x/10}\right]_0^5 = \frac{1}{10}(-10)\left[e^{-x/10}\right]_0^5$

$= -\left[(e^{-5/10}) - (e^0)\right] = -e^{-1/2} + e^0 = \boxed{1 - e^{-1/2} = 0{,}3}$

② $P(5 \leq X \leq 10) = \int_5^{10} f_X(x)\,dx = \int_5^{10} \frac{1}{10} e^{-x/10}\,dx = \frac{1}{10}\int_5^{10} e^{-x/10}\,dx$

Déjà calculer

$= -\left[e^{-x/10}\right]_5^{10} = -\left[e^{-10/10} - e^{-5/10}\right]$

$= -e^{-1} + e^{-1/2} = \boxed{e^{-1/2} - e^{-1} = 0{,}239}$

Méth 2 :

$P(X \leq x) = \int_{x \in I} f_X(x)\,dx = F_X(x)$

$P(a \leq X \leq b) = F_X(b) - F_X(a)$

$$F_X(x) = \int_0^x f_X(k)\, dk = \int_0^x \frac{1}{10} e^{-k/10}\, dk$$

$$= \frac{1}{10}\int_0^x e^{-k/10}\, dk = \frac{1}{10}(-10)\left[e^{-k/10}\right]_0^x$$

$$= -\left[\left(e^{-x/10}\right) - e^0\right] = -e^{-x/10} + 1$$

$$\boxed{F_X(x) = 1 - e^{-x/10}}$$

$$\boxed{F_X(x) = \begin{cases} 1 - e^{-x/10} & \text{si } x \in [0, +\infty[\\ 0 & \text{sinon} \end{cases}}$$

Donc

① $P(X \leq 5) = F_X(5) = 1 - e^{-5/10} = 1 - e^{-1/2} = \boxed{0{,}393}$ ✓

② $P(5 \leq X \leq 10) = F(10) - F(5) = \left(1 - e^{-10/10}\right) - \left(1 - e^{-5/10}\right)$

$= \cancel{1} - e^{-1} \cancel{- 1} + e^{-1/2} = \boxed{e^{-1/2} - e^{-1} = 0{,}239}$ ✓

Ex5 :

X : Durée de vie d'une RAM (Temps d'echec)

$X \sim Exp(\lambda)$ Donc : $\boxed{f_X(x) = \begin{cases} \lambda e^{-\lambda x} & \text{si } x > 0 \\ 0 & \text{si } x \leq 0. \end{cases}}$

$$P(X \leq x) = F_X(x) = \int_0^x f(t)\, dt = \int_0^x \lambda e^{-\lambda t}\, dt = \lambda \int_0^x e^{-\lambda t}\, dt$$

$$= \lambda\left(\frac{-1}{\lambda}\right)\left[e^{-\lambda t}\right]_0^x = \left(-e^{-\lambda x}\right) - \left(-e^{-\lambda(0)}\right) = -e^{-\lambda x} + 1$$

$$\boxed{F_X(x) = \begin{cases} 1 - e^{-\lambda x} & \text{si } x > 0 \\ 0 & \text{si } x \leq 0 \end{cases}}$$

① $P(X > 10^4) = e^{-1}$ $\boxed{\lambda ?}$

$$P(X > 10^4) = 1 - P(X \leq 10^4) = 1 - F_X(10^4) = e^{-1}$$

$$= 1 - \left(1 - e^{-\lambda(10^4)}\right) = e^{-1} \cancel{= e^{-\lambda(10^4)} = e^{-1}}$$

$$= e^{-\lambda(10^4)} = e^{-1}$$

$$-\lambda(10^4) = -1 \Rightarrow \lambda = \frac{1}{10^4} = 10^{-4}$$

$$\boxed{\lambda = 10^{-4}}$$

Donc, $f_X(x) = 10^{-4} e^{-10^{-4} x} \mathbb{1}_{]0,+\infty[}(x)$

② $P(X \leq x_0) \leq 0.05$, dans Pire des cas $P(X \leq x_0) = 0{,}05$

$$F_X(x_0) = 0{,}05 \Rightarrow 1 - e^{-10^{-4} x_0} = 0{,}05$$

$$1 - e^{-10^{-4} x_0} = 0{,}05 \Rightarrow e^{-10^{-4} x_0} = 0{,}95$$

$$-10^{-4} x_0 = \ln(0{,}95)$$

$$\Rightarrow x_0 = -\frac{\ln(0{,}95)}{10^{-4}} = -10^{+4}(\ln(0{,}95))$$

$$x_0 = \boxed{513 \text{ heures}}$$