

Data Analysis and Matrix Computations Assignment 4

Exercise 1.

Consider the subsampled randomized trigonometric transform:

$$\Phi := \sqrt{\frac{n}{s}} \mathbf{R} \mathbf{D} \mathbf{F} \in \mathbb{R}^{s \times n}$$

where $\mathbf{R} \in \mathbb{R}^{s \times n}$ subsamples rows, $\mathbf{D} \in \mathbb{R}^{n \times n}$ is random diagonal, and $\mathbf{F} \in \mathbb{R}^{n \times n}$ is a DCT2 matrix. More precisely, \mathbf{R} is a uniformly random set of s rows drawn from the identity matrix \mathbf{I}_n , and the random diagonal matrix \mathbf{D} has i.i.d. $\text{uniform}\{\pm 1\}$ entries. Prove that

$$\mathbb{E} \|\Phi \mathbf{x}\|_2^2 = \|\mathbf{x}\|_2^2.$$

Exercise 2.

Consider a sparse random matrix of the form

$$\Phi = [\varphi_1 \quad \cdots \quad \varphi_n] \in \mathbb{R}^{s \times n},$$

where $\varphi_i \in \mathbb{R}^s$ are i.i.d. sparse vectors. More precisely, each column φ_i contains exactly ζ nonzero entries, equally likely to be $\pm 1/\sqrt{\zeta}$, in uniformly positions. Prove that

$$\mathbb{E} \|\Phi \mathbf{x}\|_2^2 = \|\mathbf{x}\|_2^2.$$