

# Data Analysis and Matrix Computations Assignment 2

## Exercise 1.

Let  $\mathcal{T}$  denote a random subset of  $\{1, 2, \dots, N\}$  whose cardinality depends on the random integer variable  $T$  that takes values from 1 to  $N$ , where  $\mathcal{T}$  is sampled uniformly. Let  $\mathbf{D}_{\mathcal{T}}$  denote the diagonal random matrix formed by the summation of  $T$  canonical outer products

$$\mathbf{D}_{\mathcal{T}} = \sum_{i \in \mathcal{T}} \mathbf{e}_i \mathbf{e}_i^\top.$$

Prove that

$$\mathbb{E}(\mathbf{D}_{\mathcal{T}}) = \frac{\mathbb{E}(T)}{N} \mathbf{I}.$$

## Exercise 2.

Consider the following relaxed RK algorithm.

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**Algorithm:** Relaxed RK for  $\mathbf{Ax} = \mathbf{b}$

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Initialize  $\mathbf{x}^0 \in \mathbb{R}^n$  and  $0 < \alpha < 2$

**for**  $k = 1, 2, \dots$ , **do**

Pick  $i \in [m]$  with probability  $\frac{\|\mathbf{A}_{i,:}\|_2^2}{\|\mathbf{A}\|_F^2}$

Set  $\mathbf{x}^k = \mathbf{x}^{k-1} - \alpha \frac{\mathbf{A}_{i,:} \mathbf{x}^{k-1} - \mathbf{b}_i}{\|\mathbf{A}_{i,:}\|_2^2} (\mathbf{A}_{i,:})^\top$

**end**

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Prove a convergence result of the relaxed RK for consistent linear systems.

## Reading.

- Straggler-tolerant stationary methods for linear systems

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<https://epubs.siam.org/doi/10.1137/24M1673346>