

# Numerical Linear Algebra Assignment 16

## Exercise 1. (TreBau Exercise 37.1, 10 points)

The standard recurrence relation for Legendre polynomials is

$$P_n(x) = \frac{2n-1}{n}xP_{n-1}(x) - \frac{n-1}{n}P_{n-2}(x) \quad (1)$$

with initial values  $P_0(x) = 1$ ,  $P_1(x) = x$ .

(a) Compute  $P_2(x)$  and  $P_3(x)$ .

(b) Since  $\{P_n(x)\}$  and  $\{q_{n+1}(x)\}$  are normalized differently, (1) is not the same as the recurrence

$$xq_n(x) = \beta_{n-1}q_{n-1}(x) + \alpha_n q_n(x) + \beta_n q_{n+1}(x)$$

with coefficients

$$\alpha_n = 0, \quad \beta_n = \frac{1}{2}(1 - (2n)^{-2})^{-1/2}.$$

Write down the two tridiagonal matrices corresponding to these formulas, and derive the relationship between them.

(c) Use the result of (b) to determine a formula for  $q_{n+1}(1)$ , or equivalently, for  $\|P_n\|$ .

## Exercise 2. (Programming, TreBau Exercise 37.4, 10 points)

- (a) Write a six-line MATLAB program that computes the nodes and weights for the  $n$ -point Gauss–Legendre quadrature formula and applies these numbers to compute the approximate integral of the function  $f$ .
- (b) Taking  $f(x) = e^x$  and  $n = 4$ , confirm the example in the text. Then plot  $|I(e^x) - I_n(e^x)|$  on a log scale for  $n = 1, 2, \dots, 40$  and comment on the results.
- (c) Produce a similar plot for  $f(x) = e^{|x|}$ , and comment.