

Numerical Linear Algebra Assignment 10

Exercise 1. (Zhihao Cao, 10 points)

Let

$$\mathbf{A} = \begin{bmatrix} a_1 & b_1 & & & \\ c_1 & a_2 & b_2 & & \\ & c_2 & a_3 & \ddots & \\ & & \ddots & \ddots & b_{m-1} \\ & & & c_{m-1} & a_m \end{bmatrix} \in \mathbb{R}^{m \times m}, \quad c_j b_j > 0.$$

Prove that there exists a diagonal matrix \mathbf{D} satisfying that $\mathbf{D}^{-1}\mathbf{A}\mathbf{D}$ is symmetric.

Exercise 2. (10 points)

How many eigenvalues does $\mathbf{A} = \begin{bmatrix} -2 & 2 & & \\ 2 & 2 & 1 & \\ & 1 & 2 & -1 \\ & & -1 & 1 \end{bmatrix}$ have in the interval $[1, 2]$? Determine the answer via Sturm sequences.

Exercise 3. (Demmel Question 5.16, 10 points)

Let $\mathbf{A} = \mathbf{D} + \mathbf{u}\mathbf{u}^\top$, where $\mathbf{D} = \text{diag}\{d_1, d_2, \dots, d_n\} \in \mathbb{R}^{n \times n}$ and $\mathbf{u} = [u_1 \ u_2 \ \dots \ u_n]^\top \in \mathbb{R}^n$. Show that if $d_i = d_{i+1}$ or $u_i = 0$, then d_i is an eigenvalue of \mathbf{A} . If $u_i = 0$, show that \mathbf{e}_i is an eigenvector corresponding to d_i .

Exercise 4. (TreBau Exercise 30.3, 10 points)

Show that if the largest off-diagonal entry is annihilated at each step of the Jacobi algorithm, then the sum of the squares of the off-diagonal entries decreases by at least the factor $1 - 2/(m^2 - m)$ at each step.

Exercise 5. (TreBau Exercise 31.3, 10 points)

Show that if the entries on both principal diagonals of a bidiagonal matrix are all nonzero, then the singular values of the matrix are distinct.

Compulsory requirement for programming: Use Matlab's publish to save all your code, comments, and results to a PDF file. You must use the programming format files: example_format.zip.

Programming 1. (TreBau Exercise 30.5, 10 points)

Programming 2. (TreBau Exercise 31.4, 10 points)

Programming 3. (TreBau Exercise 31.5(b), 10 points)

Programming 4. (10 points)

Write a matlab function `r = mysecular(d,beta,w)` to compute the roots of the secular equation

$$f(\lambda) = 1 + \beta \sum_{j=1}^m \frac{w_j^2}{d_j - \lambda} = 0.$$

Design an experiment to test it.