Data Analysis and Matrix Computations Assignment 6

Exercise 1.

Let

$$\pi_{\mathcal{C}}(\mathbf{x}) := \underset{\mathbf{y}\in\mathcal{C}}{\operatorname{argmin}} \|\mathbf{x}-\mathbf{y}\|_2$$

denote the projection of $\mathbf{x} \in \mathbb{R}^n$ onto the set $\mathcal{C} \subseteq \mathbb{R}^n$. Prove the following: If $\mathcal{C}$ is closed convex, then for all $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$,

$$\|\pi_{\mathcal{C}}(\mathbf{x}) - \pi_{\mathcal{C}}(\mathbf{y})\|_2 \leq \|\mathbf{x}-\mathbf{y}\|_2.$$

Exercise 2.

If

$$\mathbb{S}_n = \{\mathbf{A} \in \mathbb{R}^{n\times n} : \mathbf{A} = \mathbf{A}^\top\}$$

is the set of symmetric matrices and, for $\mathbf{A} \in \mathbb{S}_n$, $f : \mathbb{S}_n \mapsto \mathbb{R}$ given by $f(\mathbf{A}) = \lambda_{\max}(\mathbf{A})$ (maximum eigenvalue of $\mathbf{A}$), show that f is convex. Hint: for convexity, show $\mathbb{S}_n$ is convex and f satisfies Jensen's inequality.