Numerical Linear Algebra Assignment 14

Exercise 1. (10 points)

Prove Proposition 2 of Lecture 14.

Exercise 2. (10 points)

Show that an orthonormal basis for the Krylov subspace

$$\mathcal{K}_{2j}\left(\begin{bmatrix}\mathbf{I} & \mathbf{A}\\ \mathbf{A}^* & \mathbf{0}\end{bmatrix}, \begin{bmatrix}\mathbf{b}\\ \mathbf{0}\end{bmatrix}\right)$$

is given by

$$\left\{\begin{bmatrix}\mathbf{u}_1\\ \mathbf{0}\end{bmatrix}, \begin{bmatrix}\mathbf{0}\\ \mathbf{q}_1\end{bmatrix}, \begin{bmatrix}\mathbf{u}_2\\ \mathbf{0}\end{bmatrix}, \begin{bmatrix}\mathbf{0}\\ \mathbf{q}_2\end{bmatrix}, \dots, \begin{bmatrix}\mathbf{u}_j\\ \mathbf{0}\end{bmatrix}, \begin{bmatrix}\mathbf{0}\\ \mathbf{q}_j\end{bmatrix}\right\},$$

where $\{\mathbf{u}_i\}_{i=1}^j$ and $\{\mathbf{q}_i\}_{i=1}^j$ are the vectors generated by Golub–Kahan bidiagonalization for $\begin{bmatrix}\mathbf{b} & \mathbf{A}\end{bmatrix}$. (Here we assume all the vectors are well-defined.)

Exercise 3. (Programming, 10 points)

Write two matlab functions, `[U,B,V]=hb(A)` and `[U,B,V]=gkb(A)`, to implement Householder bidiagonalization and Golub–Kahan bidiagonalization for the bidiagonal decomposition in Proposition 1 of Lecture 14. For simplicity, we only consider the case $m = n$. Test the 4×4 complex matrix ($\mathrm{i} = \sqrt{-1}$)

$$\mathbf{A} = \begin{bmatrix} 1+1\mathrm{i} & -1\mathrm{i} & 0 & 1\mathrm{i}\\ 1 & 1+1\mathrm{i} & 1-1\mathrm{i} & 1+3\mathrm{i}\\ 0 & 1\mathrm{i} & -1\mathrm{i} & -1\mathrm{i}\\ 2\mathrm{i} & 1 & 0 & 0\end{bmatrix}.$$

Exercise 4. (Programming, 10 points)

Write matlab codes to implement the approach (i.e., via a sequence of Givens rotations) introduced in Lecture 14 for the least squares problem with bidiagonal structure. Design a numerical experiment to verify your codes.