

# Numerical Linear Algebra Assignment 17

## Exercise 1. (10 points)

Assume that  $n$  is even. Let  $\omega_n = e^{-i2\pi/n}$ ,  $\mathbf{F}_n = \left[ \omega_n^{ij} \right]_{i,j=0}^{n-1}$ , and

$$\mathbf{D} = \text{diag}\{1, \omega_n, \dots, \omega_n^{n/2-1}\}.$$

Construct a matrix  $\mathbf{M}$  with entries 0 or 1 such that

$$\mathbf{F}_n = \begin{bmatrix} \mathbf{I} & \mathbf{D} \\ \mathbf{I} & -\mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{F}_{n/2} & \\ & \mathbf{F}_{n/2} \end{bmatrix} \mathbf{M}.$$

## Exercise 2. (10 points)

Prove Lemma 5 of Lecture 17.

## Exercise 3. (10 points)

Prove Theorem 6 of Lecture 17.

## Exercise 4. (10 points)

Prove Theorem 10 of Lecture 17.

## Exercise 5. (Programming, 10 points)

Write a matlab function (`g = myfft(f)`) to implement FFT and test its performance. For simplicity, you can assume that  $\mathbf{f} \in \mathbb{R}^n$  with  $n = 2^k$ .