Data Analysis and Matrix Computations Assignment 5

Exercise 1.

Let $f(\mathbf{x}) = \mathbf{x}^\top \mathbf{A}\mathbf{x} + 2\mathbf{b}^\top \mathbf{x} + c$, where $\mathbf{A}$ is an $n \times n$ symmetric positive definite matrix, $\mathbf{b} \in \mathbb{R}^n$, and $\mathbf{c} \in \mathbb{R}$. Let $\mathbf{x}_k \in \mathbb{R}^n$ and let $\mathbf{d}_k \in \mathbb{R}^n$ be a descent direction of f at $\mathbf{x}_k$. Compute the stepsize generated by exact line search.

Exercise 2.

(1) Let $f(\mathbf{x}) = \mathbf{x}^\top \mathbf{A}\mathbf{x} + 2\mathbf{b}^\top \mathbf{x} + c$, where $\mathbf{A}$ is an $n \times n$ symmetric matrix, $\mathbf{b} \in \mathbb{R}^n$, and $\mathbf{c} \in \mathbb{R}$. Show that the *smallest* Lipschitz constant of ∇f is $2\|\mathbf{A}\|$.

(2) Let $f \in C_L^{1,1}(\mathbb{R}^m)$, and let $\mathbf{A} \in \mathbb{R}^{m \times n}$, $\mathbf{b} \in \mathbb{R}^m$. Show that the function $g : \mathbb{R}^n \to \mathbb{R}$ defined by $g(\mathbf{x}) = f(\mathbf{A}\mathbf{x} + \mathbf{b})$ satisfies $g \in C_{\widetilde{L}}^{1,1}(\mathbb{R}^n)$, where $\widetilde{L} = L\|\mathbf{A}\|^2$.