

# Numerical Linear Algebra Assignment 13

## Exercise 1. (TreBau Exercise 39.1, 10 points)

Consider a problem  $\mathbf{Ax} = \mathbf{b}$  for the matrix  $\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_{m-1} \\ 1 & \mathbf{0} \end{bmatrix}$ .

- (a) Show that the singular values are all 1 and that this implies that CGN converges in one step.
- (b) Show that the eigenvalues are the  $m$ th roots of unity and that this implies that GMRES requires  $m$  steps to converge for general  $\mathbf{b}$ .
- (c) This matrix  $\mathbf{A}$  has so much structure that one does not need to consider eigenvalues or singular values to understand its convergence behavior. In particular, explain by elementary argument why GMRES takes  $m$  steps to converge for the right-hand side  $\mathbf{b} = [1 \ 0 \ \cdots \ 0]^\top$ .

## Exercise 2. (TreBau Exercise 39.2, 10 points)

As a converse to Exercise 1, devise an example of a matrix of arbitrary dimension  $m$  with almost the opposite property: GMRES converges in two steps, but CGN requires  $m$  steps.

## Exercise 3. (Programming, 10 points)

- (1) Write matlab code to generate matrices for the cases (a)–(g) in TreBau Exercise 39.5.
- (2) Compare CG, GMRES, CGN, and Bi-CG for linear systems with these matrices and right-hand-side  $\mathbf{b} = [1 \ \cdots \ 1]^\top$ .
- (3) Explain your numerical results.