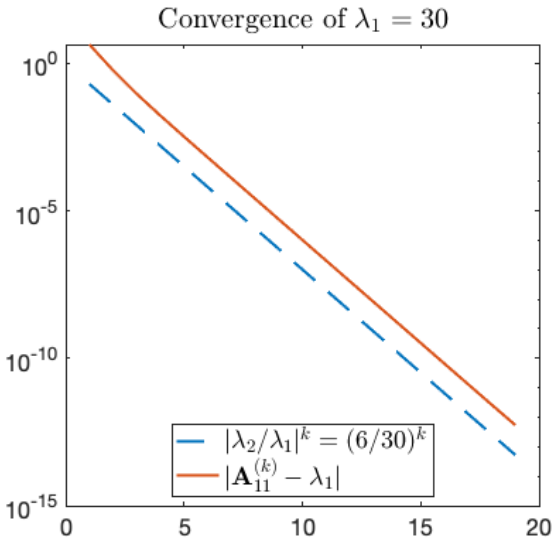
Contents

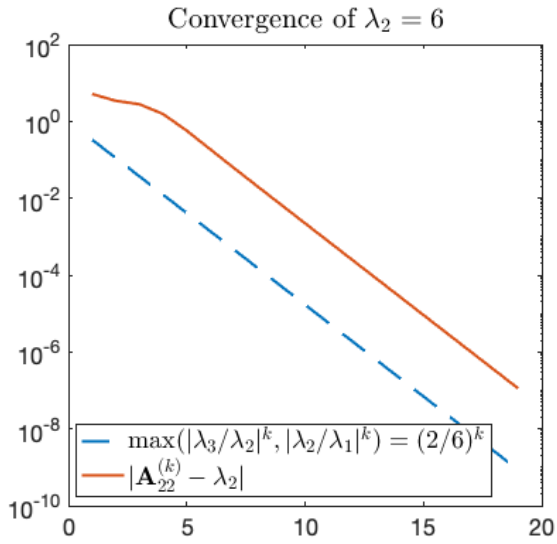
Convergence of subspace iteration

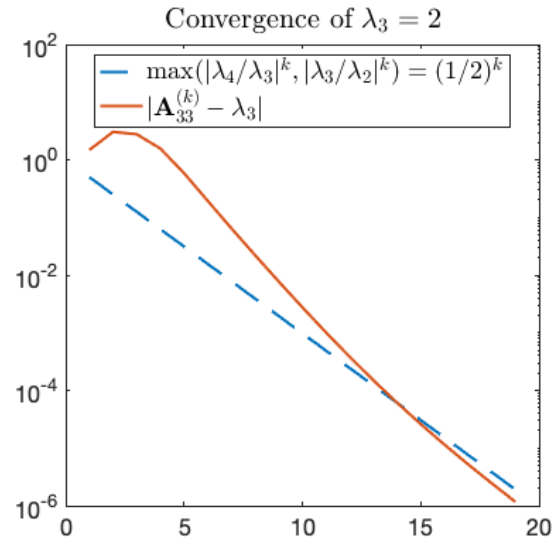
```
clear; clc; format short; format compact;
L=diag([1 2 6 30]);
S=randn(4); %condnumberS=cond(S)
A=S*L/S;
Q=eye(4);
lambda_1cr=zeros(19,1);
lambda_2cr=zeros(19,1);
lambda_3cr=zeros(19,1);
lambda_4cr=zeros(19,1);
Ablock_1=zeros(19,1);
Ablock_2=zeros(19,1);
Ablock_3=zeros(19,1);
for k=1:19
    Z=A*Q;
    [Q,R]=qr(Z);
    Ak=Q'*A*Q;
    lambda_1cr(k)=abs(Ak(1,1)-30);
    lambda_2cr(k)=abs(Ak(2,2)-6);
    lambda_3cr(k)=abs(Ak(3,3)-2);
    lambda_4cr(k)=abs(Ak(4,4)-1);
    Ablock_1(k)=norm(Ak(2:4,1:1));
    Ablock_2(k)=norm(Ak(3:4,1:2));
    Ablock_3(k)=norm(Ak(4:4,1:3));
end
A19=Ak
set(0, 'defaultaxeslinewidth',  1);
set(0, 'defaultaxesfontsize',   16);
figure('Position',[380 320 1000 400]);
subplot(1,2,1)
cr1=(6/30).^(1:19);
semilogy(1:19,cr1,'--',1:19,lambda_1cr,'-','LineWidth',2)
legend({'$|\lambda_2/\lambda_1|^k=(6/30)^k$',...
        '$|{\bf A}^{(k)}_{11}-\lambda_1|$'},...
        'Location','Best','Interpreter','latex','FontSize',18);
title('Convergence of $\lambda_1=30$','Interpreter','latex',...
      'FontSize',20)
subplot(1,2,2)
cr2=(2/6).^(1:19);
semilogy(1:19,cr2,'--',1:19,lambda_2cr,'-','LineWidth',2)
legend(...
{'$\max(|\lambda_3/\lambda_2|^k,|\lambda_2/\lambda_1|^k)=(2/6)^k$',...
    '$|{\bf A}^{(k)}_{22}-\lambda_2|$'},...
        'Location','Best','Interpreter','latex','FontSize',18);
title('Convergence of $\lambda_2=6$','Interpreter','latex','FontSize',20)
set(0, 'defaultaxeslinewidth',  1);
set(0, 'defaultaxesfontsize',   16);
figure('Position',[380 320 1000 400]);
subplot(1,2,1)
cr3=(1/2).^(1:19);
semilogy(1:19,cr3,'--',1:19,lambda_3cr,'-','LineWidth',2)
legend(...
{'$\max(|\lambda_4/\lambda_3|^k,|\lambda_3/\lambda_2|^k)=(1/2)^k$',...
    '$|{\bf A}^{(k)}_{33}-\lambda_3|$'},...
        'Location','Best','Interpreter','latex','FontSize',18);
title('Convergence of $\lambda_3=2$','Interpreter','latex','FontSize',20)
subplot(1,2,2)
cr4=(1/2).^(1:19);
semilogy(1:19,cr4,'--',1:19,lambda_4cr,'-','LineWidth',2)
legend({'$|\lambda_4/\lambda_3|^k=(1/2)^k$',...
    '$|{\bf A}^{(k)}_{44}-\lambda_4|$'},...
        'Location','Best','Interpreter','latex','FontSize',18);
title('Convergence of $\lambda_4=1$','Interpreter','latex','FontSize',20)
set(0, 'defaultaxeslinewidth',  1);
set(0, 'defaultaxesfontsize',   16);
figure('Position',[380 320 1000 400]);
subplot(1,2,1)
Acr1=(6/30).^(1:19);
semilogy(1:19,Acr1,'--',1:19,Ablock_1,'-','LineWidth',2)
legend({'$|\lambda_2/\lambda_1|^k=(6/30)^k$',...
    '$\|{\bf A}^{(k)}(2:4,1)\|_2$'},...
        'Location','Best','Interpreter','latex','FontSize',18);
title('Convergence of ${\bf A}^{(k)}(2:4,1)$',...
    'Interpreter','latex','FontSize',20)
subplot(1,2,2)
Acr2=(2/6).^(1:19);
semilogy(1:19,Acr2,'--',1:19,Ablock_2,'-','LineWidth',2)
legend({'$|\lambda_3/\lambda_2|^k=(2/6)^k$',...
    '$\|{\bf A}^{(k)}(3:4,1:2)\|_2$'},...
        'Location','Best','Interpreter','latex','FontSize',18);
title('Convergence of ${\bf A}^{(k)}(3:4,1:2)$','Interpreter',...
    'latex','FontSize',20)
set(0, 'defaultaxeslinewidth',  1);
set(0, 'defaultaxesfontsize',   16);
```

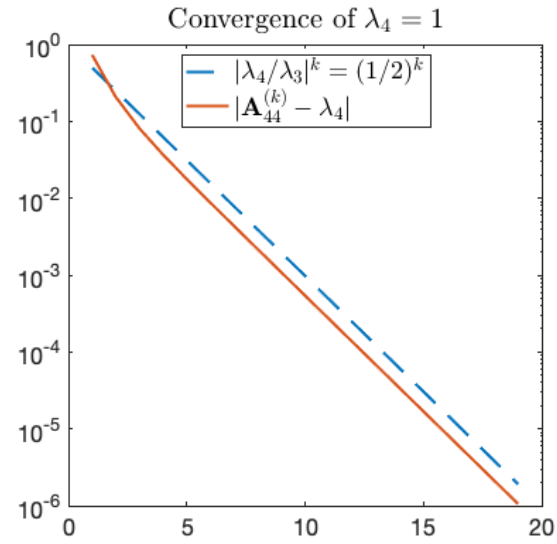
```
figure('Position',[380 320 1000 400]);
subplot(1,2,1)
Acr3=(1/2).^(1:19);
semilogy(1:19,Acr3,'--',1:19,Ablock_3,'-','LineWidth',2)
legend({'$|\lambda_4/\lambda_3|^k=(1/2)^k$',...
    '$\|{\bf A}^{(k)}(4,1:3)\|_2$'},...
        'Location','Best','Interpreter','latex','FontSize',18);
title('Convergence of ${\bf A}^{(k)}(4,1:3)$','Interpreter',...
    'latex','FontSize',20)
```

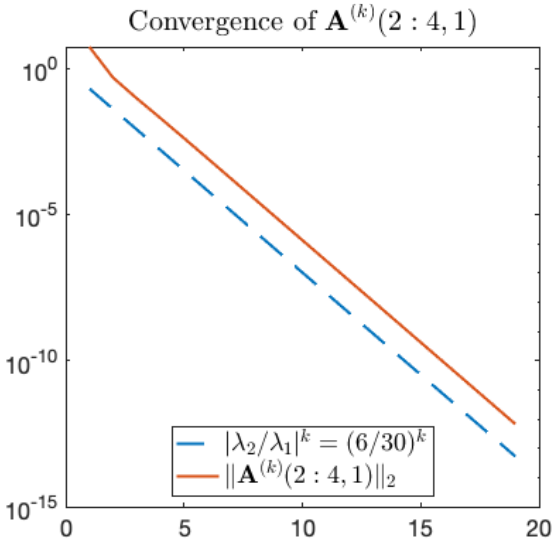
```
A19 =
   30.0000  -18.9573   28.6412   13.3758
    0.0000    6.0000   -2.5801   -7.5521
   -0.0000   -0.0000    2.0000   -1.9024
   -0.0000    0.0000    0.0000    1.0000
```

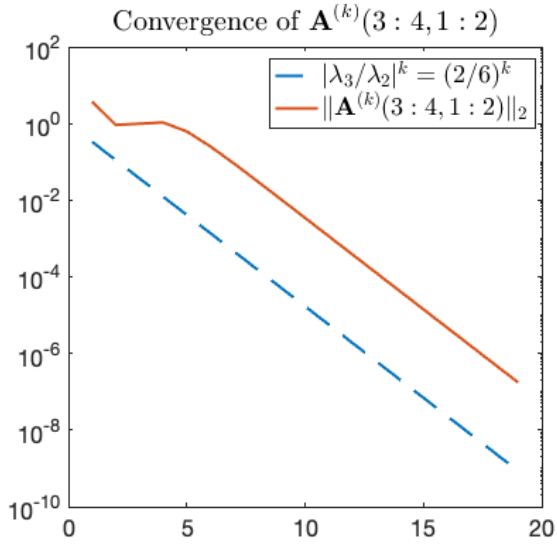


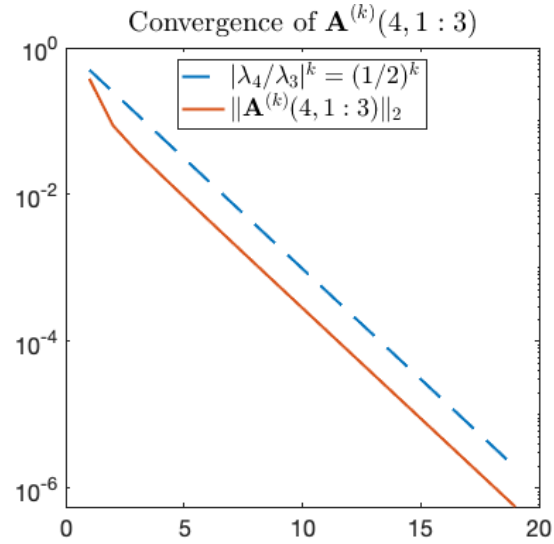












Convergence of QR with Rayleigh quotient shift

```
clear; clc; format short e; format compact;
L=diag([1 2 6 30]);
S=randn(4); condnumberS=cond(S)
A=S*L/S;
for k=1:10
    [Q,R]=qr(A-A(4,4)*eye(4));
    A=R*Q+A(4,4)*eye(4);
    A4r=A(4,:)
end
A10=A
```

```
condnumberS =
   7.7164e+00
A4r =
  -1.2729e+00   4.5561e-01   3.8986e-03   1.4929e+00
A4r =
  -2.1179e-02  -4.4886e-02   5.4344e-01   1.4829e+00
A4r =
   3.9417e-04   9.2845e-03   5.1617e-01   1.6695e+00
A4r =
  -4.7830e-06  -7.6033e-04   2.9481e-01   1.8499e+00
A4r =
   2.8480e-08   3.1060e-05   6.4659e-02   2.0047e+00
A4r =
   4.7342e-12   3.6200e-08  -2.9796e-04   2.0000e+00
A4r =
   6.9469e-18   3.7197e-13   1.2237e-08   2.0000e+00
A4r =
   4.1911e-28   1.5708e-22  -2.0673e-17   2.0000e+00
A4r =
   4.2728e-47   1.1211e-40   5.9012e-35   2.0000e+00
A4r =
   1.2434e-83   2.2836e-76  -4.8084e-70   2.0000e+00
A10 =
   3.0000e+01  -3.2532e+01   2.2698e+01  -5.0885e+00
   1.4845e-07   6.0000e+00  -6.6469e+00   3.2627e+00
```

```
   -3.5683e-14  -1.9507e-06   1.0000e+00   1.3808e-01
    1.2434e-83   2.2836e-76  -4.8084e-70   2.0000e+00
```

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