Numerical Linear Algebra Assignment 10

Exercise 1. (10 points)

How many eigenvalues does $\mathbf{A} = \begin{bmatrix} -2 & 2 & & \\ 2 & 2 & 1 & \\ & 1 & 2 & -1 \\ & & -1 & 1 \end{bmatrix}$ have in the interval $[1, 2]$? Determine the answer via Sturm sequences.

Exercise 2. (Demmel Question 5.16, 10 points)

Let $\mathbf{A} = \mathbf{D} + \mathbf{u}\mathbf{u}^\top$, where $\mathbf{D} = \mathrm{diag}\{d_1, d_2, \ldots, d_n\} \in \mathbb{R}^{n \times n}$ and $\mathbf{u} = \begin{bmatrix} u_1 & u_2 & \cdots & u_n \end{bmatrix}^\top \in \mathbb{R}^n$. Show that if $d_i = d_{i+1}$ or $u_i = 0$, then d_i is an eigenvalue of $\mathbf{A}$. If $u_i = 0$, show that $\mathbf{e}_i$ is an eigenvector corresponding to d_i.

Exercise 3. (TreBau Exercise 30.3, 10 points)

Show that if the largest off-diagonal entry is annihilated at each step of the Jacobi algorithm, then the sum of the squares of the off-diagonal entries decreases by at least the factor $1 - 2/(m^2 - m)$ at each step.

Exercise 4. (TreBau Exercise 31.3, 10 points)

Show that if the entries on both principal diagonals of a bidiagonal matrix are all nonzero, then the singular values of the matrix are distinct.

Exercise 5. (Programming, TreBau Exercise 30.5, 10 points)