Data Analysis and Matrix Computations Assignment 4

Exercise 1.

Consider the subsampled randomized trigonometric transform:

$$\mathbf{\Phi} := \sqrt{\frac{n}{s}}\mathbf{RFD} \in \mathbb{R}^{s\times n}$$

where $\mathbf{R} \in \mathbb{R}^{s\times n}$ subsamples rows, $\mathbf{F} \in \mathbb{R}^{n\times n}$ is a DCT2 matrix, and $\mathbf{D} \in \mathbb{R}^{n\times n}$ is random diagonal. More precisely, $\mathbf{R}$ is a uniformly random set of s rows drawn from the identity matrix $\mathbf{I}_n$, and the random diagonal matrix $\mathbf{D}$ has i.i.d. uniform$\{\pm 1\}$ entries. Prove that

$$\mathbb{E}\|\mathbf{\Phi x}\|_2^2 = \|\mathbf{x}\|_2^2.$$

Exercise 2.

Consider a sparse random matrix of the form

$$\mathbf{\Phi} = \begin{bmatrix} \boldsymbol{\varphi}_1 & \cdots & \boldsymbol{\varphi}_n \end{bmatrix} \in \mathbb{R}^{s\times n},$$

where $\boldsymbol{\varphi}_i \in \mathbb{R}^s$ are i.i.d. sparse vectors. More precisely, each column $\boldsymbol{\varphi}_i$ contains exactly ζ nonzero entries, equally likely to be $\pm 1/\sqrt{\zeta}$, in uniformly positions. Prove that

$$\mathbb{E}\|\mathbf{\Phi x}\|_2^2 = \|\mathbf{x}\|_2^2.$$