Numerical Linear Algebra Assignment 17

Exercise 1. (10 points)

Assume that n is even. Let $\omega_n = \mathrm{e}^{-\mathrm{i}2\pi/n}$, $\mathbf{F}_n = \left[\omega_n^{ij}\right]_{i,j=0}^{n-1}$, and

$$\mathbf{D} = \mathrm{diag}\{1, \omega_n, \ldots, \omega_n^{n/2-1}\}.$$

Construct a matrix $\mathbf{M}$ with entries 0 or 1 such that

$$\mathbf{F}_n = \begin{bmatrix} \mathbf{I} & \mathbf{D} \\ \mathbf{I} & -\mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{F}_{n/2} & \\ & \mathbf{F}_{n/2} \end{bmatrix} \mathbf{M}.$$

Exercise 2. (10 points)

Prove Lemma 5 of Lecture 17.

Exercise 3. (10 points)

Prove Theorem 6 of Lecture 17.

Exercise 4. (10 points)

Prove Theorem 10 of Lecture 17.

Compulsory requirement for programming: Use Matlab's `publish` to save all your code, comments, and results to a PDF file. You must use the programming format files: example_format.zip.

Programming 1. (10 points)

Write a matlab function (`g = myfft(f)`) to implement FFT and test its performance. For simplicity, you can assume that $\mathbf{f} \in \mathbb{R}^n$ with $n = 2^k$.

Programming 2. (10 points)

Write a matlab function (`v = circmvp(C,u)`) to implement Fast algorithm 1 of Lecture 17 and test its performance.

Programming 3. (10 points)

Write a matlab function (`v = toepmvp(T,u)`) to implement Fast algorithm 2 of Lecture 17 and test its performance.