Numerical Linear Algebra Assignment 18

Exercise 1. (10 points)

Consider the FD system $\mathbf{A}_h \mathbf{u}^h = \mathbf{f}^h$ on the fine grid, i.e.,

$$\frac{1}{h^2}\begin{bmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ & & & -1 & 2 \end{bmatrix}\begin{bmatrix} u_1^h \\ u_2^h \\ \vdots \\ u_{n-1}^h \\ u_n^h \end{bmatrix} = \begin{bmatrix} f_1^h \\ f_2^h \\ \vdots \\ f_{n-1}^h \\ f_n^h \end{bmatrix}.$$

Let $H = 2h$ and define

$$\mathbf{I}_H^h = \frac{1}{2}\begin{bmatrix} 1 & & \\ 2 & & \\ 1 & 1 & \\ & \vdots & \vdots & \vdots \\ & & 1 \end{bmatrix}, \quad \mathbf{I}_h^H = \frac{1}{4}\begin{bmatrix} 1 & 2 & 1 & & \\ & & 1 & \cdots & \\ & & & \cdots & \\ & & & \cdots & 1 \end{bmatrix}.$$

Let $\mathbf{A}_H$ be the FD discretization matrix on the coarse grid. Prove that

$$\mathbf{A}_H = \mathbf{I}_h^H \mathbf{A}_h \mathbf{I}_H^h.$$

Exercise 2. (Programming, 10 points)

Write matlab codes to verify your conclusion in Exercise 1.