

Numerical Linear Algebra Assignment 14

Exercise 1. (10 points)

Prove Proposition 2 of Lecture 14.

Exercise 2. (10 points)

Show that an orthonormal basis for the Krylov subspace

$$\mathcal{K}_{2k} \left(\begin{bmatrix} \mathbf{I} & \mathbf{A} \\ \mathbf{A}^* & \mathbf{0} \end{bmatrix}, \begin{bmatrix} \mathbf{b} \\ \mathbf{0} \end{bmatrix} \right)$$

is given by

$$\left\{ \begin{bmatrix} \mathbf{u}_1 \\ \mathbf{0} \end{bmatrix}, \begin{bmatrix} \mathbf{0} \\ \mathbf{q}_1 \end{bmatrix}, \begin{bmatrix} \mathbf{u}_2 \\ \mathbf{0} \end{bmatrix}, \begin{bmatrix} \mathbf{0} \\ \mathbf{q}_2 \end{bmatrix}, \dots, \begin{bmatrix} \mathbf{u}_k \\ \mathbf{0} \end{bmatrix}, \begin{bmatrix} \mathbf{0} \\ \mathbf{q}_k \end{bmatrix} \right\},$$

where $\{\mathbf{u}_i\}_{i=1}^k$ and $\{\mathbf{q}_i\}_{i=1}^k$ are the vectors generated by Golub–Kahan bidiagonalization for $\begin{bmatrix} \mathbf{b} & \mathbf{A} \end{bmatrix}$. (Here we assume all the vectors are well-defined.)

Exercise 3. (Programming, 10 points)

Write two matlab functions, `[U,B,V]=hb(A)` and `[U,B,V]=gkb(A)`, to implement Householder bidiagonalization and Golub–Kahan bidiagonalization for the bidiagonal decomposition in Proposition 1 of Lecture 14. For simplicity, we only consider the case $m = n$. Test the 4×4 complex matrix ($i = \sqrt{-1}$)

$$\mathbf{A} = \begin{bmatrix} 1 + i & -i & 0 & i \\ 1 & 1 + i & 1 - i & 1 + 3i \\ 0 & i & -i & -i \\ 2i & 1 & 0 & 0 \end{bmatrix}.$$

Exercise 4. (Programming, 10 points)

Write matlab codes to implement the approach (i.e., via a sequence of Givens rotations) introduced in Lecture 14 for the least squares problem with bidiagonal structure. Design a numerical experiment to verify your codes.