Numerical Linear Algebra Assignment 17

Exercise 1. (10 points)

Assume that n is even. Let $\omega_n = \mathrm{e}^{-\mathrm{i}2\pi/n}$, $\mathbf{F}_n = \left[\omega_n^{ij}\right]_{i,j=0}^{n-1}$, and

$$\mathbf{D} = \operatorname{diag}\{1, \omega_n, \ldots, \omega_n^{n/2-1}\}.$$

Construct a matrix $\mathbf{M}$ with entries 0 or 1 such that

$$\mathbf{F}_n = \begin{bmatrix} \mathbf{I} & \mathbf{D} \\ \mathbf{I} & -\mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{F}_{n/2} & \\ & \mathbf{F}_{n/2} \end{bmatrix} \mathbf{M}.$$

Exercise 2. (10 points)

Prove Lemma 5 of Lecture 17.

Exercise 3. (10 points)

Prove Theorem 6 of Lecture 17.

Exercise 4. (10 points)

Prove Theorem 10 of Lecture 17.

Exercise 5. (10 points)

Write down all the eigenpairs of the $n \times n$ tridiagonal Toeplitz matrix

$$\mathbf{T}_n = \begin{bmatrix} b & c & & \\ a & \ddots & \ddots & \\ & \ddots & \ddots & c \\ & & a & b \end{bmatrix}.$$

Exercise 6. (Programming, 10 points)

Write a MATLAB function (`g = myfft(f)`) to implement FFT and test its performance. For simplicity, you can assume that $\mathbf{f} \in \mathbb{R}^n$ with $n = 2^k$.