Numerical Linear Algebra Assignment 8

Exercise 1. (TreBau Exercise 25.2, 10 points)

Let $e_1, e_2, e_3, \cdots$ be a sequence of nonnegative numbers representing errors in some iterative process that converge to zero, and suppose there are a constant C and an exponent α such that for all sufficiently large k, $e_{k+1} \leq C(e_k)^\alpha$. Then, (1) *linear convergence* or *geometric convergence*: $\alpha = 1$ and $C < 1$; (2) *quadratic convergence*: $\alpha = 2$; (3) *cubic convergence*: $\alpha = 3$.

(a) Suppose we want an answer of accuracy $\mathcal{O}(\varepsilon_{\text{machine}})$. Assuming the amount of work for each step is $\mathcal{O}(1)$, show that the total work requirement in the case of linear convergence is $\mathcal{O}(|\log(\varepsilon_{\text{machine}})|)$. How does the constant C enter into your work estimate?

(b) Show that in the case of superlinear convergence, i.e., $\alpha > 1$, the work requirement becomes $\mathcal{O}(\log(|\log(\varepsilon_{\text{machine}})|))$. (Hint: The problem may be simplified by defining a new error measure $f_k = C^{1/(\alpha-1)}e_k$.) How does the exponent α enter into your work estimate?

Exercise 2. (Programming, 10 points)

Construct a 4×4 matrix $\mathbf{A}$ by the following Matlab scripts:

```
L=diag([1 2 6 30]); S=randn(4); A=S*L*inv(S);
```

Compare the convergence of Power iteration, Inverse iteration, and Rayleigh quotient iteration. You must use Matlab's `semilogy` to draw three pictures: the x-axis is the iteration index k, and the y-axis is the absolute error of the computed approximate eigenvalues, i.e., $|\lambda^{(k)} - \lambda|$. For each method, stop at the 10th iteration.