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CS270-002
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Induction Homework

*referring to Theorem in question number when saying Theorem number

1.

Base Case:

The smallest value of n is n = 1.

$\sum_{k=1}^{n=1}(1)^2 = \frac{1(1+1)(2(1)+1))}{6}$

$1 = \frac{6}{6}$

1=1

Thus, the claim of Theorem 1 holds for the Base Case.

Inductive Hypothesis:

Assume that for any integer $1 \leq n \leq n$, it is true that

$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} = \frac{2n^3+3n^2+n}{6}$

Inductive Proof:

Show that when the theory holds for n then it must hold for n + 1.

$(\sum_{k=1}^{n+1} k^2) = \frac{(n+1)((n+1)+1)(2(n+1)+1)}{6}$ Restate problem with n + 1

$(\sum_{k=1}^{n} k^2) + (n+1)^2 = \frac{(n+1)(n+2)(2n+3)}{6}$ Expand Sum and Simplify Right

$(\frac{2n^3+3n^2+n}{6}) + (n+1)^2 = \frac{(n^2+3n+2)(2n+3)}{6}$ Apply Inductive Hypothesis and Simplify Right

$\frac{(2n^3+3n^2+n)}{6} + (n^2+2n+1) = \frac{(2n^3+6n^2+4n)+(3n^2+9n+6)}{6}$ Simplify Left and Right

$$\frac{(2n^3+3n^2+n+6n^2+12n+6)}{6} = \frac{(2n^3+6n^2+4n+3n^2+9n+6)}{6}$$

$$\frac{2n^3+9n^2+13n+6}{6} = \frac{2n^3+9n^2+13n+6}{6}$$

This shows that under the Inductive Hypothesis, Theorem 1 holds.

Conclusion:

Both the Base Case and Inductive Case hold, therefore Theorem 1 holds ∀n ∈ N

Q.E.D

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2.

Base Case:

The smallest value of n is n = 1 since $\forall n \geq 1$.

$T(1) \leq 2$ is true since T(1) is defined as the maximum number of terms in a Boolean expression in CNF with only one variable, x_1, which is 1. Furthermore, $1 \leq 2$ is true.

Thus, the claim of Theorem 2 holds for the Base Case.

Inductive Hypothesis:

Assume that for any integer $1 \leq n \leq n$, it is true that

$T(n) \leq 2^n$

Inductive Proof:

Show that when the theory holds for n then it must hold for n + 1.

Assume that there are n + 1 variables $x_1, x_2, x_3, \ldots, x_{n+1}$

By Inductive Hypothesis, the maximum number of terms in a Boolean expression in CNF that contains $x_1, x_2, x_3, \ldots, x_n$ is less than or equal to 2^n.

Since we assume that the CNF expression is written without repetitions of symbols within the terms, and without identical repetitions of terms, there are two possibilities for x_{n+1}; if it exists or does not exist.

Therefore,

$T(n+1) = 2 \times 2^n \leq 2^{n+1}$

$T(n+1) = 2^{n+1} \leq 2^{n+1}$

This shows that under the Inductive Hypothesis, Theorem 2 holds.

Conclusion:

Both the Base Case and Inductive Case hold, therefore, Theorem 2 holds $\forall n \geq 1$

Q.E.D

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3.

Base Case:

n = 0 since we want to start at the first odd integer ($\forall a \geq 1$). We get the first odd integer using $a = 2 \times n + 1$ and n. a represents the odd integer.

$$\frac{\left((2 \times 0) + 1\right)^2 - 1}{8} = 0$$

0 is an integer, therefore $a^2 - 1$ is divisible by 8

Thus, the claim of Theorem 3 holds for the Base Case.

Inductive Hypothesis:

Assume that for any integer $0 \leq n \leq n$, it is true that

$\frac{a^2-1}{8} = \frac{\left((2 \times n)+1\right)^2-1}{8} = \frac{4n^2+4n+1-1}{8} = \frac{4(n^2+4n)}{8} = I$, where I is an integer which shows that $a^2 - 1$ is divisible by 8

Inductive Proof:

Show that when the theory holds for n then it must hold for n + 1.

$\frac{\left((2 \times (n+1))+1\right)^2-1}{8} = I$ Restate problem with n + 1

$\frac{4n^2+12n+9-1}{8} = I$ Factor Left

$\frac{4(n^2+n)+8n+8}{8} = I$ Simplify Left

$I + \frac{8n+8}{8} = I$ Apply Inductive Hypothesis

$I + n + 1 = I$ Simplify Left

Since I, n, and 1 are integers, $I + n + 1 = I$ is true.

This shows that under the Inductive Hypothesis, Theorem 3 holds.

Conclusion:

Both the Base Case and Inductive Case hold, therefore, Theorem 3 holds ∀a ≥ 1

Q.E.D

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4.

Base Cases:

The smallest values of n are 1 and 2 since $N \geq 1$

$S(1): F(1) = 2(1) + 1 = 3$

$S(2): F(2) = 2(2) + 1 = 5$

The conds in the function F tell us that $F(1) = 3$ and $F(2) = 5$

Thus, the claim of Theorem 4 holds for the Base Cases.

Inductive Hypothesis:

Assume that for any integer $1 \leq n \leq n$, it is true that

$S(n): F(n) = 2n + 1$

Inductive Proof:

Show that when the theory holds for n then it must hold for n + 1

$S(n+1): F(n+1) = 2(n+1) + 1$	Restate problem with n + 1
$F(n+1) = 2n + 3$	Simplify Right
$2F(n+1-1) - F(n+1-2) = 2n + 3$	Recursive Definition of $F(n+1)$
$2F(n) - F(n-1) = 2n + 3$	Simplify Left
$2(2n+1) - (2(n-1)+1) = 2n + 3$	Apply Inductive Hypothesis
$4n + 2 - 2n + 2 - 1 = 2n + 3$	Simplify Left
$2n + 3 = 2n + 3$	

This shows that under the Inductive Hypothesis, Theorem 4 holds.

Conclusion:

Both the Base Cases and Inductive Case hold, therefore Theorem 4 holds ∀n ∈ N

Q.E.D

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5.

<u>Base Cases:</u>

v = 12, 13, 14, 15 since any amount $v > 11$

$(12) = ((3)\ \times 4)$

$(13) = ((2)\ \times 4) + ((1)\ \times 5)$

$(14) = ((1)\ \times 4) + ((2) \times 5)$

$(15) = ((3) \times 5)$

<u>Inductive Hypothesis:</u>

Assume that for any integer $12 \leq P \leq c, c \geq 15$, it is true that

$P = (k \times 4) + (n \times 5) = 4k + 5n$

<u>Inductive Proof:</u>

Show that when the theory holds for P, it must hold for c + 1

$(c + 1)$ = 4a + 5b   Restate problem with c + 1

$(c + 1\ )$ = 4a + 5b

If $k \geq 1$ then $a = k - 1$ and $b = n + 1$,   Apply Inductive Hypothesis

$(c + 1) = 4(k - 1) + 5(n + 1)$

$(c + 1) = 4k - 4 + 5n + 5$ Simplify

$(c + 1) = 4k + 5n + (5 - 4)$

This shows that under the Inductive Hypothesis, Theorem 5 holds.

<u>Conclusion:</u>

Both the Base Cases and Inductive Case hold, therefore Theorem 5 holds ∀v > 11

Q.E.D