CS 270 - Lab 10

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Week 5

Directions

Download this worksheet from Blackboard, and fill it out. Then upload the completed worksheet into the submission bin on blackboard. This should be completed by **Sunday 11:59pm , May 5** 2019. Be sure to list the names of of the team. Only three or four people per team, as a limit. Credit will not be given (grader's choice) to those in excess of four. The instructor will ask you to list the names in your team before you leave class on Thursday, May 2.

1 Introduction

Assign Roles:

Each member of your group must be assigned a new role. Roles change for each activity period. If you have 3 people in your group, combine Facilitator and Developer roles.

Facilitator: reads the questions aloud and makes sure everyone contributes. Reports on team members participation.

Spokesperson: talks to the instructor, TA, and other teams. Only the spokesperson may raise their hand to ask a question.

Scribe: Records the team's answers on the Activity Sheet.

Developer: Writes and executes code. Uses computers to calculate answers. The developer is the only group member that may use a device (computer, phone, etc.) for entering or running your group's code during an activity.

Enter the name of the student in each role

Facilitator: ______________________________

Spokesperson: ______________________________

Scribe: ______________________________

Developer: ______________________________

Score (out of 56): __________ **Graded By**: ______________________________

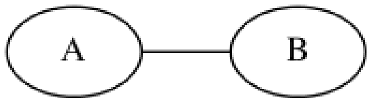
2 3 Color Problem

Any Boolean Formula can be satisfied by coloring in a graph. This is called the 3-Color Problem.

In this section, you will see how a Boolean Formula is related to a Graph Coloring.

2.1 Rules of the 3-Color Problem

The below imagine contains a graph. The circles are called **nodes** and the lines are called **edges**. Each **node** may have a **label** written inside it.



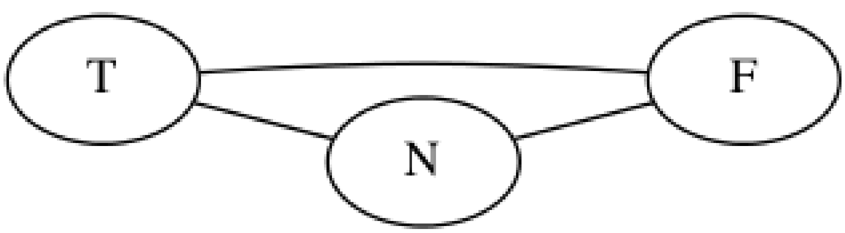
In the 3-Color Problem, we want to color in the **nodes**. We have exactly three colors. For example, we could use red, yellow, and blue. If two nodes are connected by an edge, they cannot be the same color. The goal is to color all the nodes in.

- Select 3 Colors.
- Try to color in all the nodes.
- If two nodes are connected by an edge, they **cannot** be the same color.

We will show in this exercise that if you can color in the nodes, it will tell you the true/false values of the variables in a Boolean Expression. If you cannot, it means the Boolean Expressions is contradictory.

Question 1 : 4 points

Color in the following graph using the rules of the 3-Color Problem. The color you pick for node T means true and the color you pick for node F means False. Be consistent about colors in future questions.



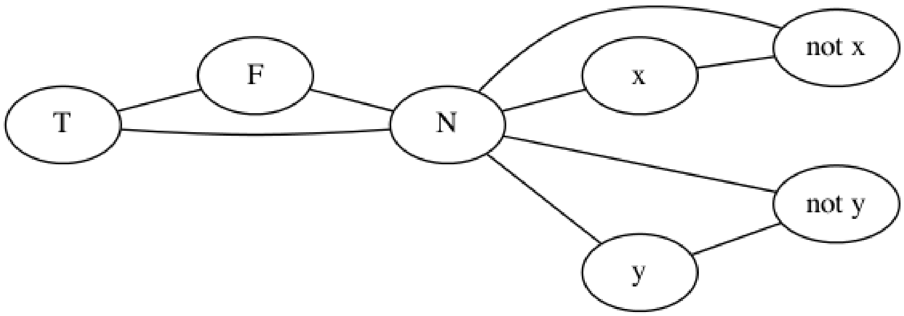
(a) (2 points) Color in above graph correctly.

(b) (1 point) What Color did you pick for **True** (the T node)?

(c) (1 point) What Color did you pick for **False** (the F node)?

Question 2 : 7 points

Color in the following graph using the rules of the 3-Color Problem.



Question 3 : 6 points

Select the Boolean value assigned to each node label by your coloring.

These answers should be clear from your coloring. For example, the node "not x" should match the color of either the T node or the F node. If it's color matches the T node, then we know $\neg x = True$.

(a) (1 point) x
- ◯ True
- ◯ False

(b) (1 point) $\neg x$
- ◯ True
- ◯ False

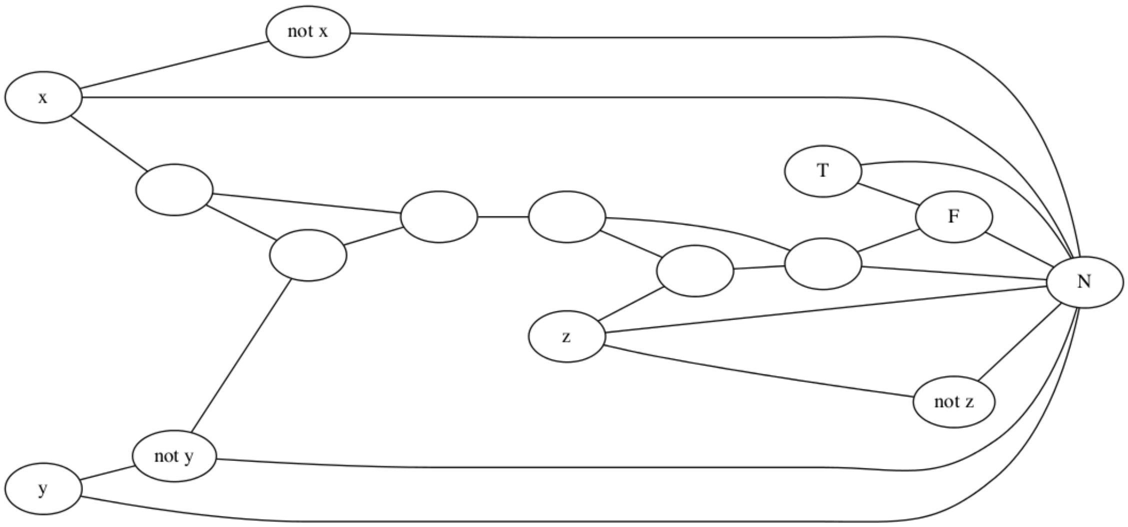
(c) (1 point) y
- ◯ True
- ◯ False

(d) (1 point) $\neg y$
- ◯ True
- ◯ False

(e) (2 points) In the above graph, what color did you apply to the N node? A variable node is a node labeled with a variable (x and y in this example). Is it possible for any variable node to have the same color as the N node?

Question 4 : 4 points

Color in the following graph using the rules of the 3-Color Problem.



Question 5 : 3 points

Select the Boolean value assigned to each node label by your coloring. Use the coloring above to answer these questions. The answers should be clear from the colors you assigned the nodes.

(a) (1 point) x

◯ True

◯ False

(b) (1 point) $\sim y$

◯ True

◯ False

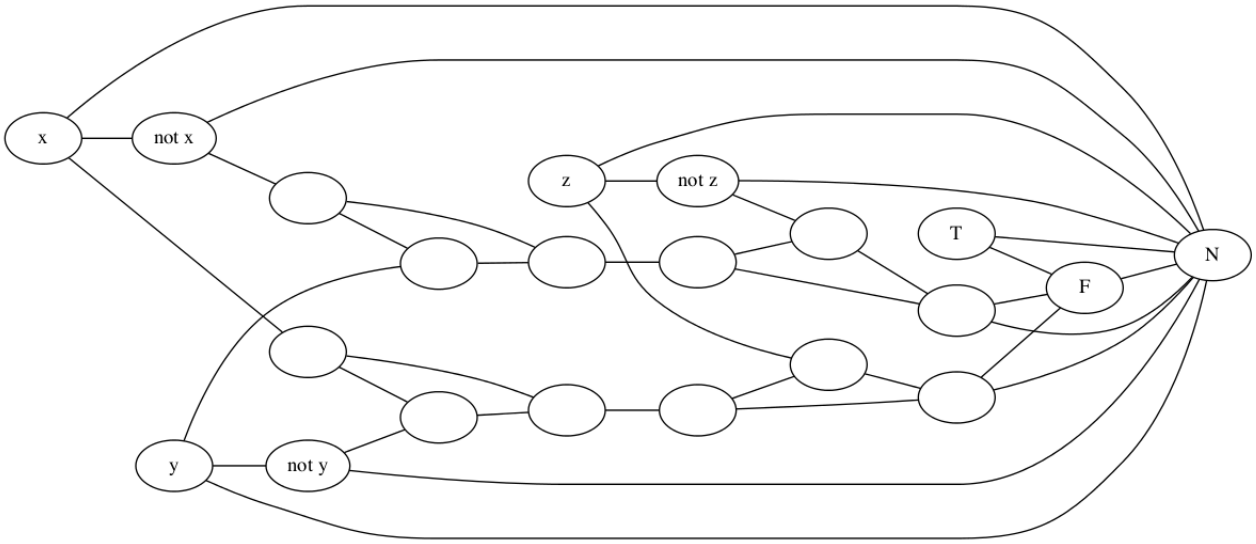
(c) (1 point) z

◯ True

◯ False

Question 6 : 4 points

Color in the following graph using the rules of the 3-Color Problem.



Question 7 : 3 points

Give the Boolean values that make the expression below true.

If you found a coloring of the graph that worked, that coloring is the answer to this question. Just match the colors with the variable values to answer this question.

$$(\sim y \vee x \vee z) \wedge (\sim x \vee y \vee \sim z)$$

3 Graph into Boolean Expression

This problem can also be solved in the other direction. We can start with a graph, then build a boolean expression that will tell us how to color the graph. In this section, we will find a coloring using logic.

Let $G = (V, E)$ be a graph with a set of vertices V and edges E. Assume the number of nodes in the graph is n.

Define edge(i, j) to be true if there is an edges from node i to node j.

When edge(i,j) is true i is said to be adjacent to j.

Define color(i, c) to be true if node i has color c.

Given a set of m colors, and m-coloring of a graph G is a map from nodes to colors such that every node has a unique color and no two adjacent nodes have the same color.

We want to reduce 3-colorability to SAT.

Question 8 : 9 points

We will use predicate logic (that is, formulas that may have Boolean functions and quantifiers) to specify the general conditions to be satisfied.

Use predicate logic to specify that a graph is 3-colored.

(a) (3 points) Use predicate logic to write "Every node has at least one color."

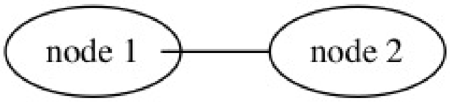
(b) (3 points) Use predicate logic to write "No node has more then one color'.

(c) (3 points) Use predicate logic to write "No two adjacent nodes have the same color".

Question 9 : 6 points

Given a specific graph there is a finite number of nodes and hence the predicates edge(i, j) and color(i, c) can be replaced by variables $E_{i,j}$ and $C_{i,c}$ respectively and the quantifiers forall and exists can be replaced by conjunctions and disjunctions.

Use this idea to convert the predicate logic specification to a propositional logic formula.



Write a propositional logic formula to color the above graph with 2 colors.

(a) (2 points) Write a Boolean expression that states "Node 1 is either Color 1 or Color 2". Simplify to CNF if not already in this form.

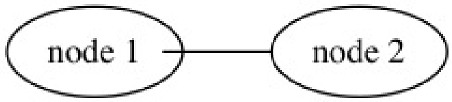
(b) (2 points) Write a Boolean expression that states "if Node 1 is color 1 then it cannot be color 2". Simplify to CNF if not already in this form.

(c) (2 points) There is an edge from node 1 to node 2.
Write a Boolean expression that states "if Node 1 is color 1 then Node 2 cannot be color 1". Simplify to CNF if not already in this form.

Question 10 : 10 points

DIMACS is the format you saw last week for MiniSAT.

In question 9, you came up with formulas for some of the constrains of the below graph.



Use the patterns you found in Question 9 to give all the constraints on the graph. Write a DIMACS format expression for this graph.

Use MiniSat to find an assignment of variables that solves the 2-color problem for the given graph.