Comparison of Orthogonal and Biorthogonal Wavelets for Multicarrier Systems

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Abstract **–Wavelets are constructed from the basis sets of their parent scaling functions of the two-scale dilation equation (1). Whereas orthogonal wavelets come from one orthogonal basis set, the biorthogonal wavelets project from different basis sets. Each basis set is correspondingly weighted to form filters, either highpass or lowpass, which form the constituents of quadrature mirror filter (QMF) banks. Consequently, these filters can be used to design wavelets, the differently weighted parameters contributing respective wavelet properties which influence the performance of the transforms in applications, for example multicarrier modulation. This study investigated applications for onward multicarrier modulation applications. The results show that the optimum choice of particular wavelet adopted in digital multicarrier communication signal processing may be quite different from choices in other areas of wavelet applications, for example image and video compression.**

Index Words – **wavelets, orthogonal, biorthogonal, digital filters, multipath.**

I. INTRODUCTION

In [1], it was shown that orthogonal wavelets cannot combine orthogonality and symmetry, leading to the invention of biorthogonal wavelets. It was further identified that biorthogonal wavelets are preferred over orthogonal wavelets in image compression applications because the orthogonal wavelet lack the symmetric filters needed to properly resolve image borders [2]. Thus, biorthogonal wavelets are more symmetric in nature the orthogonal wavelets are more periodic [2]. In fact the periodicity of wavelets is more suitable for multicarrier modulation applications, where multipath effects arise. Whereas the length of the synthesis and analysis filters affects the quality of the image in biorthogonal wavelets [2], and can also affect the performance in multicarrier modulation, the length of the filters in orthogonal wavelets does not affect the performance of the multicarrier system, explicitly in terms of BER. Since application specific wavelets can be constructed by changing the basis functions of the wavelets function or by design of the filters [3], more symmetric filters or more periodic filters can be used to design the respective needed wavelet. In [3] and [4], it was observed that no single wavelet has universally ideal properties that suit all applications. Consequently, if biorthogonal wavelets best fit image processing then orthogonal wavelets can be excellent for other applications such multicarrier modulation. Although Daubechies in her work [5] identified that wavelets can be good for signal processing, compression, detection and denoising, it takes some comparative tests to validate which wavelet suits a particular application most and whether there is a need to construct a new specific wavelet. Biorthogonal and orthogonal wavelets are differentiated in this study for onward applications in multicarrier digital signal processing.

The basic concept of wavelet transform is presented in Section II with orthogonal and biorthogonal wavelets examined in Sections III and IV respectively. The simulation environments with results are discussed in Section V. Finally, conclusions are presented in Section VI with the references following.

II. BASIC WAVELET TRANSFORM CONCEPT

Wavelets are constructs understood to give detailed insight into signals of interest using multiresolution. They are used to investigate time-varying signals and they elucidate the prevailing frequency component at some desired time. Other than in video compression studies, the multiresolution property can as well be exploited for digital communication signal processing merit. Wavelets resolve signals into high frequency component parts called details and low frequency components parts called approximates. There are similarities to be traced from the windowing of short-time Fourier transforms of signals.

A. *The Discrete and Packet Wavelets*

Wavelets use windowing that is scaled and translated in multiresolution, thus permitting a signal to be represented in both frequency and time domains. This sedates the well-known Heisenberg Uncertainty principle. By this method, it is possible to analyse the frequency component of a signal at each time instant. Given the parent two-scale dilation equation given as;

$$\varphi(t) = \sum_{n} h(n)\sqrt{2}\varphi(2t-n) \qquad (1)$$

in discrete form, wavelets can be obtained as;

$$\psi(t) = \sum_{n} g(n)\sqrt{2}\varphi(2t-n) \qquad (2)$$

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where $\varphi(t)$ and $\psi(t)$ are the scaling and wavelet functions respectively, $h(n)$ and $g(n)$ are the lowpass and highpass filters respectively with n as the periodic shift which implements the filter coefficient index. Both filters are related by $g(n) = (-1)^n h(L+1-n)$ with L as the filter length. It is a requirement that $\sum h(n) = \sqrt{2}$ for all wavelets. As stated, one advantage of the Discrete Wavelet Transform (DWT) is that it allows signals of both high and low frequencies to be observed simultaneously [6], but trades resolution in time for resolution in frequency and vice-versa. If the signal is shifted and translated by some equal coefficients such that k=n, with "k" as the shift parameter along the time-axis and "n" as the scale parameter along the frequency-axis, then the dilation-contraction problem is balanced, and this form of wavelet is named the packet wavelet.

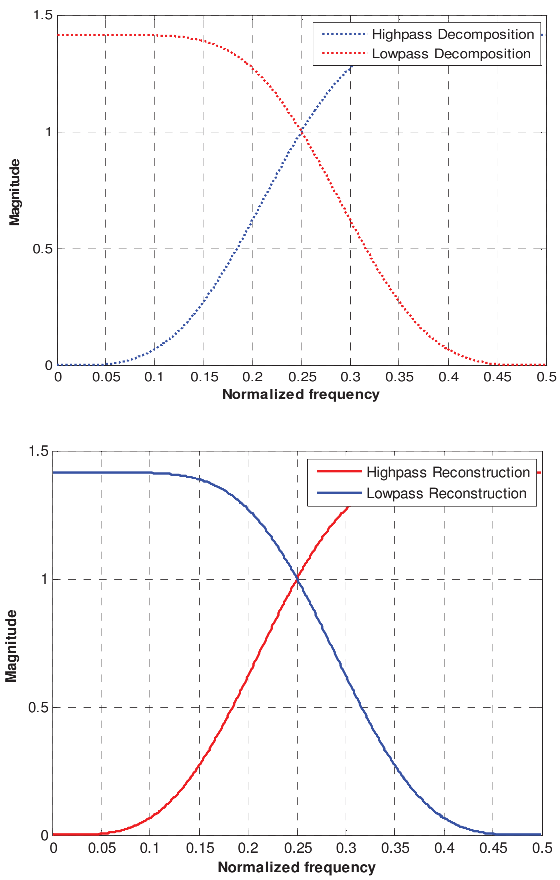

Figure 1: Reconstruction and decomposition filters of db2 (orthogonal wavelet). Note that they are similar both for reconstruction and decomposition.

III. Orthogonal Wavelets

Wavelets that use the same filters for decomposition (analysis) and reconstruction (synthesis) belong to the orthogonal wavelet family. For instance, if $\varphi_n(t)$ spans some n-spaces, with some input signal d_n, then the transmit sequence can be obtained as follows [7], [8];

$$s(t) = \sum_n d_n \varphi_n(t) \tag{3}$$

Eq. 3 is the synthesis part of Figure 3. In a reverse form, d_n can be obtained as the inner product of $s(t)$ and $\varphi(t)$;

$$d_n = \langle s(t), \quad \varphi(t) \rangle \tag{4}$$

Eq. 4 is the analysis part of Figure 1, with the interpretation that $\varphi_n(t)$ spans the space **R** and is the basis set of **R** if the set of $\{d_n\}$ differs for any given $s(t) \in R$.

If $\langle \varphi_n(t), \quad \varphi_m(t) \rangle = 0$, then the basis sets are orthogonal, and wavelets constructed from this form of scaling function are orthogonal wavelets. Examples of orthogonal base wavelets in literature are the Daubechies wavelets [9].

In use, for example, within the MATLAB environment, the Daubechies wavelets are designated as 'dbN" where "N" stands for the effective filter length. These wavelets are both orthogonal and orthonormal according to the following;

$$\langle \varphi_n(t), \quad \varphi_m(t) \rangle = \delta(n-m) \tag{5}$$

It can be shown from Eq. 5 that the functions $\varphi_m(t)$ are the spanning set of the original $\varphi_n(t)$.

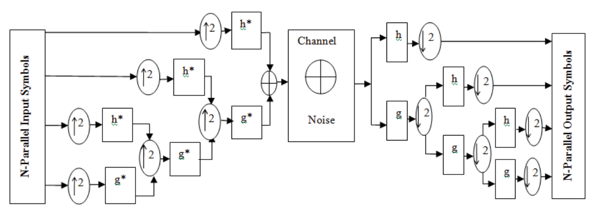

Figure 2: Baseband architecture of the discrete wavelet based multicarrier modulation

IV. Biorthogonal Wavelets

If the scaling functions basis set arises from different sources but still satisfy the orthogonal and orthonormal properties such as;

$$\langle \varphi_n(t), \quad \xi_m(t) \rangle = \delta(n-m) \tag{6}$$

where $\varphi_n(t)$, and $\xi_m(t)$ are different orthogonal basis sets, then the scaling function basis sets are said to be biorthogonal. The basis sets are among themselves biorthogonal but not orthogonal to each other. Wavelets projecting from such basis sets are described as biorthogonal wavelets.

Analysing signals using biorthogonal wavelets would involve a modification to Eq. 4 as follows;

$$d_n = \langle s(t), \quad \varphi(t) \rangle \delta(t) \tag{7}$$

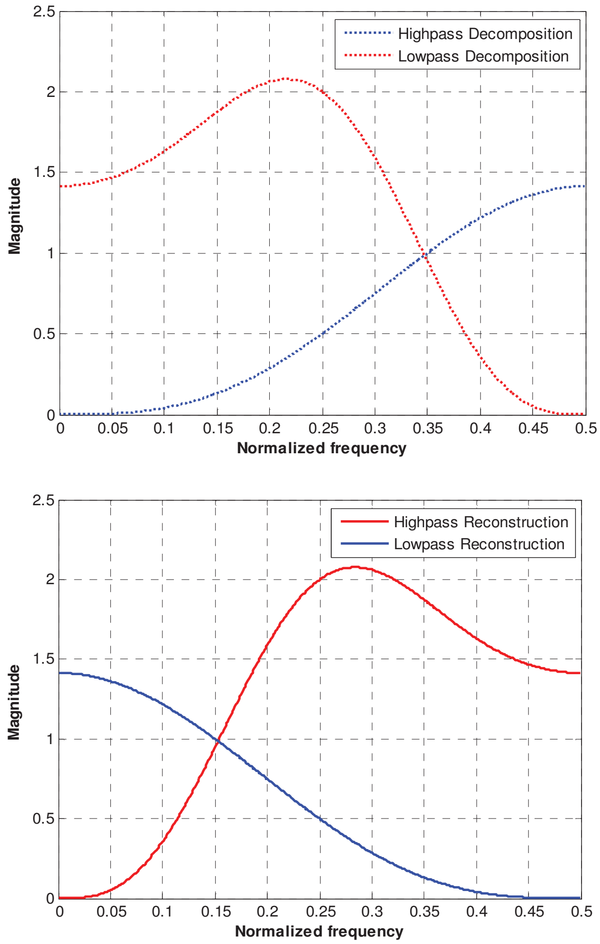

Figure 3: Reconstruction and decomposition filters of bior3.3 (biorthogonal wavelet). Note again that in this case the filters are not similar for reconstruction and decomposition. This is the case for other biorthogonal wavelets.

Examples of such wavelets are the biorthogonal and reverse-biorthogonal wavelets. In MATLAB notations, the biorthogonal wavelets are designated in use as "biorN_R.N_D" where "bior" stands for biorthogonal, "N_R" and "N_D" stand for the effective number of the reconstruction filters and effective number of the decomposition filters respectively. Because the wavelets project from different sources, filters which are on the other hand weightings of the scaling and wavelet functions must as well be different in each case so that biorthogonal wavelets differ from the orthogonal wavelets that use the same filters.

For different filter lengths, the performance of wavelets responds to the linearity of the phase. For instance, in choosing the solution for a filter in wavelet construction, if a non-minimum phase filter is chosen in the synthesis side of the wavelet design, perfect reconstruction requires that a non-minimum phase be chosen at the analysis side [10]. Doing this will introduce poorer phase response. From coding theory, wavelets would require that there be more linear-phase in the analysis side (the receiver) to reduce the effect of early quantization (as in the hard decision case). There are many biorthogonal wavelets of this sort although their use of unequal filters is a major challenge in deploying them for signal processing.

V. Simulation Environment And Results

6.4 million input binary signals were randomly generated, mapped unto 16-constellation points using the 16-QAM mapping scheme such that each constellation point consists of 4-bits. These bits are however converted to parallel form to obtain the parallel input symbols. By the inverse discrete wavelet transform (IDWT), the signal is wavelet modulated and passed through an AWGN channel and Rayleigh multipath channels respectively. The system model considered does not involve channel training. In this case, the receiver is assumed to have the knowledge of the channel. If the transmitted signal is given by s[n] and the channel through which the signal traverses is received had some impulse p[n], then we can write the received signal as;

$$y[n] = s[n] * p[n] + k[n] \qquad (8)$$

where $k[n]$ is the one-sided added white noise with zero mean.

The effect of the channel can be removed by zero-forcing (ZF) as y_{eq}=y[n]/p[n]. Using ZF-scheme is not strongly advised when better BER performance is required from communication systems since it does not deeply equalize the effect of the channel.

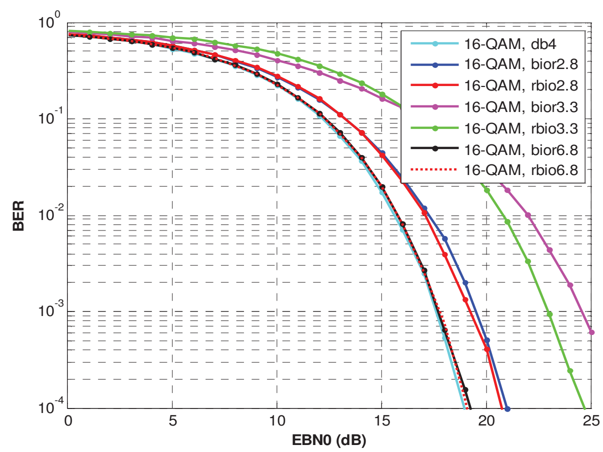

Figure 4: Orthogonal and reverse-biorthogonal wavelets compared over AWGN channel

The received parallel symbols are then scaled and translated by the reconstruction highpass and lowpass filters using the following orthogonal (db4) and biorthogonal wavelets (bior3.1, rbio3.1, bior2.8, rbio2.8, bior3.3, bior3.9 rbio3.9, bior6.8 and rbio6.8). This is done by the DWT in the receiver. In the model, no coding has been applied. Notice that the birothogonal wavelets are characterized by two filters – the decomposition (**D**) and the reconstruction (**R**) filters – hence the designation "bior$\mathbf{N_R}$.N_D" or "rbio$\mathbf{N_R}$.N_D".

In Figure 4, db4 is seen to perform best of the biorthogonal wavelets with bior3.3 performing worst. Though the filters of the QMF bank of the biorthogonal wavelets are symmetric according to [1], we do not find the reconstruction required to obtain a good BER in multicarrier modulation over an AWGN channel. Meanwhile, it may be misleading to characterize the performance of the biorthogonal and reverse-biorthogonal wavelets based on the number of decomposition or reconstruction filters. In fact, it can be shown that the number of filters in orthogonal wavelets does not affect performance.

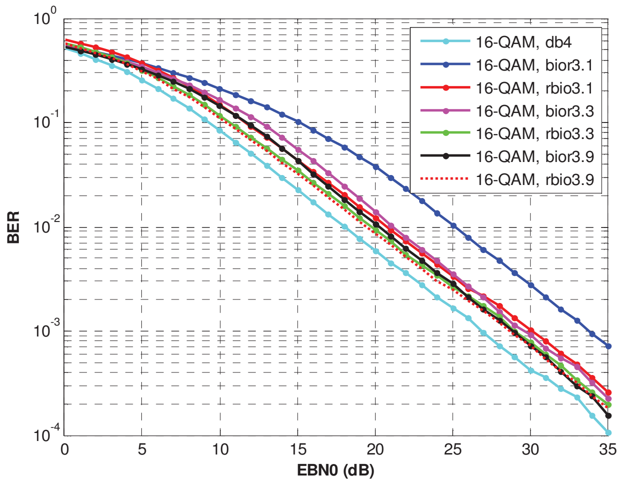

Figure 5: Orthogonal, biorthogonal and reverse-biorthogonal wavelets compared, using a multipath channel with AWGN.

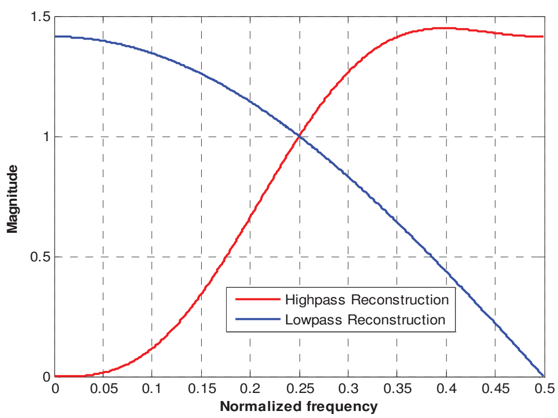

Figure 6: Reconstruction filters of bior1.3 (biorthogonal wavelet)

Some biorthogonal wavelets (results not shown), namely bior4.4, bior5.5 and bior6.8, possess similar decomposition and reconstruction bases functions and filters [5], consequently should perform alike. Over the multipath channel, the orthogonal wavelet consistently performed better than the biorthogonal wavelets for all wavelets considered.

It is possible that lower order biorthogonal wavelets (Figure 6) may perform nearly equally with the orthogonal wavelets. These wavelets have nearly equal magnitude filters and show less aliasing during sampling and reconstruct the transmitted signal rather better than some higher order members. Hence, it can be inferred that orthogonal based wavelets tend to perform better than biorthogonal wavelets over AWGN and multipath channels in terms of BER.

VI. Conclusion

In this work, biorthogonal and orthogonal wavelets were investigated for their comparative performance in multicarrier digital communications applications. The study supports the assertion that all wavelets are optimally suited for all fields of engineering applications. The orthogonal wavelets can be constructed from a single basis set while the biorthogonal wavelets are constructed from different basis sets. So, constructing wavelet filters are different and of unequal length (or magnitude) only for biorthogonal wavelets. Consequently more self-noise due to aliasing can be present in the biorthogonal case than in the orthogonal case so that complete perfect reconstruction can not be achieved. It follows therefore that orthogonal wavelets are preferable in digital communication signal processing over AWGN channels, with or without multipath.

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