On the Construction of Multiresolution Analysis Compatible with General Subdivisions

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Plan

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Multiresolution Transform

Multi-scale transform of Harten [Harten, 1996]

Successive spaces : $\cdots \subset V^{j-1} \subset V^j \subset V^{j+1} \subset \cdots$

$$
\begin{tikzcd}
V^{j+1}(f^{j+1}) \arrow[d, shift right, "\tilde{h}"'] \arrow[dr, shift right, "\tilde{g}"'] & \\
V^j(f^j) \arrow[u, shift right, "h"'] & W^j(d^j) \arrow[ul, shift right, "g"']
\end{tikzcd}
$$

$$\begin{cases} f^j &= \tilde{h} f^{j+1} \\ d^j &= \tilde{g} f^{j+1} \end{cases}$$ Decomposition

$$f^{j+1} = h f^j + g d^j$$ Reconstruction

$$f^{j+1} \longleftrightarrow \{f^{j_0}, d^{j_0}, \ldots, d^j\}$$

Subdivision and Decimation Schemes

- Subdivision

$$h : \begin{cases} l^\infty(\mathbb{Z}) \to l^\infty(\mathbb{Z}) \\ (f_k^j)_{k\in\mathbb{Z}} \mapsto (f_k^{j+1})_{k\in\mathbb{Z}} \end{cases}, \quad \text{linear : } f_l^{j+1} = \sum_{k\in\mathbb{Z}} h_{l-2k} f_k^j$$

- Decimation

$$\tilde{h} : \begin{cases} l^\infty(\mathbb{Z}) \to l^\infty(\mathbb{Z}) \\ (f_k^{j+1})_{k\in\mathbb{Z}} \mapsto (f_k^{j})_{k\in\mathbb{Z}} \end{cases}, \quad \text{linear : } f_k^{j} = \sum_{l\in\mathbb{Z}} \tilde{h}_{l-2k} f_l^{j+1}$$

where $(f_k^j)_{k\in\mathbb{Z}}$ associated to grid $X_j = k2^{-j}, k, j \in \mathbb{Z}$

Non-linear, Non-uniform, Non-stationary

Multiresolution Transform

One-scale framework :

$$\begin{tikzcd}
 & f^j = \tilde{h} f^{j+1} \arrow[dr, "h"] & \\
f^{j+1} \arrow[ur, "\tilde{h}"] \arrow[dr, "\tilde{g}"'] & & h f^j + g d^j = f^{j+1} \\
 & d^j = \tilde{g} f^{j+1} \arrow[ur, "g"'] &
\end{tikzcd}$$

Reconstruction :

$$h\tilde{h} + g\tilde{g} = I.$$

Compatibility :

$$\tilde{h}h = I, \quad \tilde{g}g = I, \quad \tilde{h}g = 0, \quad \tilde{g}h = 0.$$

Prediction Errors and Details

Consistent decimation :

$$\tilde{h}h = I \ .$$

Prediction error :

$$e^{j+1} = f^{j+1} - h\tilde{h}f^{j+1} \ .$$

Detail decimation operator $\tilde{g}$ and detail subdivision operator g,
Detail :

$$d^j = \tilde{g}f^{j+1} \ ,$$

satisfying

$$e^{j+1} = g\tilde{g}f^{j+1}.$$

Plan

1. Introduction
   - Multiresolution Framework
   - Subdivision and Decimation
   - Prediction Error and Detail
2. Construction of Multiresolution
   - Linear Uniform Consistent Decimation
   - General Consistent Decimation
   - Detail and Prediction Error
3. Analysis of Multiresolution
   - Stability
   - Polynomial Reproduction
   - Decay of Prediction Error
   - Numerical Tests

Linear Uniform Case

Consistency Condition :

$$\tilde{h}h = I \quad \Longrightarrow \quad \forall j \in \mathbb{Z},\ \sum_{i \in \mathbb{Z}} h_i \tilde{h}_{i+2j} = \delta_{j,0} \ .$$

Theorem (Elementary Operators)

Let h be a uniform subdivision operator given by

$$\{h_{-n}, h_{-n+1}, \ldots, h_{n-2}, h_{n-1}\}.$$

Introducing square matrix [1] of dimension $(2n-2) \times (2n-2)$

$$H = \begin{bmatrix} h_{n-2} & h_{n-4} & \cdots & h_{-n} & 0 & \cdots & 0 \\ h_{n-1} & h_{n-3} & \cdots & h_{-n+1} & 0 & \cdots & 0 \\ 0 & h_{n-2} & h_{n-4} & \cdots & h_{-n} & \cdots & 0 \\ 0 & h_{n-1} & h_{n-3} & \cdots & h_{-n+1} & \cdots & 0 \\ \vdots & & & \vdots & & & \\ 0 & 0 & \cdots & h_{n-2} & h_{n-4} & \cdots & h_{-n} \\ 0 & 0 & \cdots & h_{n-1} & h_{n-3} & \cdots & h_{-n+1} \end{bmatrix}.$$

If $det(H) \neq 0$, there exists $2(n-1)$ consistent decimation operators of length $2(n-1)$. They are given by each row of H^{-1}.

1. If $h_{-n} = 0$, dimension $(2n-3) \times (2n-3)$ without last column and row

Proposition (Decimation Operator Generation)

Let h be a given subdivision operator, denote $\{\tilde{h}^i\}_{i\in\mathcal{I}}$ a set of decimation operators which are consistent with h, then a general consistent decimation operator can be constructed as

$$\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}} c_{i,t} T_{2t}(\tilde{h}^i)$$

with

$$\forall t\in\mathcal{T},\ \sum_{i\in\mathcal{I}} c_{i,t} = \delta_{t,0},\ 0\in\mathcal{T}\subset\mathbb{Z}\ .$$

Proposition (Completeness of the Method)

All the consistent decimation operators can be constructed by combining elementary decimation operators with the Decimation Operator Generation formula.

Refinement Matrices

Relation with refinement matrices [Dyn, 1992]

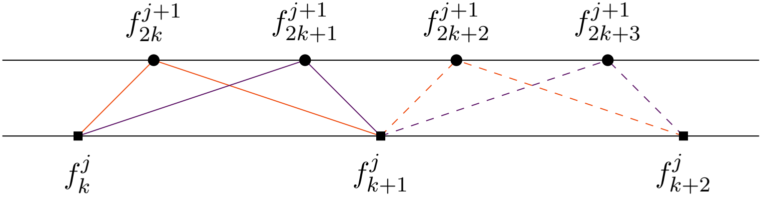
$$A_0 = \begin{bmatrix} H & 0 \\ U & h_{-n} \end{bmatrix}, \quad A_1 = \begin{bmatrix} h_{n-1} & U' \\ 0 & H \end{bmatrix}$$

with $U = [h_{n-2}, h_{n-4}, \cdots, h_{-n-2}]$, $U' = [h_{n-3}, h_{n-3}, \cdots, h_{-n+1}]$.

Proposition (Subdivision and Refinement Matrices)

All the consistent elementary decimation operators can be deduced by inverting one of the refinement matrix.
Moreover, the eigenvalues of each refinement matrix are the eigenvalues of the subdivision matrix H plus the first or last non-zero values of the subdivision mask.

Example : Quarter (B-spline, Lagrange) [Dyn, 1992]



$$\tilde{H} = H^{-1} = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} \end{bmatrix}^{-1} = \begin{bmatrix} \tilde{h}^0 \\ \tilde{h}^2 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{bmatrix}.$$

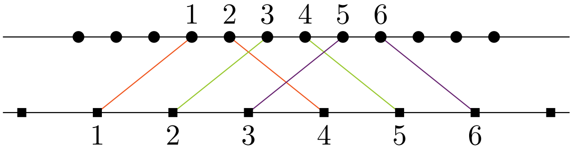
$$\tilde{h} = \frac{1}{2}\tilde{h}^2 + \frac{1}{2}\tilde{h}^0 = \{-\frac{1}{4}, \frac{3}{4}, \frac{3}{4}, -\frac{1}{4}\}.$$

Consistent decimation, Biorthogonal Spline [Cohen et al. 1992] :

$$\tilde{h} = \frac{1}{2}\tilde{h}^2 + \frac{1}{2}\tilde{h}^0 - \frac{3}{32}T_{-2}(h^2) + \frac{3}{32}T_{-2}(h^0) + \frac{3}{32}T_2(h^2) - \frac{3}{32}T_2(h^0)$$

$$= \{\frac{3}{64}, -\frac{9}{64}, -\frac{7}{64}, \frac{45}{64}, \frac{45}{64}, -\frac{7}{64}, -\frac{9}{64}, \frac{3}{64}\}.$$

Example : 4-point Shifted Lagrange Scheme [Dyn et al., 2005]



$$\begin{cases} f_{2k}^{j+1} & = & -\frac{7}{128} f_{k-1}^j + \frac{105}{128} f_k^j + \frac{35}{128} f_{k+1}^j - \frac{5}{128} f_{k+2}^j \\ f_{2k+1}^{j+1} & = & -\frac{5}{128} f_{k-1}^j + \frac{35}{128} f_k^j + \frac{105}{128} f_{k+1}^j - \frac{7}{128} f_{k+2}^j \end{cases}$$

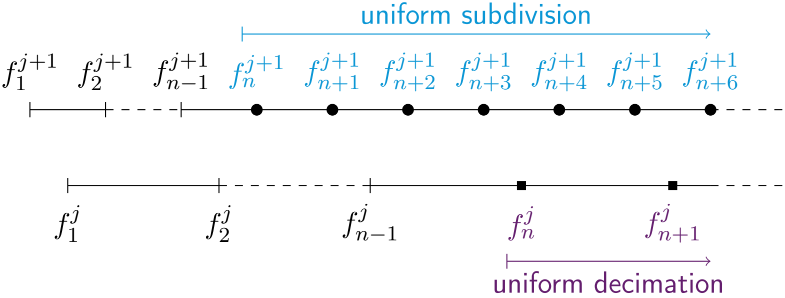
Consistent decimation of length 8 :

$$\{\frac{95}{2304}, -\frac{133}{2304}, -\frac{35}{256}, \frac{1505}{2304}, \frac{1505}{2304}, -\frac{35}{256}, -\frac{133}{2304}, \frac{95}{2304}\}.$$

Length 12 :

$$\{\frac{19}{16128}, -\frac{19}{11520}, \frac{19}{576}, -\frac{19}{576}, -\frac{2623}{16128}, \frac{7639}{11520}, \frac{7639}{11520}, \ldots, \frac{19}{16128}\}.$$

Finite length Sequence



Following the position-dependent strategy [Baccou et al. 2007] :

$$\begin{bmatrix} f_1^{j+1} \\ f_2^{j+1} \\ \vdots \\ f_n^{j+1} \end{bmatrix} = M_0 \begin{bmatrix} f_1^{j} \\ f_2^{j} \\ \vdots \\ f_n^{j} \end{bmatrix}, \quad \begin{bmatrix} f_{2^{j+1}-n+1}^{j+1} \\ f_{2^{j+1}-n+2}^{j+1} \\ \vdots \\ f_{2^{j+1}}^{j+1} \end{bmatrix} = M_1 \begin{bmatrix} f_{2^{j}-n+1}^{j} \\ f_{2^{j}-n+2}^{j} \\ \vdots \\ f_{2^{j}}^{j} \end{bmatrix} .$$

Extension to Edge

Given subdivision matrix H^j of dimension $2^{j+1} \times 2^j$,
find decimation matrix $\tilde{H}^j$ of dimension $2^j \times 2^{j+1}$ such that

$$\begin{bmatrix} f_1^{j+1} \\ f_2^{j+1} \\ \vdots \\ f_{2^{j+1}}^{j+1} \end{bmatrix} = H^j \begin{bmatrix} f_1^{j} \\ f_2^{j} \\ \vdots \\ f_{2^{j}}^{j} \end{bmatrix}, \quad \begin{bmatrix} f_1^{j} \\ f_2^{j} \\ \vdots \\ f_{2^{j}}^{j} \end{bmatrix} = \tilde{H}^j \begin{bmatrix} f_1^{j+1} \\ f_2^{j+1} \\ \vdots \\ f_{2^{j+1}}^{j+1} \end{bmatrix}.$$

Proposition (Extension Uniform Scheme to Edge)

Let H^j be a subdivision matrix constructed using the uniform subdivision operator h. Given $\tilde{h}$ a consistent uniform decimation operator of length far smaller than 2^j, if $det(M_0) \cdot det(M_1) \neq 0$, a consistent decimation matrix $\tilde{H}^j$ can be constructed by $\tilde{h}$ with M_0^{-1}, M_1^{-1} and H^{-1}.

Linear Global Method

$$H^j \sim \begin{bmatrix} H_1 \\ H_2 \end{bmatrix}, \quad \tilde{H}^j \sim \begin{bmatrix} \Lambda H_1^{-1} & (I-\Lambda)H_2^{-1} \end{bmatrix}, \quad \tilde{H}^j H = I.$$

Proposition (Linear Global Method)

Given h^L a linear subdivision operator, if there exist a left inverse for the operators $h^e : (h^e f)_k = (h^L f)_{2k}$, and $h^o : (h^o f)_k = (h^L f)_{2k+1}$, denoted $(h^o)^{-1}$ and $(h^e)^{-1}$ (i.e. $(h^o)^{-1} h^o = I, (h^e)^{-1} h^e = I$), then for all scaling operator λ,

$$\tilde{h}^L = \lambda (h^o)^{-1} \sigma + (I-\lambda)(h^e)^{-1} \sigma'$$

defines a consistent decimation scheme with h^L.

Theorem (General Method)

Let h be a subdivision operator, if there exists a linear decimation $\tilde{h}^L$ so that $\tilde{h}^L h - I$ is contractive, then for any $f^{j+1} \in l^\infty(\mathbb{Z})$, the fixed-point equation

$$f^j = \tilde{h}^L f^{j+1} - (\tilde{h}^L h - I) f^j$$

has a unique solution. Moreover,
$\tilde{h} : f^{j+1} \mapsto f^j$ is a decimation operator consistent with h.

$f^j = \tilde{h} f^{j+1} = \lim_{n\to\infty} (f^j)_n$ can be constructed by induction :

$$\left\{ \begin{array}{lcl} (f^j)_0 & = & \tilde{h}^L f^{j+1} \\ (f^j)_{n+1} & = & \tilde{h}^L f^{j+1} - (\tilde{h}^L h - I)(f^j)_n \end{array} \right. .$$

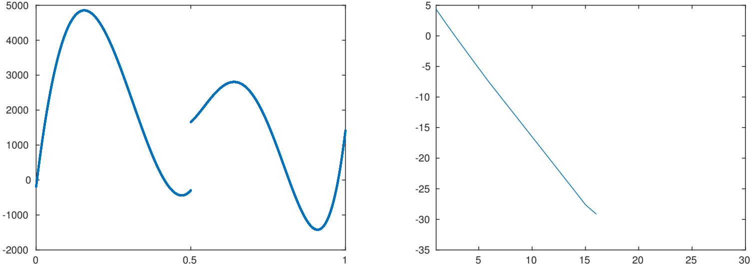


Figure: Left : Discontinuous test function, Right : Construction of the non-linear decimation operator : $\log_2 ||f_n^j - f_{n-1}^j||_\infty$ versus n for the fixed-point algorithm.

Example : 4-point Shifted PPH [Amat et al. 2011]

If $|\Delta^2 f_k| \leq |\Delta^2 f_{k+1}|$,

$$\left\{ \begin{array}{lll} f_{2k}^{j+1} & = & -\frac{7}{128} f_{k-1}^j + \frac{105}{128} f_k^j + \frac{35}{128} f_{k+1}^j - \frac{5}{128} f_{k+2}^j - \frac{5}{64} DHA_k \\ f_{2k+1}^{j+1} & = & -\frac{5}{128} f_{k-1}^j + \frac{35}{128} f_k^j + \frac{105}{128} f_{k+1}^j - \frac{7}{128} f_{k+2}^j - \frac{7}{64} DHA_k \end{array} \right. ,$$

if $|\Delta^2 f_k| > |\Delta^2 f_{k+1}|$,

$$\left\{ \begin{array}{lll} f_{2k}^{j+1} & = & -\frac{7}{128} f_{k-1}^j + \frac{105}{128} f_k^j + \frac{35}{128} f_{k+1}^j - \frac{5}{128} f_{k+2}^j - \frac{7}{64} DHA_k \\ f_{2k+1}^{j+1} & = & -\frac{5}{128} f_{k-1}^j + \frac{35}{128} f_k^j + \frac{105}{128} f_{k+1}^j - \frac{7}{128} f_{k+2}^j - \frac{5}{64} DHA_k \end{array} \right. ,$$

where

$$\Delta^2 f_k = (f_{k+1}^j - f_k^j) - (f_k^j - f_{k-1}^j),$$

$$DHA_k = H(\Delta^2 f_k, \Delta^2 f_{k+1}) - A(\Delta^2 f_k, \Delta^2 f_{k+1}).$$

Example : 4-point Shifted PPH

If $h = h^L + h^N$, the contractivity of $\tilde{h}^L h - I = \tilde{h}^L h^N$ is needed,

- Consistent decimation of Quarter of length 4 :

$$||\tilde{h}^L h^N u - \tilde{h}^L h^N v||_\infty \leq \frac{48}{64} ||u - v||_\infty.$$

- Consistent decimation of Shifted Lagrange of length 8 :

$$||\tilde{h}^L h^N u - \tilde{h}^L h^N v||_\infty \leq \frac{307}{384} ||u - v||_\infty.$$

- Consistent decimation of Shifted Lagrange of length 12 :

$$||\tilde{h}^L h^N u - \tilde{h}^L h^N v||_\infty \leq \frac{15481}{20160} ||u - v||_\infty.$$

Detail and Prediction Error

Linear case :

$$e^{j+1} = (I - h\tilde{h})f^{j+1} \quad \Longrightarrow \quad \tilde{h}e^{j+1} = 0.$$

Proposition (Kernel of Decimation)

Let h be a general subdivision operator and $\tilde{h}$ be a consistent decimation operator given by General Method with $\tilde{h}^L$ the involved linear decimation operator. The associated prediction error e^{j+1} verifies

$$\tilde{h}^L e^{j+1} = 0,$$

and

$$\tilde{h}e^{j+1} = 0.$$

Detail Operators

$$\tilde{H}^L E = 0 \implies \tilde{H}^e E^e = -\tilde{H}^o E^o \implies E^e = -(\tilde{H}^e)^{-1} \tilde{H}^o E^o.$$

Theorem (Detail Operators)

Let h be a subdivision operator and $\tilde{h}$ a consistent decimation. Introducing $\tilde{h}^o : (\tilde{h}^o f)_k = \sum_l \tilde{h}^L_{2l+1-2k} f_{2l+1}$, $\tilde{h}^e : (\tilde{h}^e f)_k = \sum_l \tilde{h}^L_{2l-2k} f_{2l}$. If there exists a linear left inverse operator of $\tilde{h}^e$, denoted $(\tilde{h}^e)^{-1}$, then $(g, \tilde{g})$ defined as

$$\begin{cases} \tilde{g} = \sigma(I - h\tilde{h}) \\ g = \tau(\cdot, -(\tilde{h}^e)^{-1} \tilde{h}^o \cdot) \end{cases}$$

are detail operators compatible with $(h, \tilde{h})$.

Plan

Stability

Definition (Stability of Decimation Schemes)

A decimation scheme $\tilde{h}$ is stable if there exists a constant $C \in \mathbb{R}$ such that for all $f, f_\epsilon \in l^\infty(\mathbb{Z})$,

$$\forall i \in \mathbb{N}, \quad ||\tilde{h}^i f - \tilde{h}^i f_\epsilon||_\infty \leq C||f - f_\epsilon||_\infty.$$

Proposition

The decimation operator $\tilde{h}$ is stable if and only if there exists $i \in \mathbb{N}^*$, such that the subdivision h constructed from sequence $2(\tilde{h}_l^i)_{l\in\mathbb{Z}}$ is stable.

Convergence of subdivision [Dyn, 2002].

$$\tilde{h} = \{\frac{95}{2304}, -\frac{133}{2304}, -\frac{35}{256}, \frac{1505}{2304}, \frac{1505}{2304}, -\frac{35}{256}, -\frac{133}{2304}, \frac{95}{2304}\}.$$

Subdivision $2(\tilde{h}_l^3)_{l\in\mathbb{Z}}$ is convergent,

$$\forall i_1, ..., i_9 \in \{0,1\}, \quad max\left((\frac{1}{2})^9 ||A_{i_1}^{(1)} A_{i_2}^{(1)} \cdots A_{i_9}^{(1)}||\right) = 0.983338 < 1.$$

$$\tilde{h} = \{\frac{3}{64}, -\frac{9}{64}, -\frac{7}{64}, \frac{45}{64}, \frac{45}{64}, -\frac{7}{64}, -\frac{9}{64}, \frac{3}{64}\}.$$

Subdivision $2(\tilde{h}_l^2)_{l\in\mathbb{Z}}$ is convergent,

$$\forall i_1, ..., i_5 \in \{0,1\}, \quad max\left((\frac{1}{2})^5 ||A_{i_1}^{(1)} A_{i_2}^{(1)} \cdots A_{i_5}^{(1)}||\right) = 0.86584 < 1,$$

where $A_0^{(1)}, A_1^{(1)}$ are associated refinement matrices for differences.

Numerical Stability : Non-linear

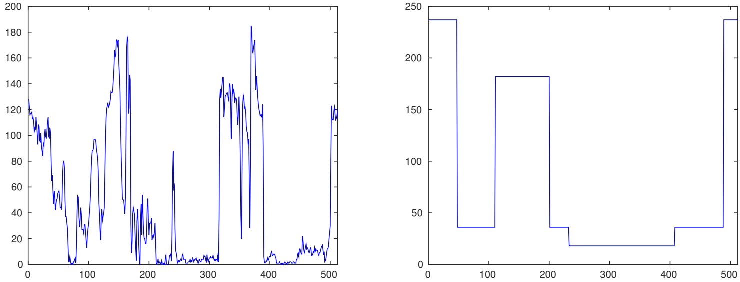


Figure: Sequence constructed from the 260-th column of image *man* (left) and image *geometric* (right)

Numerical Stability : Non-linear

Stability constant : $$C_s = \frac{||f^j - \hat{f}^j||_1}{||f^{j_0} - \hat{f}^{j_0}||_1 + \sum_{i=j_0}^{j-1} ||d^i - \hat{d}^i||_1}.$$

column	50	120	190	260	330	400
C_s- SLAG	1.3256	1.3646	1.3420	1.3456	1.2602	1.3248
C_s- SPPH	1.2337	1.2668	1.2307	1.1794	1.2353	1.2920

Table: Estimation of the stability constant for the decomposition associated to the shifted PPH and the shifted Lagrange schemes based on image *man*

column	50	120	190	260	330	400
C_s- SLAG	1.3011	1.2060	1.3170	1.2617	1.2334	1.3235
C_s- SPPH	1.2049	1.2296	1.2053	1.1848	1.2748	1.2904

Table: Estimation of the stability constant for the decomposition associated to the shifted PPH and the shifted Lagrange schemes based on image *geometric*

Polynomial Reproduction

Proposition (Reproduction for Subdivision)

A subdivision operator h quasi-reproduces polynomials up to degree p if and only if $\forall n \in \{0, 1, 2, \dots, p\}, \exists t_n \in \mathbb{R}$,

$$\sum_{l \in \mathbb{Z}} h_{2l}(2l)^n = \sum_{l \in \mathbb{Z}} h_{2l+1}(2l+1)^n = (t_n)^n,$$

If t_n independent on n, it is polynomial reproduction.

Theorem

Given a subdivision operator h and a consistent decimation operator $\tilde{h}$, if h quasi-reproduces polynomials up to degree p, then $h\tilde{h}$ reproduces polynomials up to degree p.

Proposition (B-spline Quasi-Reproduction)

Let h be the B-spline subdivision scheme of order m (length $m+1$), then h quasi-reproduces polynomials up to degree $m-1$. Moreover, it is the only operator of length $m+1$ that leads to the quasi-reproduction of polynomials up to degree $m-1$.

Proposition (Lagrange Reproduction)

Let h be a p-point Lagrange subdivision scheme (length $2p$), then h reproduces polynomials up to degree $p-1$. Moreover, it is the only operator of length $2p$ which leads to the reproduction of polynomials up to degree $p-1$.

Proposition

Let h be a shifted $2q$-point symmetric Lagrange subdivision, then h reproduces polynomials up to degree $2q-1$ and quasi-reproduces polynomials up to degree $2q$.

Decay of Prediction Error

Proposition (Linear)

Let h be a linear uniform stable subdivision operator and $\tilde{h}$ be a linear stable decimation operator. If $h\tilde{h}$ reproduces polynomial up to degree p, then for sufficiently large $j \in \mathbb{Z}$, $||e^j|| \leq C2^{-(p+1)j}$, where C does not depend on j. $p+1$ is called the decay of prediction error.

Proposition (Non-Linear)

Let h be a non-linear subdivision scheme with $h = h^L + h^N$ where h^L is linear subdivision quasi-reproduce polynomial of degree p. For all $f^j \in l^\infty(\mathbb{Z})$, there exists constant C independent on j such that $||h^N f^j|| \leq C2^{-q(j+1)}$. If $\tilde{h}$ is a stable consistent decimation operator constructed by General Method, then the decay rate of the associated prediction error is at least $min(p+1, q)$.

Example : Decay of Prediction Error

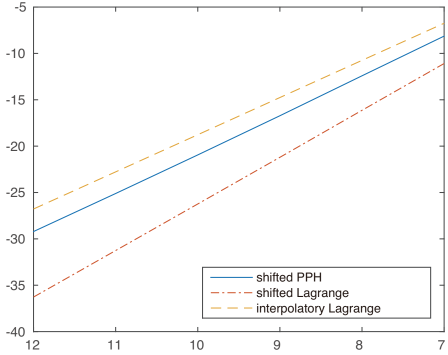


Figure: log of prediction error versus scale from 12 to 7, slope for 4-point interpolatory Lagrange, 4-point shifted Lagrange and 4-point shifted PPH scheme are 4.00717, 5.0379 and 4.21979

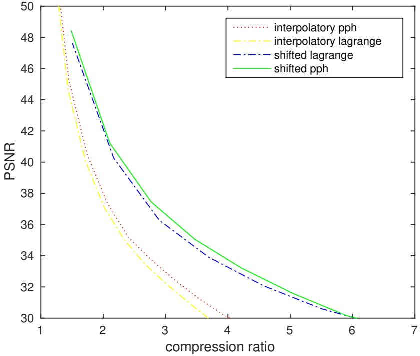


Figure: Left : test image *man*, Right : PSNR versus compression ratio for interpolatory Lagrange, shifted Lagrange, interpolatory PPH and shifted PPH multiresolutions.

Numerical Result : Image Compression

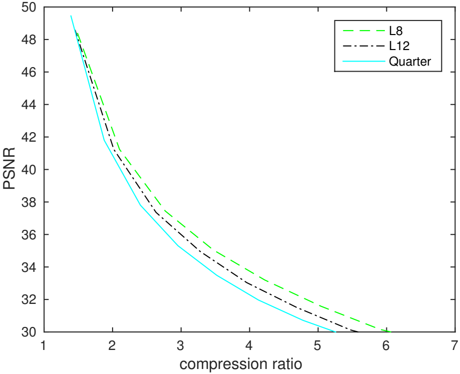


Figure: PSNR versus compression ratio for the 4-point shifted PPH subdivision scheme with three different consistent decimation operators, *man* image

Conclusion and Perspectives

Main contributions :

- Construction of consistent decimations for linear uniform subdivision ;
- Construction of consistent decimations for linear subdivision applied to finite length sequence ;
- Construction of consistent decimations for general subdivision ;
- Construction of a complete compatible multiresolution framework ;
- Application of new multiresolution in image compression ;
- Analyses of the multiresolution.

Perspectives :

- Full analysis of multiresolution in the non-linear framework ;
- Choice of the linear decimation operator in General Method ;
- Coupling with wavelet framework and lifting schemes ;
- Further application and comparison for image compression.

Publications

Z. Kui, J. Baccou, J. Liandrat,

- On the coupling of decimation operator with subdivision schemes for multi-scale analysis.
  Lecture Notes in Comput. Sci.
- Subdivision schemes and multiresolution analysis : Focus on the shifted lagrange and shifted PPH schemes.
  SEMA SIMAI Springer Series.
- On the construction of multiresolution analysis associated to general subdivision schemes.
  Submitted to *Appl. and Comput. Harmon. Anal.*
- Construction and Application of Finite-Dimensional Biorthogonal Multiresolution based on Subdivision.
  Submitted to *J. Comput. Appl. Math.*

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