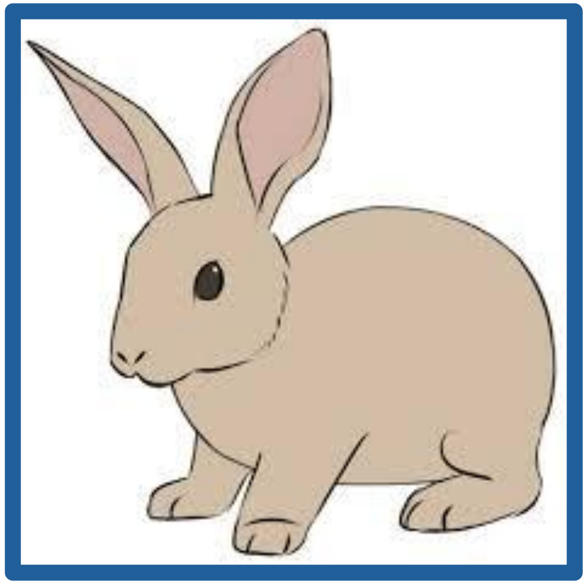
TOR VERGATA
UNIVERSITÀ DEGLI STUDI DI ROMA



Prey-Predator model by Volterra-Lotka

Introduction **History** **Applications**	(slides)
Model equations **Model properties**	(whiteboard)
Model simulation	(Matlab)
Wrap-up	(slides)

Introduction

- Ecological model describing predation **dynamics within 2 species** (prey-predator)
- Developed independently by Alfred J. **Lotka** (1925) and by Vito **Volterra** (1926)
- Highly simplified model, but it's the basis for more advanced and sophisticated models

Alfred J. Lotka

Vito Volterra

History

- Volterra's son in law (D'Ancona, a biologist) was looking at fish catched before and during WWI.

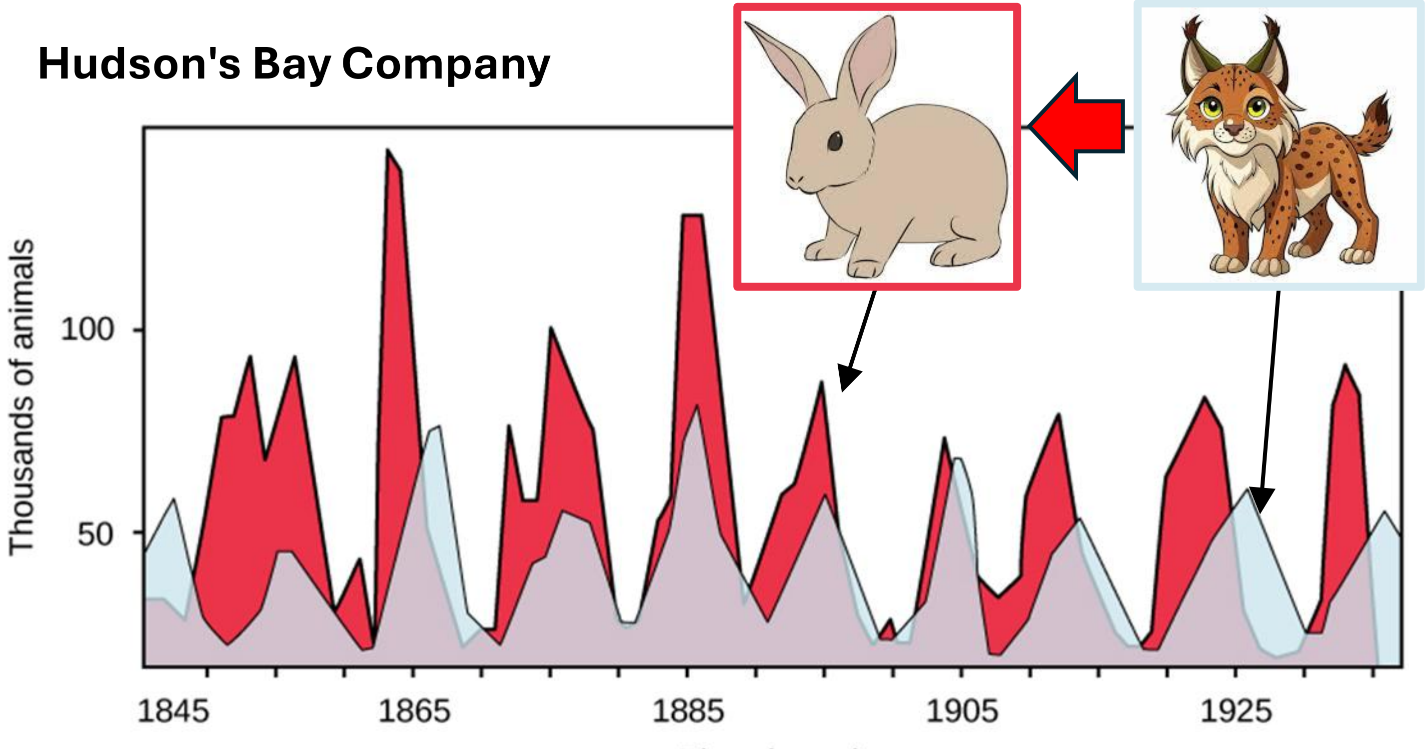
- He noticed 2 aspects that he couldn't explain:

  1) **CYCLICAL BEHAVIOUR**

  2) **MORE SHARKS CATCHED THAN NORMAL FISHES**

Other observed cases

Hudson's Bay Company



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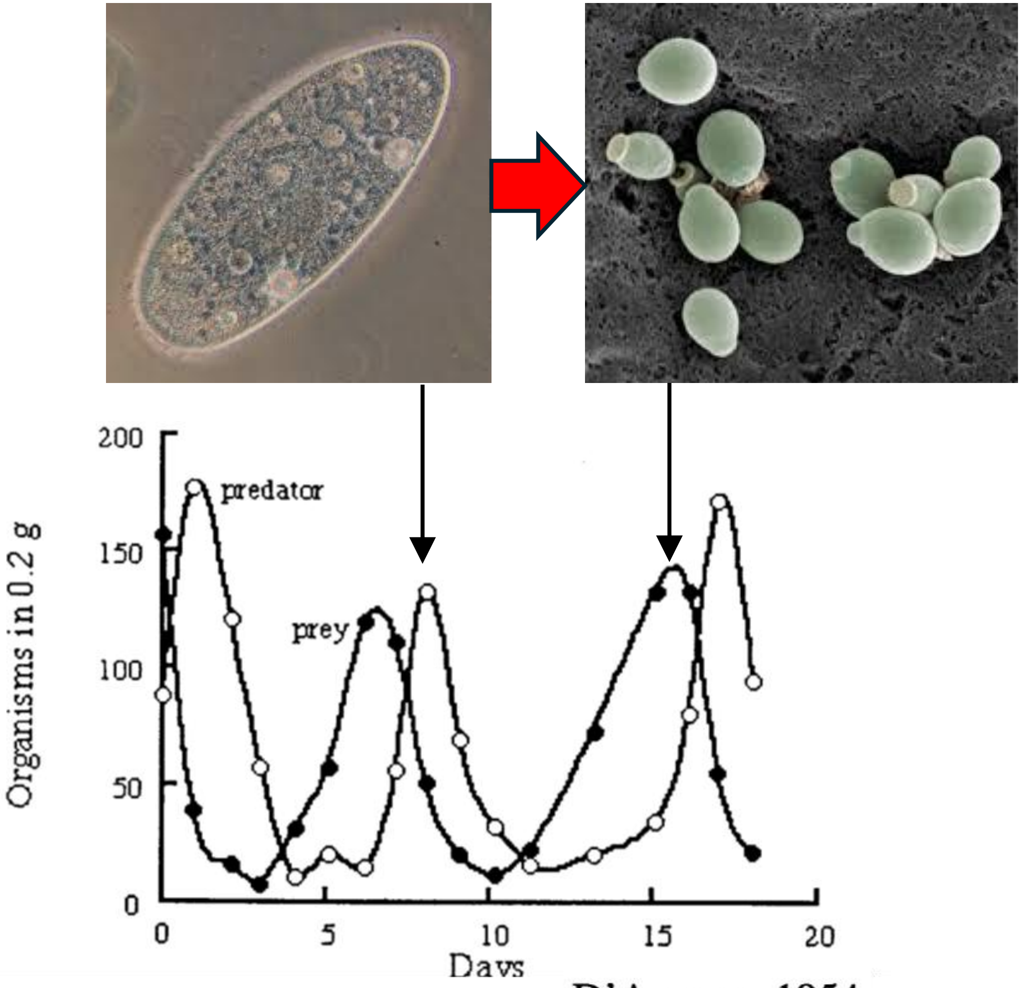
PARAMECIUM **YEAST**



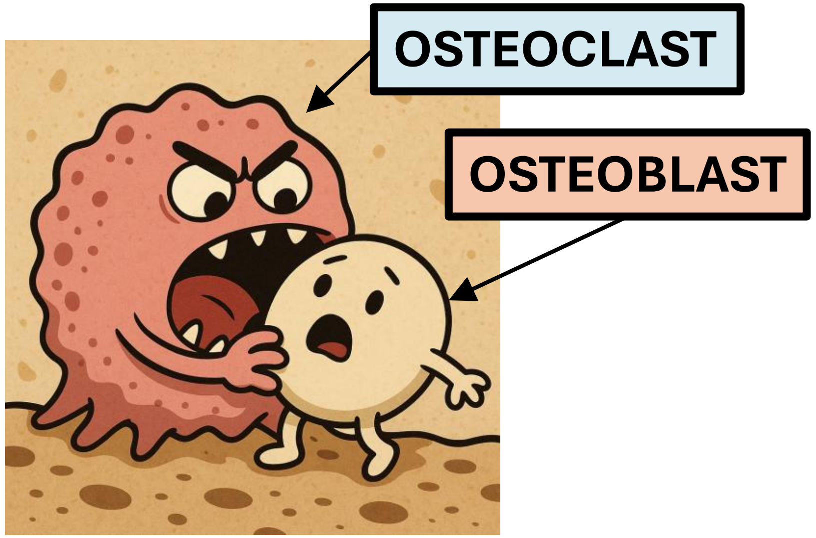
D'Ancona, 1954

Examples of applications to physiology

FLEXIBLE MODELS -> MANY DIFFERENT APPLICATIONS

1) Bone formation remodelling

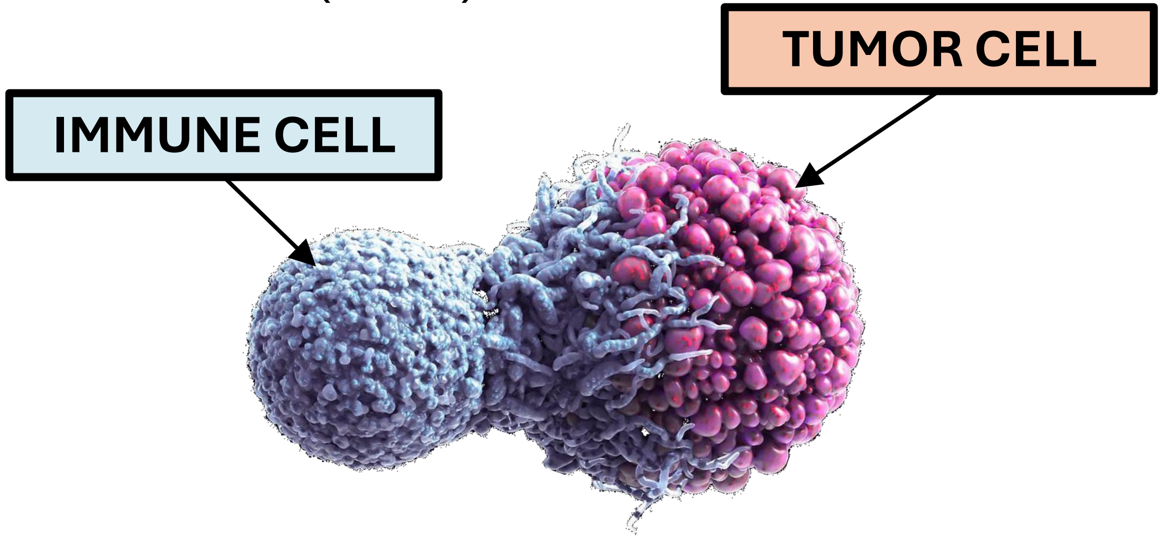
Osteoblasts (PREY) promote the growth of **osteoclasts** (PREDATOR).



A. Nutini, Theoretical model of bone remodelling, RJBIO (2015)

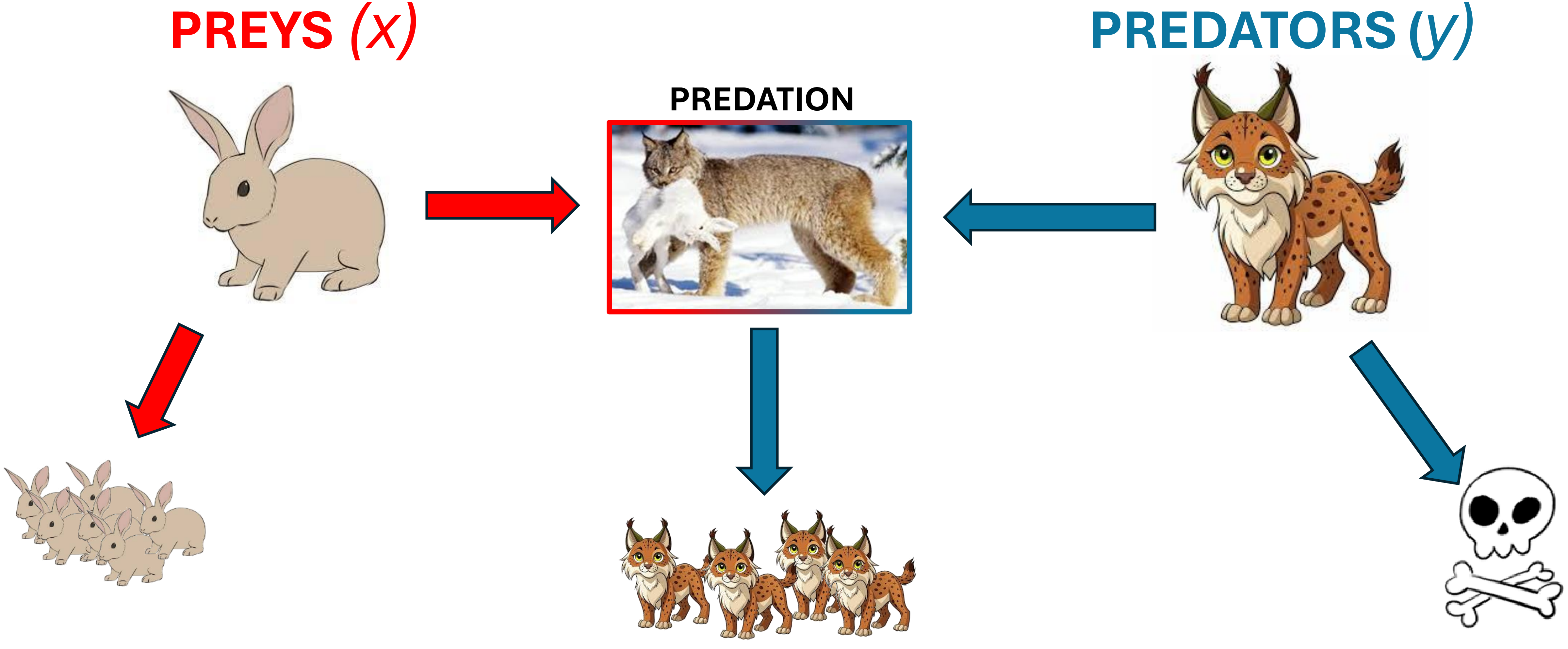
2) Tumor–Immune System Dynamics

Immune cells (PREDATOR) "hunt" or suppress **tumor cells** (PREY).



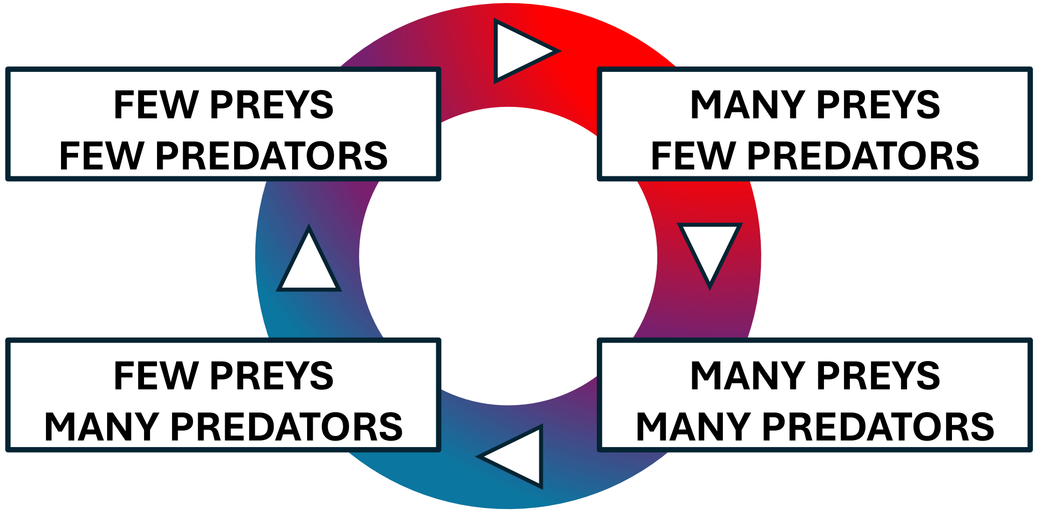
L.G De Pillis, A Radunskaya, The dynamics of an optimally controlled tumor model: A case study, Mat Com Mod (2003)

Model setup

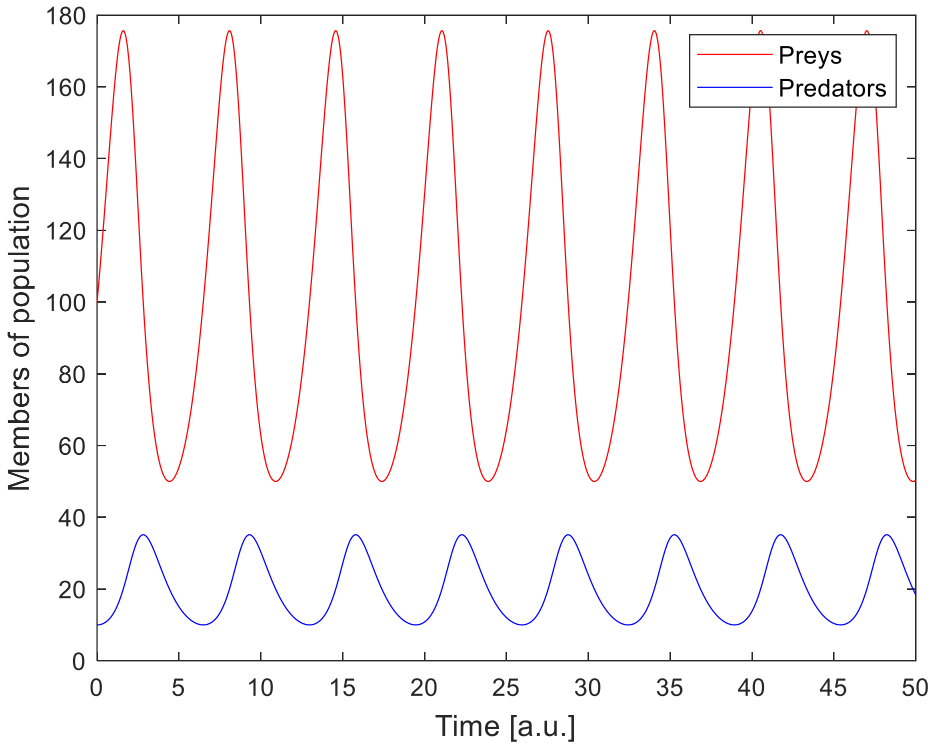


Volterra-Lotka model

$$\begin{cases} \dfrac{dx}{dt} = \alpha x - \beta xy \\ \dfrac{dy}{dt} = -\gamma y + \delta xy \end{cases}$$



Predicts oscillations in populations

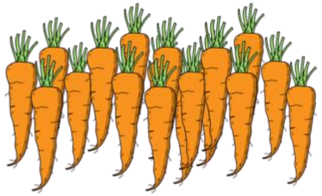


Some model assumptions

FOOD

UNLIMITED FOOD AVAILABILITY

Preys can always find food



EXCLUSIVE PREDATION

Predators only eats preys

HUNGRY PREDATORS

Predators will always eat when they can

UNIFORMITY

SPATIAL

Preys/predators are distributed homogeneously

TEMPORAL

Preys and predators don't get old

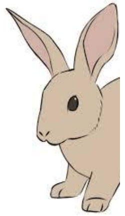
OTHER

NO EVOLUTION

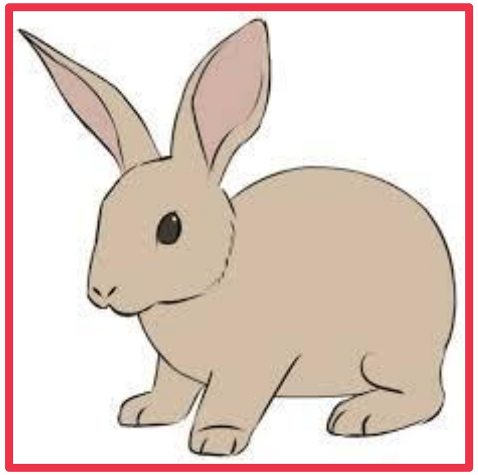
Predators will eat preys with the same efficiency over time

NO EXTINCTION

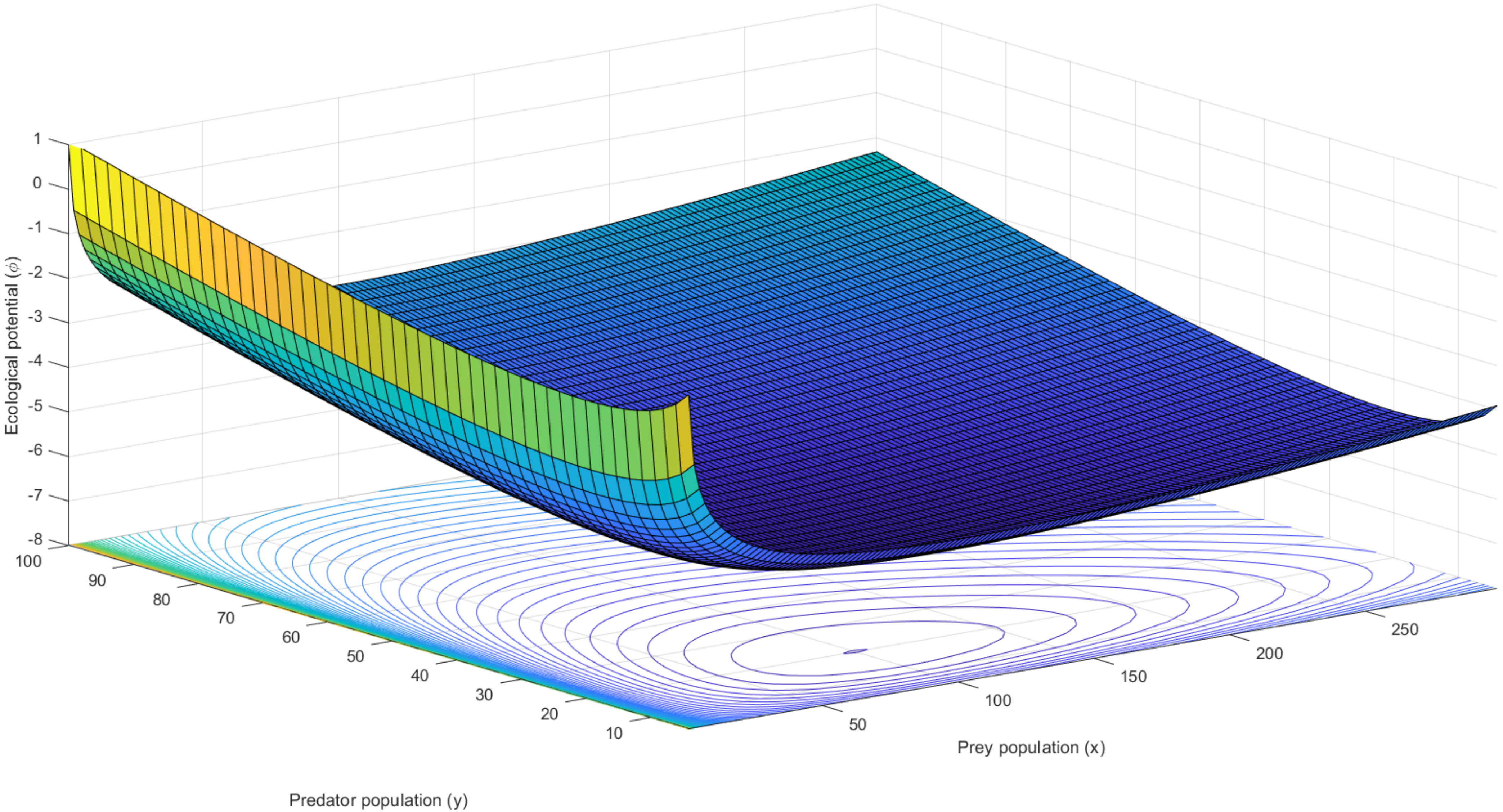
There can be fractional numbers of preys and predators



Matlab model simulation



Prey-Predator model (Volterra-Lotka)



Matlab model simulation

GOALS

1. Compute equilibrium point (x*,y*)
2. Generate time-domain curves for several (x0, y0)
3. Generate phases-space orbits for several (x0, y0)
4. Compute ecological potential over time
8. Find duration of 1 cycle
9. Compute average values over 1 cycle
10. Generate surface ecological potential ϕ vs x,y
11. Draw orbits as levels of ϕ

NICE PARAMETERS

x0=20:40:100	α = 1
y0=10:10:30	β = 0.05
t0=0	γ = 1
tf=300	δ = 0.01

SUGGESTIONS

1. Create model as function with input: t, y, α, β, γ, δ
2. Solve using ODE45

MODEL

$$\begin{cases} \dfrac{dx}{dt} = \alpha x - \beta xy \\ \dfrac{dy}{dt} = -\gamma y + \delta xy \end{cases}$$

EQUILIBRIUM

$$x^* = \frac{\gamma}{\delta}, y^* = \frac{\alpha}{\beta}$$

ECOLOGICAL POTENTIAL

$$\phi = -\gamma ln(x) - \alpha ln(y) + \delta x + \beta y$$

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Extension to *n* species

$$\frac{dx_i}{dt} = x_i \sum_{j=1}^{n} A_{ij}(1 - x_j)$$

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

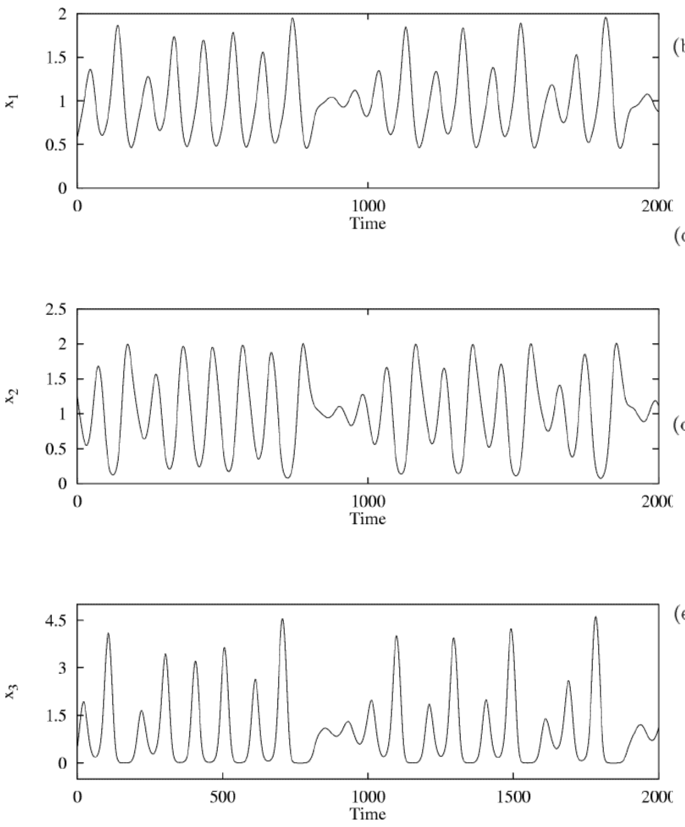
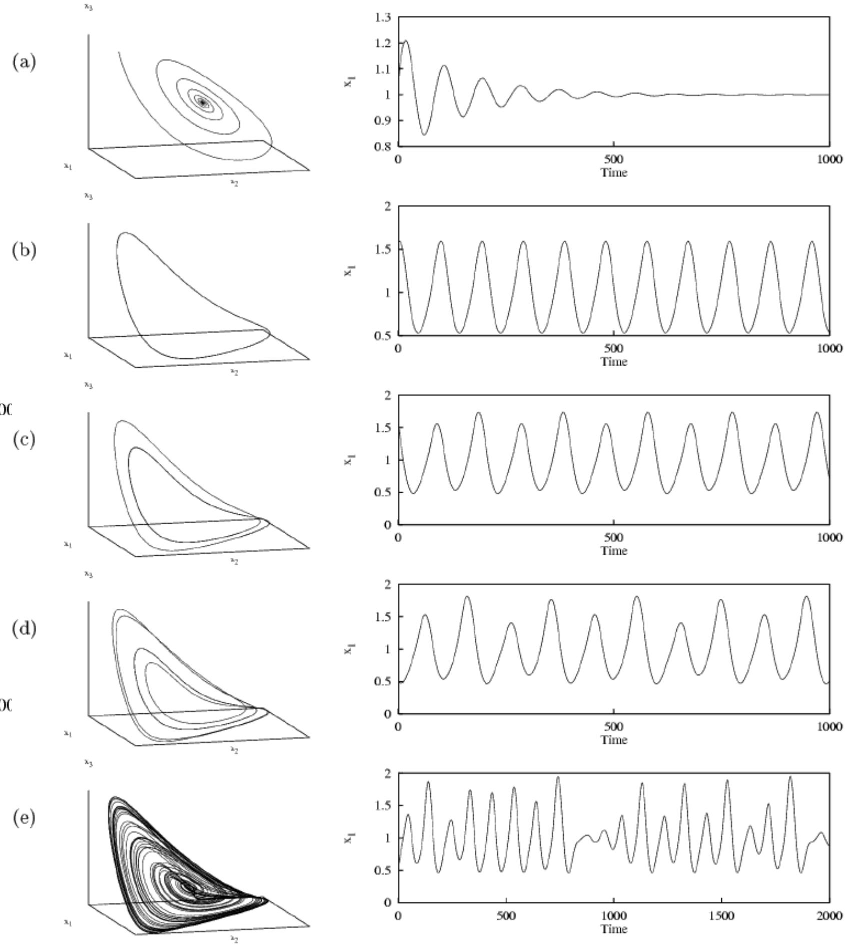


Figure 12.3 Population levels for the three-species Lotka-Volterra system



STABLE EQUILIBRIUM

OSCILLATIONS

OSCILLATIONS WITH HIGHER PERIODS

CHAOTIC BEHAVIOUR

Figure 12.4 Period doublings in a three-species Lotka-Volterra system: phase space is on the left and x_1 is plotted on the right. (a) spiral fixed point, (b) simple periodic orbit, (c) period-2 orbit, (d) period-4 orbit, (e) chaos