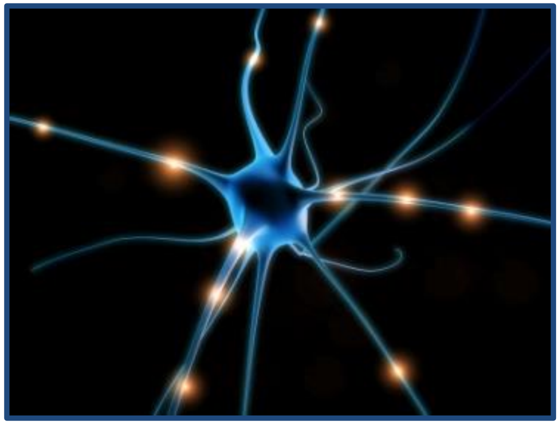
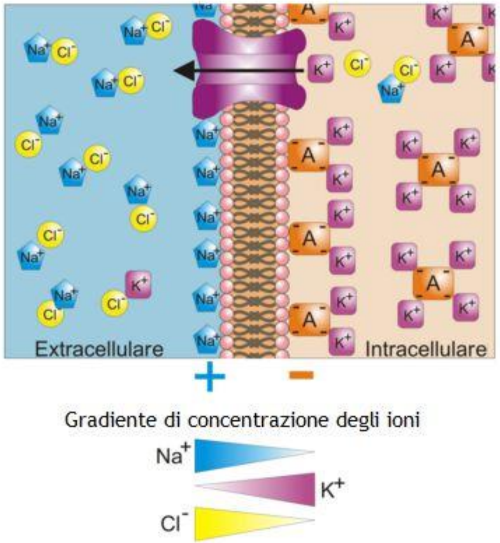
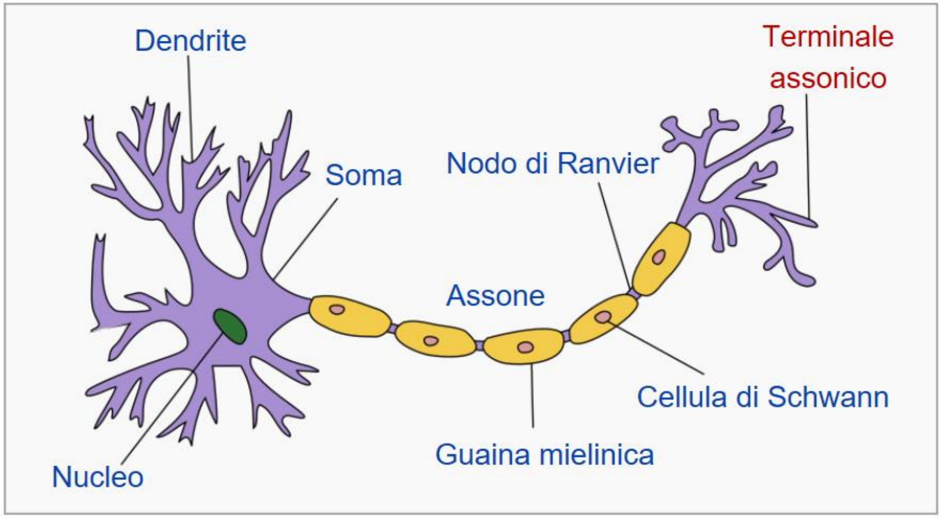
Neuron modeling



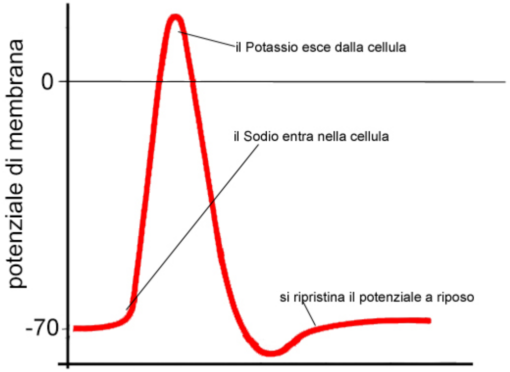
Neurons



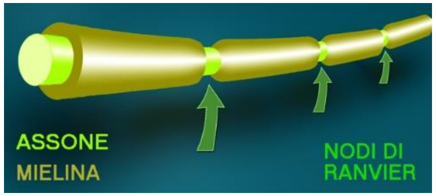
Extracellulare
Intracellulare
Gradiente di concentrazione degli ioni
Na^+
K^+
Cl^-



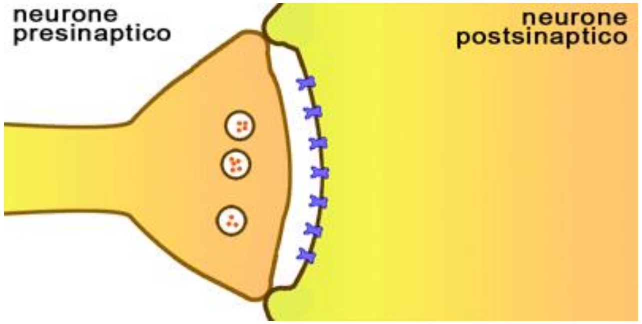
Dendrite
Terminale assonico
Soma
Nodo di Ranvier
Assone
Cellula di Schwann
Guaina mielinica
Nucleo



potenziale di membrana
0
-70
il Potassio esce dalla cellula
il Sodio entra nella cellula
si ripristina il potenziale a riposo



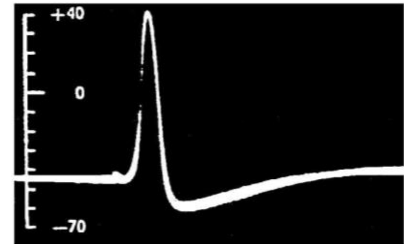
ASSONE
MIELINA
NODI DI RANVIER



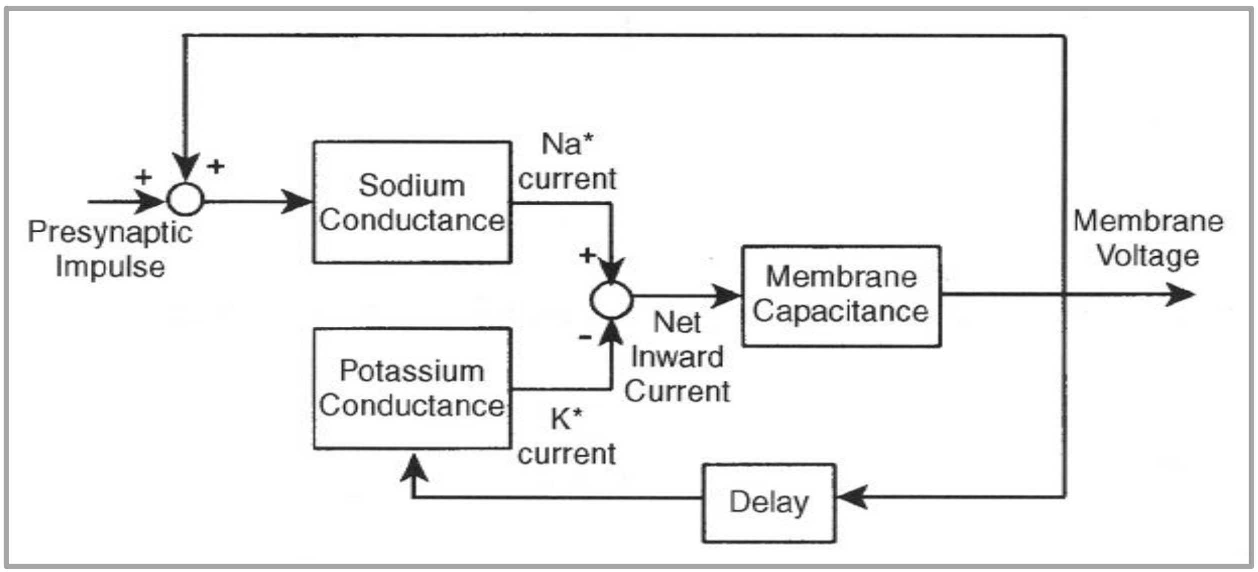
neurone presinaptico
neurone postsinaptico

Action potential generation

Il **potenziale d’azione** è un evento di breve durata (ms) in cui il potenziale elettrico di membrana di una **cellula eccitabile** aumenta rapidamente e scende seguendo una traiettoria coerente



Control system with positive and negative feedback



From Khoo M.C.K., **Physiological Control Systems**, IEEE Press, 2000

Hodgkin & Huxley model

J. Physiol. (1952) 117, 500–544

A QUANTITATIVE DESCRIPTION OF MEMBRANE CURRENT AND ITS APPLICATION TO CONDUCTION AND EXCITATION IN NERVE

BY A. L. HODGKIN AND A. F. HUXLEY

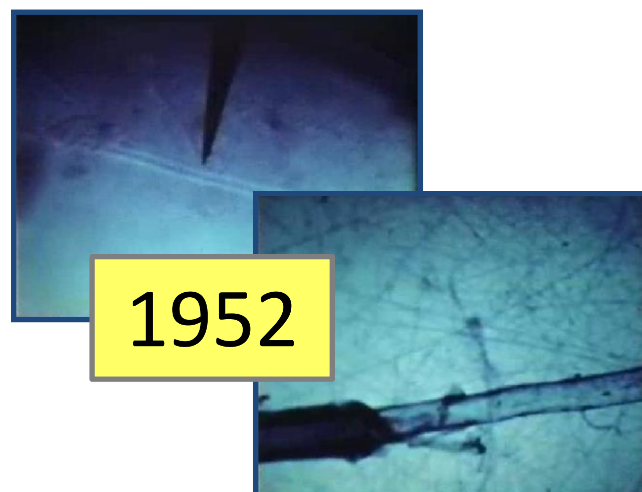
From the Physiological Laboratory, University of Cambridge

(*Received* 10 *March* 1952)

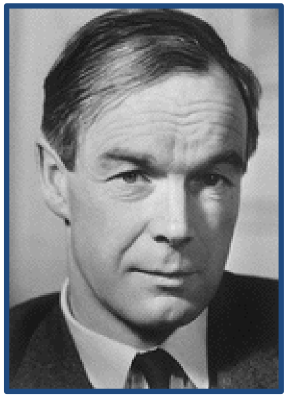
This article concludes a series of papers concerned with the flow of electric current through the surface membrane of a giant nerve fibre (Hodgkin, Huxley & Katz, 1952; Hodgkin & Huxley, 1952 *a–c*). Its general object is to discuss the results of the preceding papers (Part I), to put them into mathematical form (Part II) and to show that they will account for conduction and excitation in quantitative terms (Part III).

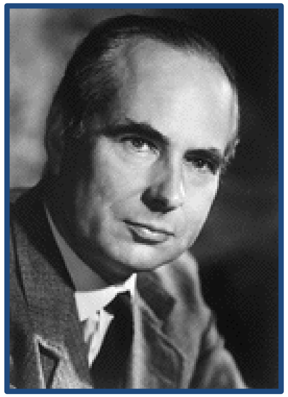
Hodgkin & Huxley model

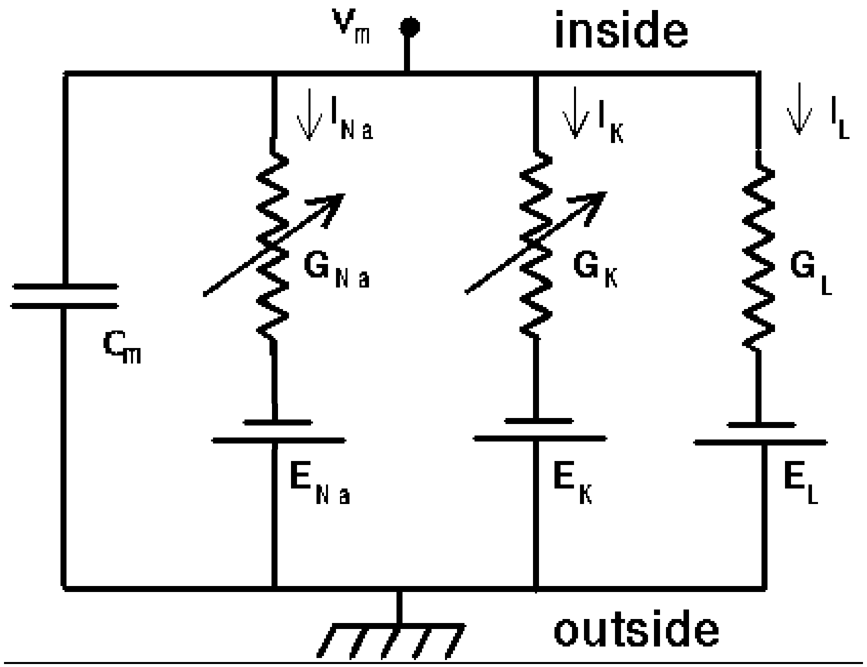
Theory, Experiments, Calculus, Insight!



Hodgkin & Huxley model

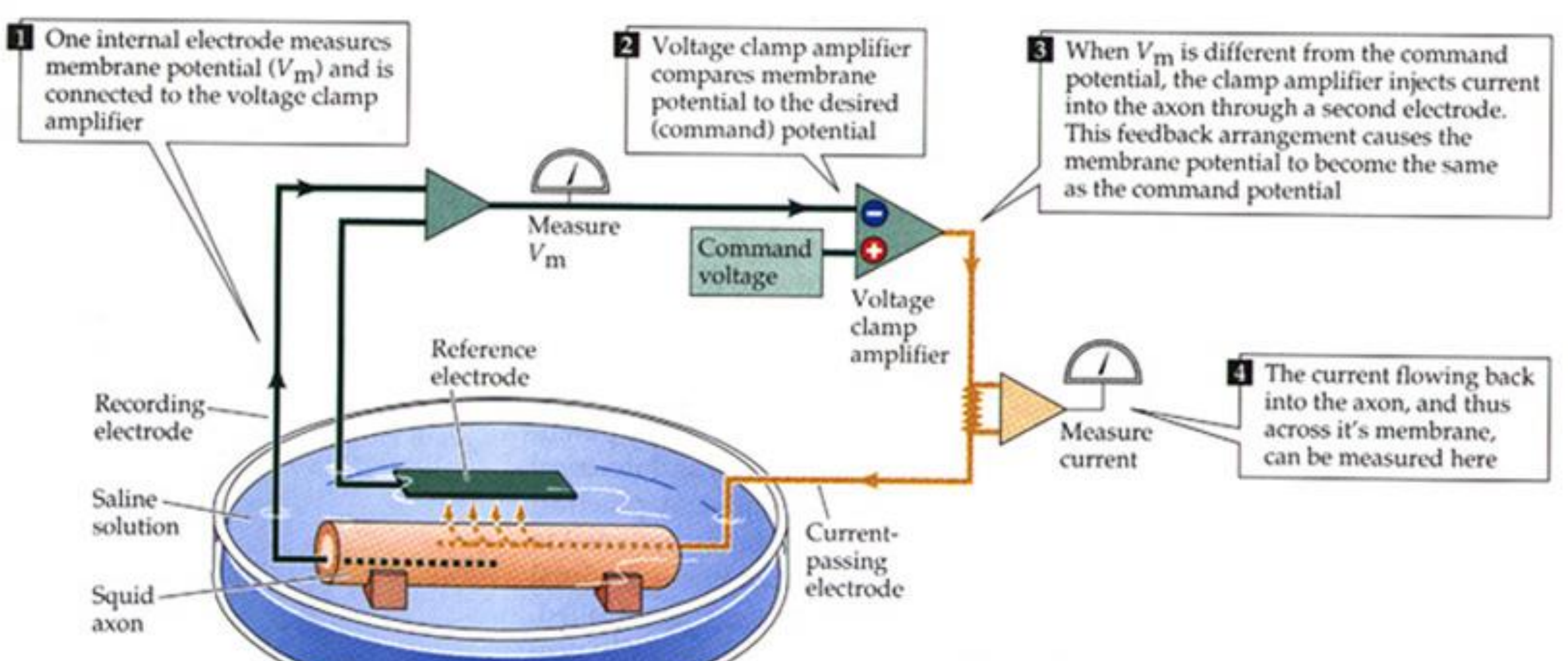






Equivalent circuit

Hodgkin & Huxley model



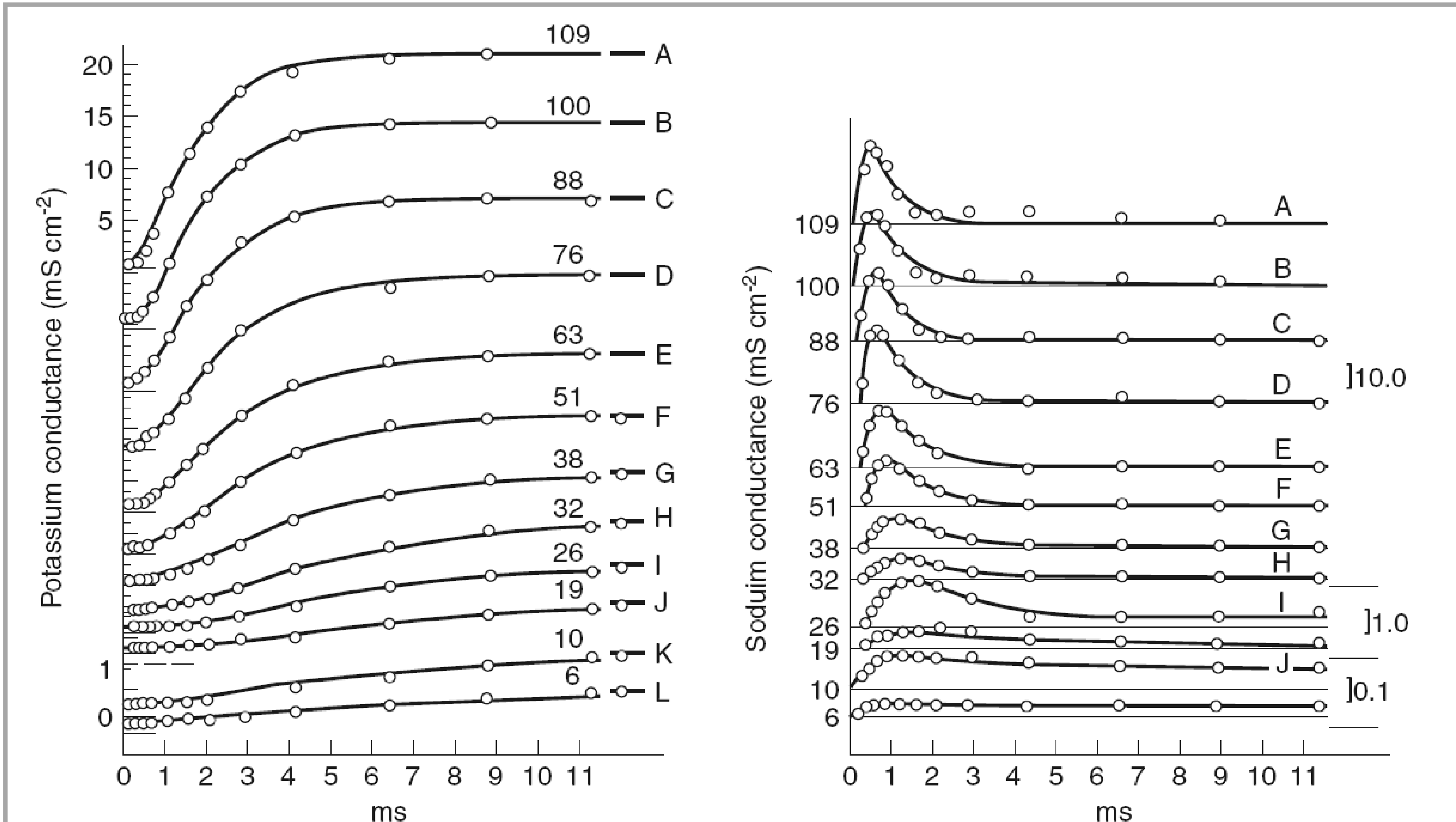
Voltage clamp technique for studying membrane currents of a squid axon.

Voltage clamp (to open the loop)

Cole & Marmont

Hodgkin & Huxley model

Ionic channel conductance



From Keener J. and Sneyd J., **Mathematical Physiology**, Springer

Hodgkin & Huxley model

Equations

$$C_m \frac{dv}{dt} = -\bar{g}_{\mathrm{K}} n^4 (v - v_{\mathrm{K}}) - \bar{g}_{\mathrm{Na}} m^3 h (v - v_{\mathrm{Na}}) - \bar{g}_{\mathrm{L}} (v - v_{\mathrm{L}}) + I_{\mathrm{app}},$$

$$\frac{dm}{dt} = \alpha_m (1 - m) - \beta_m m,$$

$$\frac{dn}{dt} = \alpha_n (1 - n) - \beta_n n,$$

$$\frac{dh}{dt} = \alpha_h (1 - h) - \beta_h h.$$

$$\alpha_m = 0.1 \frac{25 - v}{\exp\left(\frac{25-v}{10}\right) - 1},$$

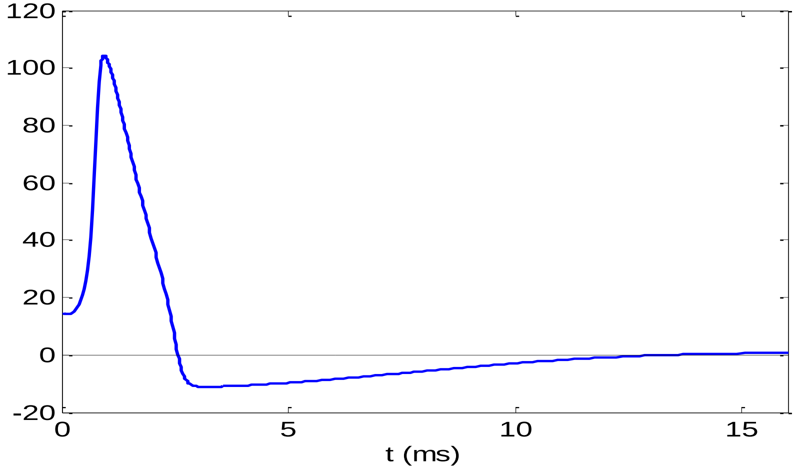
$$\beta_m = 4 \exp\left(\frac{-v}{18}\right),$$

$$\alpha_h = 0.07 \exp\left(\frac{-v}{20}\right),$$

$$\beta_h = \frac{1}{\exp\left(\frac{30-v}{10}\right) + 1},$$

$$\alpha_n = 0.01 \frac{10 - v}{\exp\left(\frac{10-v}{10}\right) - 1},$$

$$\beta_n = 0.125 \exp\left(\frac{-v}{80}\right).$$



Hodgkin & Huxley model

NUMERICAL METHODS

Membrane action potentials

Integration procedure. The equations to be solved are the four simultaneous first-order equations (26), (7), (15), and (16) (p. 518). After slight rearrangement (which will be omitted in this description) these were integrated by the method of Hartree (1932–3). Denoting the beginning and end of a step by t_0 and t_1 $(=t_0+\delta t)$ the procedure for each step was as follows:

(1) Estimate V_1 from V_0 and its backward differences.

(2) Estimate n_1 from n_0 and its backward differences.

(3) Calculate $(\mathrm{d}n/\mathrm{d}t)_1$ from eqn. 7 using the estimated n_1 and the values of α_n and β_n appropriate to the estimated V_1.

(4) Calculate n_1 from the equation

$$n_1 - n_0 = \frac{\delta t}{2}\left\{\left(\frac{\mathrm{d}n}{\mathrm{d}t}\right)_0 + \left(\frac{\mathrm{d}n}{\mathrm{d}t}\right)_1 - \frac{1}{12}\left[\Delta^2\left(\frac{\mathrm{d}n}{\mathrm{d}t}\right)_0 + \Delta^2\left(\frac{\mathrm{d}n}{\mathrm{d}t}\right)_1\right]\right\};$$

$\Delta^2(\mathrm{d}n/\mathrm{d}t)$ is the second difference of $\mathrm{d}n/\mathrm{d}t$; its value at t_1 has to be estimated.

(5) If this value of n_1 differs from that estimated in (2), repeat (3) and (4) using the new n_1. If necessary, repeat again until successive values of n_1 are the same.

(6) Find m_1 and h_1 by procedures analogous to steps (2)–(5).

(7) Calculate $\bar{g}_{\mathrm{K}} n_1^4$ and $\bar{g}_{\mathrm{Na}} m_1^3 h_1$.

(8) Calculate $(\mathrm{d}V/\mathrm{d}t)_1$ from eqn. 26 using the values found in (7) and the originally estimated V_1.

(9) Calculate a corrected V_1 by procedures analogous to steps (4) and (5). This result never differed enough from the original estimated value to necessitate repeating the whole procedure from step (3) onwards.

The step value had to be very small initially (since there are no differences at $t=0$) and it also had to be changed repeatedly during a run, because the differences became unmanageable if it was too large. It varied between about 0·01 msec at the beginning of a run or 0·02 msec during the rising phase of the action potential, and 1 msec during the small oscillations which follow the spike.

Hodgkin & Huxley model

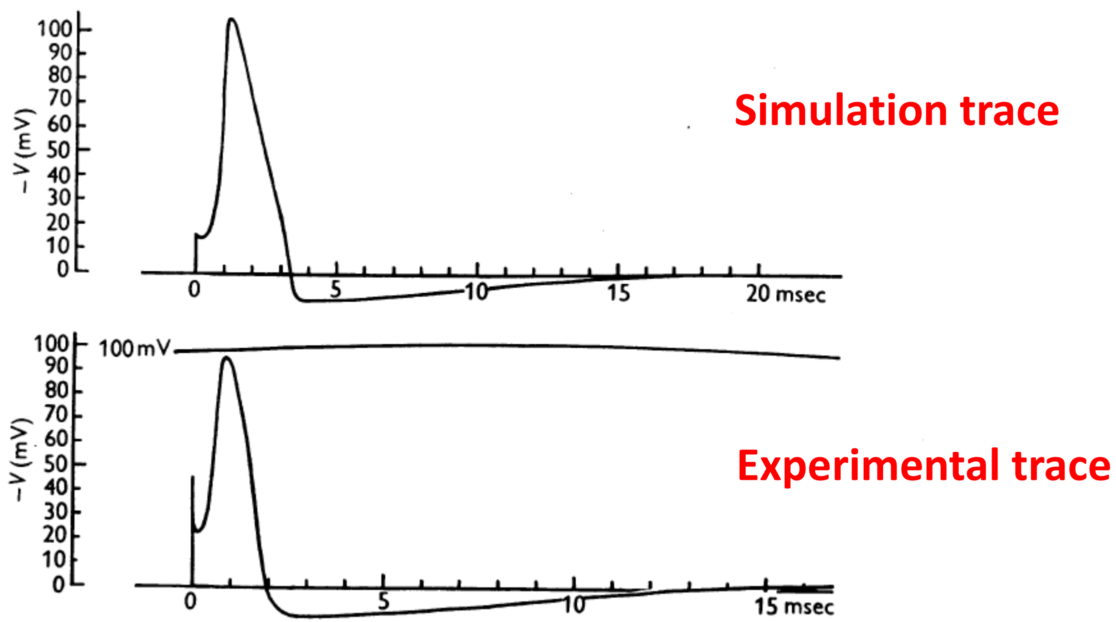
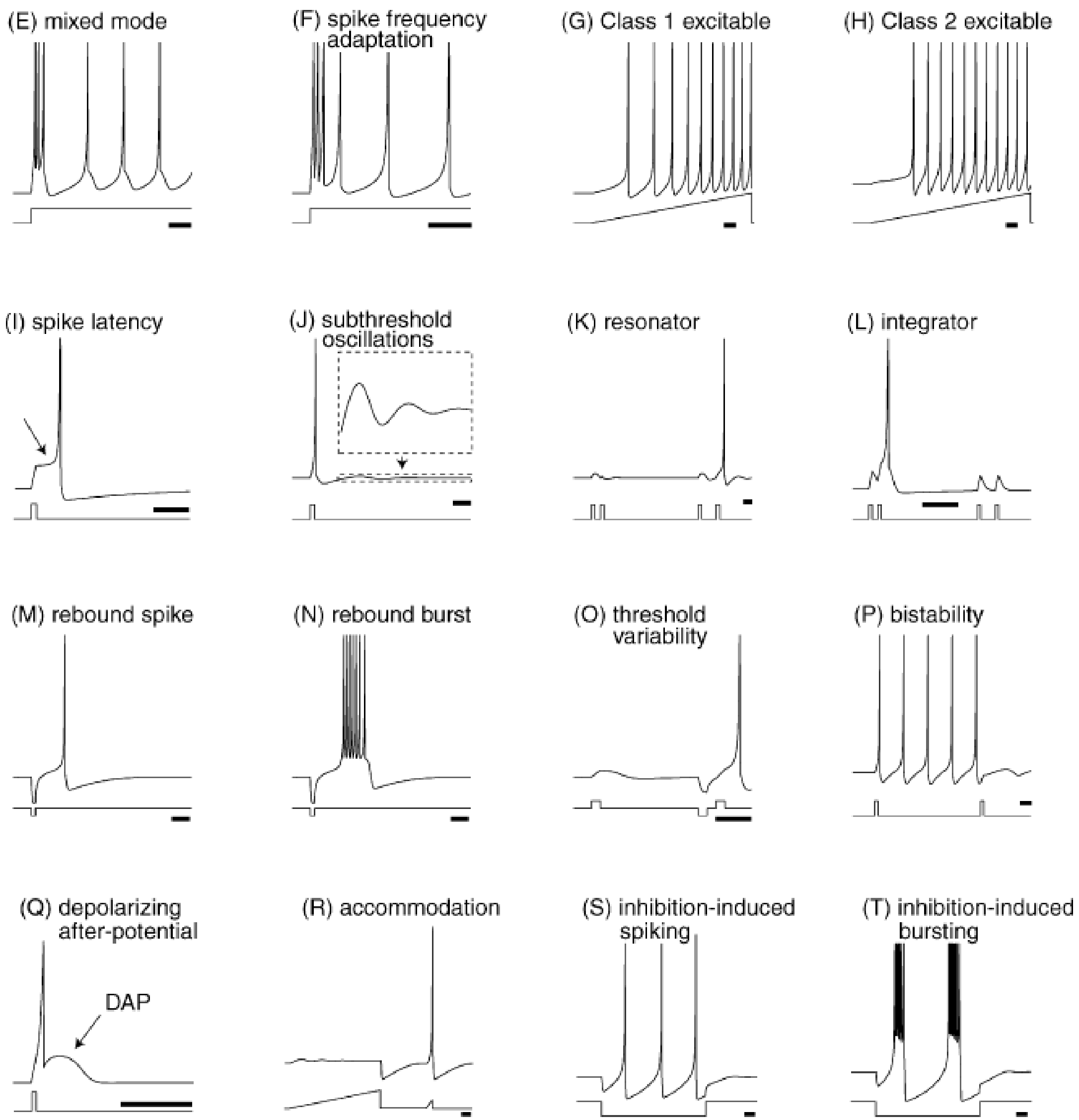


Fig. 13. Upper curve: solution of eqn. (26) for initial depolarization of 15 mV, calculated for 6° C. Lower curve: tracing of membrane action potential recorded at 9·1° C (axon 14). The vertical scales are the same in both curves (apart from curvature in the lower record). The horizontal scales differ by a factor appropriate to the temperature difference.

Hodgkin & Huxley model

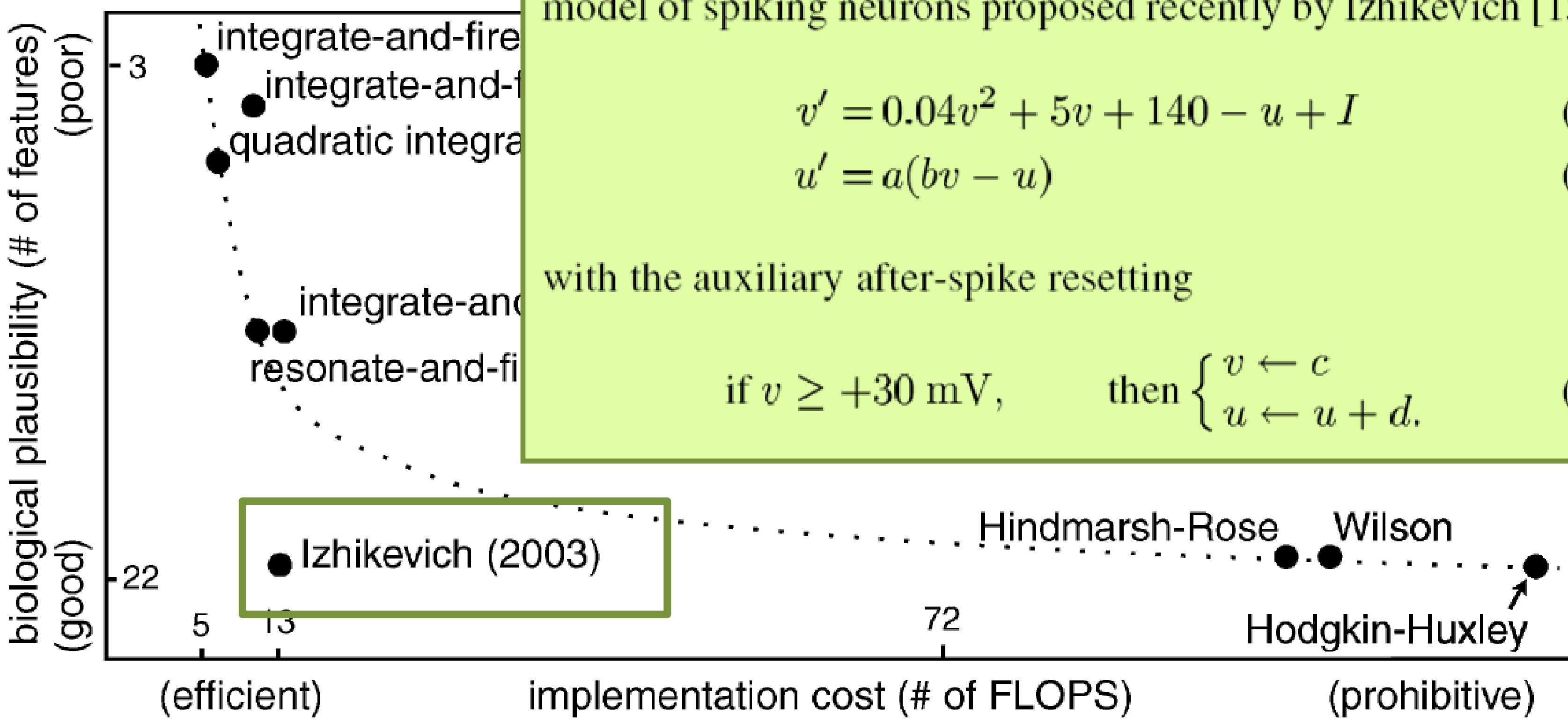
https://www.youtube.com/watch?v=k48jXzFGMc8

Not only spikes



From Izhikevich EM, IEEE Trans. Neural Networks, 2004

Many models



F. Spiking Model by Izhikevich (2003)

All of the responses in Fig. 1 were obtained using a simple model of spiking neurons proposed recently by Izhikevich [15]

$$v' = 0.04v^2 + 5v + 140 - u + I \tag{1}$$

$$u' = a(bv - u) \tag{2}$$

with the auxiliary after-spike resetting

$$\text{if } v \geq +30 \text{ mV}, \qquad \text{then } \begin{cases} v \leftarrow c \\ u \leftarrow u + d. \end{cases} \tag{3}$$

From Izhikevich EM, IEEE Trans. Neural Networks, 2004

Many models

Models	biophysically meaningful	tonic spiking	phasic spiking	tonic bursting	phasic bursting	mixed mode	spike frequency adaptation	class 1 excitable	class 2 excitable	spike latency	subthreshold oscillations	resonator	integrator	rebound spike	rebound burst	threshold variability	bistability	DAP	accommodation	inhibition-indiced spiking	inhibition-induced bursting	chaos	# of FLOPS
integrate-and-fire	-	+	-	-	-	-	-	+	-	-	-	-	+	-	-	-	-	-	-	-	-	-	5
integrate-and-fire with adapt.	-	+	-	-	-	-	+	+	-	-	-	-	+	-	-	-	-	+	-	-	-	-	10
integrate-and-fire-or-burst	-	+	+		+	-	+	+	-	-	-	-	+	+	+	-	+	+	-	-	-		13
resonate-and-fire	-	+	+	-	-	-	-	+	+	-	+	+	+	+	-	-	+	+	+	-	-	+	10
quadratic integrate-and-fire	-	+	-	-	-	-	-	+	-	+	-	-	+	-	-	+	+	-	-	-	-	-	7
Izhikevich (2003)	-	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	13
FitzHugh-Nagumo	-	+	+	-		-	-	+	-	+	+	+	-	+	-	+	+	-	+	+	-	-	72
Hindmarsh-Rose	-	+	+	+			+	+	+	+	+	+	+	+	+	+	+	+	+	+		+	120
Morris-Lecar	+	+	+	-		-	-	+	+	+	+	+	+	+		+	+	-	+	+	-	-	600
Wilson	-	+	+	+			+	+	+	+	+	+	+	+	+	+		+	+				180
Hodgkin-Huxley	+	+	+	+			+	+	+	+	+	+	+	+	+	+	+	+	+	+		+	1200

From Izhikevich EM, IEEE Trans. Neural Networks, 2004