CHAPTER 10

Modern Portfolio Theory: The Efficient Frontier and Portfolio Optimization

Investors want the best possible return for a given amount of risk. In other words, they want to maximize *risk-adjusted* returns. Portfolio design is about the creation of a collection of assets whose combined risk level may be lower than that for any individual component. This apparent magic was first formalized by Harry Markowitz in the 1950s. He shared a Nobel Prize for this work, and his original ideas have been extended by other academics. These ideas are known collectively as modern portfolio theory (MPT). In this chapter we will use a stylized example to make the core elements of MPT clear (Figure 10.1).

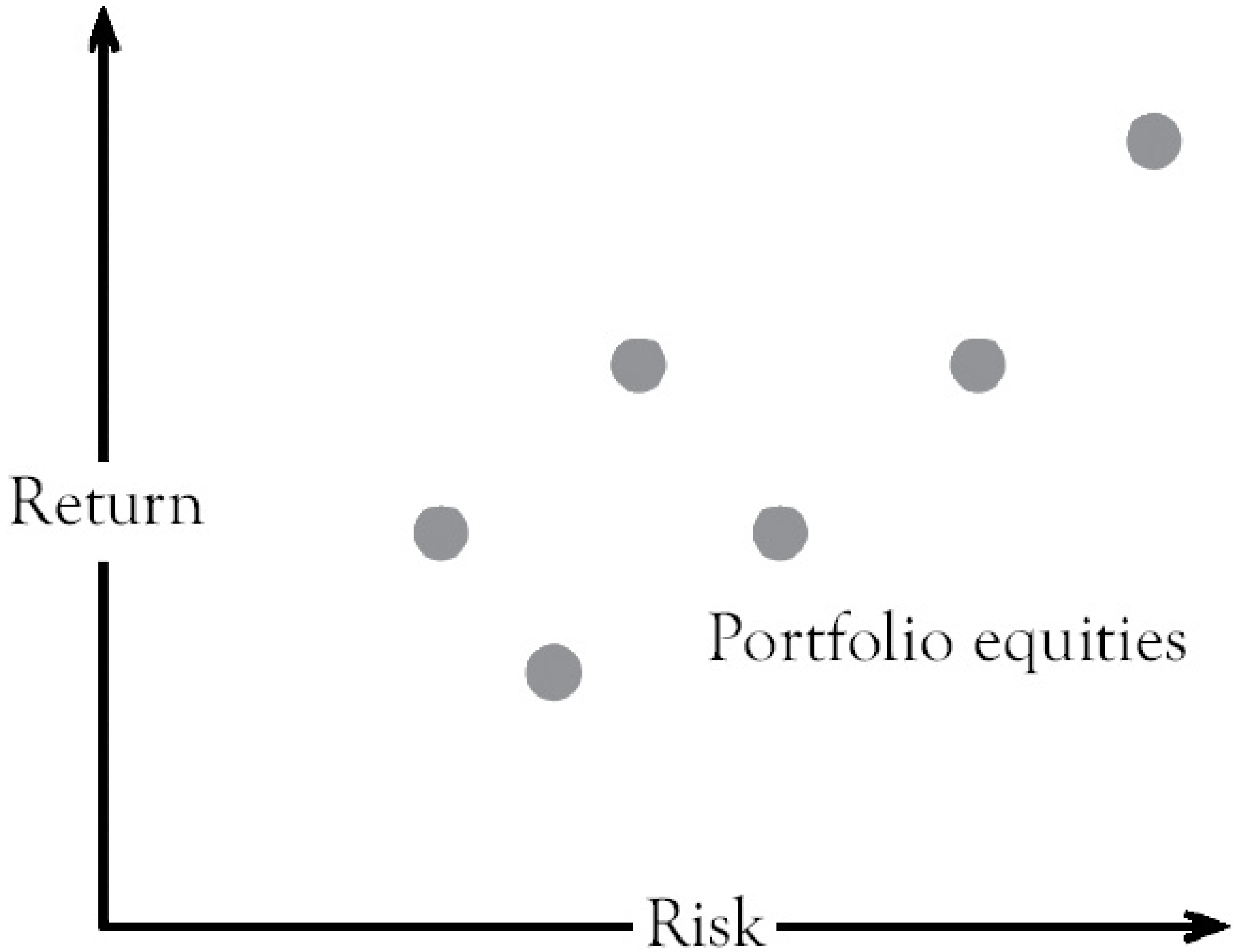

Figure 10.1 Risk and returns for asset classes

Risk and Return, Again

Assume your portfolio can be composed of a range of specific assets, such as company stocks or individual bonds. Each of these can be classified in risk and return space based on your expectations for each. Expectations may be based on a forecast, or on past experience (implicitly assuming that the asset's future behavior will resemble its past behavior). Figure 10.2 illustrates a typical scatter plot for a range of assets. Note that individual stocks' returns and risks seem to be randomly distributed, but with an upward bias: Generally, it isn't possible to enhance return without accepting more risk. Looking at it another way, in an efficient market, investors will price low risk and high return assets highly in the first place, thereby depressing future returns. Only volatile (risky) assets will be priced low and produce superior returns. This *risk and return trade-off* is one of the fundamental assumptions in modern finance.

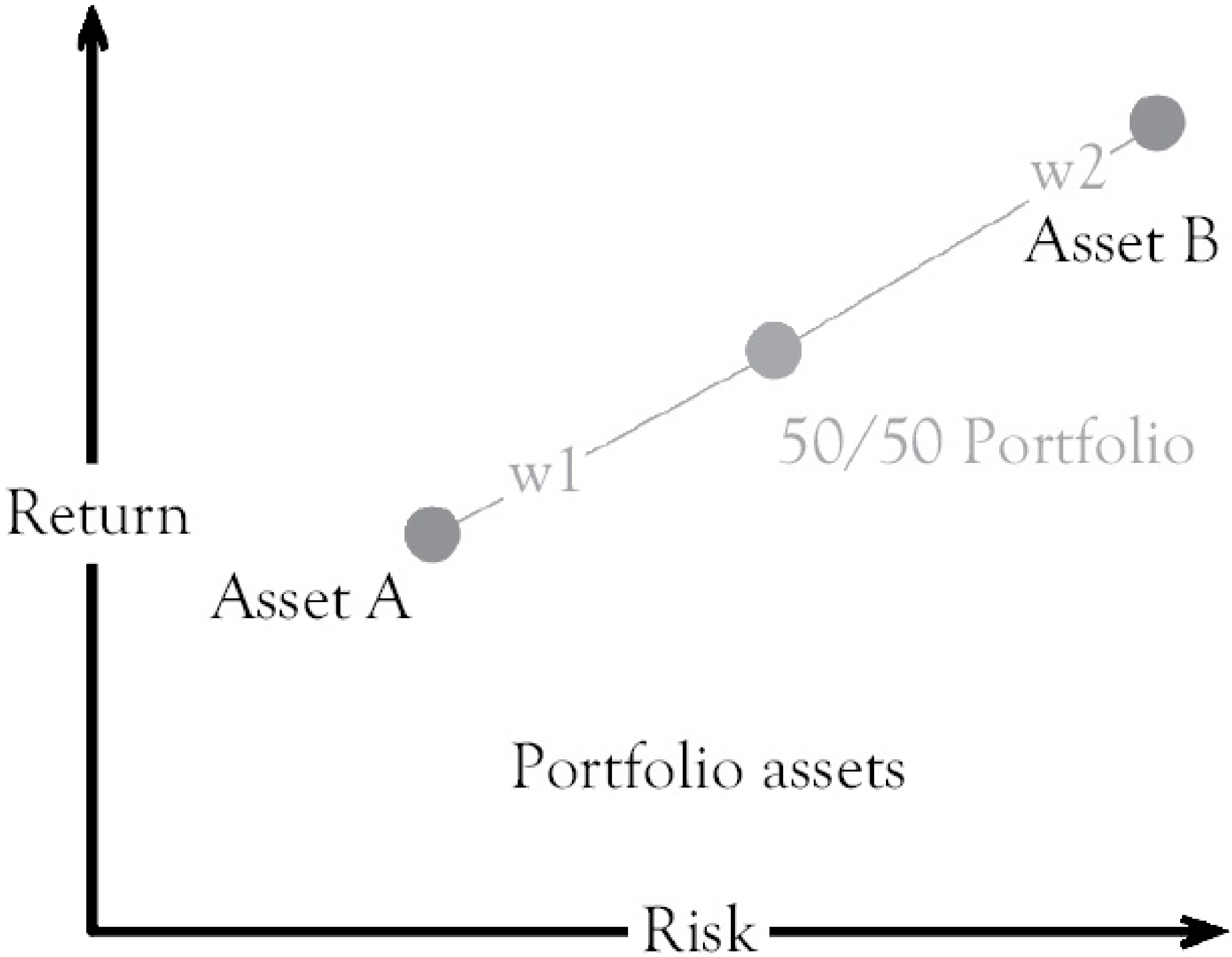

Figure 10.2 Risk and returns for two pure asset classes and one 50/50 portfolio

Once assets are selected (e.g., which stocks to buy) a key decision in the construction of a portfolio is to select the proportions we use of each asset, called portfolio weights or allocations. The *efficient frontier* connects the assets that lie generally above and to the left of all other assets on the scatter plot. If 100 percent of our portfolio is in Asset A, its risk and return are those of point A in Figure 10.2. A 50/50 mix of Assets A and B will usually produce risk and return about halfway between points A and B. In principle, it seems we should be able to extrapolate this to a portfolio with any number of assets: We would expect its risk and return to fall roughly in the center of the included points (assets included in the portfolio), closer to the heaviest-weighted assets. An example is Figure 10.3. But this need not be the case: The right combination of assets can reduce risk at little cost in return. This apparent magic is known as portfolio construction, specifically portfolio optimization.

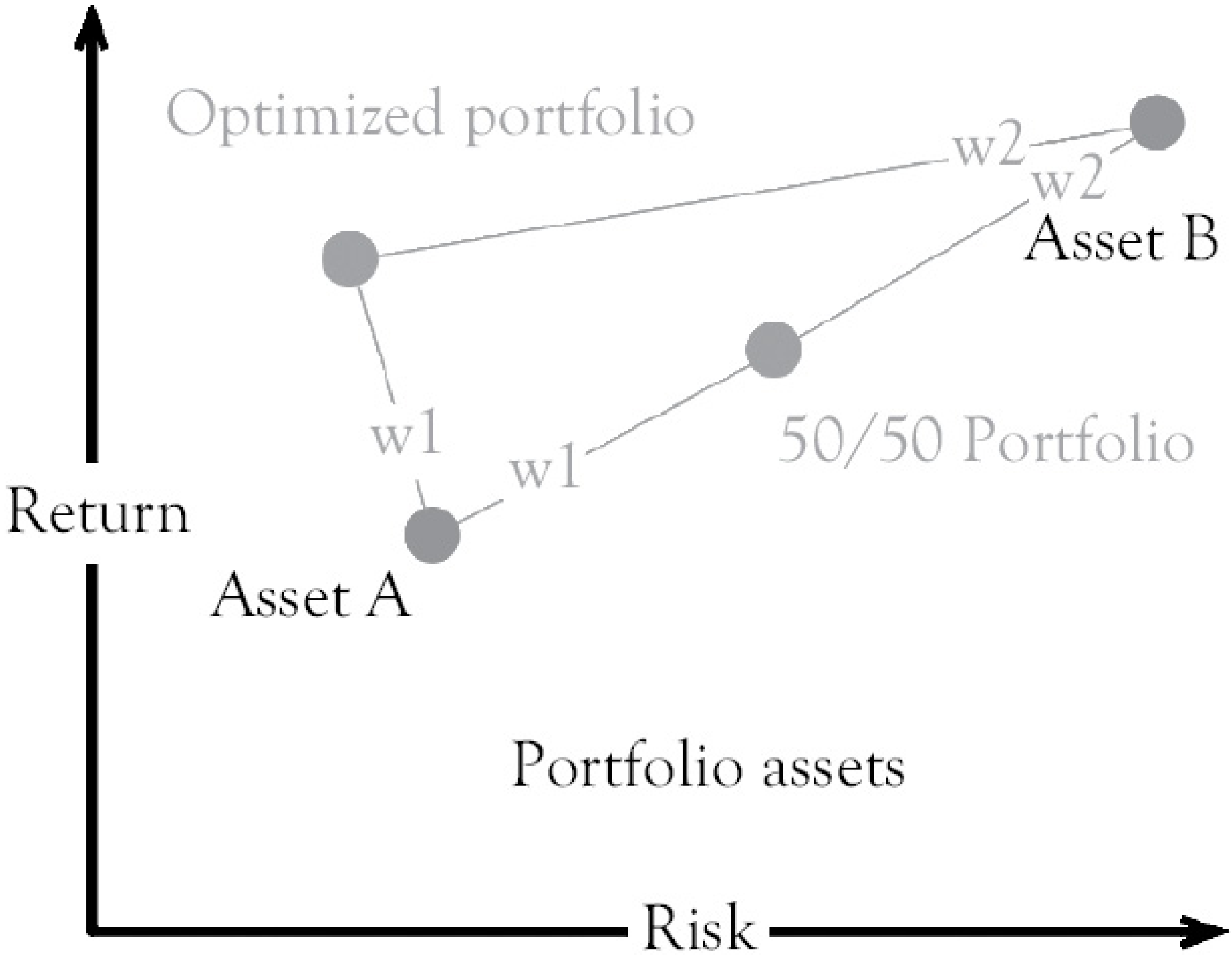

Figure 10.3 Risk and returns for mixed and optimized portfolios

To explain this magic entails a brief digression into the profound importance of correlations in a portfolio.

Why Low Correlation Is Prized

Figure 10.4 shows the price history of three stocks, with tickers ABC, DEF, and GHI. Each achieves about the same return over the period, and appears to have similar volatility (standard deviation), based on the graph alone. You can see that ABC and DEF move virtually in tandem, so events appear to affect them similarly. These two stocks have a very high positive correlation (say, 0.9). Buying both does not really diversify your portfolio. In contrast, GHI seems to move opposite to ABC and DEF: it rises when they fall, and vice versa. GHI appears to be highly *negatively correlated* with ABC and DEF (say, –0.9).

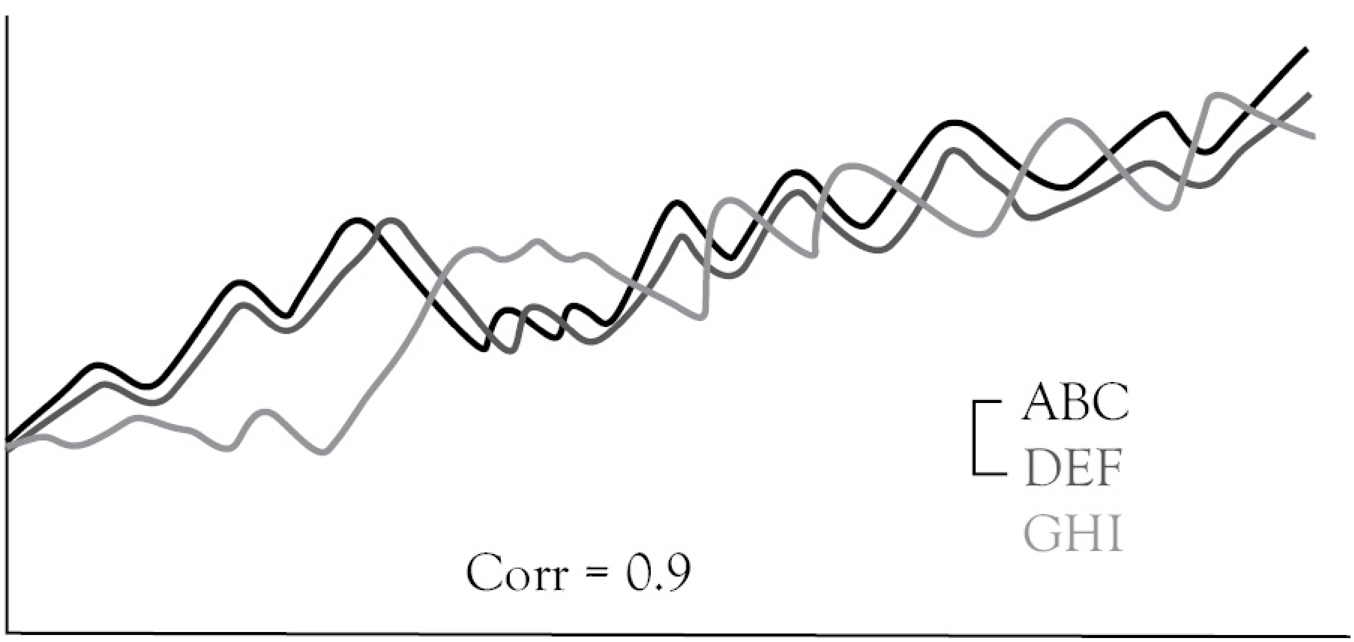

Figure 10.4 Price history of stocks ABC, DEF, and GHI

Creating a portfolio combining GHI for 50 percent with each of ABC (25 percent) and DEF (25 percent) will produce Figure 10.5, the portfolio's price history over the same period. Its total return will be the weighted average of the returns of the three stocks, as expected. But what is surprising is that *the portfolio's volatility is far less than any of its components.*

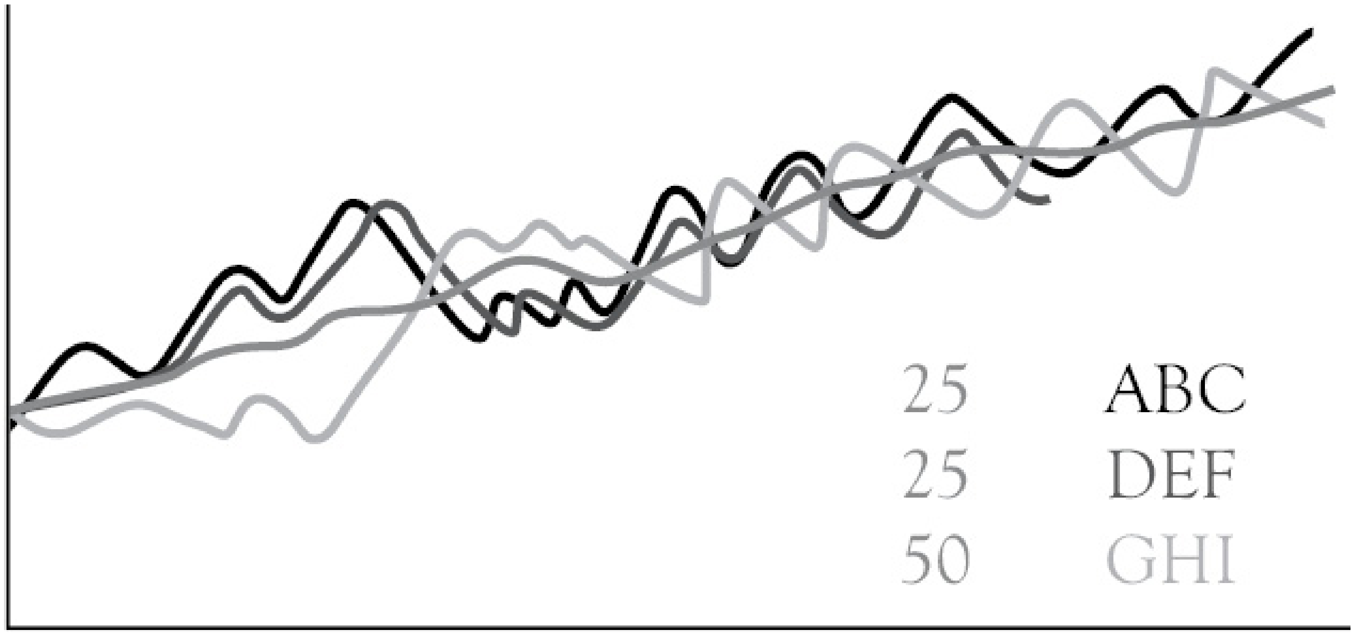

Figure 10.5 Price history of portfolio of stocks ABC (25 percent), DEF (25 percent), and GHI (50 percent)

The reason for this is that the portfolio combines assets that are negatively correlated. This dampens the oscillations in return and produces much less volatility than any of the three stocks achieve by themselves.

Figure 10.6 now returns us to the scatter plot of portfolio components in terms of risk and return, but with two important additions:

- Since our measure of risk-adjusted return, the Sharpe ratio, is the ratio of the vertical (return) divided by the horizontal (risk) in Figure 10.6, the slope of a ray from the origin outward that passes through a portfolio's location is that portfolio's Sharpe ratio. The point on the efficient frontier that intersects with the steepest-sloped ray has the highest Share ratio—it delivers the most return per unit of risk.
- Combining assets with negative correlations but similar returns can lower risk without sacrificing return. In practice, few assets are negatively correlated, so we will have to settle for low positive correlations. But the same principle applies, if not as strongly.

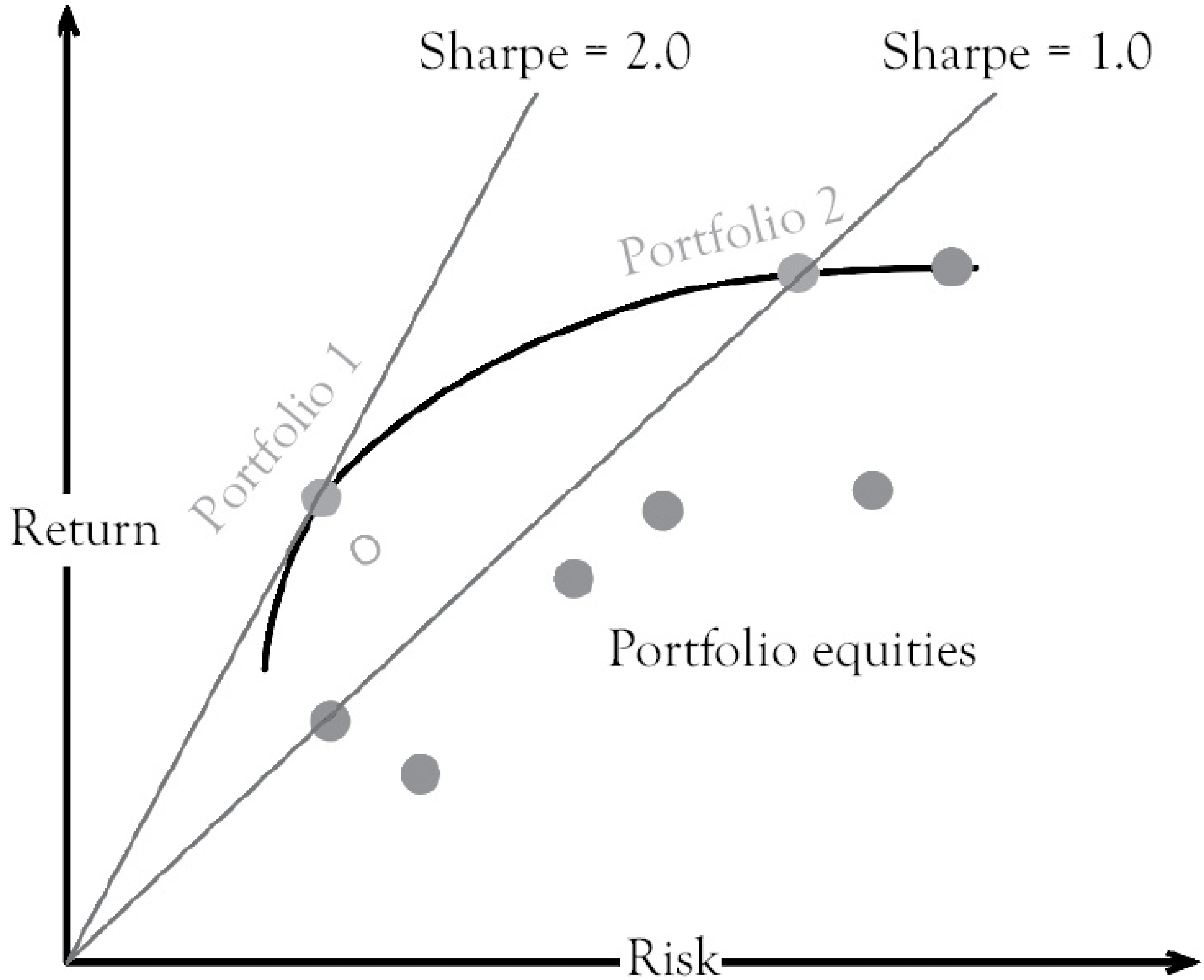

Figure 10.6 Scatter plot of asset risks and returns

Assume for the moment that Portfolio 1 in Figure 10.6 is our designed portfolio, found by combining several weakly correlated asset choices. You can see that it is above and to the left of most assets in the scatter plot. It will have the highest Sharpe ratio (the steepest-sloped ray from the origin). This is the goal of portfolio optimization: to design the combination of assets that produces the lowest risk for a specified target return.

Optimization Basics

Optimization means identifying the best choice among a number of options. In the context of a portfolio, this means choosing what fraction of the portfolio should comprise each of the available assets. Assets could be specific issues such as individual companies' common stock, or asset classes such as large cap, midcap, small cap, and microcap equities; short-, intermediate-, or long-term bonds; residential, commercial, or industrial real estate, and so forth—whatever asset class trades on a major exchange.

Most optimization problems have common components. They are listed here, with explanation specific to portfolio optimization.

1. *Decision variables*: Proportion of portfolio devoted to each asset (portfolio weights).
2. *Objective function*: Goal you wish to maximize or minimize. Commonly, portfolio optimizers minimize risk, but they could maximize total portfolio return. The objective function has a coefficient on each portfolio element that reflects that element's contribution to the goal. For an investment portfolio, such coefficients would be each asset's return.
3. *Constraints*: Limits you impose on the set the optimizer should consider. If the objective function was to minimize risk, one constraint might be to achieve a total return equal or above a threshold amount. You may also set minimum and maximum weights on some elements of the portfolio, such as holding no more than 10 percent in bonds rated below investment grade; or no more than 5 percent in any single issue. One definitional constraint is that the sum of all weights in a long-only portfolio cannot exceed 100 percent.
4. *A search procedure*: The optimization algorithm uses some procedure to efficiently search for the best combination of decision variables. In some cases, it may be brute force—trying one combination after another. Generally, it uses some efficient or semiefficient procedure. For example, if the function being maximized is convex, trying alternatives an increment above and below the last trial will determine which weights should be increased and which decreased. Some optimization problems have very efficient solutions; for example, any function for which there is a calculable derivative (remember your high school calculus) will have an immediately identifiable optimum. Similarly, for linear programming problems (optimization problems in which the objective and constraints can be expressed as linear functions), optimal solutions will always be corner solutions, so it is only necessary to compute the objective value in corners (intersections between constraints). Binary search algorithms and similar procedures likewise speed up the process of seeking an optimum.

A publicly available optimizer is available as part of QSTK in the public domain, thanks to the efforts of UCLA. You will find it at http://wiki.quantsoftware.org/index.php?title=QSTK_Tutorial_8. Microsoft Excel also includes a simple optimizer within its macros.

Portfolio Optimization and the Efficient Frontier

Figure 10.7 repeats Figure 10.6 from before, with small additions. You can see that the most desirable portfolios will be located in the upper-left portion of the scatter plot. The point (combination of assets) that intersects with the steepest ray emanating from the origin (Sharpe ratio) will maximize the ratio of return divided by risk. In this figure, two straight lines have been drawn, a horizontal line indicating the target return, and a vertical line indicating the minimum risk portfolio discovered by an optimizer that provides that return.

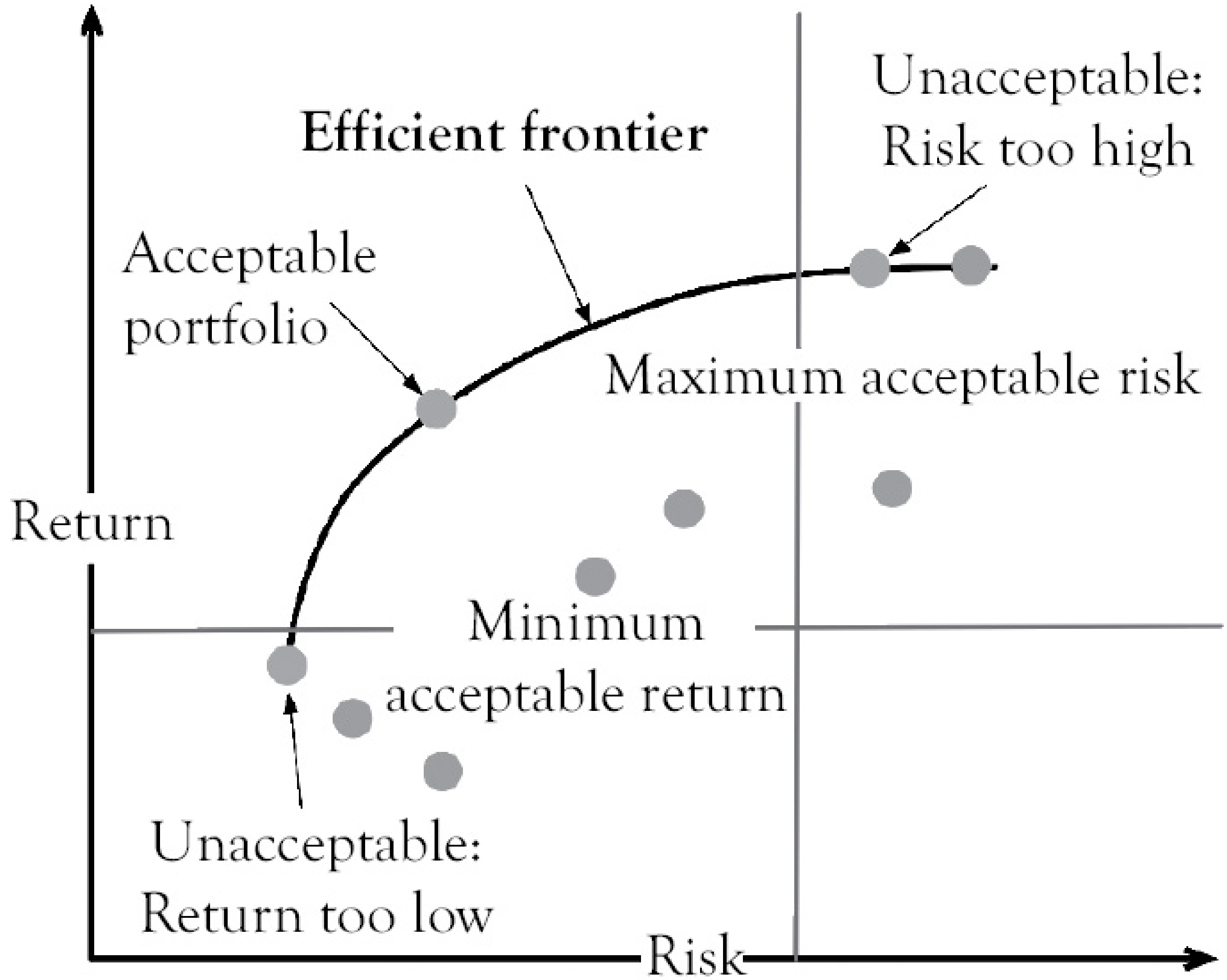

Figure 10.7 The lowest risk portfolios for each level of return lie along a line called the efficient frontier

For each level of target return, there is a set of weights that provides the lowest-risk portfolio for that return. If we chart all these possible portfolios, by optimizing for each target return they form a curve called the *efficient frontier*. Note that the efficient frontier provides lower-risk portfolios (further left) than individual assets with similar returns.

A Dynamic Process

You can see that the optimal portfolio weights for any asset will be greatly affected by its expected return, and its correlation with other assets that are candidates for the portfolio. *These are not stable values.* Returns are cyclical: When an asset becomes popular, buyers bid up its price, which reduces future returns. Correlations can change: If a global event affects two apparently unrelated assets, uncorrelated assets can suddenly become highly correlated. This is common in a market crisis; A Wall Street joke is “in a market meltdown, nothing rises except correlations.” When investors feared Depression conditions in the fall of 2008, they rushed to liquidate risky assets—of all types—to move into cash and Treasuries. Bonds and stocks, which are usually relatively uncorrelated, were suddenly highly correlated. So portfolios need to be optimized on a recurring basis.

No Panacea

While optimization can reduce some of the guesswork of portfolio construction, its apparent rigor and scientific basis can be seductive. Any model-based construction suffers from the following limitations:

- *Outputs are only as good as inputs.* Asset allocations depend on forecast returns, and forecasts can be noisy and erroneous. Return data may have biases and errors, such as the survivorship bias in hedge fund returns mentioned later. Quantitative outputs can appear overly rigorous if they are based on flawed inputs.
- *Standard deviation of return is not the only measure of risk.* Other metrics that focus mainly or exclusively on downside deviations in asset returns, such as the Sortino ratio or downside capture, may better reflect what risk means to you.
- *Beware tail risk.* Many hedge fund blowups, such as LTCM’s in the summer of 1998 (described in the case study later), occurred because managers underestimated the likelihood of extreme negative events. Portfolios that were designed based on only a few decades of data omitted “black swan” events that last occurred before the data series began. In addition, it is arguable that tails are getting fatter—extreme events are occurring more frequently, and with wider consequences as assets become more closely correlated. In essence, risk was misestimated.

The upshot is that, as with any quantitative decision support tool, it is only a tool, not an oracle.

Bio: Ray Dalio, Bridgewater

Born: 1950

Firm: Bridgewater Associates, Westport, Connecticut

Founded: 1975

Style: Global macro

How it differentiates: (1) Intensive research, (2) exceptional transparency with clients and (3) corporate culture places strong emphasis on introspection and self-criticism—almost cultlike.

Annual return: 18 percent, *since 1991*

Assets under management (AUM): $145 billion (March 2013)

Dalio's background: Ray Dalio grew up in a middle-class family on Long Island, a self-proclaimed "below average" student. He became interested in investing as a preteen golf caddy. It was the early 1960s and the stock market was first emerging from a generation-long slumber following the 1929 Crash and the Great Depression. After college, Dalio went to the Harvard Business School for his MBA, graduating in 1973. He worked as a commodities trader only briefly before being fired for insubordination, leaving to found Bridgewater in 1975. The firm was originally a contract research shop, which moved into money management in the early 1990s. Bridgewater's culture is unique in its emphasis on self-criticism and other principles at the core of Dalio's philosophy.

Colorful quotes (from Dalio's self-published book, Principles*):*

"The consensus is often wrong, so I have to be an independent thinker. To make any money, you have to be right when they're wrong."

"I believe that you can probably get what you want out of life if you can suspend your ego and take a no-excuses approach to achieving your goals with open-mindedness, determination, and courage, especially of you rely on the help of people who are strong in areas where you are weak."

"Create a culture in which it is OK to make mistakes but unacceptable not to identify, analyze, and learn from them."