Training Sequence Assisted Channel Estimation for MIMO OFDM

Sumei Sun*, Ingo Wiemer†, C. K. Ho*, and T. T. Tjhung*
*Institute for Communications Research, 20 Science Park Road, #02-34/37 TeleTech Park,
Science Park II, Singapore 117674
Email: sunsm, hock, tjhungtt@icr.a-star.edu.sg
†Faculty of Electrical Engineering, IEE, Dresden University of Technology,
Mommsenstr. 13, 01062 Dresden, Germany
Email: wiemer@iee1.et.tu-dresden.de

Abstract--- **In this paper we present several results of our study on training sequence assisted channel estimation for Multiple-Input Multiple-Output (MIMO) Orthogonal Frequency Division Multiplexing (OFDM) systems. After developing a linear matrix algebraic model for the cyclic prefix based MIMO OFDM systems, we will define a generalised preamble structure which is a simple extension of the preambles used in single-input single-output (SISO) OFDM so that its good properties, such as low peak to average power ratio, can be maintained. We then derive the least squares (LS) and linear minimum mean squared error (LMMSE) channel estimation algorithms based on the proposed preamble design. In order to reduce the preamble length, we further propose a switched subcarrier preamble scheme in which the transmit antennas are divided into groups, and preambles are transmitted in alternative subset of subcarriers in each group. A LMMSE filter-based interpolation scheme and a DFT-based LS interpolation scheme will then be used to obtain the channel estimates for all the subcarriers of interest. In all the proposed schemes in this paper, the filter parameters can be fixed and robust performance are obtained even when mismatched SNR and channel statistics are used in the filter parameter calculations.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) has been adopted in several high speed wireless communciation standards due to its capability to effectively combat intersymbol interference (ISI), and its spectral efficiency achieved by spectrum overlapping through the adoption of fast Fourier transform (FFT) and inverse fast Fourier transform (IFFT) in the implementation [1]. For example, IEEE 802.11a [2] and ETSI Hiperlan/2 [3] have specified to transmit up to 54 Mbps data rate with a total of 20 MHz bandwidth at 5 GHz by using OFDM, and IEEE 802.16.1 is drafting an OFDM-based standard [4] to transmit up to 155 Mbps data rate for broadband wireless access (BWA) in the frequency band of 2∼11 GHz.

Two recent information theoretic studies have shown that rich scattering wireless channels have enormous capacities if the multipaths are properly exploited [5], [6]. This can be achieved by deploying multi-element antenna arrays at both the transmitter and the receiver, hence creating a multiple-input multiple-output (MIMO) communication system. In [7], it has been shown that by applying a simple VBLAST (Vertical Bell Laboratories Layered Space-Time) MIMO architecture in a quasi-static narrowband indoor radio channel, spectral efficiency of 20-40 bits/sec/Hz can be achieved at average SNR's ranging from 24 to 34 dB. MIMO architecture can also be used to exploit diversity gain from the spatial domain. In this context, space-time trellis coded modulation (STTCM) has been proposed by Tarokh *et. al.* [8], [9], and space-time block code (STBC) by Alamouti [10] and Tarokh *et. al.* [11].

In the original development of the BLAST system and the space-time codes (STC), a narrow band quasi-static flat fading channel has been assumed. For wideband signals and frequency selective channels, the multipath interference can be easily alleviated by combining OFDM with the MIMO structure, as suggested by [12].

Both coherent detection in BLAST and STC decoding need channel information, hence channel estimation is essential in a MIMO detector. In this paper, we will focus on training sequence assisted channel estimation for packet-based MIMO OFDM systems in wireless local area networks (LAN). Due to the low mobility in this network, a quasi-static channel can be assumed for each packet. Training signals are thus needed only at the beginning of the packet, as in IEEE 802.11a [2] and Hiperlan/2 [3]. We will first develop a linear matrix algebraic model for cyclic prefix based MIMO OFDM systems, in Section II. Using this model, we will then define the basic preamble structure in Section III. It is a simple extension of the conventional single-input single-output (SISO) OFDM preambles, hence the properties of low peak to average power ratio (PAPR), easy time and frequency synchronisation, and so on, can be maintained. It also does not require the transmission of training signals in all the subcarriers. Therefore it is very suitable for deployment in practical systems which usually allocate guard subcarriers. We then derive the least squares (LS) and linear minimum mean squared error (LMMSE) channel estimation algorithms, which compute the channel estimates by filtering the received frequency domain signal with fixed parameters. However, one drawback with this scheme is that the preamble period has to be at least equal to the number of transmit antennas. The transmission efficiency can thus be severely degraded when a large number of transmit antennas are deployed in the system. In order to overcome

0-7803-7700-1/03/$17.00 (C) 2003 IEEE

this problem, we propose in Section III a switched subcarrier preamble scheme in which the transmit antennas are divided into groups, and preambles are transmitted in alternative subset of subcarriers in each group. Three interpolation algorithms are then considered, namely, linear interpolation which assumes correlation between only neighbouring subcarriers, LMMSE interpolation which makes use of a more realistic channel correlation information among the different subcarriers, and DFT-based LS interpolation which assumes a fixed number of multipaths in the MIMO radio channel and makes use of the time- and frequency-domain relationship of the channel parameters. The simulated performance of the different algorithms will be presented in Section IV, and conclusions will be drawn in Section V.

Throughout this paper, the time domain data are represented with lower-case, frequency-domain data with upper-case, vectors and matrices with bold face letters, and calligraphy letters are used to depict signals in a MIMO system. The symbols $(\cdot)^T$, $(\cdot)^H$, and $(\cdot)^{-1}$ represent matrix transposition, Hermitian, and inversion, respectively. All vectors are defined as column vectors with row vectors represented by transposition.

II. System Model

In this section, we will derive a mathematical model for MIMO OFDM systems with cyclic prefix (CP). We denote N as the total number of subcarriers, or the FFT size, P as the number of subcarriers used to transmit data (or pilot signals during the training period) where $P \leq N$, L as the cyclic prefix length, and we assume a sample-spaced multipath channel for each of the MIMO channels defined by the transmit-receive antenna pairs with multipath delay values taken from $\{0,\ 1, \cdots, L-1\}$. It has been shown in [13] that the SISO OFDM symstem can be modelled by the following equation:

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{V}, \tag{1}$$

where $\mathbf{Y}$ is the received frequency domain signal vector of size N, $\mathbf{H} = \mathrm{diag}\left(H_0, H_1, \cdots, H_{N-1}\right)$ is the frequency response of the SISO channel with the ith diagonal element expressed as $H_i = \sum_{l=0}^{L-1} h_l \exp(-j\frac{2\pi}{N}il)$, and h_l is the complex amplitude of the lth multipath, and $\mathbf{X}$ is the frequency domain signal vector of size N. When $P = N$, $\mathbf{X}$ is the signal to be transmitted; when $P < N$, some subcarriers at the borders of the allocated bandwidth are used as guard band, and $\mathbf{X}$ is formed as follows:

$$\mathbf{X}^T = \left(\begin{array}{ccc} \mathbf{X}_h^T & \mathbf{0}_{N-P}^T & \mathbf{X}_l^T \end{array} \right),$$

where $\mathbf{X}_h$ denotes the frequency domain signal at the higher frequency subcarriers (compared to direct current, or 0th subcarrier), $\mathbf{X}_l$ denotes the signal at the lower frequency subcarriers, and $\mathbf{0}_{N-P}$ denotes the all zero vector of size $(N-P)$. $\mathbf{V}$ is the frequency domain noise vector of size N which is AWGN.

With the SISO-OFDM model defined in Equaion (1), we will now derive the MIMO OFDM system model as follows:

$$\mathcal{Y} = \mathcal{H}\mathcal{X} + \mathcal{V}, \tag{2}$$

where $\mathcal{X}$ is an order $n_T N$ column vector comprising of n_T stacked vectors $\mathbf{X}_i$, $i = 1,\ 2, \cdots, n_T$ and n_T is the number of transmit antennas, $\mathcal{Y}$ is an order $n_R N$ column vector comprising of n_R stacked vectors $\mathbf{Y}_j$, $j = 1,\ 2, \cdots, n_R$ and n_R denotes the number of receive antennas, and $\mathcal{H}$ is the frequency domain MIMO-OFDM channel of size $n_R N \times n_T N$ which is written as:

$$\mathcal{H} = \begin{bmatrix} \mathbf{H}_{1,1} & \mathbf{H}_{1,2} & \cdots & \mathbf{H}_{1,n_T} \\ \mathbf{H}_{2,1} & \mathbf{H}_{2,2} & \cdots & \mathbf{H}_{2,n_T} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_{n_R,1} & \mathbf{H}_{n_R,2} & \cdots & \mathbf{H}_{n_R,n_T} \end{bmatrix},$$

where $\mathbf{H}_{m,n}$ is a $N \times N$ diagonal matrix corresponding to the SISO frequency domain channel defined by the nth-transmit mth-receive antenna pair, $\mathcal{V}$ is a stacked vector of the AWGN noise at the receive antennas of order $n_R N$.

Therefore, the received signal at receive antenna m, $m = 1,\ 2, \cdots, n_T$ and subcarrier k, $k = 0,\ 1, \cdots, N-1$ can be expressed as:

$$\begin{aligned} \mathcal{Y}_{m,k} &= \mathcal{Y}_{(m-1)\times N+k} \\ &= \sum_{n=1}^{n_T} \mathcal{H}_{(m-1)\times N+k,(n-1)\times N+k}\mathcal{X}_{(n-1)\times N+k} \\ &= \sum_{n=1}^{n_T} H_{m,n,k} X_{n,k}\ , \end{aligned} \tag{3}$$

where $H_{m,n,k}$ depicts the kth diagonal element of $\mathbf{H}_{m,n}$, and $X_{n,k}$ is the kth element of the transmitted signal vector $\mathbf{X}_n$ from the nth transmit antenna.

Defining

$$\begin{aligned} \mathcal{R}_k &= \left[\begin{array}{cccc} \mathcal{Y}_{1,k} & \mathcal{Y}_{2,k} & \cdots & \mathcal{Y}_{n_R,k} \end{array} \right]^T, \\ \mathcal{W}_k &= \begin{bmatrix} \mathcal{H}_{k,k} & \mathcal{H}_{k,2k} & \cdots & \mathcal{H}_{k,n_T k} \\ \mathcal{H}_{2k,k} & \mathcal{H}_{2k,2k} & \cdots & \mathcal{H}_{2k,n_T k} \\ & \cdots & & \\ \mathcal{H}_{n_R k,k} & \mathcal{H}_{n_R k,2k} & \cdots & \mathcal{H}_{n_R k,n_T k} \end{bmatrix} \\ &= \begin{bmatrix} H_{1,1,k} & H_{1,2,k} & \cdots & H_{1,n_T,k} \\ H_{2,1,k} & H_{2,2,k} & \cdots & H_{2,n_T,k} \\ & \cdots & & \\ H_{n_R,1,k} & H_{n_R,2,2k} & \cdots & H_{n_R,n_T,k} \end{bmatrix}, \\ \mathcal{S}_k &= \left[\begin{array}{cccc} \mathcal{X}_k & \mathcal{X}_{2k} & \cdots & \mathcal{X}_{n_T k} \end{array} \right]^T \\ &= \left[\begin{array}{cccc} X_{1,k} & X_{2,k} & \cdots & X_{n_T,k} \end{array} \right]^T, \end{aligned}$$

and

$$\mathcal{N}_k = \left[\begin{array}{cccc} \mathcal{V}_k & \mathcal{V}_{2k} & \cdots & \mathcal{V}_{n_R k} \end{array} \right]^T,$$

we can write Equation (3) as follows:

$$\mathcal{R}_k = \mathcal{W}_k \mathcal{S}_k + \mathcal{N}_k. \tag{4}$$

III. Preamble Design and Channel Estimation

A. The LS Channel Estimation

Excluding the AWGN term in Equation (4), we can observe a linear relation between the channel parameters and the received signals. For training sequence assisted channel estimation, solving this linear equation will lead to the least squares (LS) channel estimates. In order to do this, pilot signal with length n_T OFDM symbols is needed. The received signal at subcarrier k for all the receive antennas during the training period of n_T OFDM symbols can be written as:

$$\underline{\mathcal{R}}_k = \mathcal{W}_k \underline{\mathcal{S}}_k + \underline{\mathcal{N}}_k, \tag{5}$$

where $\underline{\mathcal{R}}_k = \begin{bmatrix} \mathcal{R}_{k,1} & \mathcal{R}_{k,2} & \cdots & \mathcal{R}_{k,n_T} \end{bmatrix}$ is an $n_R \times n_T$ matrix representing the received signal at the n_R antennas, subcarrier k during the training period, $\underline{\mathcal{S}}_k = \begin{bmatrix} \mathcal{S}_{k,1} & \mathcal{S}_{k,2} & \cdots & \mathcal{S}_{k,n_T} \end{bmatrix}$ is an $n_T \times n_T$ square matrix representing the training signals at subcarrier k with a period of n_T OFDM symbols, and $\underline{\mathcal{N}}_k = \begin{bmatrix} \mathcal{N}_{k,1} & \mathcal{N}_{k,2} & \cdots & \mathcal{N}_{k,n_T} \end{bmatrix}$ of size $n_R \times n_T$ represents the AWGN noise. The LS channel estimates are then obtained by right multiplying $\underline{\mathcal{S}}_k^{-1}$ with $\underline{\mathcal{R}}_k$, as follows:

$$\hat{\mathcal{W}}_{k,LS} = \underline{\mathcal{R}}_k \underline{\mathcal{S}}_k^{-1}. \tag{6}$$

As long as $\underline{\mathcal{S}}_k$ is a non-singular matrix, $\underline{\mathcal{S}}_k^{-1}$ exists and it can be calculated off-line. Channel estimation is just a linear combination of the received signals at the different antennas. Re-organising all the elements in $\hat{\mathcal{W}}_{k,LS}$, $k = 0,\ 1, \cdots, N-1$, we are able to obtain the LS channel estimates $\hat{\mathcal{H}}$.

We have to point out that a non-singular matrix $\underline{\mathcal{S}}_k$ with elements taken from the modulation constellation may not lead to a ''good'' preamble design for each SISO OFDM. Some other requirements, e.g., low PAPR, easy time and frequency synchronisation, etc., may not be satisfied. In order to avoid this problem, we may use a ''good'' preamble sequence designed for SISO OFDM, and extend it to the MIMO OFDM system as follows:

$$\underline{\mathcal{S}}_k = T_k \underline{\mathcal{M}}, \tag{7}$$

where T_k is the training signal at the kth subcarrier for SISO OFDM, $\underline{\mathcal{M}}$ is a non-singular matrix with elements of $+1$ and -1[1]. Therefore $\underline{\mathcal{S}}_k^{-1} = \frac{1}{T_k}\underline{\mathcal{M}}^{-1}$. We can easily see that in this preamble design, each SISO OFDM in the MIMO system uses the same preamble with or without a sign reverse. Therefore, not only is the preamble optimised for each SISO OFDM, MIMO channel estimation for each subcarrier can be obtained by the same linear matrix filter. The implementation is thus very simple. In addition, training signals are only needed in the P subcarriers. This is an advantage over the training signal scheme proposed in [14], in which pilot signals have to be transmitted in all the N subcarriers, otherwise performance degradation will be incurred.

The mean-squared error (MSE) of the LS estimates is:

$$\begin{aligned}
\text{MSE} &= E(||\hat{\mathcal{W}}_{k,LS} - \mathcal{W}_k||^2) \\
&= E(||\underline{\mathcal{N}}_k \underline{\mathcal{S}}_k^{-1}||^2) \\
&= \frac{N_o}{n_T} E\{\frac{1}{|T_k|^2}\} \text{trace}\left((\underline{\mathcal{M}}^{-1})^H \underline{\mathcal{M}}^{-1}\right) \\
&= \frac{\beta}{\text{SNR} n_T} \text{trace}\left((\underline{\mathcal{M}}^{-1})^H \underline{\mathcal{M}}^{-1}\right),
\end{aligned} \tag{8}$$

[1]In general, the elements of $\underline{\mathcal{M}}$ can take values of $\exp(j\theta_i)$, where $\{\theta_i\}$ are some discrete phase values taken from $[0, 2\pi)$.

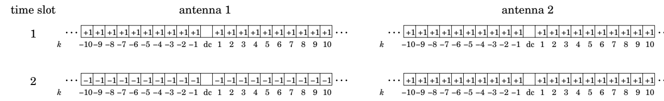

Fig. 1. Orthogonal preamble design for 2 transmit antennas.

where $\text{SNR} = \frac{E(|T_k|^2)}{N_o}$ is the signal to noise ratio per transmit antenna, and $\beta = E\{|T_k|^2\}E\{\frac{1}{|T_k|^2}\}$.

Performing eigen-decomposition on $\underline{\mathcal{M}}$ as $\underline{\mathcal{M}} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$, where $\mathbf{U}\mathbf{U}^H = \mathbf{U}^H\mathbf{U} = \mathbf{I}$, we have $\underline{\mathcal{M}}^{-1} = \mathbf{U}\mathbf{\Lambda}^{-1}\mathbf{U}^H$, and hence

$$\begin{aligned}
\text{MSE} &= \frac{\beta}{\text{SNR} n_T} \text{trace}\left(\mathbf{U}(\mathbf{\Lambda}^{-1})^H \mathbf{U}^H \mathbf{U} \mathbf{\Lambda}^{-1} \mathbf{U}^H\right) \\
&= \frac{\beta}{\text{SNR} n_T} \text{trace}\left((\mathbf{\Lambda}^{-1})^H \mathbf{\Lambda}^{-1}\right) \\
&= \frac{\beta}{\text{SNR} n_T} \sum_{i=1}^{n_T} \frac{1}{|\lambda_i|^2},
\end{aligned} \tag{9}$$

where λ_i is the ith eigenvalue of $\underline{\mathcal{M}}$. Therefore, minimum MSE is obtained when $\sum_{i=1}^{n_T} \frac{1}{|\lambda_i|^2}$ is minimised.

As

$$\begin{aligned}
\sum_{i=1}^{n_T} |\lambda_i|^2 &= \text{trace}\left(\mathbf{\Lambda}^H \mathbf{\Lambda}\right) \\
&= \text{trace}\left(\mathbf{U}\mathbf{\Lambda}^H \mathbf{U}^H \mathbf{U} \mathbf{\Lambda} \mathbf{U}^H\right) \\
&= \text{trace}\left(\underline{\mathcal{M}}^H \underline{\mathcal{M}}\right) \\
&= n_T^2,
\end{aligned}$$

the minimum MSE is obtained when $|\lambda_i|^2 = n_T$, $i = 1,\ 2,\ \cdots,\ n_T$, which is

$$\min(\text{MSE}) = \frac{\beta}{\text{SNR} n_T}. \tag{10}$$

When n_T is 1, 2, 4, or a multiple of 4, $\underline{\mathcal{M}}$ can be set to the Walsh Hadamard matrix of size n_T. In this case, $\underline{\mathcal{M}}^{-1} = \frac{1}{n_T}\underline{\mathcal{M}}^T$, $|\lambda_i|^2 = n_T$, $i = 1,\ 2,\ \cdots,\ n_T$. The minimum MSE as shown in Equation (10) is obtained for each subcarrier in each SISO OFDM defined by the transmit-receive antenna pairs. In Figure 1, such a preamble design for $n_T = 2$ case is illustrated, in which k denotes the subcarrier index, $+1$ means that the original SISO pilot signal T_k is transmitted in this subcarrier, and -1 denotes that $-T_k$ is transmitted. In the first time slot (OFDM symbol), both antennas transmit T_k. In the second time slot, antenna 1 transmits $-T_k$ and antenna 2 transmits T_k so as to obtain the orthogonality.

B. The LMMSE Channel Estimation

The LS channel estimates are obtained by using only the knowledge of the training signals, which can be further improved by making use of the frequency domain correlation of the multipath channel defined by each transmit-receive antenna pair. We then obtain the LMMSE channel estimates, as follows [15]:

$$\hat{\mathbf{H}}_{m,n,lmmse} = \mathbf{R}_{HH}\left(\mathbf{R}_{HH} + \frac{\beta}{\text{SNR}}\mathbf{I}\right)^{-1} \hat{\mathbf{H}}_{m,n,ls}, \tag{11}$$

where subscipts m and n denotes the receive and transmit antenna index, respectively, SNR is the signal to noise ratio of the training signals, and β is a constant depending on the training signal's constellation. For MPSK training signals, $\beta = 1$. $\mathbf{R}_{HH} = E(\mathbf{H}_{mn}\mathbf{H}_{mn}^H)$ is the channel autocorrelation matrix. When same statistical properties are assumed for each SISO channel, $\mathbf{R}_{HH}$ is independent of m and n and can be computed off-line. It has been given in [15] that its diagonal element $r_{k,k} = 1$, and the off-diagonal element r_{k_1,k_2} ($k_1 \neq k_2$) is:

$$r_{k_1,k_2} = \frac{1 - \mathrm{e}^{-L(\frac{1}{\tau_{rms}} + \frac{2\pi j(k_1-k_2)}{N})}}{\tau_{rms}\left(1 - \mathrm{e}^{-\frac{L}{\tau_{rms}}}\right)\left(\frac{1}{\tau_{rms}} + j2\pi\frac{k_1-k_2}{N}\right)},$$

for an exponentially decaying power delay profile with root-mean squared delay of τ_{rms} and maximum excess delay of L. Here N is the FFT size, and the multipath delays are assumed to be uniformly distributed over $[0, L-1]$.

The LMMSE channel estimator in Equation (11) is robust to power delay profile mismatch if $\mathbf{R}_{HH}$ for the least correlated channel is used, as indicated in [15]. As for the SNR mismatch, [15] showed that a design for a high SNR will be preferable as channel estimation errors will be concealed in noise for low SNR, and they will tend to dominate for high SNR where the noise is low. Therefore, the LMMSE matrix filter $\mathbf{R}_{HH}\left(\mathbf{R}_{HH} + \frac{\beta}{\text{SNR}}\mathbf{I}\right)^{-1}$ can be calculated off-line. This will greatly reduce the computation load in the receiver.

C. Interpolation-based Channel Estimation

As discussed in Section III-A and III-B, LS and LMMSE channel estimations can be obtained if training signals with n_T OFDM symbols are sent and each subcarrier's training signal matrix $\underline{\mathcal{S}}_k = T_k\underline{\mathcal{M}}$ is a non-singular matrix. When the number of transmit antennas is large, this preamble scheme could decrease the system throughput severely. We therefore in this section consider a switched subcarrier preamble scheme in which the transmit antennas and the subcarriers are both divided into n_G groups, and training signals are transmitted in different subset of subcarriers in each transmit antenna group. Therefore, the preamble period needed can be effectively reduced, from n_T to $\frac{n_T}{n_G}$. For example, if there are four antennas at the transmitter, we can divide them into two groups and send training signals at even number subcarriers for the first antenna group, and odd number subcarriers for the second group. As there are two antennas in each group, we can set $\underline{\mathcal{M}}$ equal to the Walsh Hadamard matrix of dimension 2, and the training signal period is reduced from four to two. LS channel estimates can be obtained for even and odd number subcarriers for 1st and 2nd transmit antenna groups according to Equation (6), respectively. Channel estimates for odd number subcarriers of the 1st antenna group and even number subcarriers of the 2nd transmit antenna group will be obtained by interpolation. The training signal period can be further reduced to one OFDM symbol if the transmit antennas are divided into four groups.

In this paper, we will consider three types of interpolation, namely, linear interpolation, LMMSE interpolation, and DFT-based LS interpolation.

1) Linear Interpolation: Assuming that the transmit antennas are divided into two groups, $\hat{H}_{m,n,k-1}$ and $\hat{H}_{m,n,k+1}$ are the LS estimates obtained from Equation (6), then channel estimate at the kth subcarrier can be obtained through linear interpolation as follows:

$$\tilde{H}_{m,n,k} = \frac{\hat{H}_{m,n,k-1} + \hat{H}_{m,n,k+1}}{2}, \tag{12}$$

which is a linear matrix filtering operation as follows:

$$\begin{bmatrix} \tilde{H}_{m,n,k-1} \\ \tilde{H}_{m,n,k} \\ \tilde{H}_{m,n,k+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & \epsilon & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{H}_{m,n,k-1} \\ 0 \\ \hat{H}_{m,n,k+1} \end{bmatrix},$$

where ϵ represents any complex number as it does not affect the results.

2) LMMSE Interpolation: Linear interpolation expressed in (12) assumes a channel correlation matrix $\mathbf{R}_{HH}$ with its elements defined as:

$$R_{i,j} = \begin{cases} \alpha & \text{when } |i-j| = 1 \\ 1 & \text{when } i = j \\ 0 & \text{otherwise} \end{cases},$$

where α is a real number and $\alpha \in (0, 1)$. This suggests that more accurate estimates could be obtained if the real channel correlation information is applied in the interpolation. In this case, not only the neighbouring subcarriers, but all the available subcarriers' channel estimates will be used to calculate the missing subcarriers' channel parameters, and the contribution from different subcarriers is determined by their correlation. In our study, we use $\mathbf{W}\mathbf{R}_{HH}\left(\mathbf{R}_{HH} + \frac{\beta}{\text{SNR}}\mathbf{I}\right)^{-1}$ as the interpolation filter, where $\mathbf{W}$ is a normalisation matrix, and $\mathbf{R}_{HH}\left(\mathbf{R}_{HH} + \frac{\beta}{\text{SNR}}\mathbf{I}\right)^{-1}$ is the LMMSE filter. This is the reason we call this the *LMMSE interpolation*. The simulation results presented in Section IV will show that this interpolation scheme has better performance than linear interpolation and it is also robust to the $\mathbf{R}_{HH}$ and SNR mismatches.

3) DFT-based LS Interpolation: As defined in Section II, we assume a sample-spaced channel whose excess delay is no greater than the cyclic prefix length, and the time- and frequency-domain channel parameters are related by FFT and IFFT. Taking these into consideration, we propose a DFT-based LS interpolation. The derivation is as follows.

LS channel estimates for the subcarriers with training signals can be obtained according to Equation (6), which will be denoted as $\hat{\mathbf{H}}_{m,n,pilot}$. Denoting the channel estimates for the other subcarriers as $\hat{\mathbf{H}}_{m,n,missing}$, we can express the channel estimates $\hat{\mathbf{H}}_{m,n}$ as:

$$\hat{\mathbf{H}}_{m,n} = \mathbf{P}\begin{bmatrix} \hat{\mathbf{H}}_{m,n,pilot} \\ \hat{\mathbf{H}}_{m,n,missing} \end{bmatrix},$$

where $\mathbf{P}$ represents a permutation matrix of size $N \times N$. As L multipaths are assumed in the time domain channel, we therefore have the following relation:

$$\mathbf{G}^H \hat{\mathbf{H}}_{m,n} = \mathbf{G}^H \mathbf{P} \begin{bmatrix} \hat{\mathbf{H}}_{m,n,pilot} \\ \hat{\mathbf{H}}_{m,n,missing} \end{bmatrix} = \mathbf{0}_{N-L}, \quad (13)$$

where $\mathbf{G}$ is the last $(N-L)$ columns of the Fourier transform matrix $\mathbf{F}$. Letting $\mathbf{G}^H\mathbf{P} = [\mathbf{G}_T \ \mathbf{G}_M]$ so as to re-write (13) as:

$$[\mathbf{G}_T \ \mathbf{G}_M] \begin{bmatrix} \hat{\mathbf{H}}_{m,n,pilot} \\ \hat{\mathbf{H}}_{m,n,missing} \end{bmatrix} = \mathbf{0}_{N-L}. \quad (14)$$

we will have the following relation:

$$\mathbf{G}_T \hat{\mathbf{H}}_{m,n,pilot} = -\mathbf{G}_M \hat{\mathbf{H}}_{m,n,missing}, \quad (15)$$

which leads to:

$$\hat{\mathbf{H}}_{m,n,missing} = -(\mathbf{G}_M^H \mathbf{G}_M)^{-1} \mathbf{G}_M^H \mathbf{G}_T \hat{\mathbf{H}}_{m,n,pilot}. \quad (16)$$

This is a LS estimation of $\hat{\mathbf{H}}_{m,n,missing}$ from $\hat{\mathbf{H}}_{m,n,pilot}$, which suggests the name of *LS interpolation*.

IFFT can then be applied to the above frequency domain estimates to obtain a L-tap time domain channel estimates. The final frequency domain channel estimates will be computed by applying FFT to the L-tap time domain channel estimates. This IFFT and FFT operation can filter out some AWGN noise and thus improve the estimation accuracy.

IV. Simulation Results

In this section, we will present our simulation results. For each SISO OFDM corresponding to one transmit-receive antenna pair, the system parameters defined in IEEE 802.11a [2] are used. That is, the FFT size is $N = 64$, the number of used subcarriers is $P = 52$, and the number of gaurd subcarriers is 12. The cyclic prefix length is $L = 16$. The long preamble given in [2] are used to construct the MIMO preambles. For the 20MHz channel, two channel models are used, namely, Channel A with $\tau_{rms} = 50$ ns, and Channel E with $\tau_{rms} = 250$ ns. For frequency domain channel, Channel E is thus less correlated than Channel A. Both channels assume an exponentially decaying power delay profile with 16 multipaths which are sample-spaced and independently generated using Jake's model [16]. The mean squared error (MSE) for the frequency domain channel estimates is used for performance comparison.

Depicted in Figure 2 are the MSE versus SNR per transmit antenna performances for the LS channel estimtaion algorithms with different number of transmit and receive antennas. We can observe from the figure that the MSE decreases linearly with the increasing SNR's. We can also observe that when the number of transmit antennas is the same, the MSE is the same for different channel models and different number of receive antennas, which is due to the fact that same power is transmitted per antenna. Therefore, the more the transmit antennas, the more the total power per receive antenna, which results in MSE drop when the transmit antenna number is increased.

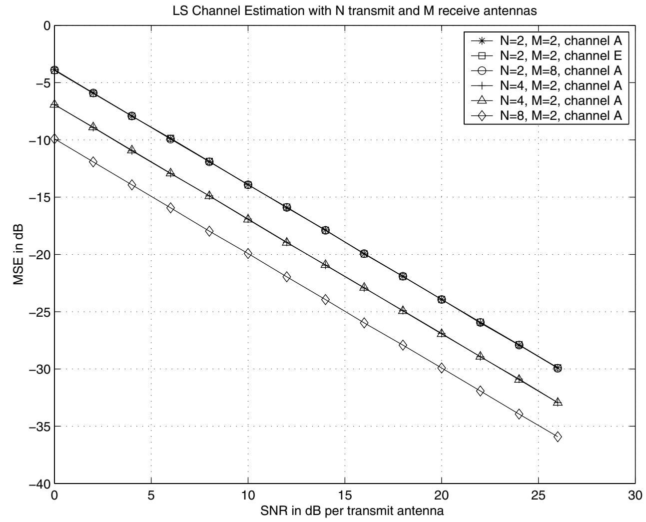

Fig. 2. MSE vs. SNR for LS channel estimation with N transmit and M receive antennas.

We then depict in Figure 3 the LMMSE performance for a 2×2 MIMO-OFDM system. Shown in the same figure is the LS performance for Channel A. A few LMMSE filters are tested, namely, the LMMSE filter designed for Channel A used for Channel A or Channel E, and LMMSE filter designed for Channel E used for estimation of Channel A or Channel E. In all these LMMSE filters, a fixed SNR of 20dB is used in the LMMSE filter calculation. Studying the curves in Figure 3, we can observe that a fixed SNR of 20dB will result in a MSE error floor in higher than 20dB SNR regions. For low SNR values, the performance is very good. Therefore, as long as we fix the SNR value to the highest possible realistic SNR's, the LMMSE estimation performance is very robust to the SNR mismatch. We can also observe from this figure that using a less correlated LMMSE filter (Channel E) to estimate a more correlated channel (Channel A), good MSE performance can still be obtained in the low to medium SNR regions. In high SNR regions, a correlation matrix mismatch of this type will result in some error floor. However, if a more correlated channel matrix (Channel A) is used to estimate a not so correlated channel (Channel E), very poor performance will be resulted, in almost all the SNR regions of interest.

We then present our simulation results based on interpolations for switched subcarrier preamble schemes in Figure 4. Comparing the three interpolation schemes for Channel A, we can observe that in the low to medium SNR regions, linear interpolation and DFT-based LS interpolation have the same performance, and in high SNR regions, the later scheme has slighly better performance. While for LMMSE interpolation, even in the mismatched case (Channel E's correlation matrix used for Channel A, fixed SNR value of 20 dB in the interpolation filter), it demonstrates better performance than the other two schemes in all the SNR regions simulated. Similar to LMMSE channel estimation, LMMSE interpolation is robust to channel model mismatch if a not so correlated

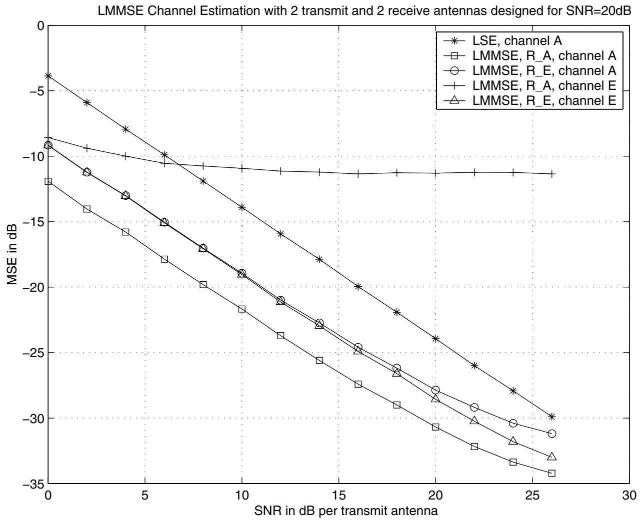

Fig. 3. MSE vs. SNR for LMMSE Channel Estimation with 2 transmit and 2 receive antennas. SNR = 20 dB is used in the LMMSE filter.

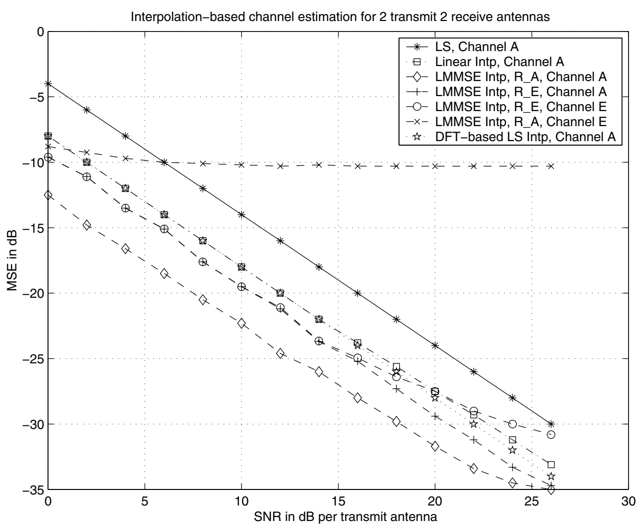

Fig. 4. Interpolation-based channel estimation for switched subcarrier scheme. SNR = 20 dB is used in the LMMSE interpolation filter.

channel is used in the correlation matrix computation, and it is robust to SNR mismatch as well if a high SNR value is used in computing the correlation matrix.

V. Conclusions

We have presented several results of our study on MIMO OFDM channel estimation. Based on a linear matrix algebraic model, we have derived a general preamble structure which is just a simple extension from the SISO OFDM preamble. Therefore, the good properties, such as low PAPR, easy time and frequency synchronisation of the SISO OFDM preamble can be maintained. We then developed the least squares and linear minimum mean squared error channel estimation algorithms for this proposed preamble scheme. We further proposed a switched subcarrier preamble scheme which needs fewer OFDM symbols in the training sequence and therefore the transmission efficiency is improved. Three interpolation schemes, namely, linear interpolation, LMMSE interpolation and DFT-based LS interpolation are proposed, among which the LMMSE interpolation scheme demonstrates the best performance, even in the mismatch case. As both LMMSE channel estimation and LMMSE interpolation can be implemented with fixed parameter values in the matrix filter, the implementation is very simple and therefore attractive for practical deployment.

Acknowledgment

The authors would like to thank Dr. P. He, Mr. Y. Wu, Mr. H. W. Fung, Dr. Z. D. Lei, Dr. Y. Li, and Ms. Y. M. Dai for their support.

References

[1] W. Y. Zou and Y. Wu, ''COFDM: An overview,'' *IEEE Trans. Broadcast*, vol. 41, pp. 1--8, March 1995.
[2] ''Part 11: wireless LAN medium access control (MAC) and physical layer (PHY) specifications: High-speed physical layer in the 5GHz band.'' IEEE std 802.11a-1999: Supplement to IEEE std 802.11-1999, Sept. 1999.
[3] ''Broadband radio access networks BRAN: HIPERLAN type 2 technical specification: physical layer.'' ETSI/OTS/BRAN 0023003, Oct. 1999.
[4] IEEE 802.16 BWA Working Group, ''OFDM mode for the IEEE 802.16a PHY draft standard.'' IEEE 802.16.3c-01/59, 30 April 2001.
[5] G. J. Foschini and M. J. Gans, ''On the limits of wireless communications in a fading environment when using multiple antennas,'' *Wireless Personal Communications*, pp. 315--335, March 1996.
[6] I. Emre Telatar, ''Capacity of multi-antenna Gaussian channels,'' *European Transactions on Telecommunications*, vol. 10, pp. 585--595, Nov/Dec 1999.
[7] P. W. Wolniansky, G. J. Foschini, G. D. Golden and R. A. Valenzuela, ''V-BLAST: An architecture for realizing very high data rates over the rich-scattering wireless channel,'' in *IEEE ISSSE-98*, (Pisa, Italy), pp. 295--300, Sept. 1998.
[8] V. Tarokh, N. Seshadri and A. R. Calderbank, ''Space-time codes for high data rate wireless communications: Performance criterion and code construction,'' *IEEE Trans. Inform. Theory*, vol. 44, pp. 744--765, March 1998.
[9] V. Tarokh, A. Naguib, N. Seshadri and A. R. Calderbank, ''Space-time codes for high data rate wireless communication: performance criteria in the presence of channel estimation errors, mobility, and multiple paths,'' *IEEE Trans. Commun.*, vol. 47, pp. 199--207, Feb. 1999.
[10] S. M. Alamouti, ''A simple transmit diversity techniques for wireless communications,'' *IEEE J. Selected Areas Commun.*, vol. 16, pp. 1451--1458, Oct. 1998.
[11] V. Tarokh, H. Jafarkhani and A. R. Calderbank, ''Space-time block codes from orthogonal design,'' *IEEE Trans. Inform. Theory*, vol. 45, pp. 1456--1467, July 1999.
[12] D. Agrawal, V. Tarokh, A. Naguib, and N. Seshadri, ''Space-time coded OFDM for high data-rate wireless communication over wideband channels,'' in *IEEE Vehicular Technology Conference*, vol. 3, pp. 2232--2236, 1998.
[13] Yan Wu, C. K. Ho and S. Sun, ''On some properties of Walsh-Hadamard transformed OFDM,'' in *VTC Fall*, vol. 4, (Vancouver, Canada), pp. 2096--2100, Sept. 2002.
[14] Y. Li, ''Simplified channel estimation for OFDM systems with multiple transmit antennas,'' *IEEE Trans. Wireless Commun.*, vol. 1, pp. 67--75, Jan. 2002.
[15] O. Edfors, M. Sandell, J.-J. van de Beek ans Sarah Kate Wilson, and P. O. Borjesson, ''OFDM Channel Estimation by Singular Value Decomposition,'' *IEEE Trans. Commun.*, vol. 46, pp. 931--939, July 1998.
[16] W. C. Jakes, *Microwave Mobile Communications*. New York: Wiley, 1974.