Fall 2024

Regression Analysis
Assignments

- **Assignment 1 due by October 2, 2024**

1. (100 pt.) Consider a simple linear regression model $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$. The least squares estimators of β_0 and β_1 are $\hat{\beta}_0$ and $\hat{\beta}_1$, respectively, where

$$\hat{\beta}_1 = \frac{\sum(x_i - \bar{x})y_i}{\sum(x_i - \bar{x})^2} \text{ and } \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}.$$

   (a) (10 pt.) Show that $\hat{\beta}_0$ is a linear combination of y_i.
   (b) (10 pt.) Show that $\mathrm{E}(\hat{\beta}_1) = \beta_1$.
   (c) (20 pt.) Show that $\mathrm{E}(\hat{\beta}_0) = \beta_0$.
   (d) (20 pt.) Show that $\mathrm{Cov}(\bar{y}, \hat{\beta}_1) = 0$.
   (e) (20 pt.) Show that $\mathrm{Var}(\hat{\beta}_1) = \frac{\sigma^2}{\sum(x_i - \bar{x})^2}$.
   (f) (20 pt.) Show that $\mathrm{Var}(\hat{\beta}_0) = \sigma^2 \left[\frac{1}{n} + \frac{\bar{x}^2}{\sum(x_i - \bar{x})^2}\right]$.