

線性代數期中考練習題 範圍：Sec1.1-Sec2.2

學號：_____ 姓名：_____ 座號：_____ (5%)

- 請於 11/01(五)12:00 前繳至 B1102 統計系系辦
- 此份作業算一次小考成績請勿抄襲，只要被發現一律 0 分
- 答案卷請雙面列印且翻轉長邊

Q1 a. $\left[\begin{array}{cccc|c} 0 & 1 & -3 & 0 & 0 \\ 1 & 1 & -1 & 4 & 0 \\ 2 & -1 & 7 & 8 & 0 \end{array} \right] \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 2 & 4 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$

Let $x_3 = s, x_4 = t, \Rightarrow x_1 = -2s - 4t, x_2 = 3s, s, t \in \mathbb{R} \neq$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2s-4t \\ 3s \\ s \\ t \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} -4 \\ 0 \\ 0 \\ 1 \end{bmatrix} t, s, t \in \mathbb{R} \neq$$

b. $W = \{u = (-2s - 4t, 3s, s, t), v = (-2p - 4q, 3p, p, q), s, t, p, q \in \mathbb{R}\}$

$u + kv = (-2(s+kp) - 4(t+kq), 3(s+kp), s+kp, t+kq) \in W$

\Rightarrow 滿足加封及乘封 \Rightarrow 為 \mathbb{R}^4 子空間 \neq

c. $(-2s - 4t, 3s, s, t) = (-2, 3, 1, 0)s + (-4, 0, 0, 1)t$ 生成/建造

$p(-2, 3, 1, 0) + q(-4, 0, 0, 1) = (0, 0, 0, 0) \Rightarrow p = q = 0$ 獨立

basis = $\{(-2, 3, 1, 0), (-4, 0, 0, 1)\} \neq$

d. dimension = 2 \neq

為 \mathbb{R}^4 空間中的一平面 \neq

Q2 (2% x 10 = 20%)

					\mathbb{R}^2		\mathbb{R}^3		\mathbb{R}^4
a	b	c	d	e	f	g	h	i	j
T	T	F	F	T	F	F	F	F	F

Q3 a. $(a, b, c) \in \mathbb{R}^3$

$$(a, b, c) = c_1 u + c_2 v + c_3 w \quad \text{生成}$$

$$\begin{bmatrix} 1 & 0 & 2 & | & a \\ 2 & -1 & 0 & | & b \\ 3 & -2 & -1 & | & c \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & | & -a+4b-2c \\ 0 & 1 & 0 & | & -2a+7b-4c \\ 0 & 0 & 1 & | & a-2b+c \end{bmatrix} \Rightarrow \begin{cases} c_1 = -a+4b-2c \\ c_2 = -2a+7b-4c \\ c_3 = a-2b+c \end{cases}$$

$$c_4 u + c_5 v + c_6 w = (0, 0, 0) \Rightarrow c_4 = c_5 = c_6 = 0 \quad \text{獨立}$$

$$B = \{u, v, w\} \text{ 為 } \mathbb{R}^3 \text{ 基底} \quad \#$$

b. By a.

$$\Rightarrow (a, b, c) = (-a+4b-2c)u + (-2a+7b-4c)v + (a-2b+c)w \quad \#$$

$$c. \|u\| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$$

$$\Rightarrow \frac{u}{\|u\|} = \left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right) \quad \#$$

$$d. \cos \theta = \frac{v \cdot w}{\|v\| \|w\|} = \frac{1}{\sqrt{5} \times \sqrt{5}} = \frac{1}{5} \quad \#$$

$$e. \|u-w\| = \|(1, 2, 4)\| = \sqrt{(-1)^2 + 2^2 + 4^2} = \sqrt{21} \quad \#$$

$$f. (p, q, r) \cdot u = (p, q, r) \cdot (1, 2, 3) = p + 2q + 3r = 0$$

$$(p, q, r) \cdot v = (p, q, r) \cdot (0, -1, -2) = -q - 2r = 0$$

$$\begin{bmatrix} 1 & 2 & 3 & | & 0 \\ 0 & -1 & -2 & | & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -1 & | & 0 \\ 0 & -1 & -2 & | & 0 \end{bmatrix}$$

$$\text{Let } r = s, p = s, q = -2s, s \in \mathbb{R}. \Rightarrow (p, q, r) = (s, -2s, s) = (1, -2, 1)s, s \in \mathbb{R} \quad \#$$

$$g. (p, q, r) = (s, -2s, s) = (1, -2, 1)s$$

$$(1, -2, 1) \cdot c_1 = (0, 0, 0) \Rightarrow c_1 = 0$$

$$\text{basis} = \{(1, -2, 1)\} \quad \#$$

$$h. \text{dimension} = 1 \quad \#$$

為 \mathbb{R}^3 空間中的一條線 $\#$.

Q4

(20% x 10 = 20%)

a	b	c	d	e	f	g	h	i	j
F	F	T	F	T	T	F	T	F	T