Fall 2024

Regression Analysis
Assignments

- **Assignment 1 due by October 2, 2024**

  1. (100 pt.) Consider a simple linear regression model $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$. The least squares estimators of β_0 and β_1 are $\hat{\beta}_0$ and $\hat{\beta}_1$, respectively, where

  $$\hat{\beta}_1 = \frac{\sum(x_i - \bar{x})y_i}{\sum(x_i - \bar{x})^2} \text{ and } \hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x}.$$

     (a) (10 pt.) Show that $\hat{\beta}_0$ is a linear combination of y_i.
     (b) (10 pt.) Show that $\mathrm{E}(\hat{\beta}_1) = \beta_1$.
     (c) (20 pt.) Show that $\mathrm{E}(\hat{\beta}_0) = \beta_0$.
     (d) (20 pt.) Show that $\mathrm{Cov}(\bar{y}, \hat{\beta}_1) = 0$.
     (e) (20 pt.) Show that $\mathrm{Var}(\hat{\beta}_1) = \frac{\sigma^2}{\sum(x_i - \bar{x})^2}$.
     (f) (20 pt.) Show that $\mathrm{Var}(\hat{\beta}_0) = \sigma^2\left[\frac{1}{n} + \frac{\bar{x}^2}{\sum(x_i - \bar{x})^2}\right]$.

- **Assignment 2 due by October 16, 2024**

  1. (75 pt.) Do Problem 2.4 using SAS.
  2. (25 pt.) Do Problem 2.5 using SAS.

- **Assignment 3 due by October 29, 2024**

  1. (70 pt.) Do Problem 2.10.
  2. (30 pt.) Do Problem 2.11.

- **Assignment 4 due by November 20, 2024**

  1. (85 pt.) Do Problem 3.5. Also interpret the estimate of β_1 and test $H_0 : \beta_1 = 0$ versus $H_1 : \beta_1 \neq 0$ using the partial F test.
  2. (15 pt.) Using the results of Problem 3.5, show numerically that the square of the simple correlation coefficient between the observed values y_i and the fitted values $\hat{y}_i$ equals R^2.