Estimating the Black Body curve

Krutarth Patel ee23b137

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Introduction

This report aims to estimate the various constants in the black body curve using raw data.

The equation for the curve is given by:

$$I(\nu, T) = \frac{2\pi hc^2}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

- $I(\lambda, T)$ is the intensity of radiation at wavelength λ and temperature T,
- h is Planck's constant,
- λ is the wavelength of radiation,
- c is the speed of light in a vacuum,
- k_B is the Boltzmann's constant,
- T is the absolute temperature of the black body.

We are interested in estimating the values of

$$h, c, k_B, T$$

Data

the following samples are provided to us:

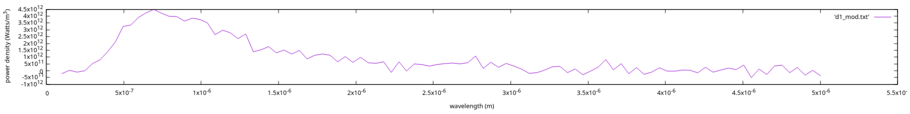


Figure 1: Sample 1

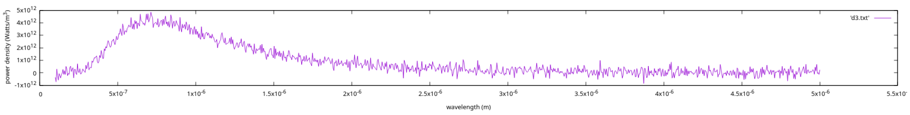


Figure 2: Sample 3

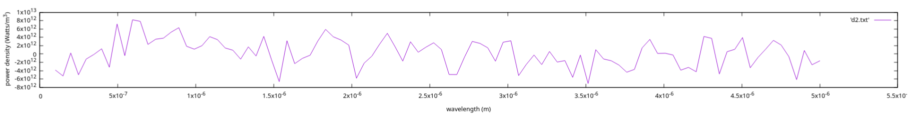


Figure 3: Sample 2

No Initial guess

To begin with we will try to fit the sample points with no prior estimation given. We quickly run into a problem though, trying to fit without an initial guess leads to a float overflow error. This is expected since Scipy assumes an initial value of 1 for all parameters. Assuming

$$h, c, k_B, T = 1$$

and

$$\lambda = 10^{-7}$$

the approximate value of

$$I(\lambda, T) = \frac{2\pi}{10^{-35}} \cdot \frac{1}{e^{10^7} - 1}$$

which for obvious reasons cannot be stored using the precision of a 32-bit floating point number. e^{10^7} is causing the overflow, thus we can **normalize** the samples. Multiplying each input value with 10^6 avoids the overflow error and allows the algorithm to begin iterating.

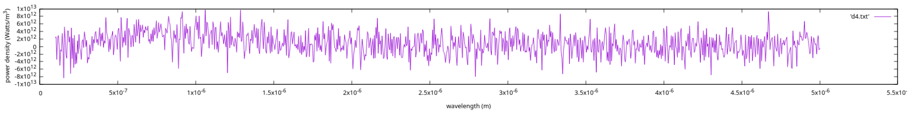


Figure 4: Sample 4

Algorithm

Now is a good time to talk about the tool I am using to fit. my first instinct was to mess with the data using `gnuplot`, it is a command-line plotting tool which is very easy to use. gnuplot has a fitting tool as well, so I tried it first.

Passing the normalized data to the fitting tool in `gnuplot` using the following command:

```
fit f(x) 'd1_normalized.txt' via h,k,c,t
```

But, this does not help much. We get the following fit:

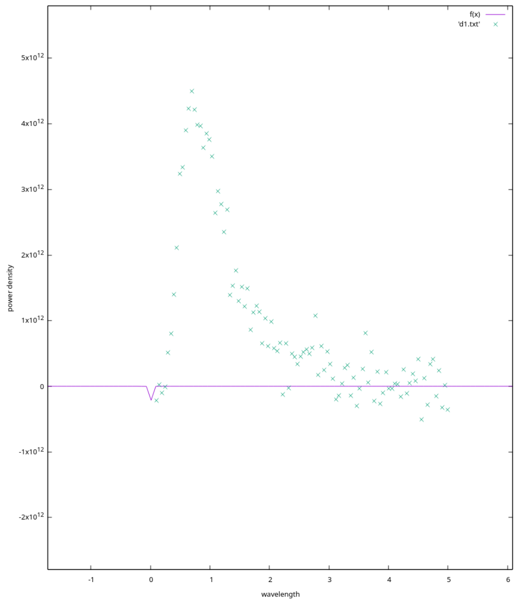


Figure 5: first attempt

Now, to converge the fit properly, I used an approximation of the black body curve:

$$I(\nu, T) = \frac{2\pi hc^2}{\lambda^5} \cdot (e^{\frac{-hc}{\lambda k_B T}} - 1)$$

This gives:

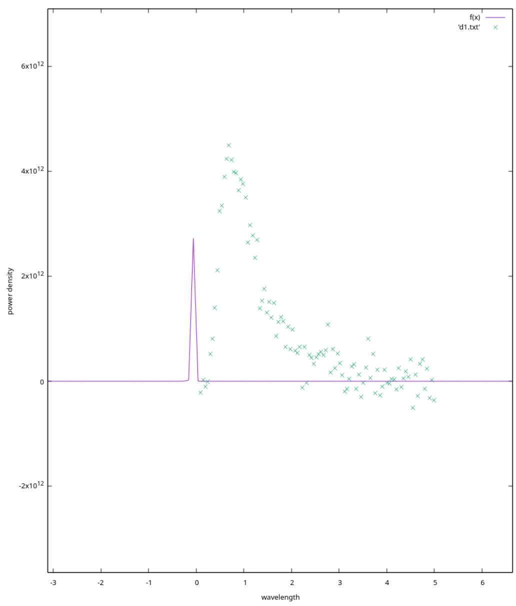


Figure 6: using approximation

Alternating between this approximation and the real formula, I was able to get the following curve:

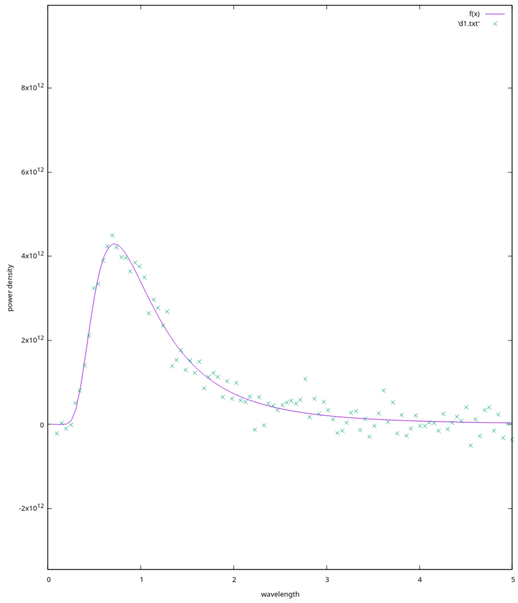


Figure 7: no initial guess(note the values in x axis range from 0-5)

NOTE: This can be accomplished using SciPy as well, The code I have submitted will approximate without a guess and with a guess and show the results.

With Initial guess

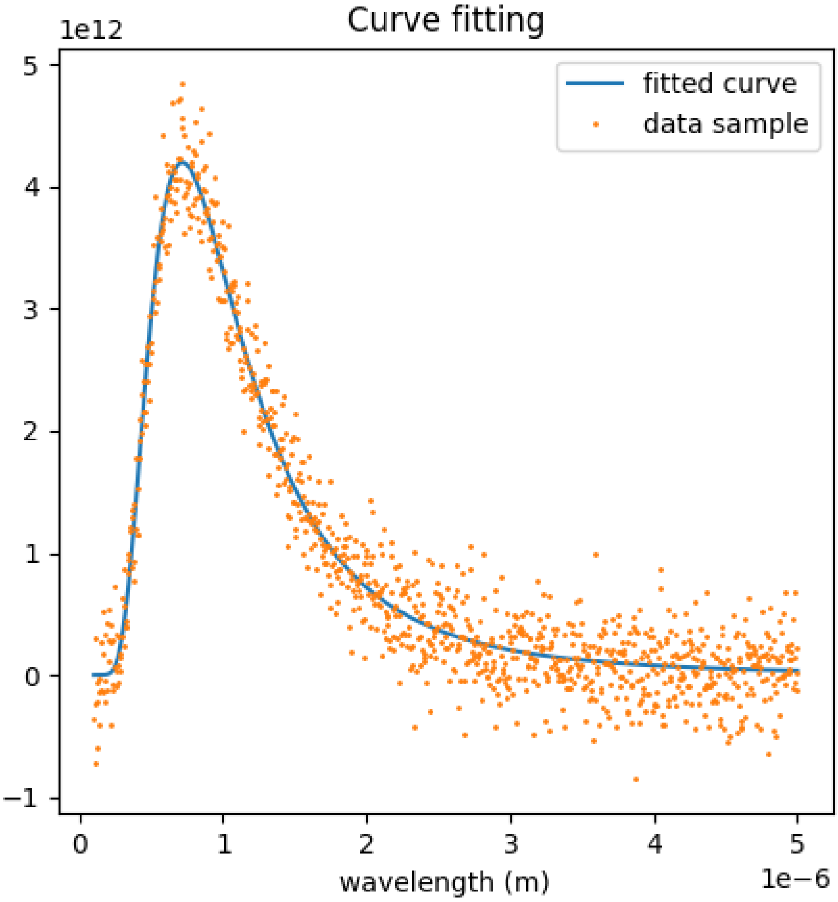
For this section I have used the SciPy libraries `curve_fit` function.

The code for the same looks like this:

```

import scipy
import numpy as np
import matplotlib.pyplot as plt


def f(x, h, c, k, t):
    return (2 * h * c * c / x**5) /
      (np.exp(1 * h * c / (x * k * t)) - 1)


def main():
    xdata = []
    ydata = []
    with open("data/d3.txt", "r") as fin:
        for line in fin.readlines():
            x, y = line.split(",")
            xdata.append(float(x))
            ydata.append(float(y))

    #fitting
    initial_guess = [6.62 * 1e-34, 3 * 1e3, 1.38 * 1e-23, 1]
    params, cov = scipy.optimize.curve_fit(
        f, xdata, ydata, initial_guess, nan_policy="omit"
    )

    #getting prediction
    ypred = []
    for val in xdata:
        ypred.append(
          f(val, params[0], params[1],
            params[2], params[3])
                    )

    #plotting
    plt.title("Curve fitting")
    plt.xlabel("wavelength (m)")
    plt.ylabel("power density (Watts/m^3)")
    plt.plot(xdata, ypred, label="fitted curve")
    plt.plot(
        xdata, ydata, markersize=2, marker=".", linestyle="None", label="data sample"
    )
    plt.legend()
    plt.show()


if __name__ == "__main__":
    main()
```

Since there are three known constants h, c, k_B, we can give the initial guess of any combination of these, starting with:

$$h = 6.62 \times 10^{-34}, c = 3 \times 10^{3}, k_B = 1.38 \times 10^{-23}$$



And the final constants are:

$$h = 2.17146884 * 10^{-31}, c = 1.61328417 * 10^{7}, k_B = -3.53152592 * 10^{-21}, t = -2.78747897 * 10^{2}$$

Using the program

You can reproduce these results with the python file I have attached with my submission. Please keep the following in mind while using the program:

- pass the filename as a positional argument
- As mentioned earlier, I normalize the input data by multiplying with a constant. To get a correct fit, One will have to play around with this normalization constant. For the purpose of this assignment please use the following values:
  - for `d1.txt` use 10^6
  - for `d3.txt` use 10^5

  Thus, running the program would look something like this:

```
$ python3 ee23b137.py d1.txt -normalize 1e6
```