

Semestrální projekt IEL 2016/2017

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Příklad 1.C

$$U_{R8} = ?$$

$$I_{R8} = ?$$

$$U_1 = 100V$$

$$U_2 = 80V$$

$$R_1 = 450 \Omega$$

$$R_2 = 810 \Omega$$

$$R_3 = 190 \Omega$$

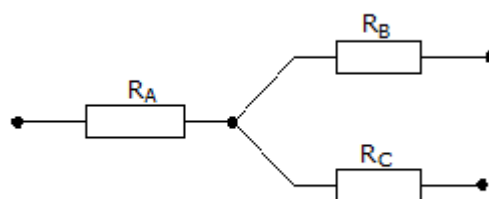
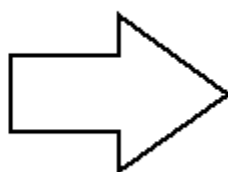
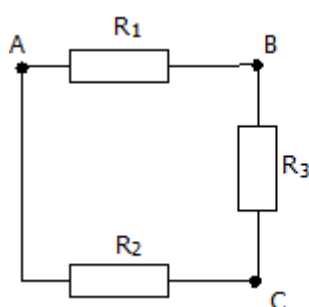
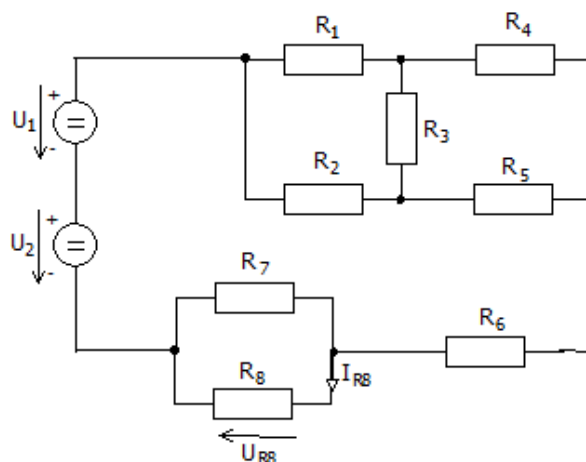
$$R_4 = 220 \Omega$$

$$R_5 = 220 \Omega$$

$$R_6 = 720 \Omega$$

$$R_7 = 260 \Omega$$

$$R_8 = 180 \Omega$$



$$R_B = \frac{R_1 * R_3}{R_1 + R_2 + R_3}$$

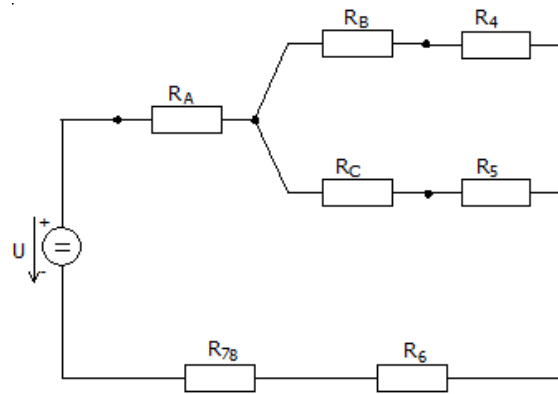
$$R_B = \frac{450 * 190}{450 + 810 + 190} = \frac{1710}{29} = 58.9655 \Omega$$

$$R_C = \frac{R_2 * R_3}{R_1 + R_2 + R_3}$$

$$R_C = \frac{810 * 190}{450 + 810 + 190} = \frac{3078}{29} = 106.1379 \Omega$$

$$R_A = \frac{R_1 * R_2}{R_1 + R_2 + R_3}$$

$$R_A = \frac{450 * 810}{450 + 810 + 190} = \frac{7290}{29} = 251.3793 \Omega$$

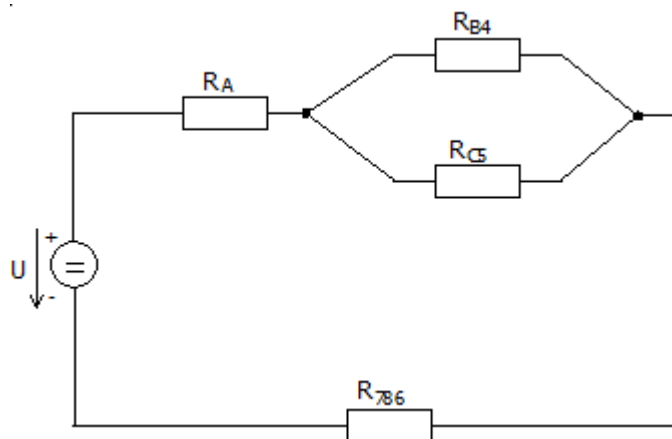


$$U = U_1 + U_2$$

$$U = 100 + 80 = 180V$$

$$R_{78} = \frac{R_7 * R_8}{R_7 + R_8}$$

$$R_{78} = \frac{260 * 180}{260 + 180} = \frac{46800}{440} \Omega = 106.3636 \Omega$$



$$R_{B4} = R_B + R_4$$

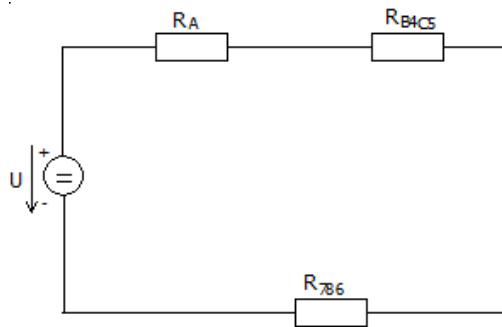
$$R_{B4} = 58.9655 + 220 = 278.9655 \Omega$$

$$R_{C5} = R_C + R_5$$

$$R_{C5} = 106.1379 + 220 = 326.1379 \Omega$$

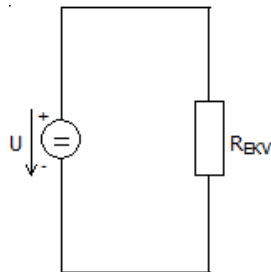
$$R_{786} = R_{78} + R_6$$

$$R_{786} = 106.3636 + 720 = 826.3636 \Omega$$



$$R_{B4C5} = \frac{R_{B4} * R_{C5}}{R_{B4} + R_{C5}}$$

$$R_{B4C5} = \frac{278.9655 * 326.1379}{278.9655 + 326.1379} = 150.3565 \, \Omega$$



$$R_{EKV} = R_A + R_{B4C5} + R_{786}$$

$$R_{EKV} = 251.3793 + 150.3565 + 826.3636 = 1228.0994 \, \Omega$$

$$I = \frac{U}{R_{EKV}}$$

$$I = \frac{180}{1228.0994} = 0.1466 \, A$$

$$U_{R78} = I * R_{78}$$

$$U_{R78} = 0.1466 * 106.3636 = 15.5929 \, V = U_{R8}$$

$$I_{R8} = \frac{U_{R8}}{R_8}$$

$$I_{R8} = \frac{15.5929}{180} = 0.08663 \, A$$

$$I_{R8} = 0.08663 \, A$$

$$U_{R8} = 15.5929 \, V$$

Příklad 2.E

$$U_{R4} = ?$$

$$I_{R4} = ?$$

$$U = 250V$$

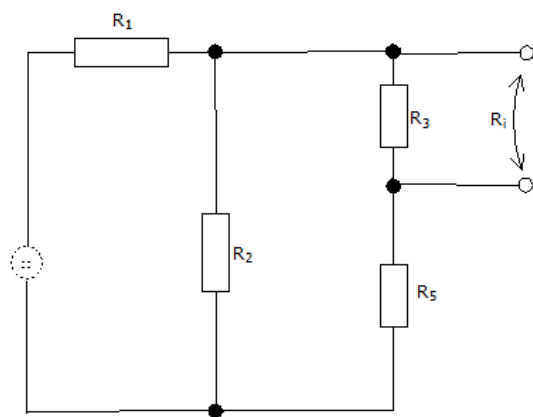
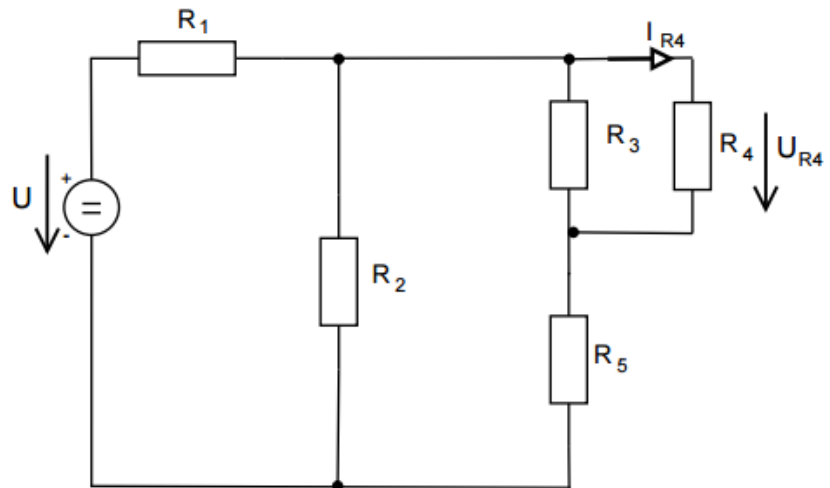
$$R_1 = 335 \Omega$$

$$R_2 = 625 \Omega$$

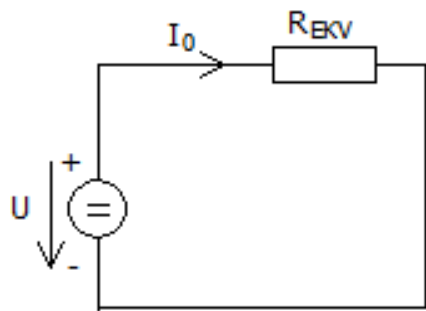
$$R_3 = 245 \Omega$$

$$R_4 = 600 \Omega$$

$$R_5 = 180 \Omega$$



Zprvė zjistíme I_0 pomocí R_{EKV} :



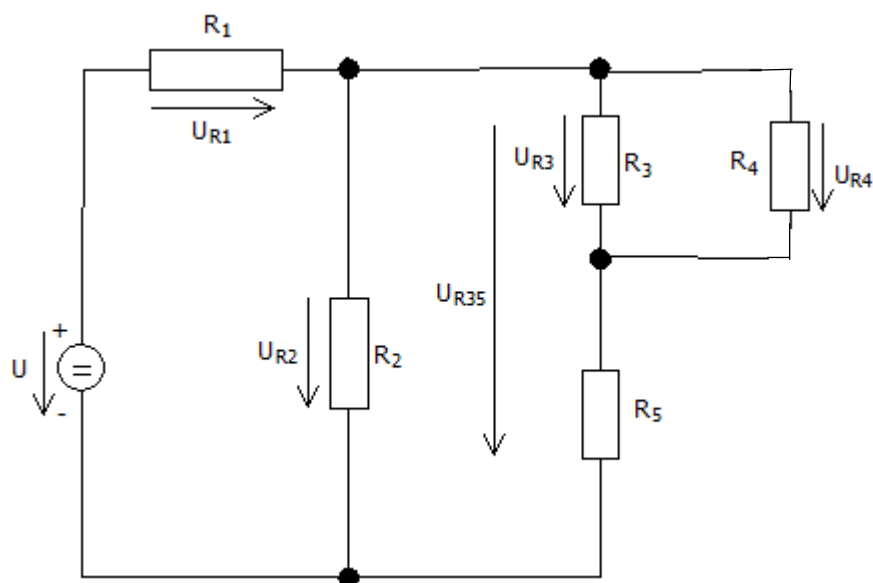
$$R_{ekv} = R_1 + \frac{R_2 * (R_3 + R_5)}{R_2 + R_3 + R_5}$$

$$R_{ekv} = 335 + \frac{625 * (245 + 180)}{625 + 245 + 180} = 587.9762 \Omega$$

$$I_0 = \frac{U}{R_{ekv}}$$

$$I_0 = \frac{250}{587.9762} = 0.4252A$$

Zjistíme U_{R1} a U_{R2} :



$$U_{R1} = R_1 * I_0$$

$$U_{R1} = 335 * 0.4252 = 142.442 \text{ V}$$

$$U_{R2} = U_{R35} = U - U_{R1}$$

$$U_{R2} = U_{R35} = 250 - 142.442 = 107.558 \text{ V}$$

Vypočteme I_{R35} a U_{R3} ($= U_i$):

$$I_{R35} = \frac{U_{R35}}{R_3 + R_5}$$

$$I_{R35} = \frac{107.558}{245 + 180} = 0.2531 \text{ A}$$

$$U_{R3} = R_3 * I_{R35}$$

$$U_{R3} = 245 * 0.2531 = 62.0095 \text{ V}$$

Vypočteme R_i :

$$R_{12} = \frac{R_1 * R_2}{R_1 + R_2}$$

$$R_{12} = \frac{335 * 625}{335 + 625} = 218.09896 \text{ } \Omega$$

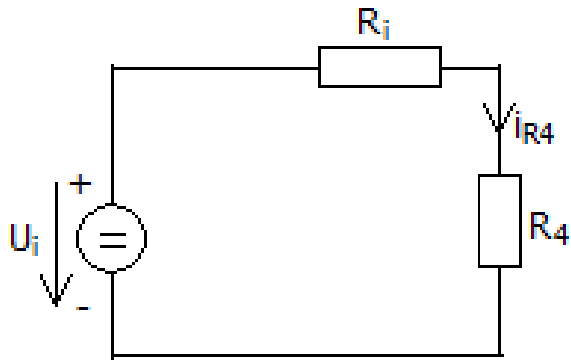
$$R_{125} = R_{12} + R_5$$

$$R_{125} = 218.09896 + 180 = 396.09896 \text{ } \Omega$$

$$R_i = R_{1235} = \frac{R_{125} * R_3}{R_{125} + R_3}$$

$$R_i = R_{1235} = \frac{396.09896 * 245}{396.09896 + 245} = 151.3717 \, \Omega$$

Zname R_i , můžeme dopočítat I_{R4} a U_{R4} :



$$I_{R4} = \frac{U_i}{R_i + R_4}$$

$$I_{R4} = \frac{62.0095}{151.3717 + 600} = 0.08253 \, A$$

$$U_{R4} = R_4 * I_{R4}$$

$$U_{R4} = 600 * 0.08253 = 49.518 \, V$$

Příklad 3.E

$$U_{R4} = ?$$

$$I_{R4} = ?$$

$$U = 135V$$

$$I_1 = 0.55A$$

$$I_2 = 0.65A$$

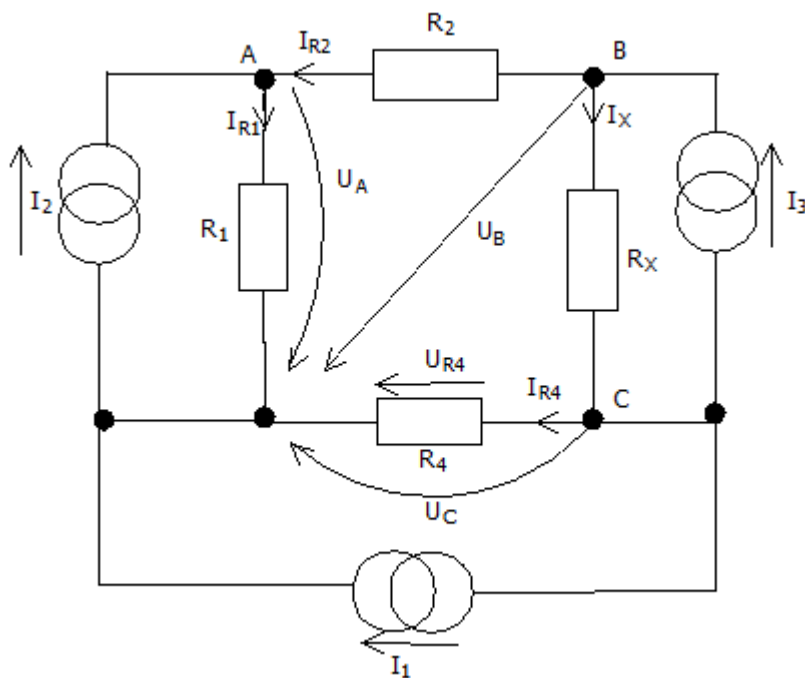
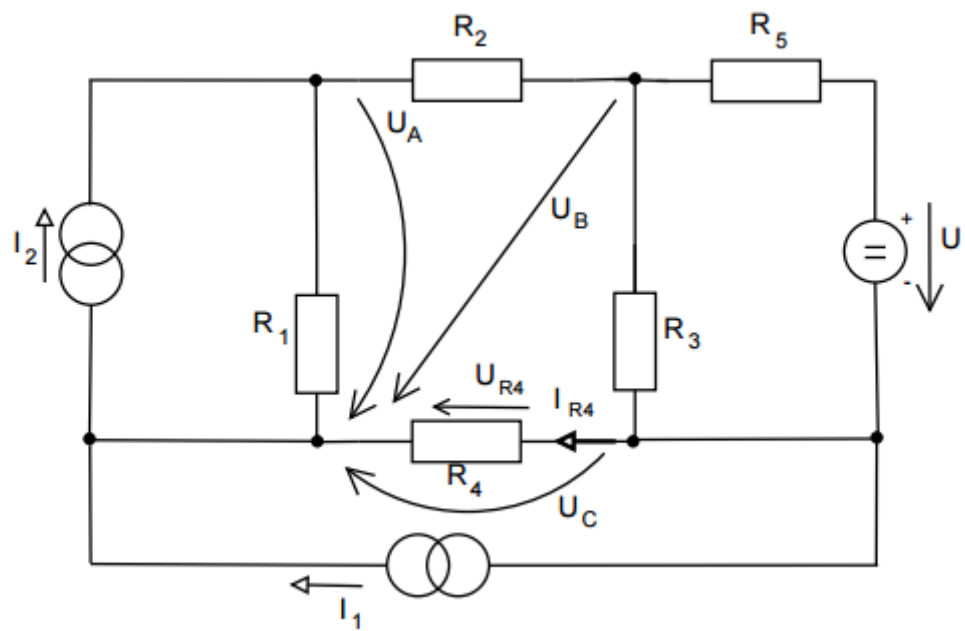
$$R_1 = 52 \Omega$$

$$R_2 = 42 \Omega$$

$$R_3 = 52 \Omega$$

$$R_4 = 42 \Omega$$

$$R_5 = 21 \Omega$$



$$R_X = \frac{R_3 * R_5}{R_3 + R_5}$$

$$R_X = \frac{52 * 21}{52 + 21} = 14.9589 \Omega$$

$$I_3 = \frac{U_3}{R_5}$$

$$I_3 = \frac{135}{21} = 6,4286 A$$

Podle Kirchhoffova zákona:

$$A: I_2 + I_{R2} - I_{R1} = 0$$

$$B: I_3 - I_{R2} - I_{RX} = 0$$

$$C: I_{RX} - I_3 - I_{R4} - I_1 = 0$$

Sestavíme rovnice pro jednotlivé proudy:

$$I_{R1} = \frac{U_A}{R_1}$$

$$I_{R2} = \frac{U_B - U_A}{R_2}$$

$$I_{R4} = \frac{U_C}{R_4}$$

$$I_{RX} = \frac{U_B - U_C}{R_X}$$

Dosadíme do rovnic pro jednotlivé uzly:

$$I_2 + \frac{U_B - U_A}{R_2} - \frac{U_A}{R_1} = 0$$

$$I_3 - \frac{U_B - U_A}{R_2} - \frac{U_B - U_C}{R_X} = 0$$

$$\frac{U_B - U_C}{R_X} - I_3 - \frac{U_C}{R_4} - I_1 = 0$$

$$0.65 + \frac{U_B - U_A}{42} - \frac{U_A}{52} = 0$$

$$6.4286 - \frac{U_B - U_A}{42} - \frac{U_B - U_C}{14.9589} = 0$$

$$\frac{U_B - U_C}{14.9589} - 6.4286 - \frac{U_C}{42} - 0.55 = 0$$

Provedeme úpravu rovnic:

$$0.65 - \frac{47U_A}{1092} + \frac{U_B}{42} = 0$$

$$6.4286 + \frac{U_A}{42} - 0.0906594U_B + 0.0668498U_C = 0$$

$$-6.9786 + 0.0668498U_B - 0.0906594U_C = 0$$

Z první rovnice vyjádříme U_A :

$$\frac{3549 - 235U_A + 130U_B}{5460} = 0$$

$$3549 - 235U_A + 130U_B = 0$$

$$235U_A = 3549 + 130U_B$$

$$U_A = \frac{26U_B}{47} + \frac{3549}{235}$$

V druhé rovnici provedeme substituci:

$$6.4286 + \frac{\frac{26U_B}{47} + \frac{3549}{235}}{42} - 0.0906594U_B + 0.0668498U_C = 0$$

$$6.4286 + \frac{13U_B}{987} + \frac{169}{470} - 0.0906594U_B + 0.0668498U_C = 0$$

$$6.78817 - 0.0774881U_B + 0.0668498U_C = 0$$

Z druhé rovnice vyjádříme U_B :

$$0.0774881U_B = 6.78817 + 0.0668498U_C$$

$$U_B = 0.86711U_C + 87.6028$$

Ve třetí rovnici provedeme substituci a dopočítáme U_C :

$$-6.9786 + 0.0668498(0.86711U_C + 87.6028) - 0.0906594U_C = 0$$

$$-6.9786 + 0.0576721U_C + 5.85623 - 0.0906594U_C = 0$$

$$-1.12237 - 0.0329873U_C = 0$$

$$0.0329873U_C = -1.12237$$

$$U_C = -34.0243 \text{ V}$$

Dosadíme U_C do druhé rovnice a dopočítáme U_B :

$$U_B = 0.86711 * (-34.0243) + 87.6028$$

$$U_B = 58.2496 \text{ V}$$

Dosadíme U_B do první rovnice a dopočítáme U_A :

$$U_A = \frac{26 * 58.2496}{47} + \frac{3549}{235}$$

$$U_A = 47.3253 \text{ V}$$

Provedeme výpočet I_{R4} a U_{R4} :

$$I_{R4} = \frac{U_C}{R_4}$$

$$I_{R4} = \frac{-34.0243}{42} = -0.8101 \text{ A}$$

$$U_{R4} = R_4 * I_{R4}$$

$$U_{R4} = 42 * (-0.8101) = -34.0242 \text{ V}$$

Příklad 5.E

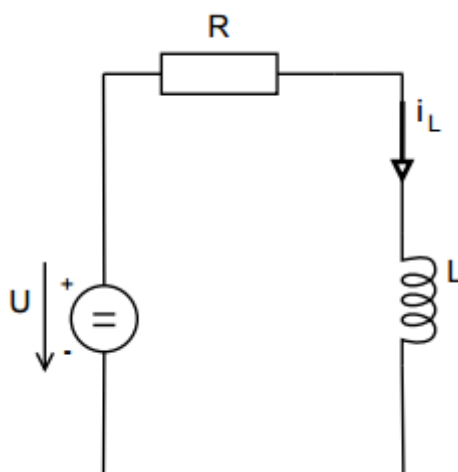
Vypočítat analytické řešení $i_L = f(t)$

$$U = 20V$$

$$L = 30H$$

$$R = 40\ \Omega$$

$$i_L(0) = 5A$$



Druhý Kirchhoffův zákon:

$$U_R + u_L - U = 0$$

$$U = U_R + u_L$$

$$u_L = U - i_L R$$

Předpokládáme, že platí:

$$i_L' = \frac{1}{L} u_L$$

Dosadíme u_L do rovnice:

$$i_L' = \frac{1}{L} * (U - i_L R)$$

$$i_L' = \frac{U}{L} - \frac{i_L R}{L}$$

$$L i_L' = U - i_L R$$

$$L i_L' + i_L R = U$$

Z charakteristické rovnice vyplývá:

$$L\lambda + R = 0$$

$$\lambda = -\frac{R}{L}$$

Dosadime λ do:

$$i_L(t) = K(t)e^{\lambda t}$$

$$i_L(t) = K(t)e^{-\frac{R}{L}t}$$

$$i_L'(t) = (K(t)e^{-\frac{R}{L}t})'$$

$$i_L'(t) = K'(t)e^{-\frac{R}{L}t} - \frac{R}{L}K(t)e^{-\frac{R}{L}t}$$

Najdeme $K'(t)$:

$$L(K'(t)e^{-\frac{R}{L}t} - \frac{R}{L}K(t)e^{-\frac{R}{L}t}) + R(K(t)e^{-\frac{R}{L}t}) = U$$

$$LK'(t)e^{-\frac{R}{L}t} - RK(t)e^{-\frac{R}{L}t} + RK(t)e^{-\frac{R}{L}t} = U$$

$$LK'(t)e^{-\frac{R}{L}t} = U$$

$$K'(t) = \frac{U}{L}e^{\frac{R}{L}t}$$

Vyjadrieme si $K(t)$:

$$\int K'(t)dt = \int \frac{U}{L}e^{\frac{R}{L}t}dt = \frac{UL}{LR}e^{\frac{R}{L}t} + C = \frac{U}{R}e^{\frac{R}{L}t} + C$$

Dosadime $K(t)$ do i_L :

$$i_L = (\frac{U}{R}e^{\frac{R}{L}t} + C)e^{-\frac{R}{L}t}$$

$$i_L = \frac{U}{R} + Ce^{-\frac{R}{L}t}$$

Mame $i_L(0) = 5 \text{ A}$:

$$5 = \frac{80}{40} + Ce^{-\frac{40}{30} \cdot 0}$$

$$5 = 2 + C$$

$$C = 3$$

$$i_L = 2 + 3e^{-\frac{4}{3}t}$$

| Tabulka výsledků | | | | |
|----------------------------------------------|---------------------------------------------|-----------------------------------------------|----|--------------------------------|
| 1. | 2. | 3. | 4. | 5. |
| C | E | E | C | E |
| $I_{R8} = 0,08663 A$ $U_{R8} = 15.5929 V$ | $I_{R4} = 0.08253 A$ $U_{R4} = 49.518 V$ | $I_{R4} = -0.8101 A$ $U_{R4} = -34.0242 V$ | | $i_L = 2 + 3e^{-\frac{4}{3}t}$ |
| | | | | |