Barra Portfolio Risk Model Implementation in KDB+/q

This document provides a detailed implementation of the Barra-style risk model in KDB+/q. It explains each step of the function `barraPortfolioRisk` and the financial concepts behind them.

Key Concepts and Formulas

1. Total Covariance Matrix

To assess how assets move together, the total risk covariance matrix is computed:

```
Σ = X F X' + D
```

- `X`: matrix of asset exposures to systematic risk factors
- `F`: factor covariance matrix (correlations and volatilities)
- `D`: diagonal matrix of asset-specific variances

2. Portfolio Variance and Volatility

This measures overall risk assuming a linear combination of asset variances and covariances:

```
σ²_p = w' Σ w,  σ_p = sqrt(σ²_p)
```

3. Systematic and Specific Risk

The total portfolio variance is decomposed into:

- **Systematic Risk**: from common factors (e.g., market, size, value)

- **Specific Risk**: unique to each asset, non-factor related

```
Systematic = w' X F X' w,  Specific = w' D w
```

4. Factor Contribution to Risk

How much each factor contributes to total portfolio variance:

```
Contribution_i = b_i * (F b)_i,  where b = w' X
```

This shows which factors dominate portfolio risk.

5. Marginal Contribution to Risk (MCR)

Measures how sensitive portfolio risk is to a small increase in asset weight:

```
MCR_i = (Σ w)_i / σ_p
```

6. Contribution to Risk (CTR)

Total contribution of each asset to risk (MCR scaled by actual holding):

```
CTR_i = w_i * MCR_i
```

7. Risk Conversion to Dollar and Basis Points

All risk values are reported in:

- **Dollars**: by multiplying by total notional
- **Basis Points (bps)**: by multiplying by 10,000

```
Risk_$ = Risk * Notional,  Risk_bps = Risk * 10,000
```

8. Value at Risk (VaR)

Calculates potential loss under normal market conditions over a 1-day horizon:

```
DailyVol = σ_p / sqrt(252)
VaR_95% = 1.645 * DailyVol * Notional
VaR_99% = 2.326 * DailyVol * Notional
```

Full KDB+/q Function Code

Below is the full implementation with no modifications. It combines all calculations described above into a dictionary output structure.

```
barraPortfolioRisk:{[weights; X; F; specVar; notional]
    w:enlist weights;
    Xmat:flip X;
    Fmat:flip F;
    D:diag specVar;

    systematicCov:Xmat mmu Fmat mmu flip Xmat;
    totalCov:systematicCov + D;

    portVar: w mmu totalCov mmu flip w;
    portVol: sqrt first portVar;

    sysVar: w mmu systematicCov mmu flip w;
    specVarTotal: w mmu D mmu flip w;
    sysPct: first sysVar % first portVar;
    specPct: first specVarTotal % first portVar;

    b: first w mmu Xmat;
    Fb: Fmat mmu enlist b;
    factorContrib: b * first each Fb;
    factorContribPct: factorContrib % first portVar;
```

```
    sigmaW: totalCov mmu flip w;
    mcr: (first each sigmaW) % portVol;
    ctr: weights * mcr;

    factorContribUSD: factorContrib * notional;
    factorContribBPS: factorContrib * 10000;

    sysRiskUSD: first sysVar * notional;
    specRiskUSD: first specVarTotal * notional;
    portVolUSD: portVol * notional;

    sysRiskBPS: first sysVar * 10000;
    specRiskBPS: first specVarTotal * 10000;
    portVolBPS: portVol * 10000;

    mcrUSD: mcr * notional;
    ctrUSD: ctr * notional;

    mcrBPS: mcr * 10000;
    ctrBPS: ctr * 10000;

    dailyVol: portVol % sqrt 252f;
    z95: 1.645;
    z99: 2.326;

    var95: z95 * dailyVol * notional;
    var99: z99 * dailyVol * notional;
    var95bps: var95 % (notional % 10000f);
    var99bps: var99 % (notional % 10000f);

    (
      `PortfolioVolatility`PortfolioVolatilityUSD`PortfolioVolatility
        ! (portVol; portVolUSD; portVolBPS);

      `SystematicRisk`SpecificRisk`SystematicRiskUSD`SpecificRiskUSD`
        ! (first sysVar; first specVarTotal; sysRiskUSD; specRiskUSD;
```

```
      `FactorContribution`FactorContributionUSD`FactorContributionBPS
        ! (factorContrib; factorContribUSD; factorContribBPS; factorC

      `MCR`MCRUSD`MCRBPS
        ! (mcr; mcrUSD; mcrBPS);

      `CTR`CTRUSD`CTRbps
        ! (ctr; ctrUSD; ctrBPS);

      `VaR_95_USD`VaR_99_USD`VaR_95_BPS`VaR_99_BPS
        ! (var95; var99; var95bps; var99bps)
    )
};
```