Exercise set 3 for CS 498

We expect all plots to be provided with your written report for the homework. Please do not just submit code, or a folder full of figures, as sometimes the code will not run on our machines due to dependencies. Submit your final report as a PDF.

Total points: 100

Question 1 (15 points)

You are surveying for a high-stake donor who wants to build a fancy building on campus. The committee is requesting you to provide accurate measurements of the area where the building will be built. You in your wisdom after taking the field robotics class choose to use RTK-GPS instead of tape measures. A GPS reading (in degrees, minutes, seconds format) in the southwest corner of the rectangular field where the building is to be built gave the coordinates 80°, 10', 14.50" west longitude, 41°, 14', 33.50" north latitude. The GPS reading in the northeast corner of the same field gave coordinates 80°, 11', 9.8" west longitude, 41°, 16', 2.1" north latitude.
Calculate

1. The field dimensions in m and (10)
2. The field area in ha. (5)

Show your work

Question 2 (15 points)

Your company executive team thinks that MEMS Inertial Measurement Sensors that cost $50 should be good enough for the robot your company is building to navigate accurately across the field. They remember from their dynamics class that position is the integral of velocity and velocity is the integral of acceleration. Your job is to convince them that the errors in an un-aided INS system will build up over time. Rummaging through your notes of the field robotics class, you decide to make your case with a very simple model of a one dimensional point-mass system:

$$\dot{p}(t) = v(t)$$
$$\dot{v}(t) = a(t)$$

where $p(t), v(t), a(t)$ are the positions, velocity, and acceleration of the one-dimensional robot

You then make a model for the accelerometer measurements $\tilde{u}(t)$, you can assume the accelerometer updates at 100Hz (i.e. dt=0.01 sec)

$$\tilde{u}(t) = a(t) + b(t) + \zeta(t)$$

where $b(t)$ is the bias of the accelerometer, and $\zeta(t)$ is the accelerometer noise.

You model the bias as a zero-mean Gaussian random walk process:

$$\dot{b} = \omega_b$$

where $\omega_b \sim N(0, \sigma_b^2)$ and $\sigma_b^2 = 1.0 \times \frac{10^{-6} m^2}{s^4}$, and $\zeta \sim N(0, \sigma_\zeta^2)$ where $\sigma_\zeta^2 = 2.5 \times \frac{10^{-3} m^2}{s^4}$

Perform the following numerical analysis:

1. Integrate forward the error $e(t) = x(t) - \hat{x}(t)$ for 500 seconds and show that it diverges, use white noise or a sinusoidal control input to excite the system
2. Integrate forward the covariance matrix for 500 seconds and plot the diagonal terms

Question 3 (30 points)

You now argue to the CEO that you can fix this issue by adding a GPS based position measurement. You make a simple model of the GPS position measurement:

$$\tilde{p}(t) = p(t) + \nu_p(t)$$

Where $\nu_p \sim N(0, \sigma_p^2)$ and $\sigma_p^2 = 3\ m^2$, also the GPS measurements come once every second, that is the sampling time of the GPS measurement is 1 second.

Write software for a Kalman filter and show the following:

1. Write out the equations of the Kalman filter, the transition matrix, the measurement matrix (5)
2. Implement the Kalman filter in python (5)
3. Plot the position and velocity states for 500 seconds, provide the plot in your homework report (10)
4. Plot the position and velocity co-variance for 500 seconds, provide the plot in your homework report (10)

Q4 (40 points)

You now have convinced the CEO that you need GPS, you want to have more budget for your project, so you set out to convince them you can integrate a gyro and an encoder which will actually enable the robot to know its position, velocity, and angle really well

You utilize the model from HW2 of your mobile robotics class:

$$\dot{x} = v\ cos(\theta),$$
$$\dot{y} = v \sin(\theta)$$
$$\dot{\theta} = \omega$$
$$\dot{v} = a$$

Where now you have measurements of the angular rate termed as $\widetilde{\omega}$

$$\widetilde{\omega} = \omega + \zeta_\omega$$

and the accelerometer measurements $\tilde{a}$ as

$$\tilde{a}(t) = a(t) + b_a(t) + \zeta_a(t)$$

Assume that the accelerometer noise and bias characteristics are as in question 2, and the gyro noise is $\zeta_\omega \sim N(0, \sigma^2)$, and $\sigma_\omega^2 = 2.5 \times 10^{-3} rad^2/s^2$

In addition you have position measurements $\tilde{x}$ and $\tilde{y}$ with the same noise characteristics as in Question 3. When you excite the system for evaluating it, use a sinusoidal input similar to the example studied in class or your own steering profile.

Implement an Extended Kalman Filter in Python

1. Linearize the dynamic equations about a fixed angle $\theta_0 = 45^0$ (that is the initial angle is 45 degrees) (5)
2. Implement the EKF in python, provide the code (35)
3. Plot the position, velocity, angle for 500 seconds, provide the plot in your homework report (5)
4. Plot the covariance for position, velocity, and angle for 500 seconds, provide the plot in your homework report (5)