

# Comparative Study of Two Switching Schemes for a Cascade Multilevel Inverter

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**Abstract**— In this paper, two switching schemes at fundamental frequency switching for a cascade multilevel inverter (CMLI) are compared. For the presented schemes, the switching angles are computed using optimization technique for different objective functions. Then, the computed switching angles are used to produce multilevel output voltage of CMLI. The comparison includes magnitudes of THD as well as individual harmonic components present in the output voltage, values of different losses associated with distorted output voltage, and different applications where CMLI can be used effectively. The results obtained have been validated through computational, simulation and experimental studies on an 11-level CMLI. It has also been observed that the results obtained by these three studies (computational, simulation and experimental) are in close agreement with each other.

**Index Terms**— Cascade multilevel inverter, modulation index, sequential quadratic programming, and total harmonic distortion (THD).

## 1. INTRODUCTION

**M**ultilevel inverters are power electronics devices and are used to synthesize a desired output voltage from several levels of input DC voltages.

They were developed in the last decade and having promising applications in the field of electrical power systems over the hard-switched two-level pulse width modulation inverters due to their capability to operate at high voltage with lower  $dv/dt$  per switching, high efficiency and low electromagnetic interference etc. [1]-[4].

A sinusoidal voltage of high magnitude with extremely low distortion at fundamental frequency can be generated at the output of multilevel inverters by connecting sufficient number of DC levels. There are mainly three types of multilevel inverters; these are a) diode- clamped, b) flying capacitor and c) cascade multilevel inverter (CMLI). Among these three, CMLI has a modular structure and requires least number of components as compared to other two topologies and as a result, it is receiving increasing attention for use in many applications such as static synchronous compensator (STATCOM), utility interfacing of renewable energy sources (solar cell, fuel cell etc), electric vehicles etc [3]. For the above applications, output voltage produced by multilevel inverters must have less distortion i.e. the output voltage must be nearly sinusoidal so that undesirable effects produced by distortion can be minimized [5]-[7].

The distortion produced in the output AC voltage, depends mainly on the switching techniques (modulation methods) employed for the switching of power electronics devices. These switching techniques can be classified into two categories, namely a) high frequency based switching techniques and b) fundamental frequency switching methods. As switching losses are low in case of the fundamental frequency switching method, generally it is preferred over the high frequency switching methods [8].

Among the fundamental frequency switching based modulation strategies, the most commonly used technique is selective harmonic elimination (SHE) method. In this method, the switching angles are computed by solving a set of transcendental equations in such a way that certain number of harmonic components can be eliminated from the output voltage, generally lower order harmonic components are eliminated. Different techniques have been suggested in the literature for solving these transcendental equations [8]-[10]. Although SHE technique effectively eliminates certain lower order harmonics, the magnitudes of the higher order harmonics increase simultaneously, thereby increasing the overall total harmonic distortion (THD) in the output voltage. One more problem associated with SHE technique is that the solutions can not be obtained for complete range of modulation indices [8]-[9]. In [8], a method is suggested to achieve low THD of output voltage by eliminating the lower order harmonics by using SHE technique, and harmonic components up to 31<sup>st</sup> order have been eliminated by increasing the switching frequency of the active devices of CMLI. However, in this method, further higher order harmonics are generated which can be eliminated by proper filter. Moreover, this method is implemented at high switching frequency thereby increasing the switching losses.

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paper extends the applications of optimization technique for computing the switching angles of a CMLI in two cases: a) all possible lower order harmonic components which can be eliminated (or minimized) are identified, and b) harmonic components up to a certain order are minimized. Specifically, in this work, an 11-level inverter has been considered. The computationally obtained results have also been compared with those obtained from simulation and experimental studies.

CASCADE MULTILEVEL INVERTER

A cascade multilevel inverter consists of a number of H-bridge inverter units with separate dc source for each unit connected in cascade or series as shown in Fig. 1. Each H-bridge can produce three different voltage levels:  $V_{dc}$ , 0 and  $-V_{dc}$  by connecting the DC source to AC output by different combinations of the four switches  $S_1$ ,  $S_2$ ,  $S_3$  and  $S_4$ . The AC output of each H-bridge is connected in series such that the synthesized output voltage waveform is the sum of all of the individual H-bridge outputs. By connecting sufficient number of H-bridges in cascade and using proper modulation scheme, a nearly sinusoidal output voltage waveform can be synthesized. The number of levels in the output phase voltage is  $2s+1$ , where  $s$  is the number of H-bridges used per phase. Fig. 2 shows an eleven-level output phase voltage waveform using five H-bridges. The magnitude of the AC output phase voltage is given by  $v_{an} = v_{a1} + v_{a2} + v_{a3} + v_{a4} + v_{a5}$  [3].

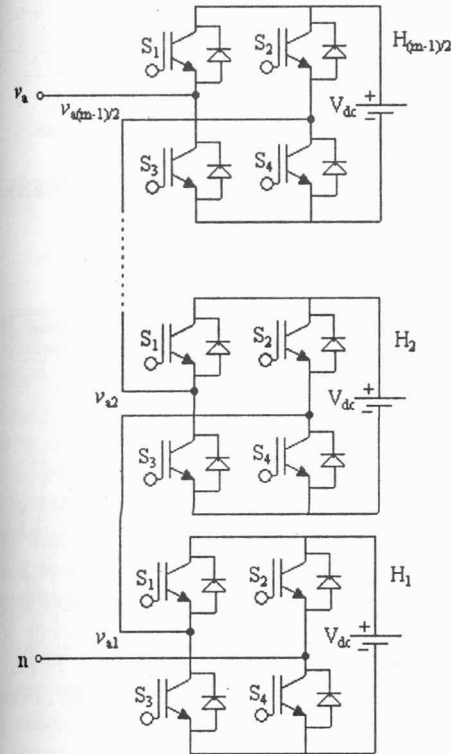


Fig. 1: Configuration of single-phase cascade multilevel inverter

The Fourier series expansion of the staircase output voltage waveform as shown in Fig. 2 is given by [4].

$$v_{an}(wt) = \sum_{k=1,3,5,\dots}^{\infty} \frac{4V_{dc}}{k\pi} (\cos(k\alpha_1) + \cos(k\alpha_2) + \dots + \cos(k\alpha_s)) \sin(k\omega t) \tag{1}$$

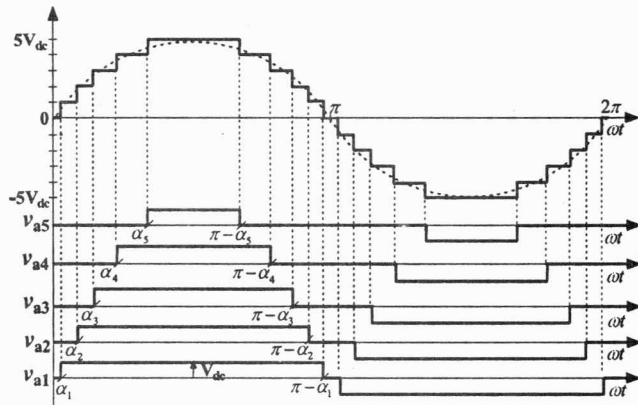


Fig. 2: Waveform of output phase voltage for an 11-level CMLI at fundamental frequency switching scheme

The percentage harmonic factor, which is a measure of individual harmonic contribution, is given as [11].

$$\psi_n = \frac{V_n}{V_1} \times 100, \quad n > 1 \tag{2}$$

Where  $V_n$  is the magnitude of  $n$ th harmonic and  $V_1$  is the fundamental component of the output voltage.

The relation between the fundamental voltage and the maximum obtainable voltage is given by modulation index. The modulation index,  $m$  is defined as the ratio of the fundamental output voltage  $V_1$  to the maximum obtainable fundamental voltage  $V_{1max}$ . The maximum fundamental voltage is obtained when all switching angles are zero.

3. OPTIMIZATION TECHNIQUE

3.1 Method I

As the harmonic content in the output voltage depends on the switching angles  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$ ,  $\alpha_4$  and  $\alpha_5$  (as shown in Fig. 2), these angles must be chosen properly. In this method, above five angles are calculated such that four dominant lower order harmonic components ( $5^{th}$ ,  $7^{th}$ ,  $11^{th}$ , and  $13^{th}$ ) are completely eliminated for the values of  $m$  for which these components can be eliminated; and for remaining values of  $m$ , these harmonic components are minimized. To compute the switching angles, following objective function is formulated:

$$\phi_1(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5) = (V_5^2 + V_7^2 + V_{11}^2 + V_{13}^2) \tag{3}$$

The objective function (3) is to be minimized such that

fundamental voltage  $V_1$  is produced as desired (for a given value of  $m$ ) and all switching angles must be

$$0 \leq \alpha_1 < \alpha_2 < \dots < \alpha_5 \leq \pi/2.$$

The above optimization problem is solved by the sequential quadratic programming (SQP) method [12] and been implemented using MATLAB optimization toolbox [13].

The switching angles computed are shown in Fig. 3. It is to be noted from the Fig. 3 that the switching angles obtained are similar to as obtained by SHE technique for those values of  $m$  where solutions exist [9]-[10], thereby establishing the feasibility of method I. From Fig. 3, it is to be noted that for some values of  $m$  there exist multiple solution sets. Among multiple solution sets, only those values of switching angles have been selected for analysis which produces least THD in output voltage.

### 3.2 Method II

Even though the lower order harmonics are eliminated or minimized by method I. It has been observed that by doing so, the higher order harmonics get enhanced. Therefore, to minimize the higher order harmonics, the switching angles have also been calculated by minimizing the aggregate contribution of all harmonics up to 31<sup>st</sup> order.

The objective function for minimizing harmonic components up to 31<sup>st</sup> order is formulated as given below:

$$\phi_2(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5) = (V_5^2 + V_7^2 + \dots + V_{31}^2) \quad (4)$$

In this case also, the switching angles are computed by minimizing the objective function (4) such that desired fundamental voltage is produced and switching angles should be  $0 \leq \alpha_1 < \alpha_2 < \dots < \alpha_5 \leq \pi/2$ .

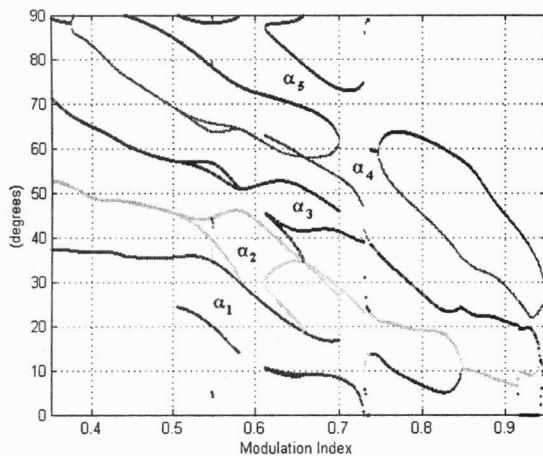


Fig. 3: Variation of switching angles for method I

It is to be noted that (4) has also been minimized through SQP technique by using MATLAB optimization toolbox.

The magnitudes of some dominant higher order harmonic components for the switching angles obtained by method I and method II are calculated using (2) and are shown in Figs. 4-6. From these figures it is observed that the magnitudes of different harmonic components are generally higher for method I as compared to those obtained by method II which corroborate the fact that elimination of lower order harmonics only enhances the magnitudes of higher order harmonics. The THD for the switching angles obtained in method I and method II has been computed according to

$$THD_\sigma = \sqrt{\sum_{n=5,7,11,\dots}^\sigma \frac{V_n^2}{V_1^2}} \quad (5)$$

Where  $\sigma = 13, 31$ , and 49. In Fig. 7 variation of  $THD_{31}$  with  $m$  is shown. From this figure it is observed that the net THD is higher for method I on account of increased magnitudes of higher order harmonic components.

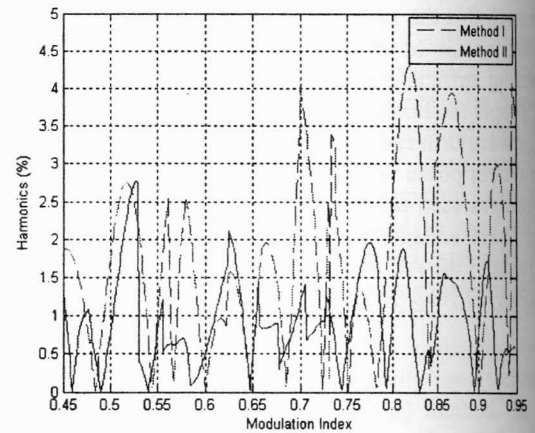


Fig. 4: Variation of 17<sup>th</sup> harmonic factor

## 4. SIMULATION AND EXPERIMENTAL RESULTS

### 4.1 Simulation Studies

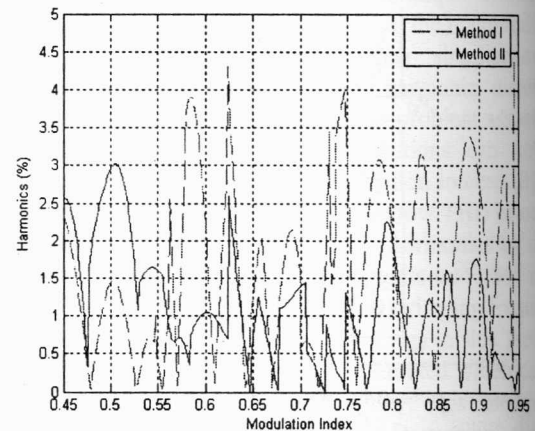


Fig. 5: Variation of 19<sup>th</sup> harmonic factor

A three-phase 11-level CMLI has been simulated on MATLAB/SIMULINK platform [13]. For each of the H-bridges in the CMLI, 12V DC source has been used. The switching angles obtained by method I and method II have been utilized for operating the CMLI. The simulation and experimental results are presented in the next part.

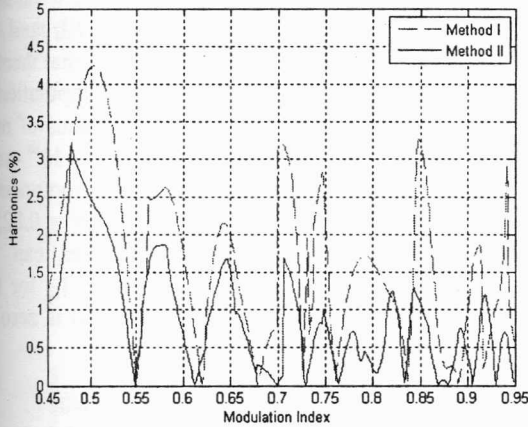


Fig. 6: Variation of 31<sup>st</sup> harmonic factor

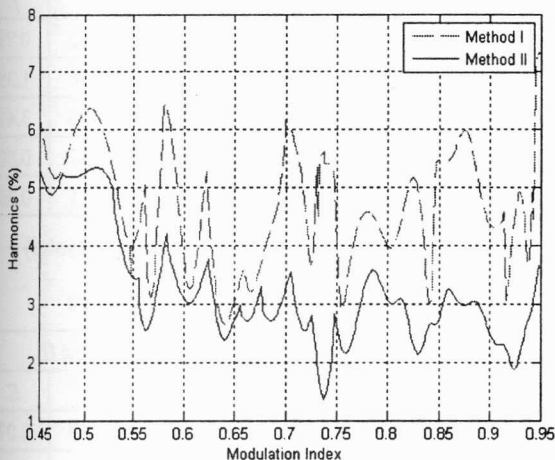


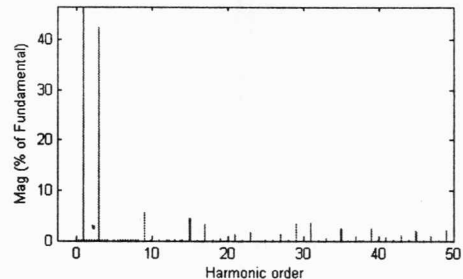
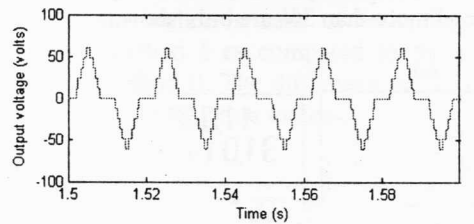
Fig. 7: Variation of THD<sub>31</sub>

## 4.2 Experimental Studies

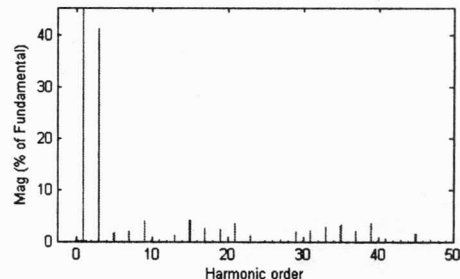
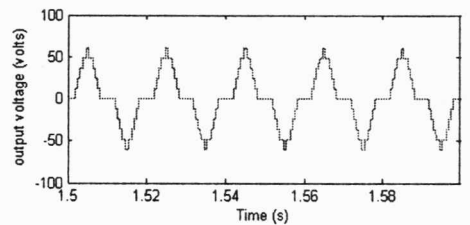
A prototype single-phase eleven-level cascade multilevel inverter has been built using 400V, 10A, MOSFET as the switching device. For the five H-bridges in the 11-level inverter, five separate dc supplies of 12V each have been obtained using step down transformers with rectifier units. The firing pulses have been generated by using a Pentium 80486 processor based PC having a clock frequency of 2MHz along with a timer I/O card.

To validate the computational and simulated results, extensive experimental studies were carried out for a large set of values of  $m$ . For each value of  $m$ , 11-level single phase output voltage was synthesized in the experimental set up for two sets of switching angles: a) angles calculated by method I and b) angles calculated by method II.

For each output voltage waveform, the individual harmonic factors (as given by 2) in percent as well as the THD values due to harmonic factors up to 13<sup>th</sup> order (THD<sub>13</sub>), 31<sup>st</sup> order (THD<sub>31</sub>) and 49<sup>th</sup> order (THD<sub>49</sub>) have been calculated. As it is not possible to include all the results in the paper due to space constraint, a few representative results are shown.



(a)



(b)

Fig. 8: Simulated plots at  $m = 0.5200$

Fig. 8 shows the simulated output voltage waveform obtained at  $m = 0.5200$ . In part a) the time domain output voltage obtained by method I and its harmonic spectrum is shown, while in part b) output voltage and its harmonic spectrum obtained by method II is shown. In Fig. 8 and other subsequent figures, the triplen harmonics are present due to the fact that the synthesized waveform is a single-phase one. In Fig. 9, experimentally obtained voltage and its harmonic spectrums for method I are shown in part (a)



and part (b), corresponding results for method II are shown in part (c) and (d) respectively.

Comparison of computational, simulated and experimental results obtained by both these methods is given in Table 1. In this as well all subsequent tables, the values of THD<sub>13</sub>, THD<sub>31</sub>, THD<sub>49</sub> and any harmonic component having a magnitude of more than 3% are only shown.

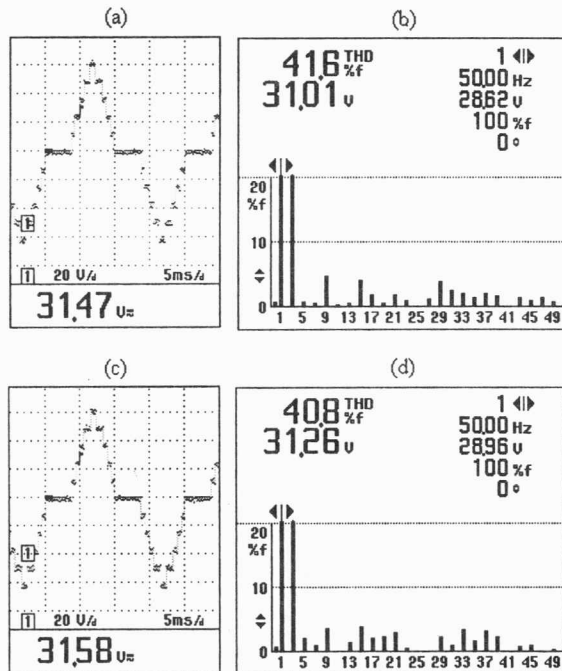


Fig. 9: Experimental plots at  $m = 0.5200$

Table 1: Comparative Result at  $m = 0.5200$

Harmonics (%)	Method I			Method II		
	C	S	E	C	S	E
THD <sub>13</sub>	0.00	0.18	1.10	2.64	2.77	2.88
THD <sub>31</sub>	5.92	5.92	5.34	5.27	5.29	5.03
THD <sub>49</sub>	6.91	6.91	6.35	6.54	6.57	6.30
29 <sup>th</sup>	3.43	3.43	3.90	1.94	1.87	2.40
31 <sup>st</sup>	3.69	3.67	2.60	2.01	1.98	0.90
35 <sup>th</sup>	2.63	2.65	1.60	3.07	3.10	1.50

(C = Computational, S = Simulated, E = Experimental)

From Table I it is observed that the results obtained with experiment are in good agreement with computational and simulated results while the computational and simulated results are almost identical to each other. Moreover, it is also observed that in method I, the value of THD<sub>13</sub> is found to be zero through computation. This is due to the fact that at  $m = 0.5200$ , all harmonic components up to 13<sup>th</sup> order can be completely eliminated. When the angles calculated by method I are used for experimental studies, the values of

THD<sub>13</sub> are found to be very small, although non-zero. This is due to the fact that the procedures and platforms used for computational and experimental studies are all different. In the experimental studies, the devices used are non-ideal.

Comparison of computational, simulated and experimental results obtained by both these methods for  $m = 0.7340$ , 0.8250, and 0.9230 is shown in Tables II, III, and IV respectively. From these tables again it is found that there is considerable agreement among the computational, simulated and experimental results. For the values of  $m = 0.7340$ , and 0.9230 harmonic components up to 13<sup>th</sup> order can not be eliminated [9]-[10], therefore these components have been minimized, while for the value of  $m = 0.8250$ , the harmonic components up to 13<sup>th</sup> order can be eliminated. Therefore, computed values of THD<sub>13</sub> for the method I is not zero in Tables 2 and 4, while it is zero in Table 3.

Table 2: Comparative Result At  $m = 0.7340$

Harmonics (%)	Method I			Method II		
	C	S	E	C	S	E
THD <sub>13</sub>	0.18	0.26	1.27	0.68	0.62	0.73
THD <sub>31</sub>	5.53	5.56	5.37	1.59	1.56	1.39
THD <sub>49</sub>	5.66	5.67	5.62	3.91	3.90	3.41
17 <sup>th</sup>	3.36	3.42	3.00	0.88	0.67	0.60
29 <sup>th</sup>	3.24	3.22	3.20	0.02	0.11	0.20

(C = Computational, S = Simulated, E = Experimental)

Table 3: Comparative Result at  $m = 0.8250$

Harmonics (%)	Method I			Method II		
	C	S	E	C	S	E
THD <sub>13</sub>	0.00	0.12	0.73	1.15	1.15	1.07
THD <sub>31</sub>	5.14	5.20	5.25	2.29	2.30	2.15
THD <sub>49</sub>	5.27	5.31	5.32	2.92	2.95	2.54
17 <sup>th</sup>	4.00	4.08	4.00	0.44	0.38	0.50
19 <sup>th</sup>	2.70	2.69	3.20	0.06	0.03	0.20

(C = Computational, S = Simulated, E = Experimental)

Table 4: Comparative Result at  $m = 0.9230$

Harmonics (%)	Method I			Method II		
	C	S	E	C	S	E
THD <sub>13</sub>	0.72	0.77	1.27	1.25	1.25	1.27
THD <sub>31</sub>	4.25	4.31	4.21	1.89	1.88	1.81
THD <sub>49</sub>	5.05	5.09	4.73	2.44	2.44	2.29
17 <sup>th</sup>	2.98	3.00	3.00	0.16	0.20	0.30

(C = Computational, S = Simulated, E = Experimental)

Simulated and experimental waveforms along with their harmonic spectrum obtained at  $m = 0.9230$  are shown in Figs. 10 and 11 respectively. The simulated waveform along with its harmonic spectrum is shown in part (a) for method I and in part (b) for method II of Fig. 10. Experimentally obtained waveform along with its harmonic spectrum for method I is shown in part (a) and (b), and for method II these are shown in part (c) and (d) of Fig. 11.

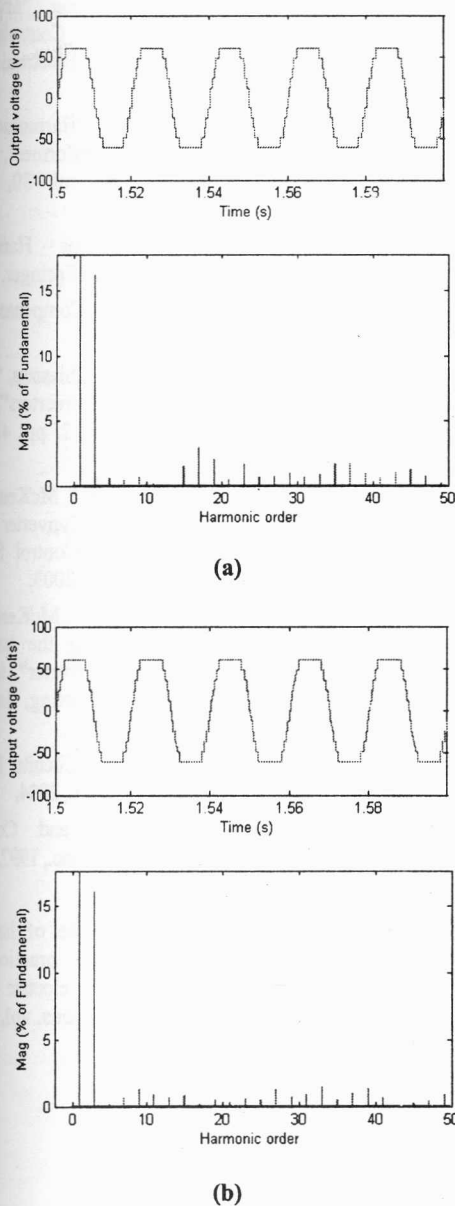


Fig. 10: Simulated plots at  $m = 0.9230$

For further assessing the relative merits of method I and method II, various harmonic losses have been calculated corresponding to the harmonics generated by these two methods according to relations given in [6]. The results are shown in Table 5.

## 5. DISCUSSION OF RESULTS

From the results presented in Section 4 following points are to be observed:

In general, individual harmonic factors ( $17^{\text{th}}$ ,  $19^{\text{th}}$  ...  $49^{\text{th}}$ ) as well as THDs due to the harmonic components up to  $31^{\text{st}}$  and  $49^{\text{th}}$  order are higher in case of switching strategy implemented in method I as compared to the switching strategy used in method II. The difference in THD is more significant at higher modulation indices.

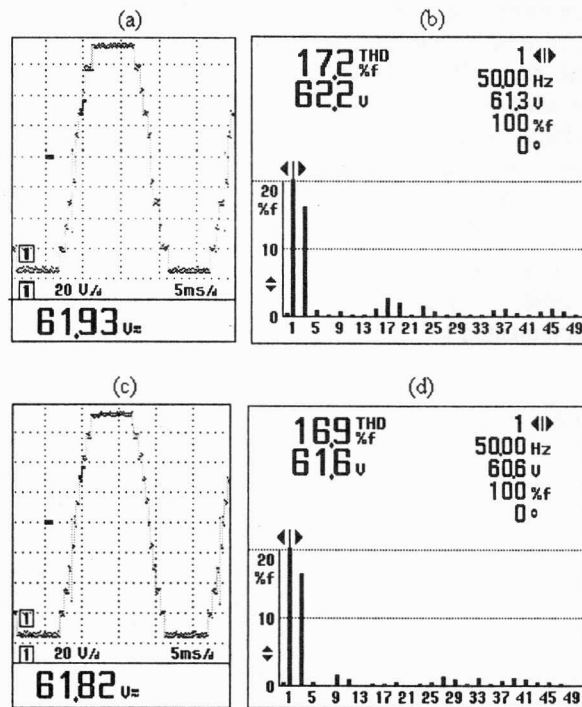


Fig. 11: Experimental plots at  $m = 0.9230$

Table 5: Comparative Results of Different Losses (pu)

Different Losses		Modulation Indices ( $m$ )			
		0.5200	0.7340	0.8250	0.9230
Copper	Method I	0.0040	0.0032	0.0028	0.0022
	Method II	0.0040	0.0012	0.0006	0.0005
Iron	Method I	0.0066	0.0055	0.0049	0.0042
	Method II	0.0073	0.0020	0.0014	0.0012
Dielectric	Method I	0.1223	0.0764	0.0527	0.0486
	Method II	0.0986	0.0462	0.0171	0.0139
Total	Method I	0.1329	0.0852	0.0604	0.0550
	Method II	0.1099	0.0494	0.0191	0.0155

- Lower order harmonic components ( $5^{\text{th}}$ ,  $7^{\text{th}}$ ... $13^{\text{th}}$ ) as given by  $\text{THD}_{13}$  for method II are comparable with method I (except at  $m = 0.5200$ ).
- Various harmonic standards [6], [14] which impose upper limits on individual as well as total harmonic

distortion for proper utilization of electric power are followed by the harmonics generated in output voltage in method II, while in case of method I these limits cross for most of the values of  $m$ .

- By looking on different harmonic losses produced due to different switching strategies in method I and method II (Table V), there is significant reduction in the losses in case of method II as compared with method I, which results higher operating efficiency, low thermal and dielectric stresses on electrical equipments, cables, transmission lines etc.
- The switching strategies used in method II is very useful in case of renewable energy applications such as solar cell, fuel cell etc. DC batteries used for CMLI can be replaced by these energy sources to generate AC voltage, as generated AC voltage contains less THD, therefore it can be used directly for electric vehicles or any electric load or can be fed to electric utility.
- In case of utility applications such as STATCOM, DC batteries can be replaced by capacitors acting as DC sources, provided that proper charging scheme is implemented to maintain DC voltage at fairly constant level.

## 6. CONCLUSION

In this paper, two optimization methods to calculate the switching angles for fundamental switching scheme for a CMLI has been presented. Based on the detailed computational and simulation studies carried out in this paper, following conclusions can be drawn:

- a) Elimination of lower order harmonics generally increases the magnitudes of higher order harmonics.
- b) As the individual harmonic components as well as the THD are well within the limits imposed by different standards, the method II is more suitable for different applications CMLI.

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