

# Selective Harmonic Elimination Technique for a Multilevel Inverter

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**Abstract**— For a multilevel inverter, switching angles at fundamental frequency are obtained by solving the selective harmonic elimination equations in such a way that the fundamental voltage is obtained as desired and certain lower order harmonics are eliminated. As these equations are nonlinear transcendental in nature, there may exist simple, multiple or even no solutions for a particular modulation index. Previous work has shown that if iterative numerical techniques are implemented to solve the transcendental equations, only one solution set is obtained even there may exist multiple solution sets; other suggested approaches such as resultants method, theory of symmetric polynomial produce all possible solution sets, but these methods are more computationally complex. In this paper a new approach is presented to implement the Newton-Raphson method for solving the transcendental equations which produces all possible solutions with any random initial guess and for any number of levels of multilevel inverter. Among multiple solution sets obtained, the solutions which produce least THD in the output voltage is chosen. As compared with the single set of solution, the decrease in the THD can be up to 3% in case of multiple solution sets. The computational results are shown graphically for better understanding and to prove the effectiveness of the method. An experimental 11-level cascade multilevel inverter is employed to validate the computational results.

## I. INTRODUCTION

A multilevel inverter is more recent and popular type of power electronic converter that synthesizes a desired output voltage from several levels of dc voltages as inputs. If sufficient number of dc sources is used, a nearly sinusoidal voltage waveform can be synthesized.

In comparison with the hard-switched two-level pulse width modulation inverter, multilevel inverter offers several advantages such as, its capabilities to operate at high voltage with lower dv/dt per switching, high efficiency and low electromagnetic interference [1]-[4].

To synthesize multilevel output ac voltage using different levels of dc inputs, semiconductor devices must be switched on and off in such a way that desired fundamental is obtained with minimum harmonic distortion. The commonly available switching technique is selective harmonic elimination (SHE)

method at fundamental frequency, for which transcendental equations characterizing harmonics are solved to compute switching angles [2], [3]. It is difficult to solve the SHE equations as these are highly nonlinear in nature and may produce simple, multiple, or even no solutions for a particular value of modulation index. A big task is how to get all possible solution sets where they exist using simple and less computationally complex method. Once these solution sets are obtained, the solutions having least THD are chosen.

In [4]-[6], iterative numerical techniques have been implemented to solve the SHE equations producing only one solution set, and even for this a proper initial guess and starting value of modulation index for which solutions exist are required. In [7], [8], theory of resultants of polynomials and the theory of symmetric polynomials has been suggested to solve the polynomial equations obtained from the transcendental equations. A difficulty with these approaches is that for several H-bridges connected in series, the order of the polynomials become very high thereby making the computations of the solutions of these polynomials very complex. Optimization technique based on Genetic Algorithm (GA) was proposed for computing switching angles for 7-level inverter in [9]. The implementation of this approach requires proper selection of certain parameters such as population size, mutation rate etc, thereby its implementation becomes also difficult for higher level inverters. To circumvent above problems, in this paper the application of the Newton-Raphson method for solving these equations is proposed. The proposed technique is implemented in such a way that all possible solutions for any number of H-bridges connected in series are computed for any arbitrary initial guess with negligible computational effort. A complete analysis for an 11-level inverter using five H-bridges per phase in series is presented, and it is shown that for a range of modulation index  $m_f$ , switching angles can be computed to produce the desired fundamental voltage  $V_1 = m_f(s \ 4V_{dc}/\pi)$  while eliminating 5<sup>th</sup>, 7<sup>th</sup>, 11<sup>th</sup>, and 13<sup>th</sup> harmonic components. For demonstrating the validity of the proposed methods the results obtained by computations are compared with experimental results.

## II. CASCADE MULTILEVEL INVERTER

Cascade Multilevel Inverter (CMLI) is one of the most important topology in the family of multilevel and multipulse inverters. It requires least number of components with compare to diode-clamped and flying capacitors type multilevel inverters and no specially designed transformer is needed as compared to multipulse inverter. It has modular structure with simple switching strategy and occupies less

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space [1], [3].

The CMLI consists of a number of H-bridge inverter units with separate dc source for each unit and is connected in cascade or series as shown in Fig. 1. Each H-bridge can produce three different voltage levels:  $+V_{dc}$ , 0, and  $-V_{dc}$  by connecting the dc source to ac output side by different combinations of the four switches  $S_1$ ,  $S_2$ ,  $S_3$ , and  $S_4$ . The ac output of each H-bridge is connected in series such that the synthesized output voltage waveform is the sum of all of the individual H-bridge outputs.

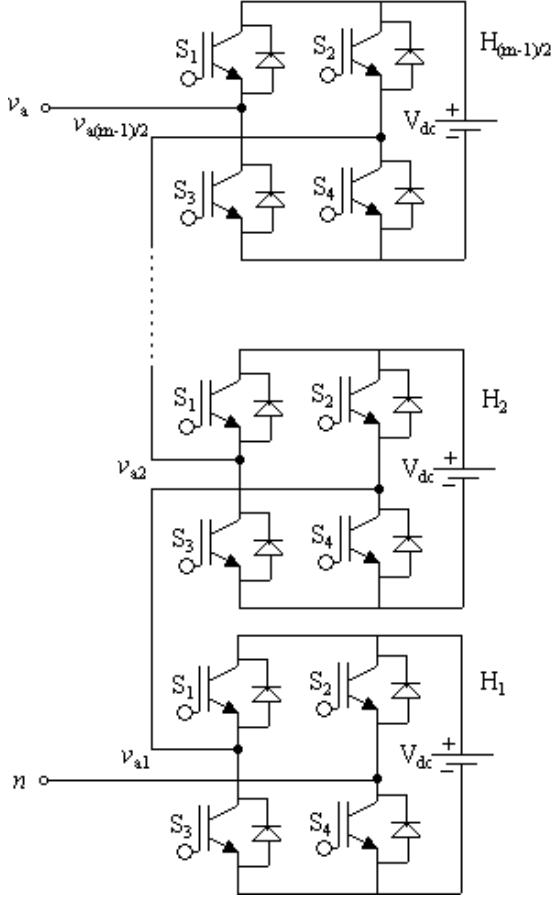


Fig. 1. Configuration of single-phase cascade multilevel inverter.

By connecting the sufficient number of H-bridges in cascade and using proper modulation scheme, a nearly sinusoidal output voltage waveform can be synthesized. The number of levels in the output phase voltage is  $2s+1$ , where  $s$  is the number of H-bridges used per phase. Fig. 2 shows an 11-level output phase voltage waveform using five H-bridges. The magnitude of the ac output phase voltage is given by  $v_{an} = v_{a1} + v_{a2} + v_{a3} + v_{a4} + v_{a5}$  [2].

### III. MATHEMATICAL MODELS OF SWITCHING ANGLES AND SHE EQUATIONS FOR A CASCADE MULTILEVEL INVERTER

In general, the Fourier series expansion of the staircase

output voltage waveform as shown in Fig. 2 is given by

$$v_{an}(\omega t) = \sum_{k=1,3,5,\dots}^{\infty} \frac{4V_{dc}}{k\pi} (\cos(k\alpha_1) + \cos(k\alpha_2) + \dots + \cos(k\alpha_s)) \sin(k\omega t) \quad (1)$$

where  $s$  is the number of H-bridges connected in cascade per phase. For a given desired fundamental peak voltage  $V_1$ , it is required to determine the switching angles such that  $0 \leq \alpha_1 < \alpha_2 < \dots < \alpha_s \leq \frac{\pi}{2}$  and some predominant lower order harmonics of phase voltage are zero. Among  $s$  number of switching angles, generally one switching angle is used for fundamental voltage selection and the remaining  $(s-1)$  switching angles are used to eliminate certain predominating lower order harmonics. In three-phase power system, triplen harmonics are cancel out automatically in line-to-line voltage as a result only non-triplen odd harmonics are present in line-to-line voltages [4], [5].

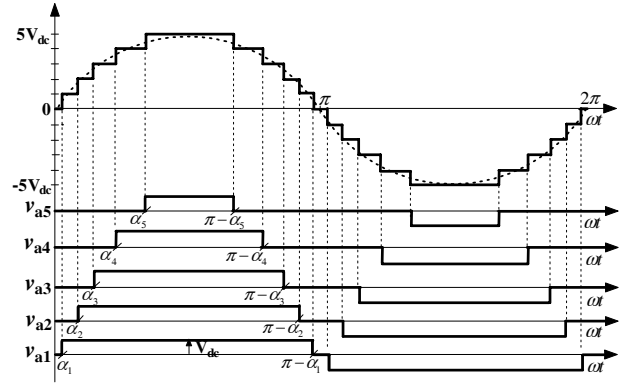


Fig. 2. Output voltage waveform of an eleven-level CMLI at fundamental frequency switching scheme.

From (1), the expression for the fundamental voltage in terms of switching angles is given by

$$\frac{4V_{dc}}{\pi} (\cos(\alpha_1) + \cos(\alpha_2) + \dots + \cos(\alpha_s)) = V_1 \quad (2)$$

Moreover, the relation between the fundamental voltage and the maximum obtainable voltage is given by modulation index. The modulation index,  $m_l$ , is defined as the ratio of the fundamental output voltage  $V_1$  to the maximum obtainable fundamental voltage  $V_{1max}$ . The maximum fundamental voltage is obtained when all the switching angles are zero i.e.

$$V_{1max} = \frac{4sV_{dc}}{\pi}. \text{ Therefore the expression for } m_l \text{ is [2];}$$

$$m_l = \frac{\pi V_1}{4sV_{dc}} \quad (3)$$

For an 11-level cascade inverter, there are five H-bridges per phase i.e.  $s = 5$  or five degrees of freedom are available; one degree of freedom is used to control the magnitude of the

fundamental voltage and the remaining four degrees of freedom are used to eliminate 5<sup>th</sup>, 7<sup>th</sup>, 11<sup>th</sup>, and 13<sup>th</sup> order harmonic components as they dominate the total harmonic distortion [5]. The above stated conditions can be written in following way by combining (1) and (3);

$$\begin{aligned}\cos(\alpha_1) + \cos(\alpha_2) + \dots + \cos(\alpha_5) &= 5m_I \\ \cos(5\alpha_1) + \cos(5\alpha_2) + \dots + \cos(5\alpha_5) &= 0 \\ \cos(7\alpha_1) + \cos(7\alpha_2) + \dots + \cos(7\alpha_5) &= 0 \\ \cos(11\alpha_1) + \cos(11\alpha_2) + \dots + \cos(11\alpha_5) &= 0 \\ \cos(13\alpha_1) + \cos(13\alpha_2) + \dots + \cos(13\alpha_5) &= 0\end{aligned}\quad (4)$$

In general, (4) can be written as

$$F(\alpha) = B(m_I) \quad (5)$$

The (4) is a system of five transcendental equations, known as selective harmonic elimination (SHE) equations, in terms of five unknowns  $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ , and  $\alpha_5$ . For the given values of  $m_I$  (from 0 to 1), it is required to get complete and all possible solutions of (4) when they exist with minimum computational burden and complexity.

In contrary to what claimed in [7], [8], here it is shown that all possible solutions for any number of levels can be computed by proper implementation of the Newton-Raphson method without knowing any specific initial guess and range of modulation index for which solutions exist.

#### IV. NEWTON-RAPHSON METHOD

The Newton-Raphson (N-R) method is one of the fastest iterative methods. This method begins with an initial approximation and generally converges at a zero of a given system of nonlinear equations [10].

The N-R method is to be implemented to compute the switching angles for the system given by (4). The Switching angles which are in the range of 0 to  $\pi/2$  producing desired fundamental voltage along with elimination of 5<sup>th</sup>, 7<sup>th</sup>, 11<sup>th</sup>, and 13<sup>th</sup> harmonic components for a given modulation index are feasible solutions of (4). The N-R method implemented previously in [4]-[6] was based on trial and error method for estimation of initial guess and  $m_I$  for which solutions exist. Once a solution set was obtained, successive solutions were computed by using previous solution set as initial guess for the next one; proceeding in this way, only one solution set was obtained. Here, the N-R method is implemented in a different way for which an arbitrary initial guess between 0 to  $\pi/2$  is assumed and switching angles (keeping all switching angles in the feasible range) along with the error (% content of 5<sup>th</sup>, 7<sup>th</sup>, 11<sup>th</sup>, and 13<sup>th</sup> harmonic components) are computed for complete range of  $m_I$  by incrementing its value in small steps (say 0.0001). The different solution sets are obtained for a particular range of  $m_I$  where they exist i.e. the error is zero for feasible solutions; after getting preliminary solution sets, complete solution sets are computed by using known solutions as initial guess. Now a basic question is how one can assure that all solution sets have been obtained. The answer is simple,

as initial guess is randomly chosen so it has nothing to do with any particular solution set i.e. there is equal chance for each solution set to occur if they exist. In support of above statement, an example is presented in which four solutions were computed for  $m_I = 0.5466$  to 0.5467 with same initial guess. As  $m_I$  was incremented in steps of 0.0001, three different solutions were obtained successively and the fourth one was same as the first one (as there exist only three solution sets for this range of  $m_I$ ) as shown in Fig. 3. It can be seen from the Fig. 3, that solutions are much sensitive to  $m_I$  rather than the initial guess and rate of convergence is very high because all switching angles are in feasible range in all iterations. It may be noted that different initial guesses may produce different solutions for a particular value of  $m_I$ , but all solution sets will be produced (when they exist) if  $m_I$  is varied sufficiently in small steps.

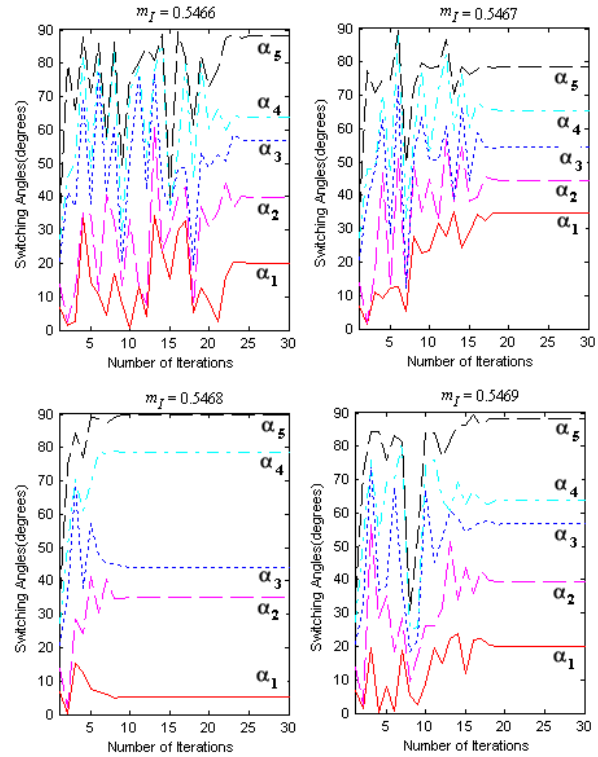


Fig. 3. Switching angles v/s number of iterations for different  $m_I$  with same initial guess.

The algorithm for the Newton-Raphson method is as follows:

- 1) Assume any random initial guess for switching angles (say  $\alpha_0$ ) such as  $0 \leq \alpha_1 < \alpha_2 < \dots < \alpha_5 \leq \frac{\pi}{2}$ .
- 2) Set  $m_I = 0$ .
- 3) Calculate  $F(\alpha_0)$ ,  $B(m_I)$ , and Jacobian  $J(\alpha_0)$ .
- 4) Compute correction  $\Delta\alpha$  during the iteration using relation,  $\Delta\alpha = J^{-1}(\alpha_0)(B(m_I) - F(\alpha_0))$ .
- 5) Update the switching angles i.e.  $\alpha(k+1) = \alpha(k) + \Delta\alpha(k)$ .

- 6) Perform  $\alpha(k+1) = \cos^{-1}(\text{abs}(\cos(\alpha(k+1))))$  transformation to bring switching angles in feasible range.
- 7) Repeat steps (3) to (6) for sufficient number of iterations to attain error goal.
- 8) Increment  $m_l$  by a fixed step.
- 9) Repeat steps (2) to (8) for whole range of  $m_l$ .
- 10) Plot the switching angles as a function of  $m_l$ . Different solution sets would be obtained.
- 11) Take one solution set at a time and compute complete solution set for the range of  $m_l$  where it exists.

By following the above steps, all possible solution sets, when they exist, can be computed without any computational complexity.

## V. COMPUTATIONAL RESULTS

### A. Seven-Level CMLI

In case of 7-level CMLI, it is required to solve three SHE equations for obtaining the same number of switching angles. As discussed previously, one switching angle is used to produce fundamental voltage while remaining two eliminates 5<sup>th</sup> and 7<sup>th</sup> order harmonic components. The solutions were computed with an arbitrary initial guess as  $m_l$  was varied from 0 to 1 in steps of 0.001. The feasible solutions thus computed are summarized in Fig. 4 (a). It can be seen from the Fig. 4 (a) that there exist two solution sets. Complete solution sets are shown in Fig. 4 (b). A few non feasible solutions appeared in very narrow range of  $m_l=0.50$  due to commencement of second solutions in this range.

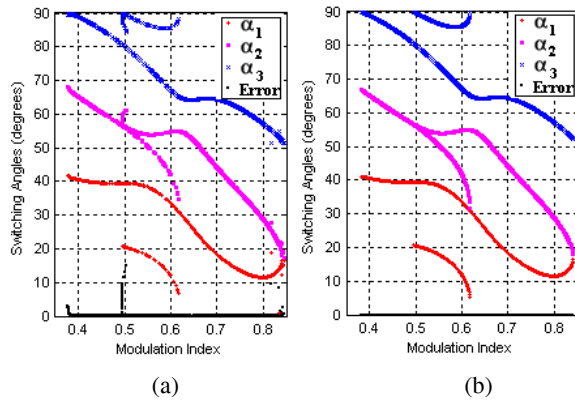


Fig. 4. (a) Solution sets computed with an arbitrary initial guess, and (b) exact solution sets.

### B. Eleven-Level CMLI

By implementing the proposed method, all possible solution sets for an 11-level CMLI were computed and a complete analysis is also presented. Starting with any random initial guess all solution sets were computed by incrementing  $m_l$  in steps of 0.001 from 0 to 1. The summary of the results

obtained are shown (only in the range of  $m_l$  where solutions occur) in Fig. 5 (a). By using preliminary computed results, complete solution sets were computed as shown in Fig. 5 (b). It can be seen from the Fig. 5 that the solutions do not exist at lower and upper ends of the modulation indices and also for  $m_l = [0.3800 \ 0.4400]$ ,  $[0.7300 \ 0.7310]$ , and  $[0.7330 \ 0.7470]$  as error is not zero at these values of  $m_l$ . Multiple solution sets exist for  $m_l = [0.5050 \ 0.5800]$ ,  $[0.6120 \ 0.7000]$ . Even some solutions existing in very narrow range of  $m_l = [0.3760 \ 0.3790]$ ,  $[0.5470 \ 0.5490]$ ,  $[0.7320]$  were also obtained by implementing the proposed method thereby demonstrating the capability of proposed method in computing all possible solution sets.

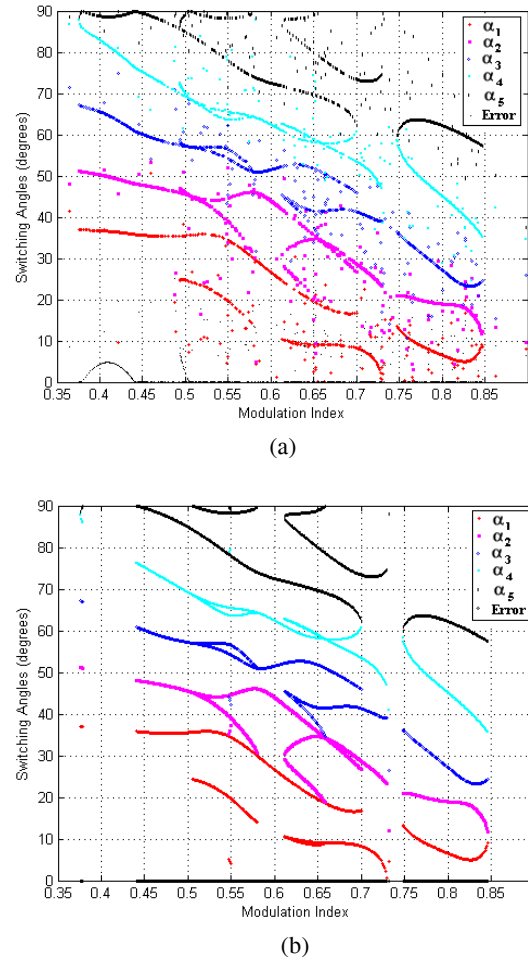


Fig. 5. (a) Solution sets computed with an arbitrary initial guess, and (b) exact solution sets for eleven-level CMLI.

For each of the multiple solution sets as computed above, total harmonic distortion (THD) in percent is computed according to (6), the set of switching angles among multiple solution sets which produce least THD is selected and termed as combined solution set. The THD plots for different solution sets along with combined solution set are plotted as a function of  $m_l$  in Fig. 6. Fig. 7 shows the plot of switching

angles which produce least THD among different solution sets. It can be seen from the Fig. 6 that there is a significant decrease in THD if one uses all possible solution sets for determining the combined solution set instead of using only one solution set as reported in [4]-[6]. For example, if one computes THD produced due to all possible solution sets at  $m_l = 0.5470$ , the difference in THD for the solution sets having highest and lowest THD is about 3%.

$$THD = \frac{\sqrt{V_{17}^2 + V_{19}^2 + \dots + V_{49}^2}}{V_1} \times 100 \quad (6)$$

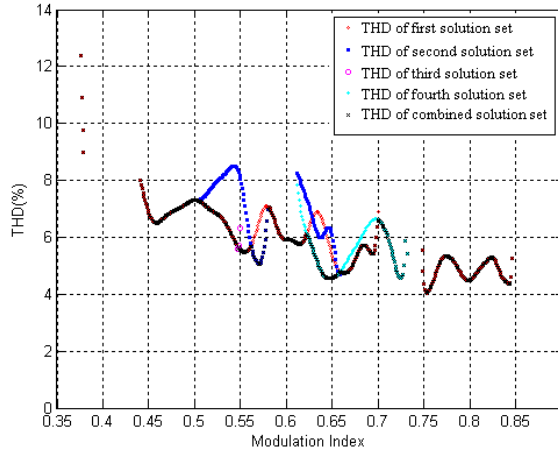


Fig. 6. THD as a function of modulation index for multiple solution sets.

For the modulation indices where solutions do not exist, different schemes can be used for fundamental voltage variation. For example one may compute THD due to 5<sup>th</sup>, 7<sup>th</sup>, 11<sup>th</sup>, and 13<sup>th</sup> order harmonic components using optimization technique or otherwise; the switching angles which produce minimum error due to the above mentioned harmonic components may be chosen.

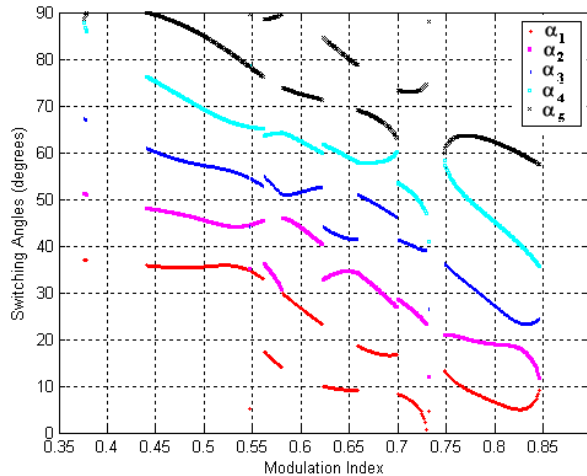


Fig. 7. Switching angles producing least THD v/s modulation index.

## VI. EXPERIMENTAL RESULTS

A prototype single-phase 11-level CMLI has been built using 400V, 10A MOSFET as the switching device. Five separate dc supply of 12V each is obtained using step down transformers with rectifier unit. Pentium 80486 processor based PC with clock frequency 2MHz with timer I/O card is used for firing pulse generation. Firing pulses to the switching devices are given through a delay circuit which provides 5μsec delay to avoid any short circuit due to simultaneous conduction of devices in the same leg of H-bridges.

In order to validate the analytical and simulated results, an 11-level single-phase output voltage at  $m_l = 0.6500$  (for this value of  $m_l$  multiple solution sets exist) was synthesized at fundamental frequency ( $f = 50\text{Hz}$ ) producing fundamental voltage  $V_1 = m_l \left(5 \times \frac{4V_{dc}}{\pi}\right) = 0.6500 \left(5 \times 4 \times \frac{12}{\pi}\right) = 49.65\text{V}$  (peak). For each of the multiple solution set, total harmonic distortion in line to line voltage was computed as per (6).

In the first experiment, the solution set corresponding to the lowest THD is considered. The experimentally produced phase voltage along with its harmonic spectrum is shown in Fig. 8 (a). In Fig. 8 (b) phase voltage along with its harmonic spectrum simulated on MATLAB/SIMULINK [11] is shown. Corresponding current waveforms are shown in Fig. 9. The harmonic spectrum of synthesized phase voltage shows that 5<sup>th</sup>, 7<sup>th</sup>, 11<sup>th</sup>, and 13<sup>th</sup> harmonics are almost absent as predicted analytically. The THD in line to line voltage as computed analytically, from simulation and experiment are: 4.57%, 4.59%, and 4.11% respectively, this shows that experimental results are in close agreement with theoretical and simulated results. The triplen harmonics are present in Figs. 8-10 due to the fact that the synthesized wave form is a single-phase one.

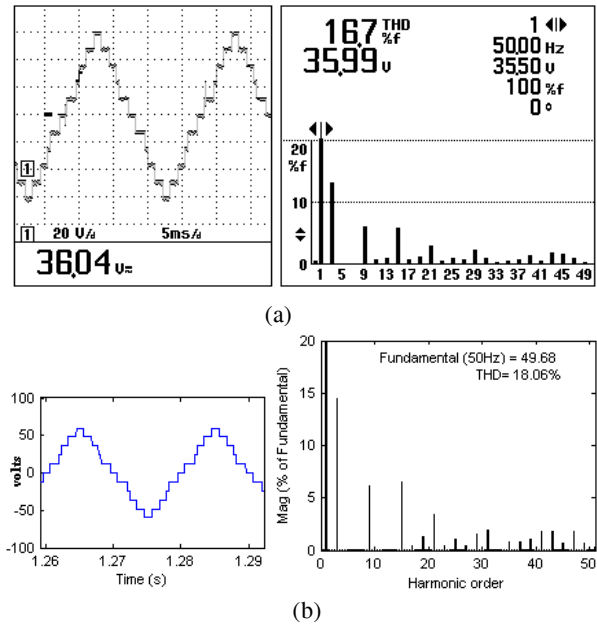


Fig. 8. Phase voltage with corresponding FFT for the solution set having smallest THD at  $m_l = 0.6500$  (a) experimental, and (b) simulated results.

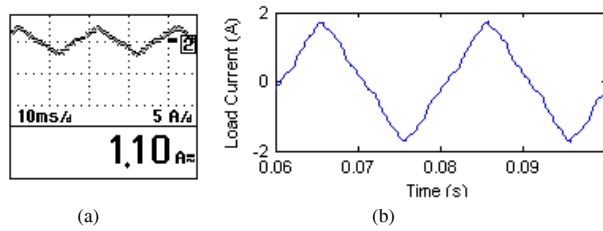


Fig. 9. Current waveform for R-L load ( $R = 34\Omega$  and  $L = 21\text{mH}$ ) for the solution set having smallest THD at  $m_f = 0.6500$  (a) experimental waveform, and (b) simulated waveform.

In the second experiment, the solution set which has highest THD among multiple solution sets is considered. The phase voltage along with its harmonic spectrum is shown in Fig. 10 (a) and corresponding simulated result is shown in Fig. 10(b). It can be seen from the Fig. 10 that experimental result is in good agreement with the simulated result. The absence of selected harmonics in the output phase voltage validates the solutions computed. The total harmonic distortions in line to line voltage as computed analytically, from simulation and experiment are 6.06%, 6.11%, and 5.56% respectively, hence, the experiment corresponds very well with the analytical and simulated results.

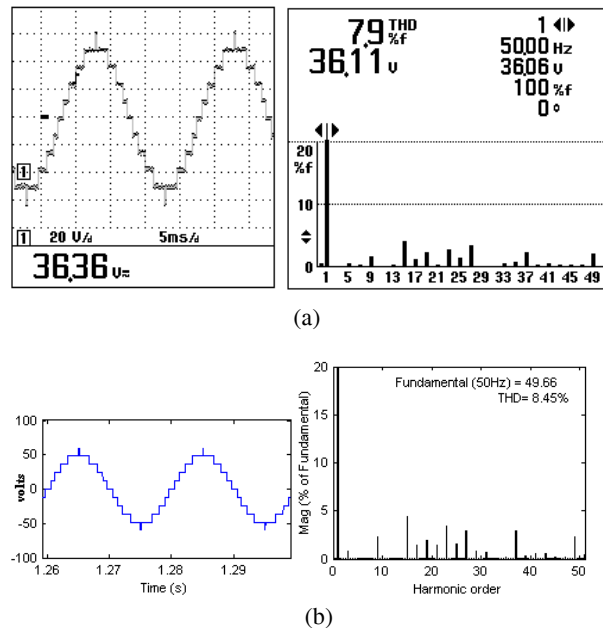


Fig. 10. Phase voltage with corresponding FFT for the solution set having highest THD at  $m_f = 0.6500$  (a) experimental results, and (b) simulated results.

## VII. CONCLUSION

The selective harmonic elimination method at fundamental frequency switching scheme has been implemented using the Newton-Raphson method that produces all possible solution

sets when they exist. In comparison with other suggested methods, the proposed technique has many advantages such as: it can produce all possible solution sets for any numbers of multilevel inverter without much computational burden, speed of convergence is fast etc. The proposed technique was successfully implemented for computing the switching angles for 7-level and 11-level CMLI. A complete analysis for 11-level inverter has been presented and it is shown that a significant amount of THD reduction can be attained if all possible solution sets are computed.

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