Control of Harmonics using 15-Level Cascade Multilevel Inverter

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***Abstract*—A fifteen level cascade multilevel inverter topology is presented in this work for reduction of total harmonic distortion produced in the output voltage of multilevel inverter. The complete switching angles computation is carried out using N-R technique. Further, it is shown that continuous and multiple solutions exist for a wide range of the modulation index. Among the multiple solution sets, switching angles which produce least harmonic distortion in the output voltage have been selected for the switching of power electronic devices in individual H-bridges of the inverter. The output voltage produced satisfies the limits set by the IEEE-519 standard for utility applications. The computed results are validated through simulation, and it is shown that there is a close agreement between these two studies (computation and simulation).**

***Index Terms*—Cascade multilevel inverter, harmonic elimination, modulation index, total harmonic distortion.**

I. Introduction

MULTILEVEL inverters have drawn increasing attention since past decade because of their promising applications in power systems. They can be efficiently used in the distributed energy systems in which, output ac voltage is obtained by connecting dc sources such as batteries, fuel cells, solar cells, rectified wind turbines etc at input side of the inverters. The ac output voltage obtained from the inverters can be fed to a load directly or interconnect to the ac grid without voltage balancing problems. In addition, the multilevel inverters are used as voltage source inverters (VSIs) in the static synchronous compensator (STATCOM), a reactive power compensating device used for voltage regulation in power systems [1]-[7].

As compared to the hard-switched two-level pulse width modulation inverters, the multilevel inverters offer several advantages such as their capabilities to operate at high voltage with lower dv/dt per switching, high efficiency , low electromagnetic interference etc [2]-[4].

A sinusoidal voltage of high magnitude with extremely low distortion at fundamental frequency can be generated at the output of multilevel inverters by connecting sufficient number of dc levels at input side. There are mainly three types of multilevel inverters; these are a) diode- clamped, b) flying capacitor and c) cascade multilevel inverter (CMLI). Among these three, CMLI has a modular structure and requires least number of components as compared to other two topologies, and as a result, it is widely used for power systems applications [1], [5].

To produce multilevel output ac voltage using different levels of dc inputs, the semiconductor devices must be switched on and off in such a way that the fundamental voltage is obtained as desired along with the elimination of certain number of higher order harmonics in order to have least harmonic distortion in the ac output voltage. For switching the semiconductor devices, proper selection of switching angles is must. The switching angles at fundamental frequency, in general, are obtained from the solution of non linear transcendental equations characterizing harmonics contents in the output ac voltage; these equations are known as selective harmonic elimination (SHE) equations. As the SHE equations are non linear transcendental in nature, their solutions may have simple, multiple and even no roots for a particular value of modulation index (m), moreover, it is difficult to solve these equations. A big challenge is how to get all possible solution sets where they exist using simple and less computationally complex method. Once these solution sets are obtained, the switching angles producing minimum total harmonic distortion (THD) in the output ac voltage are selected for switching of the power electronics devices.

In [3]-[4], iterative numerical techniques have been implemented to solve the SHE equations producing only one solution set, and even for this, a proper initial guess and starting value of m for which the solutions exist are required, in general, it is difficult to guess the initial solution and the value of m for which solution exist. In [8]-[9], theory of resultants of polynomials and the theory of symmetric polynomials has been suggested to solve the polynomial equations (these polynomial equations are obtained from the transcendental equations). A difficulty with these approaches is, for several H-bridges connected in series, the order of the polynomials become very high, thereby making the computations of the solutions of these polynomials very complex. Moreover, these techniques have been applied up to 11-level multilevel inverters only due to the computational complexity associated with these techniques. Optimization technique based on Genetic Algorithm (GA) is proposed for computing switching angles only for 7-level inverter in [6], and even for the implementation of this method, proper selection of certain parameters such as initial population size, mutation rate etc are required,

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thereby implementation of this method becomes difficult for higher level multilevel inverters. As the THD of the output ac voltage produced by 11-level multilevel inverter using SHE technique is not within the upper limits imposed by the IEEE-519 standards [10]-[11] for complete working range of *m*, and also for high power applications, implementation of high level multilevel inverter is must.

In the present work, a 15-level CMLI is employed to generate ac output voltage producing THD well within the limits imposed by IEEE-519 standards for complete working range of *m*. The switching angles are computed by the implementation of Newton Raphson (N-R) numerical technique in a particular way producing complete solutions for working range of modulation index without much computational complexity. Moreover, for the computation of the switching angles, no particular initial guess for these angles and the value of *m* for which solutions exist are required.

II. Cascade Multilevel Inverter

The cascade multilevel inverter consists of a number of H-bridge inverter units with separate dc source for each unit and is connected in cascade or series as shown in Fig. 1. Each H-bridge can produce three different voltage levels: $+V_{dc}$, 0 and $-V_{dc}$ by connecting the dc source to ac output side by different combinations of the four switches S_1, S_2, S_3, and S_4. The ac output of each H-bridge is connected in series such that the synthesized output voltage waveform is the sum of all of the individual H-bridge outputs. By connecting sufficient number of H-bridges in cascade and using proper modulation scheme, a nearly sinusoidal output voltage waveform can be synthesized.

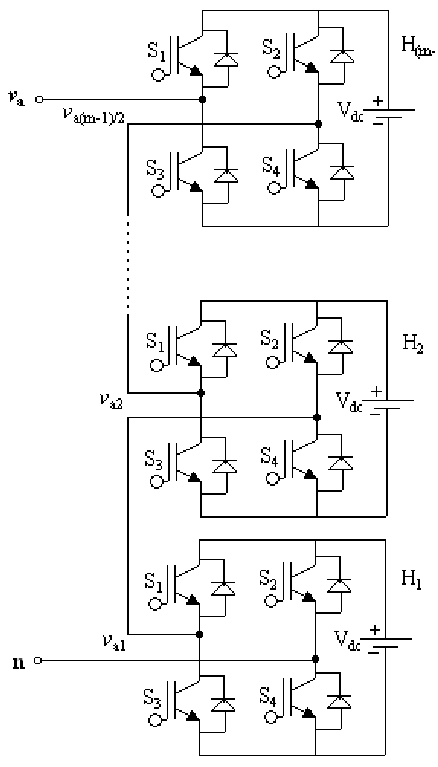

Fig. 1. Single-phase cascade multilevel inverter topology.

The number of levels in the output phase voltage and line voltage are $2s+1$ and $4s+1$ respectively, where *s* is the number of H-bridges used per phase. Fig. 2 shows a 15-level output phase voltage waveform using seven H-bridges. The magnitude of the ac output phase voltage is the sum of the voltages produced by H-bridges. In the Fig. 2, α_1, α_2, α_3, α_4, α_5, α_6, and α_7 are the respective switching angles of seven H-bridges in each phase, and β_1, β_2, β_3, β_4, β_5, β_6, and β_7 are corresponding supplementary angles for $\alpha_1, \alpha_2, \ldots \alpha_7$.

III. Selective Harmonic Elimination Equations

In general, the Fourier series expansion of the staircase output voltage waveform as shown in Fig. 2 is given by [4].

$$v_{an}(wt) = \sum_{k=1,3,5,\ldots}^{\infty} \frac{4V_{dc}}{k\pi}(\cos(k\alpha_1) + \cos(k\alpha_2) + \ldots + \cos(k\alpha_s))\sin(k\omega t) \quad (1)$$

Where *s* is the number of H-bridges connected in cascade per phase. For a given desired fundamental peak voltage V_1, it is required to determine the switching angles such that $0 \le \alpha_1 < \alpha_2 \ldots < \alpha_s \le \pi/2$ and some predominant lower order harmonics of phase voltage are zero. Among *s* number of switching angles, generally one switching angle is used for fundamental voltage selection and the remaining (s-1) switching angles are used to eliminate certain predominating lower order harmonics. In three-phase power system, triplen harmonics are canceled out automatically in line-to-line voltage, as a result, only non-triplen odd harmonics are present in line-to-line voltages [4], [5].

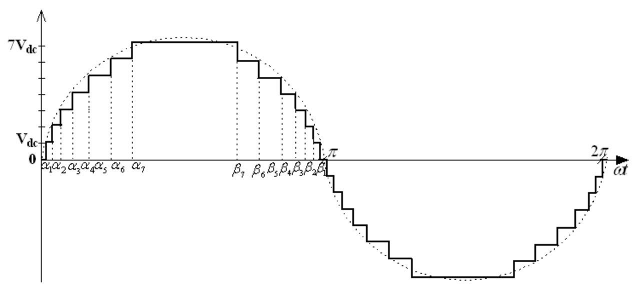

Fig. 2. Output phase voltage waveform of a 15-level CMLI.

From (1), the expression for the fundamental voltage in terms of switching angles is given by

$$\frac{4V_{dc}}{\pi}(\cos(\alpha_1) + \cos(\alpha_2) + \ldots + \cos(\alpha_s)) = V_1 \quad (2)$$

Moreover, the relation between the fundamental voltage and the maximum obtainable voltage is given by modulation index. The modulation index, *m*, is defined as the ratio of the fundamental output voltage (V_1) to the maximum obtainable fundamental voltage. The maximum fundamental voltage is obtained when all the switching angles are zero i.e. $V_{1max} = 4sV_{dc}/\pi$, therefore, $m = \pi V_1/4sV_{dc}$ [1].

For 15-level cascade inverter, there are seven H-bridges per phase i.e. s = 7 or seven degrees of freedom are available; one degree of freedom is used to control V_1 and the remaining six degrees of freedom are used to eliminate the lower order harmonic components i.e. 5th, 7th, 11th, 13th, 17th, and 19th as they dominate the total harmonic distortion [4].

From (1), the expressions for fundamental voltage in terms of m, and lower order harmonic components, when they are eliminated, can be written as:

$$
\begin{aligned}
&\cos(\alpha_1)+\cos(\alpha_2)+\ldots+\cos(\alpha_7)=7m\\
&\cos(5\alpha_1)+\cos(5\alpha_2)+\ldots+\cos(5\alpha_7)=0\\
&\cos(7\alpha_1)+\cos(7\alpha_2)+\ldots+\cos(7\alpha_7)=0\\
&\cos(11\alpha_1)+\cos(11\alpha_2)+\ldots+\cos(11\alpha_7)=0\\
&\cos(13\alpha_1)+\cos(13\alpha_2)+\ldots+\cos(13\alpha_7)=0\\
&\cos(17\alpha_1)+\cos(17\alpha_2)+\ldots+\cos(17\alpha_7)=0\\
&\cos(19\alpha_1)+\cos(19\alpha_2)+\ldots+\cos(19\alpha_7)=0
\end{aligned}
\tag{3}
$$

The (3), is a system of seven transcendental equations, known as selective harmonic elimination (SHE) equations, in terms of seven unknowns' α_1, α_2, α_3, α_4, α_5, α_6, and α_7. First equation of the set of equations given by (3) determines the magnitude of fundamental voltage for a given value of m, and the remaining six equations respectively eliminate 5th, 7th, 11th, 13th, 17th, and 19th order harmonic components [12]. The (3) is to be solved by employing N-R method in such a way that all possible solutions for a given value of m are obtained without much computational effort.

In general, (3) can be written in a compact form as

$$\Psi(\alpha)=\Phi(m) \tag{4}$$

IV. Newton-Raphson Method

The Newton-Raphson (N-R) method is one of the fastest iterative methods. This method begins with an initial approximation and generally converges at a zeros of a given system of nonlinear equations [13].

The N-R method is to be implemented to compute the switching angles for the set of equations given by (3). The switching angles which are in the range of 0 to $\pi/2$ producing desired fundamental voltage along with the elimination of 5th, 7th, 11th, 13th, 17th, and 19th order harmonic components for a given value of m are feasible solutions of (3). The N-R method implemented in [3]-[4] is based on trial and error method for estimation of initial guess and value of m for which solution exists, once starting with a solution, successive solutions are obtained by using previous solution as initial guess for the next one; proceeding this way, only one solution set is obtained. Here, the N-R method is implemented in a different way for which an arbitrary initial guess between 0 to $\pi/2$ is assumed and switching angles (keeping all switching angles in the feasible range) along with the error (% content of 5th, 7th, 11th, 13th, 17th, and 19th order harmonic components) are computed for complete range of m by incrementing its value in small steps (say 0.0001). The different solution sets are obtained for a particular range of m where they exist i.e. the error is zero for feasible solutions; after getting preliminary solution sets, complete solution sets are computed by using known solutions as initial guess.

The algorithm for the Newton-Raphson method is as follows:

1) Assume any random initial guess for switching angles (say α_0) such that $0 \le \alpha_1 < \alpha_2 \ldots < \alpha_7 \le \pi/2$.
2) Set m = 0.
3) Calculate $\Psi(\alpha_0)$, $\Phi(m)$, and Jacobian $J(\alpha_0)$.
4) Compute correction $\Delta\alpha$ during the iteration using the relation $\Delta\alpha = J^{-1}(\alpha_0)(\Phi(m)-\Psi(\alpha_0))$.
5) Update the switching angles i.e. $\alpha(k+1)=\alpha(k)+\Delta\alpha(k)$.
6) Perform $\alpha(k+1)=\cos^{-1}(abs(\cos(\alpha(k+1)))$ transformation to bring switching angles in feasible range.
7) Repeat steps (3) to (6) for sufficient number of iterations to attain error goal.
8) Increment m by a fixed step.
9) Repeat steps (2) to (8) for whole range of m.
10) Plot the switching angles as a function of m. Different solution sets would be obtained.
11) Take one solution set at a time and compute complete solution set for the range of m where it exits.

By following the above steps, all possible solution sets, when they exist, can be computed without any computational complexity.

V. Computational Result

By implementing the above algorithm, all possible solution sets for a 15-level CMLI are computed and a complete analysis is also presented. Starting with any random initial guess, all solution sets are obtained by incrementing m in steps of 0.001 from 0 to 1. By using preliminary computed results, complete switching angles are obtained, and these angles have been plotted as a function of m in Fig. 3. It can be seen from the Fig. 3, that the solutions do not exist at lower and upper ends of the modulation indices and also for m = [0.4230 0.4650], [0.7350 0.7390], and [0.7760 0.7940] as aggregate contribution due to 5th, 7th, 11th, 13th, 17th, and 19th order harmonic components is not zero at these values of m. The values of m for which solutions exist are shown by dark line along m axis. For certain range of m, multiple solutions exist as shown in the Fig. 3 (for example, at m = 0.6000, there exist four solution sets). Even some solutions existing in very narrow range of m = 0.4200, 0.7800 are also obtained by implementing the N-R technique. For the values of m where solutions do not exist, optimization technique can be employed to minimize above harmonic components instead of eliminating them [14].

For each of the multiple solution sets as computed above, total harmonic distortion (THD) in percent is computed according to (5), in (5) $V_{23}, V_{25}, \ldots V_{49}$ are the values of harmonic voltages due to 23rd, 25th … 49th order harmonics. The set of switching angles among multiple solutions which produce least THD is selected for switching of semiconductor devices, and these are termed as combined solutions. The THD plots for individual solution sets as well as for the combined solutions are plotted as a function of *m* in Fig. 4. Fig. 5 shows the plot of switching angles which produce least THD among different solution sets. It can be seen from the Fig. 4, that there is a significant decrease in THD if one uses all possible solution sets for determining the final switching angles instead of considering only one solution set as reported in [3]-[4]. For example, if one computes THD produced due to all possible solution sets at m = 0.6220, THD of output voltage for one solution set is 6.06% (maximum) while due for another solution set, its value is 2.99% (minimum); the difference in THD for these two solution sets is 3.07 %. The net reduction in THD by 3.07% is a significant value as THD produces various losses and undesirable effects in electric power systems and equipments [11], [15]-[16]. It is to be noticed that the reduction in THD is achieved just by computing multiple solutions only.

$$THD = \frac{\sqrt{V_{23}^2 + V_{25}^2 + \ldots + V_{49}^2}}{V_1} \times 100 \qquad (5)$$

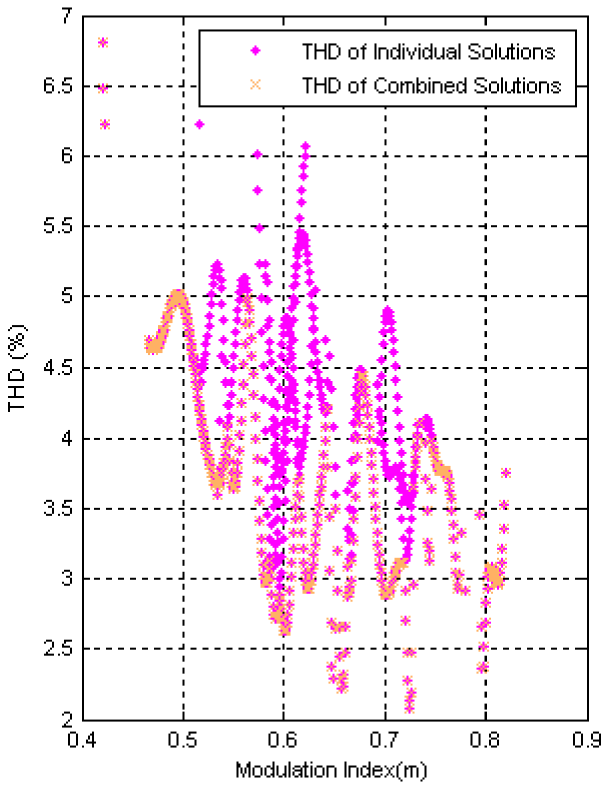

Fig. 4. THD of all solutions and combined solution.

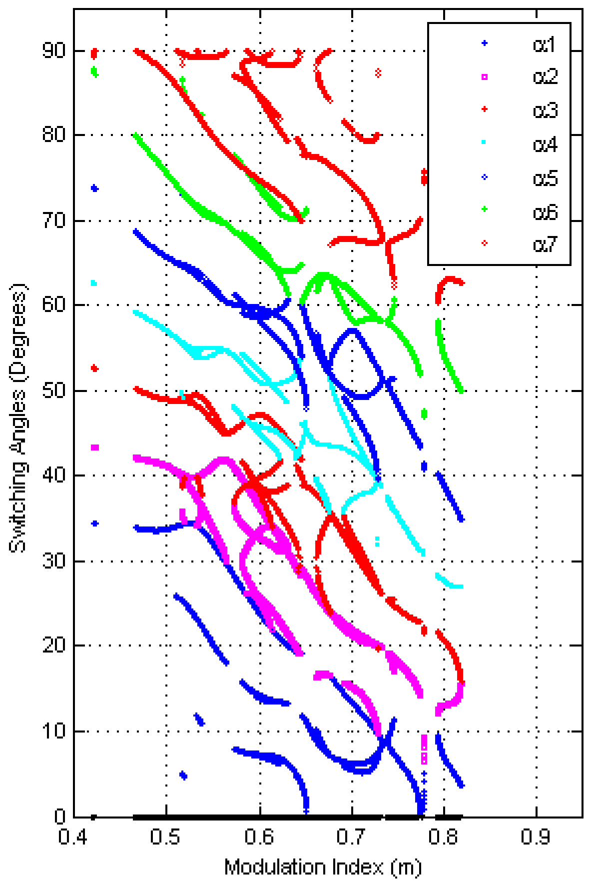

Fig. 3. Switching angles verses modulation index.

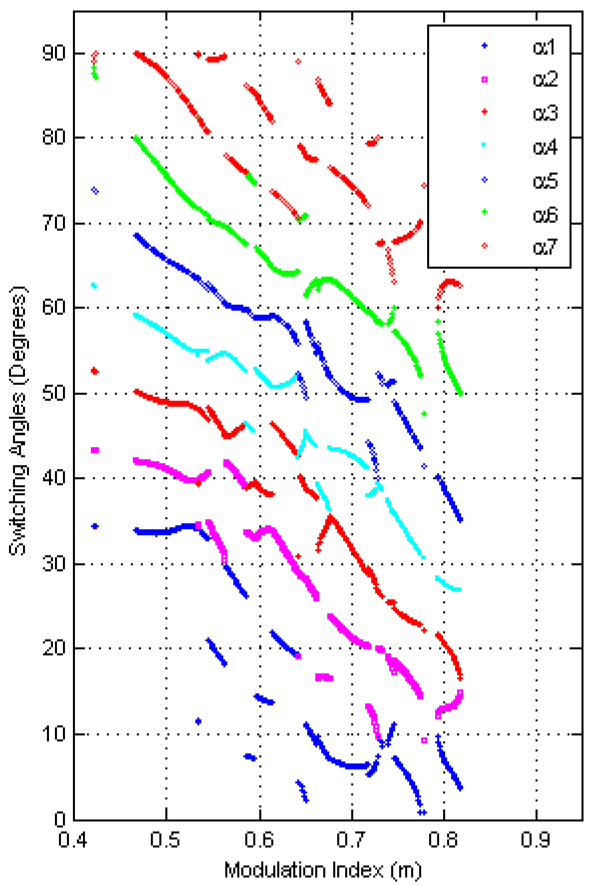

Fig. 5. Switching angles producing least THD.

VI. Simulation Results

A three-phase, 15-level CMLI has been simulated on MATLAB/SIMULINK platform [17]. For each of the H-bridges in the CMLI, 12V dc source has been used. The switching device used is 400V, 10A MOSFET. Simulation diagram for a single leg is shown in Fig. 6.

Output line voltage has been produced by simulation at m = 0.6220, using switching angles generating maximum THD, the produced line voltage along with its harmonic spectrum is shown in Fig. 7. From the harmonic spectrum, it is clear that harmonic components up to 19th order are absent as predicted analytically. The individual maximum harmonic component (29th) and THD are 4% and 6.06% respectively, these values cross the upper limits imposed by IEEE-519 standards. There is close agreement with the simulated and computed results.

In the second simulation, switching angles at m = 0.6220 has been used from a different solution set which produces least THD in the output ac voltage.

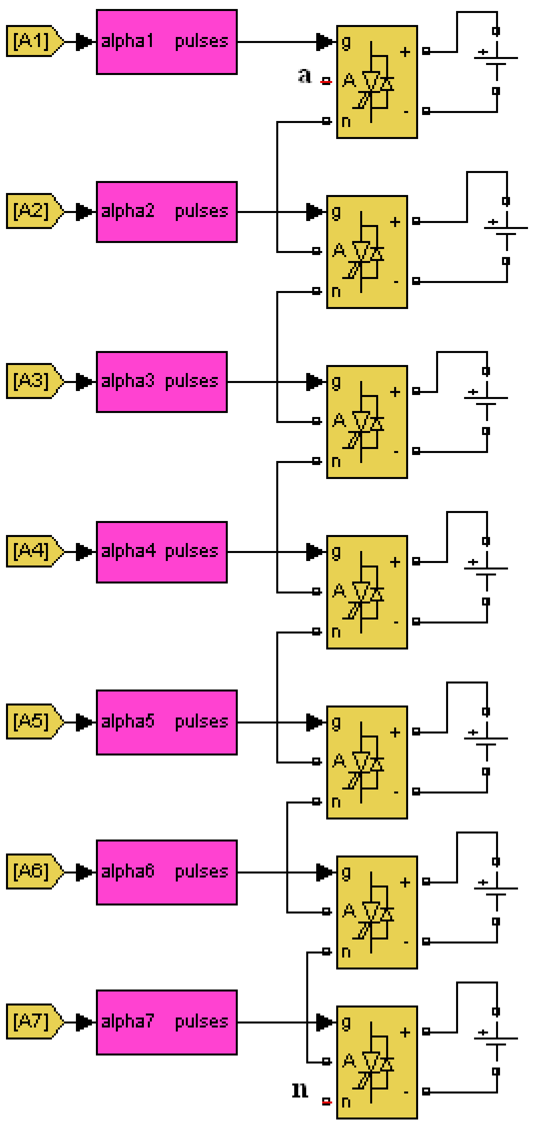

Fig. 6. Schematic diagram of single-phase, 15-level CMLI.

The simulated line voltage and its harmonic spectrum are shown in Fig. 8. The THD of the output line voltage is 3.00%; again this figure is in well correspondence with the computed value. Further, it can be observed from the harmonic spectrum of the Fig. 8, that the lower order harmonic components (i.e. 5th, 7th, 11th, 13th, 17th, and 19th) are completely absent as predicted analytically, and higher order harmonic components are well below 3%. Hence, in this case, individual harmonic as well THD are following IEEE-519 standards for power systems applications.

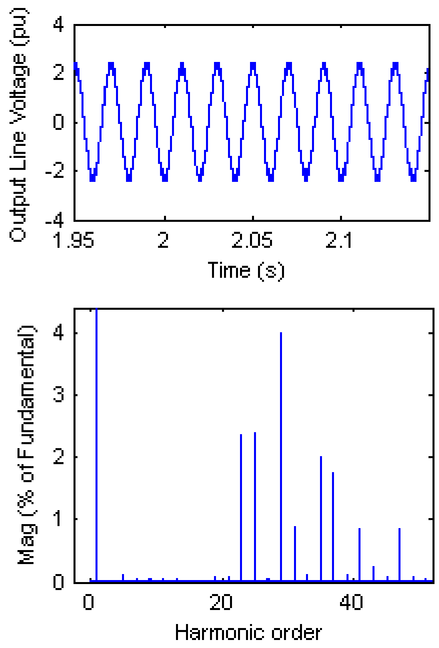

Fig. 7. Output line voltage and its harmonic spectrum for the switching angles producing maximum THD at m = 0.6220.

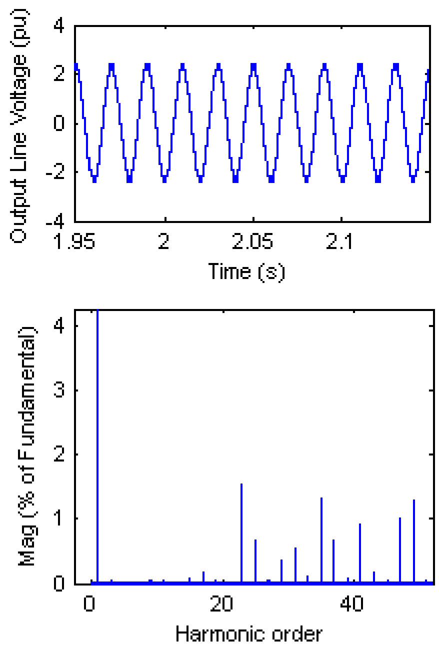

Fig. 8. Output line voltage and its harmonic spectrum for the switching angles producing minimum THD at m = 0.6220.

VII. Conclusion

A cascade multilevel inverter of 15-level has been proposed for power system applications where harmonic levels are to be kept well below a certain threshold value as defined by certain international standards. Switching angles are calculated using N-R technique in a particular way that produces all possible solutions having different range of THD as well as individual harmonic components in the output voltage. Among different solutions, switching angles have been selected which produce least THD. The THD of output voltage for complete range of modulation index is achieved below 5%, thereby satisfying IEEE-519 standards. A complete solution of switching angles for a15-level CMLI has been presented in this work which may have applications in renewable energy sources, hybrid electric drives, electric power systems etc.

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