Wilcox's k-ω model

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1 Wilcox's k-ω model

I have used the Finite Volume Method (FVM) approach. A non-uniform grid is generated using the DNS grid provided. For the given problem, full channel height (i.e. upto $y_{max} = 2\delta$) is considered.

Initial guesses are:

$k = 5\text{x}10^{-3},\ \omega = 10^{-5},\ u =$ randomly initialised between [0, 1] values

Boundary conditions applied are:

At wall: $u = 0,\ k = 0,\ \omega_{y^+ \leq 3} = 6\nu / C_2 y^2$
The model constants are:

$\beta^* = 0.09,\ \sigma_k = 2,\ \sigma_\omega = 2,\ C_1 = 5/9,\ C_2 = 3/40$

Considering a 1D fully developed turbulent channel flow, the governing equations and its discretization is given below.

1.1 U equation

$$0 = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\partial}{\partial y}\left[(\nu + \nu_t)\frac{\partial U}{\partial y}\right]$$

Discretized equation is:

$$a_P U_P = a_N U_N + a_S U_S + b_U$$
$$\text{where } a_N = \frac{(\nu + \nu_t)_n}{\delta y_n},\ a_S = \frac{(\nu + \nu_t)_s}{\delta y_s},$$
$$a_P = a_N + a_S,\ b_U = -\frac{1}{\rho}\frac{\partial P}{\partial x}\Delta y_P$$

1.2 k equation

$$0 = \frac{\partial}{\partial y}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial y}\right] + P_k - \beta^* \omega k$$
$$\text{where } P_k = \nu_t\left(\frac{\partial U}{\partial y}\right)^2$$

Discretized equation is:

$$a_P k_P = a_N k_N + a_S k_S + b_k$$

$$\text{where } a_N = \frac{\left(\nu + \frac{\nu_t}{\sigma_k}\right)_n}{\delta y_n},\ a_S = \frac{\left(\nu + \frac{\nu_t}{\sigma_k}\right)_s}{\delta y_s},$$

$$a_P = a_N + a_S - S_P \Delta y_P,\ S_P = -\beta^* \omega,\ b_k = P_k \Delta y_P$$

1.3 ω equation

$$0 = \frac{\partial}{\partial y}\left[\left(\nu + \frac{\nu_t}{\sigma_\omega}\right)\frac{\partial \omega}{\partial y}\right] + \frac{\omega}{k} C_1 P_k - C_2 \omega^2$$

Discretized equation is:

$$a_P \omega_P = a_N \omega_N + a_S \omega_S + b_\omega$$

$$\text{where } a_N = \frac{\left(\nu + \frac{\nu_t}{\sigma_\omega}\right)_n}{\delta y_n},\ a_S = \frac{\left(\nu + \frac{\nu_t}{\sigma_\omega}\right)_s}{\delta y_s},$$

$$a_P = a_N + a_S - S_P \Delta y_P,\ S_P = -C_2 \omega^*,\ b_\omega = \frac{\omega^*}{k} C_1 P_k \Delta y_P$$

1.4 ν_t expression

$$\nu_t = \begin{cases} l_m^2 \left|\frac{\partial U}{\partial y}\right| & \text{, for first 300 iterations} \\ \frac{k}{\omega} & \text{, otherwise} \end{cases}$$

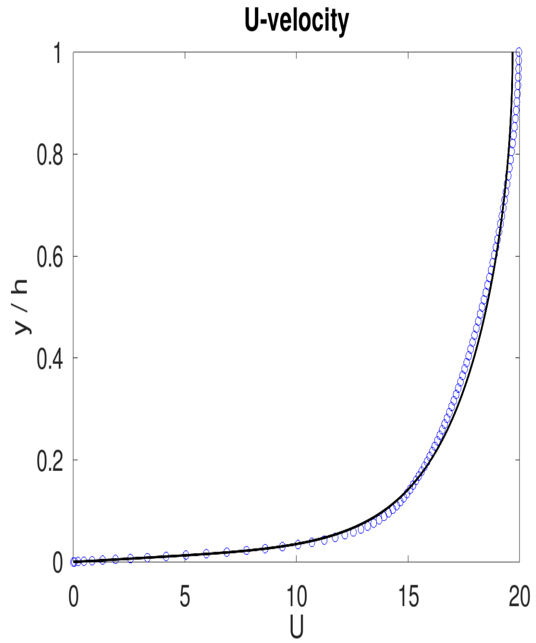
$$\text{where } l_m = \delta\left[0.14 - 0.08\left(1 - \frac{y}{\delta}\right)^2 - 0.06\left(1 - \frac{y}{\delta}\right)^4\right],\ \delta = \text{channel half width}$$

2 Results and Discussions

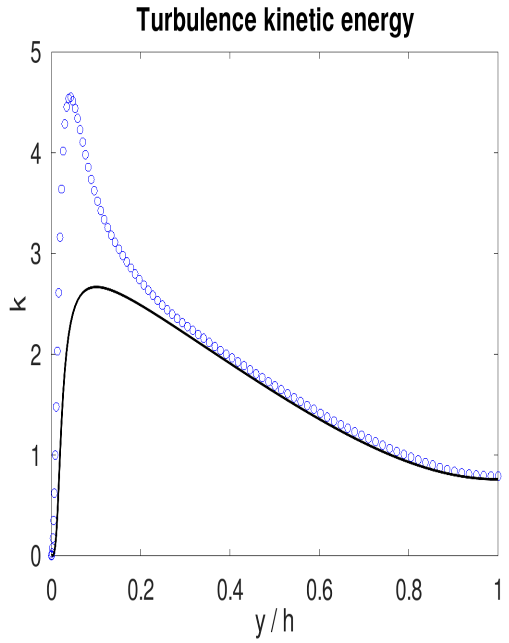
For all the plots below, the following legend style is adopted:

○ DNS data

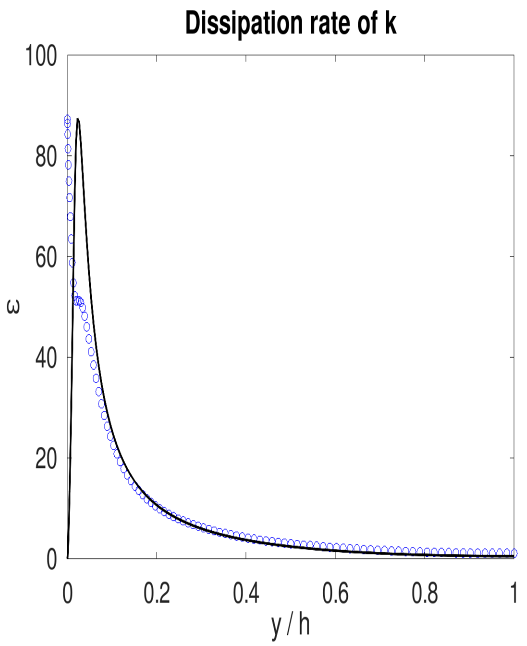
– k-ω model



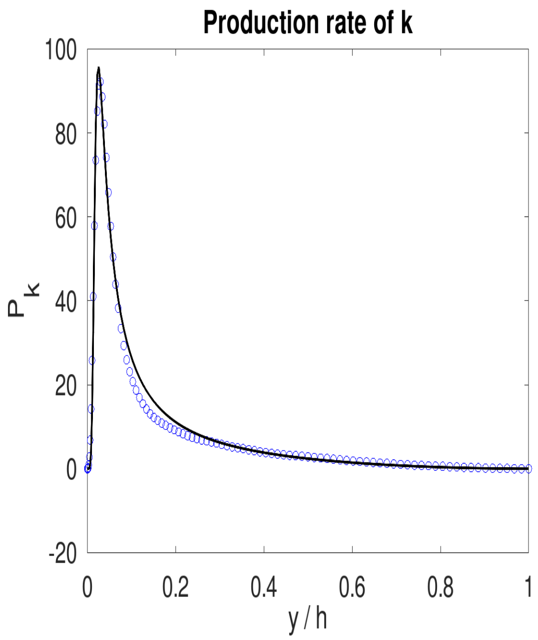
(a) U velocity



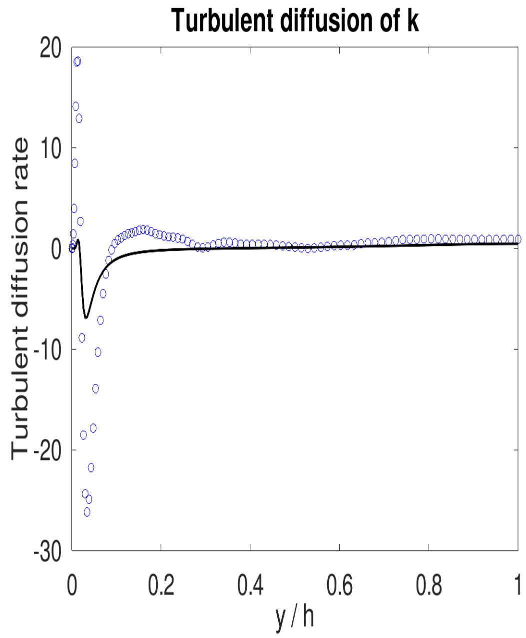
(b) Turbulence Kinetic Energy



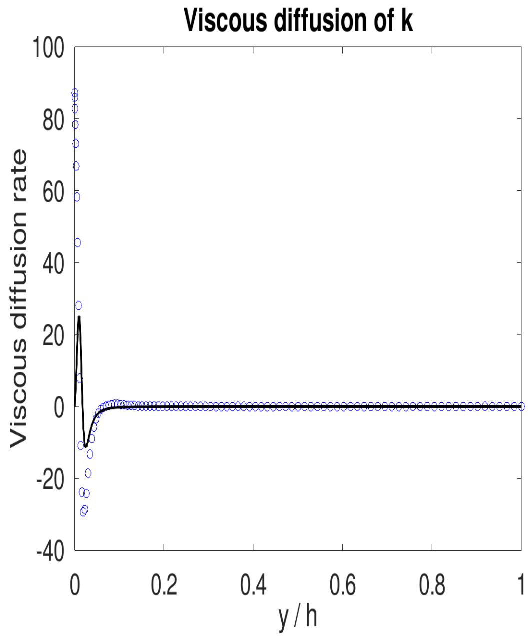
(c) Dissipation rate



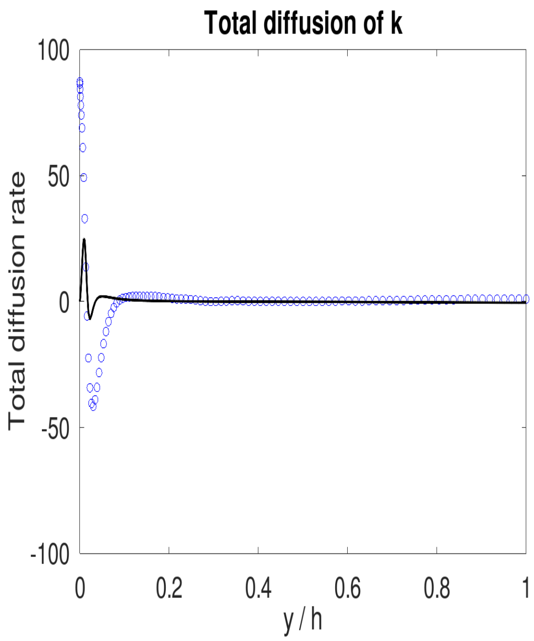
(d) Production rate



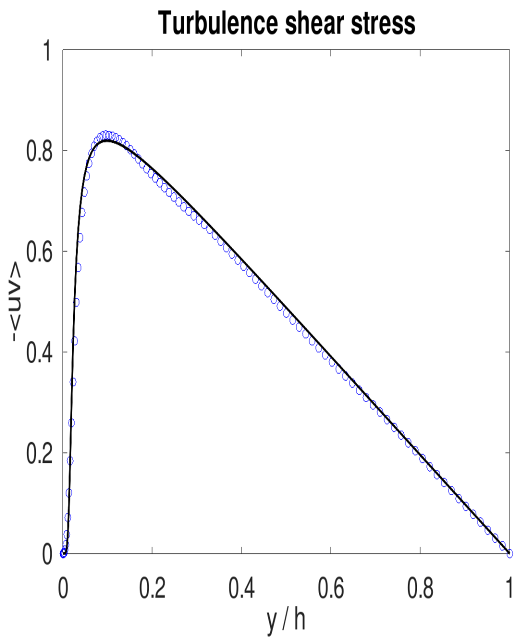
(e) Turbulent Diffusion rate



(f) Viscous Diffusion rate



(g) Total Diffusion rate



(h) Turbulent Shear stress

Wall Shear Stress

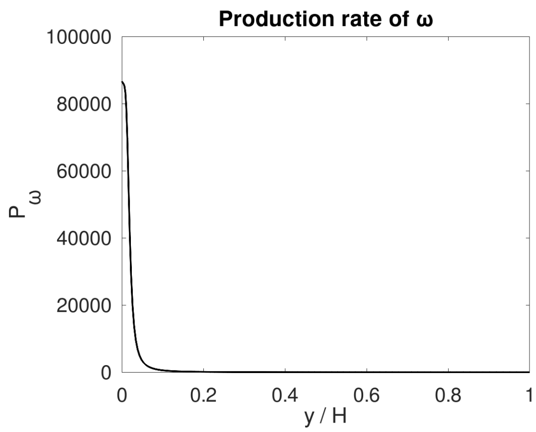
Analytical: $\tau_w = 1.0$
In k-ω model: $\tau_w = 0.9999997$

Near-Wall Region

The k-ω model has produced a good result for $U,\ P_k,\ -\langle uv\rangle$. However if we observe the dissipation rate of k (i.e. ε_k), the model has calculated it as 0 while as per the DNS data, it has some maximum value at the wall (around 87). This is because the model calculates the ε using the expression:

$$\varepsilon = \omega\beta^* k$$

So at the wall, $k = 0$ and thus, the model will calculate the ε as 0 at the wall. In the near-wall region, the turbulent diffusion rate and the viscous diffusion rate has not been accurately captured by the k-ω model. This has led to the overall under-prediction of TKE near the wall. We also neglected the pressure diffusion rate in the k modelled equation. As TKE has various transport rate terms (i.e. pressure diffusion rate, turbulent diffusion rate, and viscous diffusion rate), the under-predicted values of TKE has led to under-prediction of viscous diffusion rate ok k even though there is no modelling assumption made for this term.



(i) Production rate

From figure (i), the decoupling of k from ω is observed as we move away from the wall. This also implies that the k-ω model performs better in near-wall regions. Away from the wall, the model does not perform satisfactorily (refer figure (a), (b)) because of the decoupling phenomenon of k and ω. The need for implementing Low Reynolds Number formulation is eliminated which is not the case in the standard k-ε model. However, implementing k-ω SST model (proposed by Menter, 1994) could improve the result in the regions away from the wall.