1/10/23 Week -> 6

Date (41)

Basic Mathematics for DSA

Prime No -> Count Prime

i) Naive Approach TC -> $O(n^2)$

| 1 ———— 10 | -> i

N % p == 0
↳ Not Prime
else -> prime

ii) Sqrt Approach

Modify is Prime -> finding if N is Prime or Not

$i = 2 \longrightarrow i < N$

⇓

Less

⇓

$i = 2 \longrightarrow i < \sqrt{N}$

=> Let N is non-prime 1, | 2 n-1 | n

at least one factor

$n = a \times b$

$a = \sqrt{n}$ $a \times b > n$

$b = \sqrt{n}$ ↳ Not possible

$a \times b = n$ -> always equal

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At least one of factor must be $\sqrt{n}$
& if we can not find atleast one factor $a < \sqrt{N}$, then N is prime

→ Inner Loop
TC → $O(\sqrt{n})$ → is prime
+
outer loop → & TC → $O(n)$

→ Total → $O(n \times \sqrt{n}) = O(n^{3/2}) < O(n^2)$

(iii) Sieve of Eratosthenes →

N = 21

P	P	P	P	P	P	P
1	2	3	4	5	—	21

Mark every element Prime

(S-1) → Remove 1, n = 21 from & start from 2

(S-2) → If a is prime No, Then elements in table of a is non-prime.

ex → 2 → Prime
4, 6, 8, 10 → Non-Prime

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-> next = 3

6, 9, 12, 18,

-> next 5

10, 15

-> next -> 7

14.

-> next -> 11, 13, 17 -> Inke Multiples ka value < 21

8 prime < 21

(S.1) -> 2 -> n - 1, array of Numbers & Mark all of them prime

(S.2) -> Start from 2 till end, mark all of their multiple non - prime

(S - 3) -> Repeat (2) till (N - 1) only for prime No

(S-4) -> Rest elements marked as prime will be Counted

(iv) Segmented Sieve

↳ Same as sieve of Erastosthene

But given low & high -> have to find b/w those

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outer loop

T.C -> $n\left[\frac{n}{2}+\frac{n}{3}+\frac{n}{5}+ --- \frac{n}{11}\right]$

HP of Prime No (Tailor's Series)

$\boxed{T.C \to O(n(\log(\log n)))}$

2 -> GCD -> greatest Common Divisor

Euclid's Algo

or

HCF -> Highest Common factor

$gcd(a,b) = gcd(a-b, b)$ $a>b$

or

$gcd(b-a, b)$ $a<b$

Apply till this one of the parameter become zero

OR

$gcd(a \% b, b)$ $a>b$

$gcd(b \% a, a)$ $b>a$

can use Modulus but pc takes more time in modulus as compared to minus

e.g. $gcd(72, 24)$

$gcd(72-24, 24) = gcd(48, 24)$

$= gcd(24, 24)$

$= gcd(0, 24)$

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gcd (0, 24) -> this is the answer

LCM ->

LCM * HCF = a x b

LCM (a,b) * gcd (a,b) = a*b

Optimal Approach LCM = $\frac{gcd (a,b)}{a*b}$ -> Euclid's

8 -> Modulo Arithmetic

1 -> (a % n) -> [0, ---, n-1]

ex-> 0 % 3 = [0, 1, 2]
0 % 4 = [0, 1, 2, 3]

Properties ->

1 -> (a+b) % M = a % M + b % M

2 -> a % M - b % M = (a-b) % M

3 -> (((a % M) % M) % M) = a % M

4 -> a % M * b % M = (a * b) % M

Spiral

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4) Fast Exponentiation → a^b

i) Normal solution a^b → $O(b)$

Loop → [0 —— b-1]

↳ $a *= a$

$a \wedge b \Rightarrow O(b)$

ii) Fast soln a^b → $O(\log b)$

If b even

$a^b = (a^{b/2})^2$

odd

$a^b = (a^{b/2}) \cdot a$

eg → $2^{10} = (2^5)^2 = 2^{10}$

$2^{11} = (2^5)^2 \cdot 2 = 2^{11}$

dry-Run → ans = 1, a = 5, b = 4

⇒ a = a * a → 5*5 = 25

b = 4

b = 2, → a*a = 625

b = 1 ⇒ 1 · * a = 625

```
while (b>0)
  if (b & 1) {
    ans *= a
  }
  a *= a
  b >>= 1
}
return ans
```

Spiral

Date (47)

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Ex → (2) → $a = \cancel{2}4^{16}, b = \cancel{5}2, ans = \cancel{1}\ 2$

$b = 5$ (odd) $ans * a = 1*2 = 2$

$a = a*a = 4$

$b >> 5 \rightarrow$ half → int → 2

$b = 2$ (even) $a = a*a \Rightarrow 4*4 = 16$

$b = 1$ Odd $ans * a = 2*16 = 32$

$a*a = 16*16 = 256$

$b >> 2 \rightarrow 0$

$ans = \cancel{5}2$

2^5

$\Downarrow$

$(2^2 \cdot 2^2) \cdot 2$

$\Downarrow$

$(2 \cdot 2)(2 \cdot 2) \cdot 2$

$b >> 1 \rightarrow$ half after every Iteration

Advance Topic → 1) Pigeon Hole
2) Chinese Reminder Theorem
3) Lucas's Theorem
4) Fermat's Theorem
5) Probability Concept