

Week \rightarrow 4

Date... 25/9/23

Class \rightarrow 1

H/W \rightarrow why use $\text{Mid} = \frac{l+s}{2}$, Not $\frac{l+s}{2}$?

because $\text{int } m = \frac{l+s}{2} \rightarrow$ Then This can overflow

because firstly, + will happen then divide, if + happens then int will not able to store that, so divide will not happen.

Class \rightarrow 2 Questions \rightarrow

- (i) Find Pivot (Min Element)
- (ii) Index in sorted & Rotated Array
- (iii) Sort(A)
- (iv) BS on 2-D array

Class \rightarrow 3 \rightarrow Questions

- (i) Divide using BS
- (ii) BS on Nearly sorted array
- (iii) Find odd occurring element

Assignment

Date 25/9/23

Q1) \rightarrow k-diff pairs in an Array $k \geq 0$

[3|1|4|1|5]

$k=2$

Take abs
Difference

↓
find a & b \rightarrow pair
such that $a - b = 2$

Must be.

unique pair

e.g. (1,3) \rightarrow only one
Don't return with every 1

Approaches \rightarrow

i) Brute force

for ($i=0$; $j < n$; $i++$) {

 for ($j=i+1$; $j < n$; $j++$)

 if ($abs(a[i] - a[j]) == k$)

 Count++;

}

T C $\rightarrow O(n^2)$

y

(ii) Two Pointer
cost

1 | 3 | 3 | 4 | 5
i \rightarrow j \uparrow

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iterate j completely \rightarrow No absolute because array sorted
 $diff = a[i] - a[j]$

if ($diff = k$)

Given

* If ($i \geq j$)
 $i++$

$i++$, $j++$;

else if ($diff > k$)
 $i++$

else $\rightarrow diff < k$
 $j++$

y

DRY-Run \rightarrow i) $diff = 0$
 $0 < k$, $j++$

(ii) $\boxed{1} \ 2 \ 3 \ 4 \ 5$ $diff = 2$
 $2 = 2$, store it, $i+1, j+1$

(iii) $\boxed{1} \ 2 \ 3 \ 4 \ 5$ $diff = 3$
 ~~$3 = 2$~~ , $i+1, j+1$

To avoid same pair use set

iv) $\boxed{1} \ 2 \ 3 \ 4 \ 5$ $diff = 1$
 $1 < 2$, $i+1, j+1$

v) $\boxed{1} \ 2 \ 3 \ 4 \ 5$ $diff = 2$
 $2 = 2$, store it, $i+1, j+1$
 $j \rightarrow$ out of boundary's sign.....

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→ To avoid Duplicate pairs

ex →

$\boxed{1 \ 1 \ 3 \ 3 \ 5 \ 5}$ $k=0$

It will give $\rightarrow 2$ pairs

but there is only 1 unique pair, so \rightarrow use sets

8

ex → $\boxed{1 \ 1 \ 2 \ 3 \ 4 \ 5}$ $k=0$

1, 1

$\boxed{TC \rightarrow O(n)}$

$\begin{matrix} 2, 2 \\ 3, 3 \\ 4, 4 \end{matrix} \rightarrow i \neq j$ are on same index
so, if $i = j$ $i \neq j$

(iii) Binary Search ($O(\log n) * O(n) \rightarrow O(n \log n)$)

Sorted $\rightarrow \boxed{1 \ 1 \ 3 \ 4 \ 5}$ given $\rightarrow a - b = k$

$\theta = k + a \rightarrow arr[i]$

find that

$\boxed{1 \ 1 \ 3 \ 4 \ 5}$
 $1 \ 1 \ 1$
sorted

$b \geq a \rightarrow$ why?

$b = a + \theta \rightarrow$ adding something

So, b always a se aage milega

Search in it to nums.size()

1 | 1 | 3 | 4 | 5
i → 0 1 2 3 4

O(1) - Run

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① $\rightarrow i = 0, a[i] = 1, b + a[i] = 3$

Search(3) → found

(1, 3)

② $\rightarrow i = 1, a[i] = 1, b + a[i] = 3$

Search(3) → start from 2 → same
(1, 3)

For Distinct return use (set)

TC $\rightarrow O(n \log n)$

Q2

→ Find k-closest elements →

given

Sorted $\rightarrow [1 | 2 | 3 | 4 | 5]$ $k = 4, x = 3$ → returns that many element

an a is closer to x than b

$$|a-x| < |b-x|$$

$$|a-x| = |b-x| \quad (a < b)$$

Closer

$\rightarrow [1 | 2 | 3 | 4 | 5]$

3 → 3 → 5

3 → 2 → 2

3 → 4 → 4

3 → 1 → 1

3 → 2

3 → 2

$\frac{1}{5} < 5$

Ex → P TO

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Ex $\rightarrow k=4, x=35$

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12	16	22	30	35	39	42	45	48	50	53	55	56
23	19	13	5	0	4	14	10	13	15	18	20	21

Diff Increasing

Diff Increasing

(i) \rightarrow Sort with respect to diff

After sorting $\underline{35} \underline{39} \underline{30} \underline{42} \underline{45} \underline{22} \underline{48} \dots$
↓
Diff Sort

(ii) \rightarrow Two Pointers

12	16	22	30	35	39	42	45	48	50	53	55	56
23	19	13	15	0	4	7	10	13	15	18	20	21
(x - a[i])	1	2	3	4	5	6	7	8	9	10	11	12

If $(x - a[l] > a[h] - x)$

while

$(h-l > k) \wedge$

yellow

$O(n-k)$

$h--$

↓

TC $\rightarrow O(n)$

SC $\rightarrow O(k)$

Return ans

In Two Pointers

Lower the search upto k

(iii) Binary Search

fix in array

Search Right & left

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① → find smallest element $\geq x$

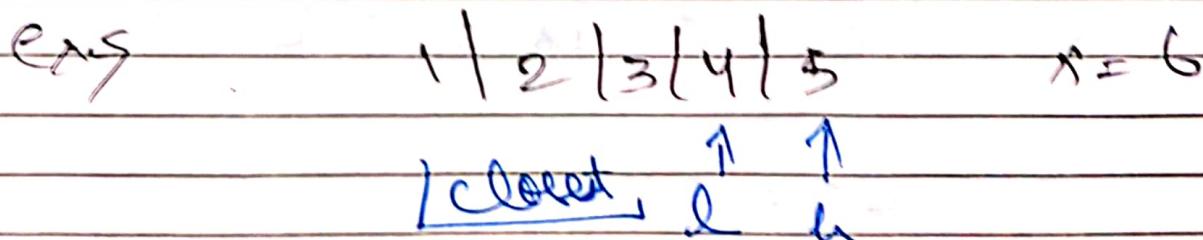
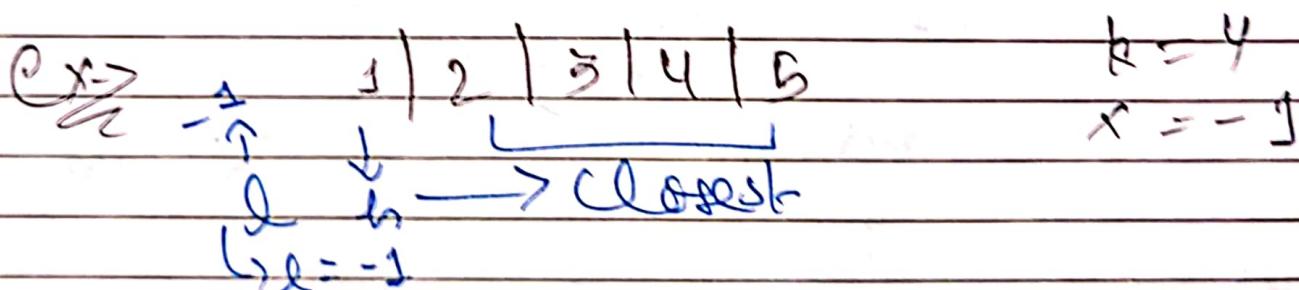
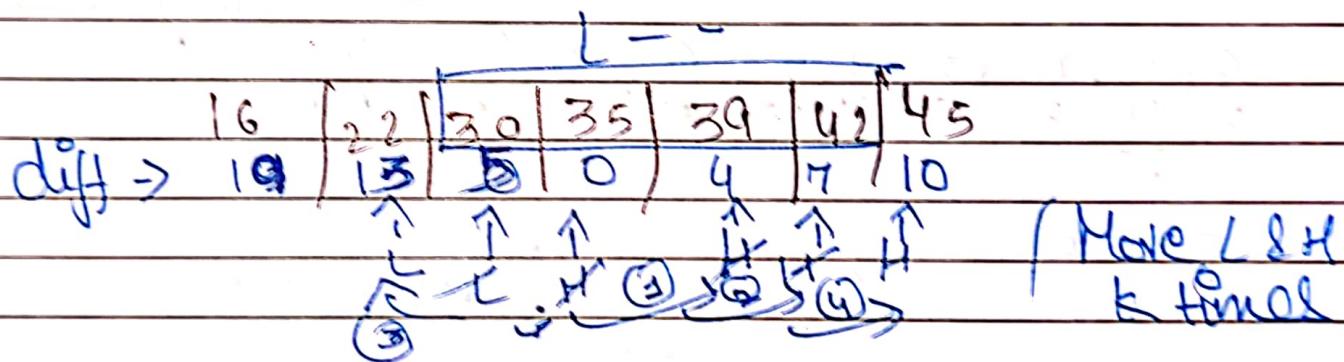
Closest element to x

② → $H \geq$ index closest Index
 $L \geq H-1$

③ → $[L, H]$ window form, size $\rightarrow k$
 \hookrightarrow expand up to k

if ($x - arr[L] > arr[H] - x$)
 $H++$

else



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Ex- $\begin{array}{|c|c|c|c|} \hline 3 & 5 & 8 & 10 \\ \hline \end{array}$

$k = 2$
 $x = 15$

lowerbound $\rightarrow -1$

ans = ~~-1~~ end kro

(b) \rightarrow Exponential Search \rightarrow

\rightarrow Sorted Array

\rightarrow doubling search

Subarray He
Binary Search

\rightarrow Streamline search

\rightarrow galloping search

$\begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|c|} \hline 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \hline 3 & 4 & 5 & 6 & 11 & 13 & 15 & 16 & 17 & 18 \\ \hline 9 & 17 & 27 & 47 & 107 & 137 & 157 & 177 & 197 & 217 \\ \hline \end{array}$

\Rightarrow step

If ($a[i] == x$) return i ;

int $i = 1$;
while ($i < n \& a[i] \leq x$) {
 $i = i * 2$;

return

$x = 15$

y

i) $i = 1, 1 < 9 \& 4 \leq 15$
 $i = 1 * 2 = 2$ 2^0

ii)

$i = 2 * 2 = 4$

2^1

iii)

$i = 4 * 2 = 8 \rightarrow 15 < 15$

2^2

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$$\frac{L}{2} = 11 \leq x$$

9th Pos $\rightarrow 5 > x$

No must be b/w them

$$ex: \frac{1}{5} - 10 \\ \text{find } x$$

$5 > 1 \rightarrow 5, 5 < b/w 1 to 10$

$$S = \frac{1}{2}$$

$$C = \min(i, n-1)$$

may be i can become

so $i > n-1$

so that min of both

$$BS(a)_{\frac{1}{2}}, \min(i, n-1)$$

Q R Y	U R m	3 4 5 6 11 13 14 15 56 70
2	0 1 2 3 4 5 6 7 8 9	

$$x = 14$$

$$i) \Rightarrow 4 \leq 13, \\ i \neq 2 = 2$$

$$(ii) S \leq 13$$

$$i \neq 2 = 4$$

$$(iii) 11 \leq 13$$

$$i \neq 2 = 8$$

$$(iv) BG \leq 13 \rightarrow \text{false out of loop}$$

$$BS \rightarrow i_2, i$$

11	13	14	15	56
4	5	6	8	

$$\frac{8+4}{2}$$

$$\text{Mid} = x \rightarrow \text{return } 6$$

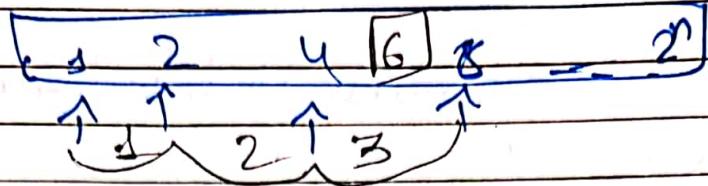
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Date: 1/12/23

TC →

Start → 2 Power & end → 2 Power



$$\log 8 = \log 2^3 = 3 \text{ steps}$$

$\log i \rightarrow \text{steps}$

while loop

$O(\log m)$

↑
i jha take jata

BS → BS on Subarray = $2^3 - 2^2 = 4$

$$= 2^{\log m} - 2^{\log m - 1}$$

$$= 2^{\log m} - 2^{\log m - 1}$$

$$= 2^{\log m} (1 - 2^{-1}) = 2^{\log m}$$

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Applications

Date: 1/10/23

- (1) → Element / Target present near start of arr
- (2) Search in ∞ array (Unbounded)
 - ↳ BS will not work because BS need end

→ expo search will find start & end

Brute force 1 2 3 4 5 6 7 - - - - - ∞

$i = 0$ while ($i <$) {
start if ($a[i] > x$)
 break;
 if ($a[i] == x$)
 ans = x
 i++

$T.C \rightarrow O(\infty)$

Expo Approach \rightarrow iterate till $a[j] < x$

1 2 3 4 5 6 7 - - - - - ∞
0 1 2 3 4 5 6
 j j j while ($a[j] < x$) {
 $i = j$
 $j = j \times 2$

So, $i \rightarrow$ become $i/2$ \Rightarrow we found subarray
 $\therefore j \rightarrow j/2$

(5) \rightarrow Book Allocation Problem Date: 1/10/23

0 1 2 3 \rightarrow array of book
 12 34 67 90 \rightarrow Aⁱ \rightarrow no of pages

M = 2

Students

(i) \rightarrow Brute force

Contingent Allocation $\leftarrow \begin{array}{c|ccc} 12 & 34 & 67 & 90 \\ \hline 0 & & 2 & \\ \end{array} \rightarrow \text{Max} \rightarrow 141$

Allocate like that $\leftarrow \begin{array}{c|ccc} 12 & 34 & 67 & 90 \\ \hline 0 & 1 & & \\ \end{array} \rightarrow \text{Max} \rightarrow 157$

$\leftarrow \begin{array}{c|ccc} 12 & 34 & 67 & 90 \\ \hline 0 & 0 & 1 & \\ \end{array} \rightarrow \text{Max} \rightarrow 113$

\rightarrow If No of student > no of book return -1

\rightarrow find all permutation in Brute force
 \hookrightarrow No

(ii) Better Solution

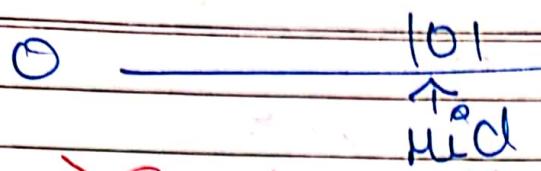
\rightarrow Define Search space

O \longrightarrow
 ↴
 ages
 book b/w
 book design
 no to
 APCO (Kiran)

Solⁿ \downarrow
 Use BS

203
 ↴
 last book
 ch li bache
 kognit
 (M&S)
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Date... 1/10/23



203

→ Try to allocate each student at most 101 pages

$$\text{I} \rightarrow 12 + 34 + 64 = 110$$

$$\text{II} \rightarrow 67 + 90 = 157$$

→ Student 1 ko $12 + 34$ kisi = 46 Tok
less than 101

→ Student 2 ko $67 + 90 = 157 > 113$
so, Not the soln

→ Ab → 101 hai half of Total pages
Ab tk bache 101

→ Maximum 101 Pages allocate Nhi ho
the Tok use kar ke hoga, so
 $c = \text{mid} + 1$

If (Not Possible Soln) $c = m + 1$,



$$\text{I} \rightarrow 12 + 34 + 67 + 96 = 203$$

$$\text{II} \rightarrow 90$$

⇒ Possible Soln
Save it &
Search

If (Possible Soln)
Save Soln

$$c = m - 1$$

$$c = m - 1$$

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(3) ~~102~~ ~~126~~
 5

$$P \rightarrow 12 + 34 + 67 + 96 = 203$$

~~II~~ $\rightarrow 90$

126 is also possible

(4) ~~102~~

~~113~~
mid

$$P \rightarrow 12 + 34 + 67 + 96 = 203$$

~~II~~ $\rightarrow 90$

113 \rightarrow Possible

(5) ~~102~~

~~113~~
mid

$$P \rightarrow 12 + 34 + 67 = 113$$

$$\text{II} \rightarrow 67 + 96 = 163$$

~~II~~ $\rightarrow 90$

$s = m + 1$

Not Possible

Possible

(6) ~~108~~

~~112~~
mid

\rightarrow Not Possible
 $s = m + 1$

(7) ~~111~~

~~112~~
mid

$s = m + 1$

P \rightarrow Not Possible

(8) ~~112~~

~~112~~
mid

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Date: 1/30/23

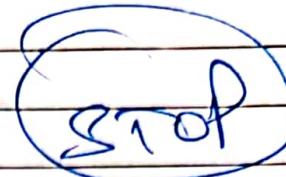
$$I = 12 + 34 + 67 = 113$$

$$II \rightarrow 67 + 96 = 157$$

$$IV \rightarrow 90 \times$$

$$\mu = 2$$

$$S = 113 \quad e = 112$$



while (se)

→ accumulate (start, end, current),
Sum ↴
Can pass
your
function

e.g. accumulate (A, A+N, 0)

6 → Painters Partition Problem

① Allocate Contingous

$$N = 5$$

Q → [5 | 10 | 30 | 20 | 15] → Length of Board

$$k = 3 \quad l_{\min} = 1 \text{ Length Paint}$$

↪ Painters find Min Time?

Easy Examples → 10 | 20 | 30 | 40 N=4
k=2

① → 10 | 20 30 40

② → 10 20 | 30 40

③ → 10 20 30 | 40 Teacher's Sign.....

(10)

90

Date... 1/10/23.....

Ex \$ 10 1 (20 30 40)

max Po work Done

Roga

Ab \rightarrow Pape Hark ko

Hin karna h

which meant

10 | (20 30 40) \rightarrow 90

10 20 | (30 40) \rightarrow 70

10 20 30 | 40 \rightarrow 60

1st Painter

Second Painter

\rightarrow Same 2nd Col Book allocation

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Q) Aggressive Cows

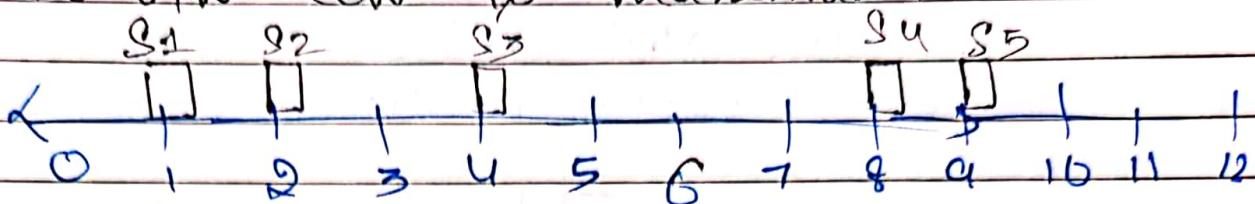
Date... 1/15/2023

Stalls
(Position) [1 2 4 8 9]

$$n = 5$$

$$k = 3 \rightarrow \text{cows}$$

Assign stalls to cows, so that min dist b/w cow is maximum



$\frac{1}{C_1}$	$\frac{2}{C_2}$	$\frac{4}{C_3}$	$\frac{8}{C_4}$	$\frac{9}{C_5}$	Min Dist $\rightarrow 1$
-----------------	-----------------	-----------------	-----------------	-----------------	--------------------------

C_1	C_2	C_3	C_4	C_5	$\rightarrow 3$
-------	-------	-------	-------	-------	-----------------

C_1	C_1	C_2	C_2	C_3	$\rightarrow 1$
-------	-------	-------	-------	-------	-----------------

C_1	C_1	C_2	C_2	C_3	$\rightarrow 2$
-------	-------	-------	-------	-------	-----------------

C_1	C_1	C_2	C_2	C_3	$\rightarrow 1$
-------	-------	-------	-------	-------	-----------------

i) Brute force $\rightarrow O(n^2) \rightarrow x$

ii) Optimal

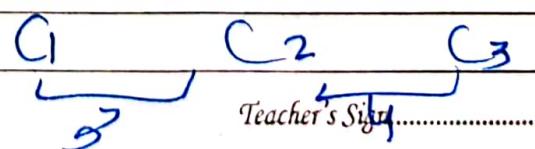
Suppose ans is $\frac{1}{3}$

Ans 1

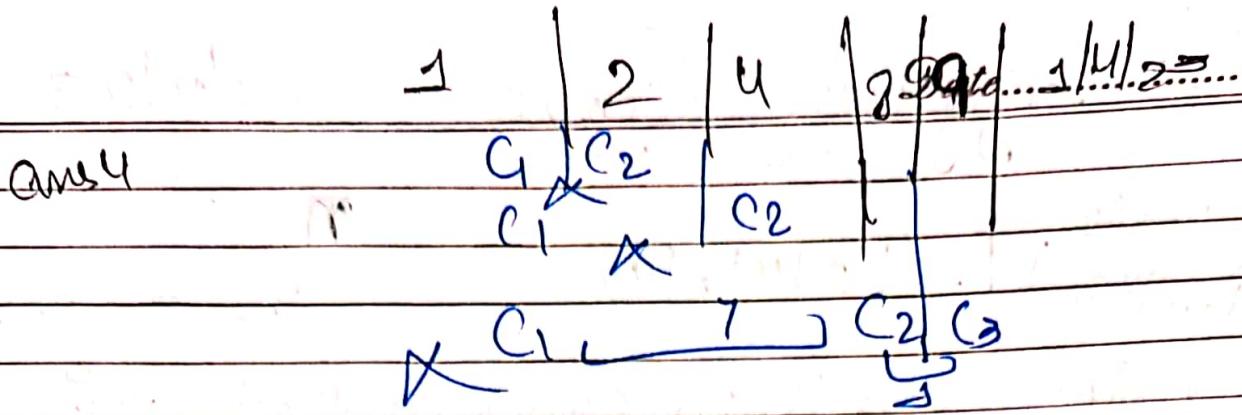
Ans 2

$\frac{1}{C_1}$	$\frac{2}{C_2}$	$\frac{4}{C_3}$	$\frac{8}{C_4}$	$\frac{9}{C_5}$
C_1	C_2	C_3	C_4	C_5

Ans 3 \rightarrow



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I am not able to place 3 cows with minimum distance

Solⁿ 1. $\dots \boxed{3} \boxed{4} \dots 10$

Search space \rightarrow min $\rightarrow 0$
 Dis b/w Cows
 max $\rightarrow \max - \min = 9 - 1 = 8$

① $\rightarrow 0 - \frac{\text{mid}}{4} 8$ At last 4 distance
 If (Not Poss)
 $c = \text{mid} - 1$

② $\rightarrow 0 - \frac{1}{2} \frac{2}{3} 8$

If (Poss)
 ans = store
 $s = \text{mid} + 1$

③ $\rightarrow 3 - \frac{4}{5}$
 mid

If (Pos)
 ans = mid
 $s = \text{mid} + 1$

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$s > cd \rightarrow$ stop loop

Step 2 (1) \rightarrow Always sort stall array

(2) If Possible \rightarrow and F.Store
 $i = mid + 1$
else $\rightarrow mid - 1$

(3) Possible soln

for ($i \rightarrow n$)
if ($a[i] - pos \geq mid$) {
 $c++$;

$pos = \text{stall}(i)$

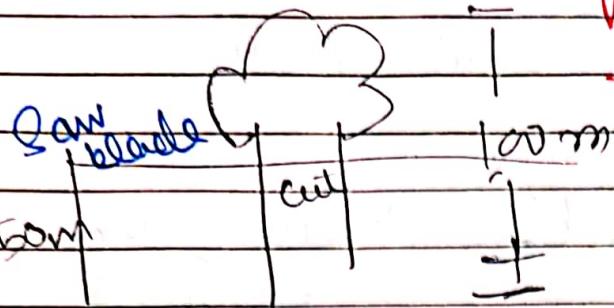
if ($c == k$)
 return true

Pattern false

Q) \rightarrow Eko (shg)

→ website

Date... 1/10/23..



Max h of
saw blade

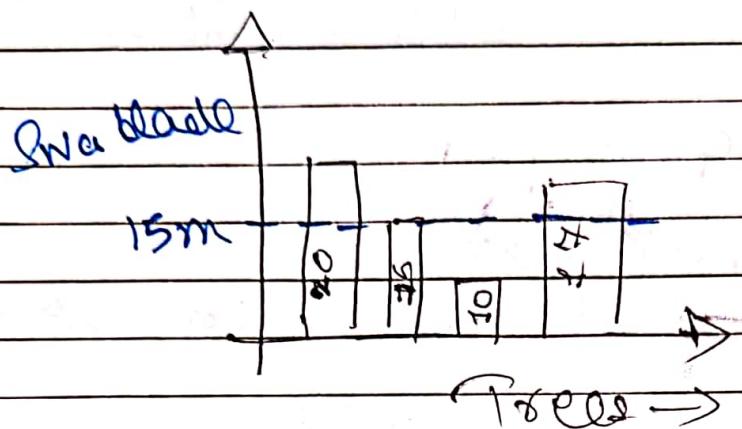
Height of
tree

20 | 15 | 10 | 17

$$M = 4 \text{ m}$$

At least \geq

1 m wood change



15 \rightarrow saw blade
↓ height

have to find this?

$$\begin{aligned} \text{Total Wood} &= (20-15) + (15-15) + (10-15) + (17-15) \\ &= 5 + 0 + 0 + 2 \\ &= 4 \end{aligned}$$

Q) Brute force \rightarrow i \rightarrow saw blade height

check at $i = 1 \text{ m}$

$$\text{Total} = 14 + 14 + 9 + 16 =$$

$$0 + 5 + 15$$

$$\text{ii) } i = 2, \text{ Total} = 18 + 13 + 8 + 15 =$$

solution

(iii) gallery for 3 to --

iv) $i = 15$

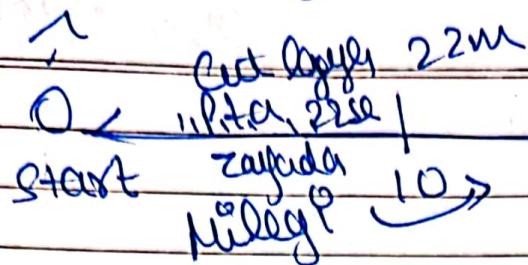
$$\text{Total} = 5 + 0 + 0 + 2 = 7$$

v) $i = 16$

$$\text{Total} = 4 + 0 + 0 + 1 = 5$$

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Wast No of wood



Date... 3/10/23

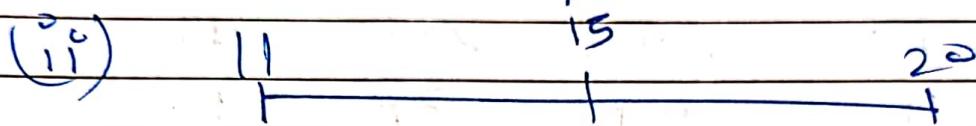
↳ Least No of wood

i) $S=0, e=20, M_id = 10$

$$\text{Total} = 10 + 5 + 0 + 7 = 22$$

Possible soln $22 > 4 \rightarrow \text{ans & store kro}$
 $s = \text{mid} + 1$

Mid

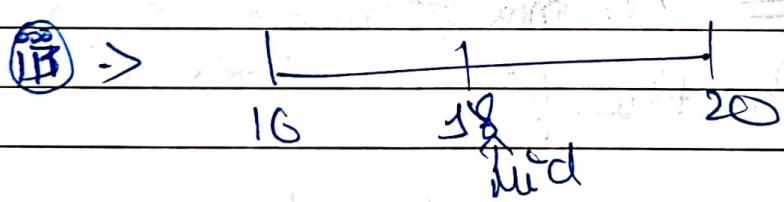


$$\text{Total} = 5 + 0 + 0 + 2 = 7$$

If (possible soln)

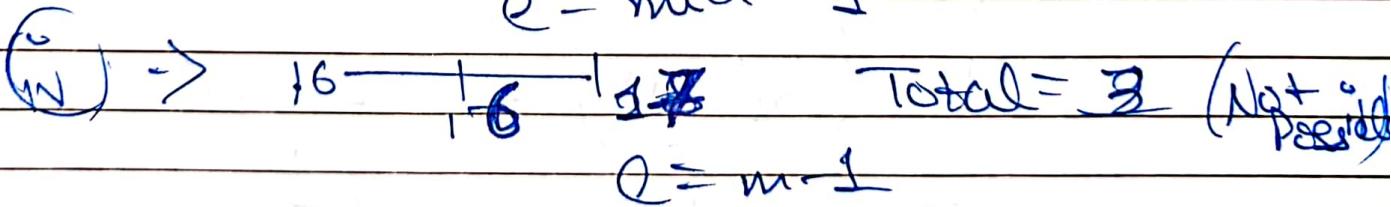
ans = mid ;

start = s+1 ;



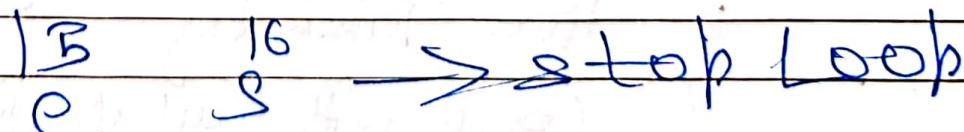
$$\text{Total} = 2 + 0 + 0 = 2 \text{ (Not possible)}$$

$s = \text{mid} - 1$



$$\text{Total} = 3 \text{ (Not possible)}$$

$s = m - 1$



→ stop Loop

Date..... 1/10/23

* \rightarrow Use long long int

(Q) \rightarrow PRATA-SPOS

P \rightarrow Prata ≤ 1000

L cooks $\Rightarrow \leq 50$

R $\Rightarrow [1, 8]$

C_i \Rightarrow 1st 2nd 3rd 4th --
R 2R 3R 4R

(i) Brute force

$$P = 10$$

4 cook

for (P \rightarrow i (max)) R $\Rightarrow 1|2|3|4$

R \rightarrow max \rightarrow (1) \rightarrow Worst Rank

man to make 10 Parathas

C₄ \Rightarrow Total

$$\begin{aligned} \text{Time} &= 1 \times 4 + 2 \times 4 + \dots + 10 \times 4 \\ &= 4 [1 + 2 + 3 + \dots + 10] \\ &= 4 \left(\frac{n(n+1)}{2} \right) = 220 \end{aligned}$$

(Max) \rightarrow Time Taken by

Cook with highest Rank if he makes all the Parathas

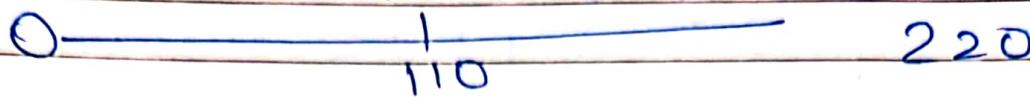
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1 | 2 | 3 | 4
 C₁ C₂ C₃ C₄

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(ii) Binary Search
 Mid

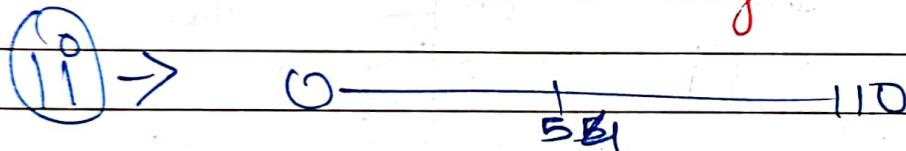


→ Can cook order of 10 pasthae
 in 110 min

$$C_1 \Rightarrow 1 + 2 \times 1 + 3 \times 1 + 4 \times 1 - \dots + 10 \\ \Rightarrow 55 \text{ min} \rightarrow 55 \text{ is Mid}$$

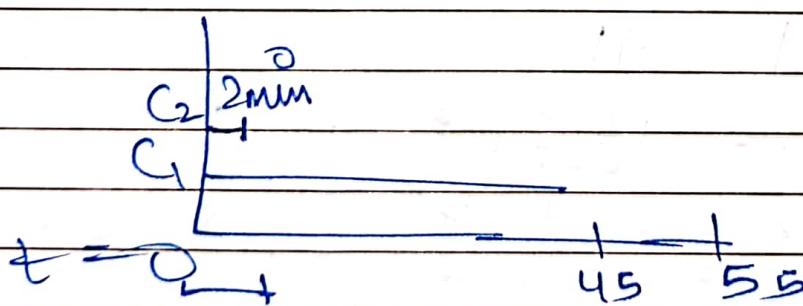
If (possible soln)
 Ans = mid, $\ell = m - 1$

→ agar cook 110 me bua ba Rha
 Jh Toh 111, 110, 112, ..., 20 min
 Ne bhi bna dega



$$C_1 = 55 \text{ min} \rightarrow 1 + 2 + 3 + 4 + \dots + 10 = 55$$

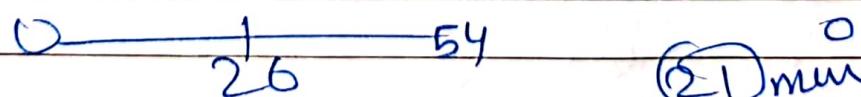
$$C_2 = ?$$



If (possible)

Ans = mid
 $\ell = m - 1$

(iii)



27 min

$$C_1 = 1 + 2 + 3 + 4 + 5 + 6 + 7$$

$$C_2 = 2 \times 4 + 6 \times 8 = 20 \text{ min}$$

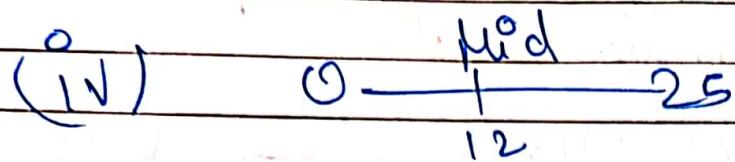
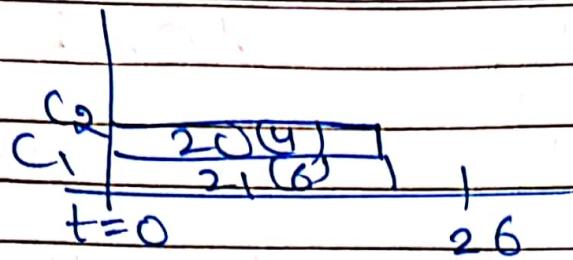
$$C_B = 13$$

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$$e = m - 1$$



$$C_1 = \boxed{2 + 21} \underset{10 \text{ min}}{+} 3 + 4 \rightarrow 4 \text{ Paths}$$

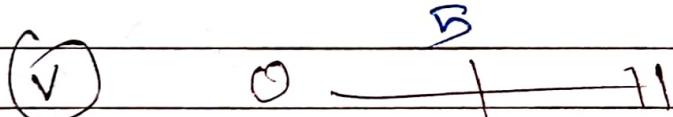
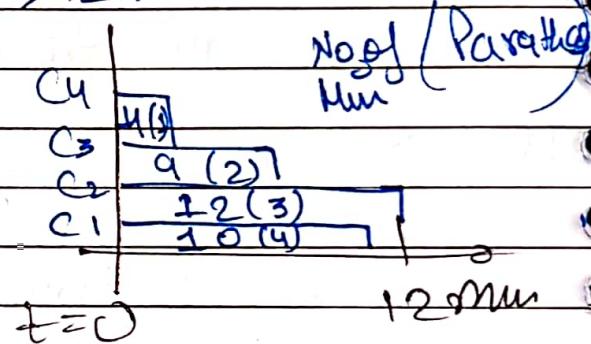
$$C_2 = \underset{12 \text{ min}}{\boxed{2 + 4 + 6}} = 3 \text{ Paths}$$

~~possible~~ $C_3 = 3 + 6 + \underset{8 \text{ min}}{\boxed{9}} \rightarrow 2 \text{ P}$

$$C_4 = 4 \rightarrow 1 \text{ P}$$

$$\text{Ans} = \text{mid}$$

$$C = m - 1$$



$$C_1 = 1 + \boxed{2 + 3} \rightarrow 2 \text{ P}$$

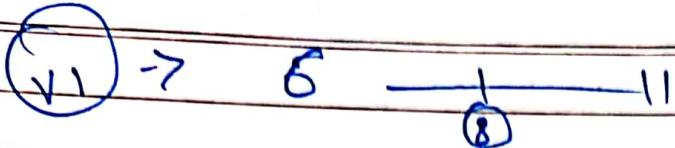
$$C_2 = 2 + \boxed{4} \rightarrow 1 \text{ P}$$

$$C_3 = 3 + \boxed{5} \rightarrow 1 \text{ P}$$

$$C_4 = 4 + \boxed{8} \rightarrow 5 \text{ P}$$

Not Possible $\rightarrow s = m + 1$

Date... 4/10/23



$$C_1 = \underbrace{1+2+3+4}_{8 \text{ min}} = 3P$$

7P = Total

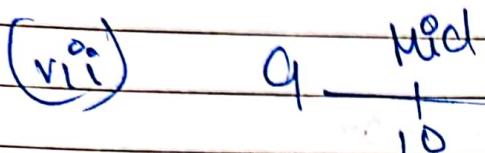
No Possible Soln

$$S = m + 1$$

$$C_2 \Rightarrow 2+4+6 = 2P$$

$$C_3 \Rightarrow 3+6 = 1P$$

$$C_4 \Rightarrow 4+8 \Rightarrow 1P$$



$$C_1 \Rightarrow 1+2+3+4+5 = 4P$$

$$C_2 \Rightarrow 2+4+6 = 2P$$

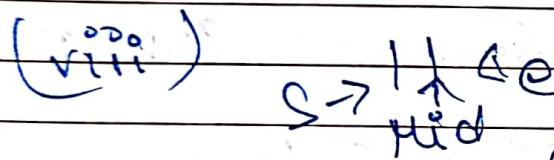
$$C_3 \Rightarrow 3+6+9 = 2P$$

$$C_4 \Rightarrow 4+8 = 1P$$

$$\left. \begin{array}{l} \\ \\ \end{array} \right\} \rightarrow 9P < 10P$$

Not Possible

$$S = m + 1$$



$$C_1 \Rightarrow 1+2+3+4+5 = 4P$$

$$C_2 \Rightarrow 2+4+6 = 2P$$

$$C_3 \Rightarrow 3+6+9 = 9P$$

$$C_4 \Rightarrow 4+8 = 1P$$

$$\left. \begin{array}{l} \\ \\ \end{array} \right\} \rightarrow 9P$$

Not Possible

$$S = m + 1$$

$$\frac{e}{11} S \geq 12$$

$S \geq e \rightarrow$ Stoop Loop

Date... 2/10/23

⑩ → Sqrt with Precision

⑪ → Divide with Precision