Design of Experiment Project / IE 6308 / Fall 2023

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I. Proposal

1. Problem Statement

Additive manufacturing is a process where a part is manufactured in a layer-by-layer fashion using a 3D printer. Mechanical properties of filament-based 3D printed parts are affected by various printing parameters such as print temperature, print speed, infill density and layer height. It is important to understand the effects of these parameters in order to optimize the additive manufacturing process to achieve desired mechanical properties. In this study, our goal is to determine the effect of two factors i.e., infill percentage (density) and print speed on the tensile strength of a 3D printed poly lactic acid (PLA) manufactured on a filament-based 3D printer.

2. Response Variable

Tensile Strength (Mega-Pascal; MPa) – Tensile strength is the amount load (Newtons) that a material can withstand before fracturing. In our study, the response variable (i.e., tensile strength) will be measured by performing a destructive test on the experimental unit. Tensile strength is also known as maximum stress and both terms will be used interchangeably in this document.

3. Factors

3.1. Infill percentage

Infill percentage represents the density of the 3D printed parts. For example, if the infill percentage is 35% then the part is 35% dense. The infill density of a 3D printed part is defined or controlled inside the machine code file (i.e., gcode file) which runs on a 3D printer meaning machine will deposit more material for higher infill percentage and less material for less infill percentage in the 3D space. The lower limit recommended for the infill percentage is 20% to avoid 3D printing failure while the upper limit is 110% to avoid part bulging at different locations.

For this experiment, the chosen two levels for this factor are 30% and 80%. Based on common knowledge, infill percentage have a significant effect on the tensile strength of 3D printed parts. This knowledge or assumption will be tested through this design of experiment.

3.2. Print speed

This is the speed at which the printer nozzle moves while depositing the material to create the specimen or sample. The print speed is controlled by the motors of the 3D printing machine. While there is no lower limit to what the motors can move, a typical filament-based 3D printing machine can run up to 6000 mm/min. However, 3600 mm/min is a recommended upper limit without compromising part quality and going at a lower speed in few hundreds' mm/min will increase the part manufacturing time unnecessarily by 5-6 folds.

For this experiment, three levels of print speed are selected: 1500 mm/min, 2000 mm/min and 2500 mm/min. Based on common knowledge, print speed has no effect on the tensile strength of 3D printed parts. This knowledge or assumption will be tested through this design of experiment.

4. Treatment levels

Table 1 shows the treatment level combinations and the treatment number that will be used in SAS for analysis. There are 3 levels for print speed and 2 levels for infill percentage resulting in 6 treatment combinations.

Table 1: Treatment combinations

Treatment #	Infill percentage	Print speed (mm/min)
1	30	1500
2	30	2000
3	30	2500
4	80	1500
5	80	2000
6	80	2500

5. Experimental Units

An experimental unit in our study is an ASTM D638 Type 5 dogbone which is a standard specimen to evaluate mechanical properties of thermo-plastic polymers such as Poly-Lactic Acid (PLA). Each treatment will be replicated on 3 units. 18 dogbones structures for 18 experimental units will be 3D printed. Table 2 shows the treatment which have been randomized using SAS and will be executed in that order. Since there will be three replications, each treatment appears three times in the table.

Table 2: Randomized experimental units and runs

Observation number	***Treatment***	***Randomization (ranno)***
1	5	0.04322
2	4	0.12269
3	3	0.15619
4	5	0.16453
5	6	0.19687
6	3	0.22233
7	5	0.24529
8	2	0.2836
9	6	0.28954
10	1	0.33959
11	3	0.42922
12	4	0.43959
13	6	0.46662
14	1	0.50966
15	4	0.56723
16	2	0.68344
17	1	0.74838
18	2	0.98817

6. Data Collection Procedure

A specific 3D printer (Flashed Forged Creator Pro) loaded with one Makerbot PLA filament, a metal scrapper, a black sharpie, and a universal tensile machine (UTM) are available in a lab.

I. Parimal creates the 6 trts. Using Simplify3D software and load them on the 3D printer.

II. 3D printing of dogbone and tensile testing in randomized order as per Table *2*.

   a. Preetam starts the 1st run by selecting the trt. File on the 3D printer display (i.e., Observation 1, trt. 5).

   b. 3D printer completes the run and 3D prints the dogbone.

   c. Preetam removes the dogbone from the 3D printer using the scrapper.

d. Preetam labels the dogbone on side according to the observation number with a black sharpie. Soulmaz loads a tensile testing program file on the trapezium software of the UTM.
e. Soulmaz sets the UTM machine to its ground zero condition (i.e Tensile strength=0, Strain=0, gage length= 1 inch)
f. Soulmaz manually loads the dogbone from observation number 1 on the UTM.
g. Soulmaz runs the tensile test and UTM destructively tests the dogbone.
h. UTM machines stops when the dogbone breaks giving out the tensile strength value.
i. Parimal records the associated tensile strength for the dogbone.
j. Soulmaz removes the dogbone from the UTM machine and position it back its zero condition (i.e Tensile strength=0, Strain=0, gage length= 1 inch).
k. Steps **a to k** are repeated for observation 2 to 18 seen in Table 2.

II. Preliminary Analysis of Model Assumptions

1. Raw Data, Plots and Model Form

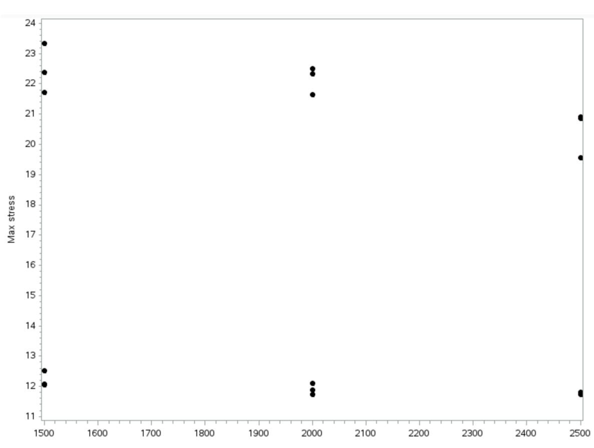
Figure 1 shows the output of the experiment where we 3D printed ASTM D638 Type 5 dog bones at various treatment settings in a randomized order.

Obs	obs	trt	infill	speed	str
1	1	5	80	2000	21.65
2	2	4	80	1500	22.39
3	3	3	30	2500	11.77
4	4	5	80	2000	22.34
5	5	6	80	2500	19.57
6	6	3	30	2500	11.80
7	7	5	80	2000	22.51
8	8	2	30	2000	11.89
9	9	6	80	2500	20.87
10	10	1	30	1500	12.07
11	11	3	30	2500	11.72
12	12	4	80	1500	23.34
13	13	6	80	2500	20.90
14	14	1	30	1500	12.51
15	15	4	80	1500	21.72
16	16	2	30	2000	11.73
17	17	1	30	1500	12.06
18	18	2	30	2000	12.11

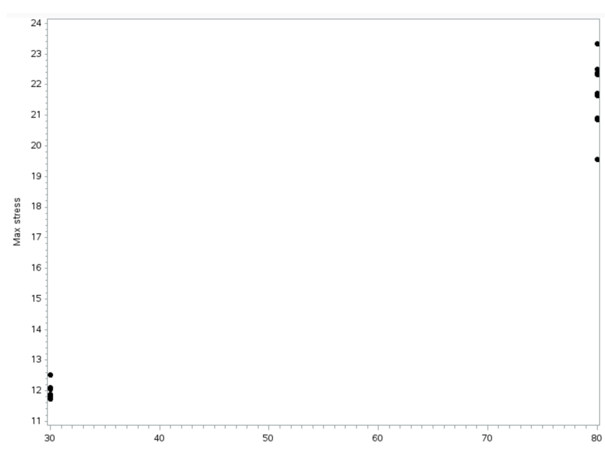
Figure 1: Raw data imported in SAS

After the samples were 3D printed, they were tested for maximum stress (i.e., tensile strength) using a universal testing machine. Maximum (max) stress at various treatment settings by factors is shown in Figure 2 (a) and Figure 2 (b), and in combination in Figure 2 (c).

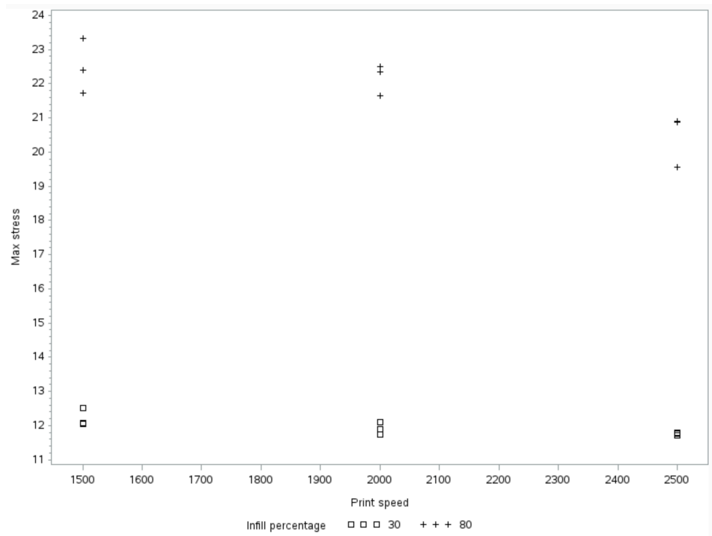
It can be seen in Figure 2 (b) and Figure 2 (c) that when the infill rate is 80%, the max. stress appears to be higher, and the spread in the data when infill rate is 80% appears to be more in comparison to the spread in the data when the infill rate is 30% irrespective of the print speed setting. Furthermore, the max. stress appears to be decreasing slightly with an increase in print speed as seen in Figure 2 (a) and Figure 2 (c).



(a)



(b)



(c)

Figure 2: Raw data plots showing (a) Max. stress vs. Print speed (b) Max. Stress vs. Infill percentage (c) Max. Stress vs. Print speed and Infill percentage in combination

Since the DOE study involves two factors, the assumed model form is as shown below (Full interaction)

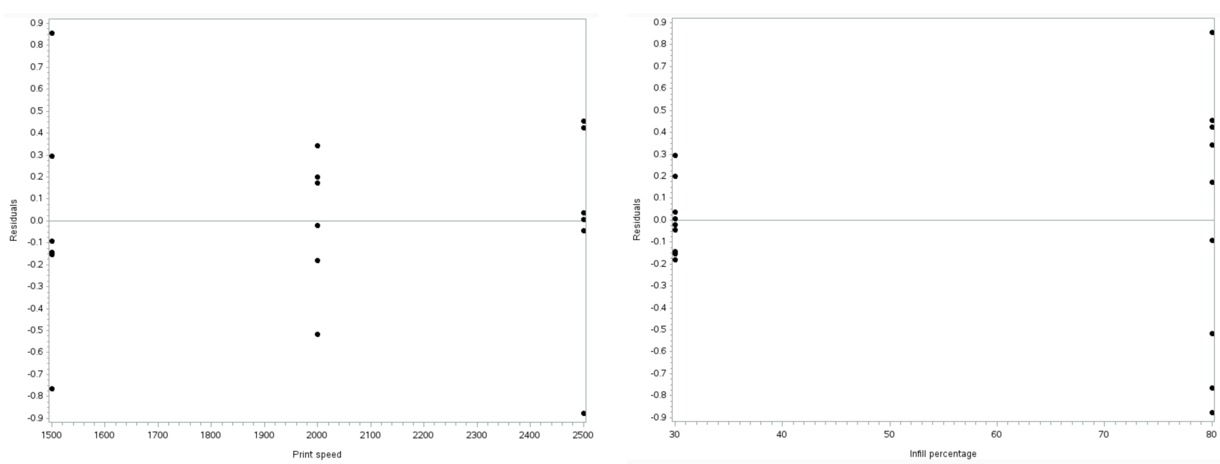
$$Y_{ijt} = \mu_{ij} + \varepsilon_{ijt} = \mu_{..} + \alpha_i + \beta_j + (\alpha\beta)_{ij}$$

Before we discuss ANOVA for a full-interaction model, we need to check the model assumptions of constant variance and residual normality for this model form to be valid. To check the model assumptions, we now review residual and NPP plots and perform various statistical tests.

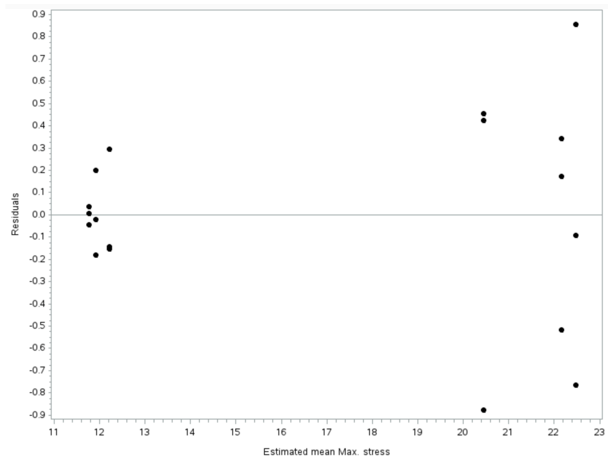
2. Checking Model Assumptions

2.1. Constant Variance

In the residual vs. factors plots shown in Figure 3 (a) and (b), the difference in the spread of residuals at various speeds and infill percentage can be clearly seen. In the residual vs. $\hat{y}$ plot a funnel shape is seen and these observations indicate non-constant variance. Hence, constant variance assumption is violated. Also, there are no outliers seen here.



(a) (b)



(c)

Figure 3: (a) Residual vs. Print speed (b) Residual vs. Infill Percentage (c) Residual vs. $\hat{y}$

2.1.1. Modified-Levene Test

This test to check for non-constant variance was conducted by splitting the population in two groups at $\hat{y} = 15$. Then the medians of the residuals and their absolute differences from all the residuals within the two groups were calculated for comparison using t-test. However, we need to know if the variance of the two groups is equal or not using an F-test.

$H_0: \sigma_1 = \sigma_2$

$H_1: \sigma_1 \neq \sigma_2$

It can be seen in Figure 4 that at a significance level of 0.05, and the p-value (0.0018) highlighted in red is less than 0.05 and hence, we Reject H_0. Which implies we should use unequal variances when conducting t-test.

The TTEST Procedure

Variable: d

group	Method	N	Mean	Std Dev	Std Err	Minimum	Maximum
1		9	0.1178	0.1042	0.0347	0	0.3167
2		9	0.4807	0.3681	0.1227	0	1.0500
Diff (1-2)	Pooled		-0.3630	0.2705	0.1275		
Diff (1-2)	Satterthwaite		-0.3630		0.1275		

group	Method	Mean	95% CL Mean		Std Dev	95% CL Std Dev	
1		0.1178	0.0377	0.1979	0.1042	0.0704	0.1997
2		0.4807	0.1978	0.7637	0.3681	0.2486	0.7051
Diff (1-2)	Pooled	-0.3630	-0.6333	-0.0926	0.2705	0.2015	0.4117
Diff (1-2)	Satterthwaite	-0.3630	-0.6501	-0.0758			

Method	Variances	DF	t Value	Pr > |t|
Pooled	Equal	16	-2.85	0.0117
Satterthwaite	Unequal	9.2745	-2.85	0.0186

Equality of Variances

Method	Num DF	Den DF	F Value	Pr > F
Folded F	8	8	12.47	0.0018

Figure 4: SAS output for modified-levene test

To check the violation of the constant variance assumption, we need to test the hypothesis shown below.

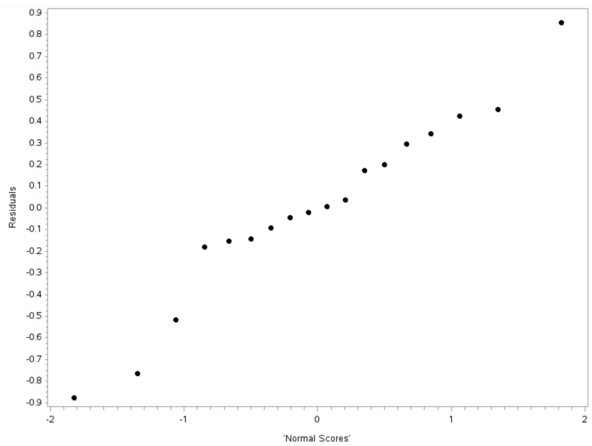
H_0: Means of the d_{ijt} populations are all equal.

H_1: Not all means are equal.

The p-value for this test highlighted in orange is 0.0186 which is less than the significance level of 0.05. Hence, we Reject H_0 and conclude that the constant variance assumption is violated.

2.2. Normality

Normality is not satisfied as seen in Figure 5 since the NPP has a shorter tail on the right side and a longer tail on the left side. Hence it is right skewed. No outliers are seen in the plot.



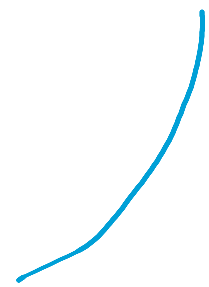


Figure 5: Normal probability plot

2.2.1 Test of Normality

H_0 – Normality is OK.

H_1 – Normality is violated.

The test is conducted at α= 0.1

Pearson Correlation Coefficients, N = 18		
	e	enrm
e Residuals	1.00000	0.97804
enrm 'Normal Scores'	0.97804	1.00000

Figure 6: Correlation between residuals and z scores

From Figure 6, $\rho_{(e,z)}$= 0.97804

$C_{(\propto=0.1;n=18)}$ = 0.957

$\rho_{(e,z)} > C_{(\propto=0.1;n=18)}$ and hence, we fail to reject H_0.

Normality is OK (Weak conclusion)

2.3. Serial Correlation

As the observations have been collected in time order, we need to check if there is any serial correlation issue by plotting residuals versus time (observation order) plot.

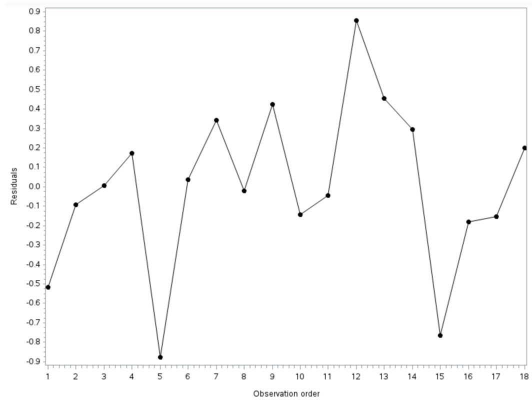


Figure 7: Residuals plotted against observation order

As seen in Figure 7, there is random jaggedness which indicates the residuals are not serially correlated.

2.4. Outliers

The table shown in Figure 8 was obtained in SAS and it shows studentized deleted residuals (tres) which need to be compared with Bonferroni cutoff value.

Obs	obs	trt	infill	speed	str	yhat	e	tres
1	1	5	80	2000	21.65	22.1667	-0.51667	-1.27697
2	2	4	80	1500	22.39	22.4833	-0.09333	-0.21573
3	3	3	30	2500	11.77	11.7633	0.00667	0.01538
4	4	5	80	2000	22.34	22.1667	0.17333	0.40273
5	5	6	80	2500	19.57	20.4467	-0.87667	-2.55095
6	6	3	30	2500	11.80	11.7633	0.03667	0.08460
7	7	5	80	2000	22.51	22.1667	0.34333	0.81549
8	8	2	30	2000	11.89	11.9100	-0.02000	-0.04613
9	9	6	80	2500	20.87	20.4467	0.42333	1.02170
10	10	1	30	1500	12.07	12.2133	-0.14333	-0.33225
11	11	3	30	2500	11.72	11.7633	-0.04333	-0.09999
12	12	4	80	1500	23.34	22.4833	0.85667	2.46015
13	13	6	80	2500	20.90	20.4467	0.45333	1.10180
14	14	1	30	1500	12.51	12.2133	0.29667	0.69931
15	15	4	80	1500	21.72	22.4833	-0.76333	-2.07752
16	16	2	30	2000	11.73	11.9100	-0.18000	-0.41846
17	17	1	30	1500	12.06	12.2133	-0.15333	-0.35569
18	18	2	30	2000	12.11	11.9100	0.20000	0.46583

(a)

Obs	tinvtres
1	3.83299

(b)

Figure 8: (a) SAS output showing studentized deleted residual (tres) (b) Bonferroni cutoff value

The Bonferroni cutoff value is $t_{n-v-1,\frac{\propto}{2n}}$. When the values of n, v and alpha are substituted, it is $t_{0.99861,11}$ which is equal to 3.83299 as seen in the SAS output shown above. None of the absolute values of ‘tres’ are greater than the cutoff value. Hence, there are no outliers.

Since the constant variance and normality assumptions are violated, variance stabilizing transformations are needed.

3. Applying Transformation on y

In this section, we would be discussing log, sqrt. and inverse transformations and identify which transformation works the best.

3.1. Log Transformation

Figures 9 (a)-(c) show the residual plots and it can be seen that applying log transform has not completely fixed the problem with non-constant variance as the funnel shape in residual versus $\hat{y}$ plot is still seen and there is a difference in the spread of residuals at various factor levels in residual versus factors plots. NPP plot in Figure 9 (d) shows that normality now appears to be reasonable after transformation.

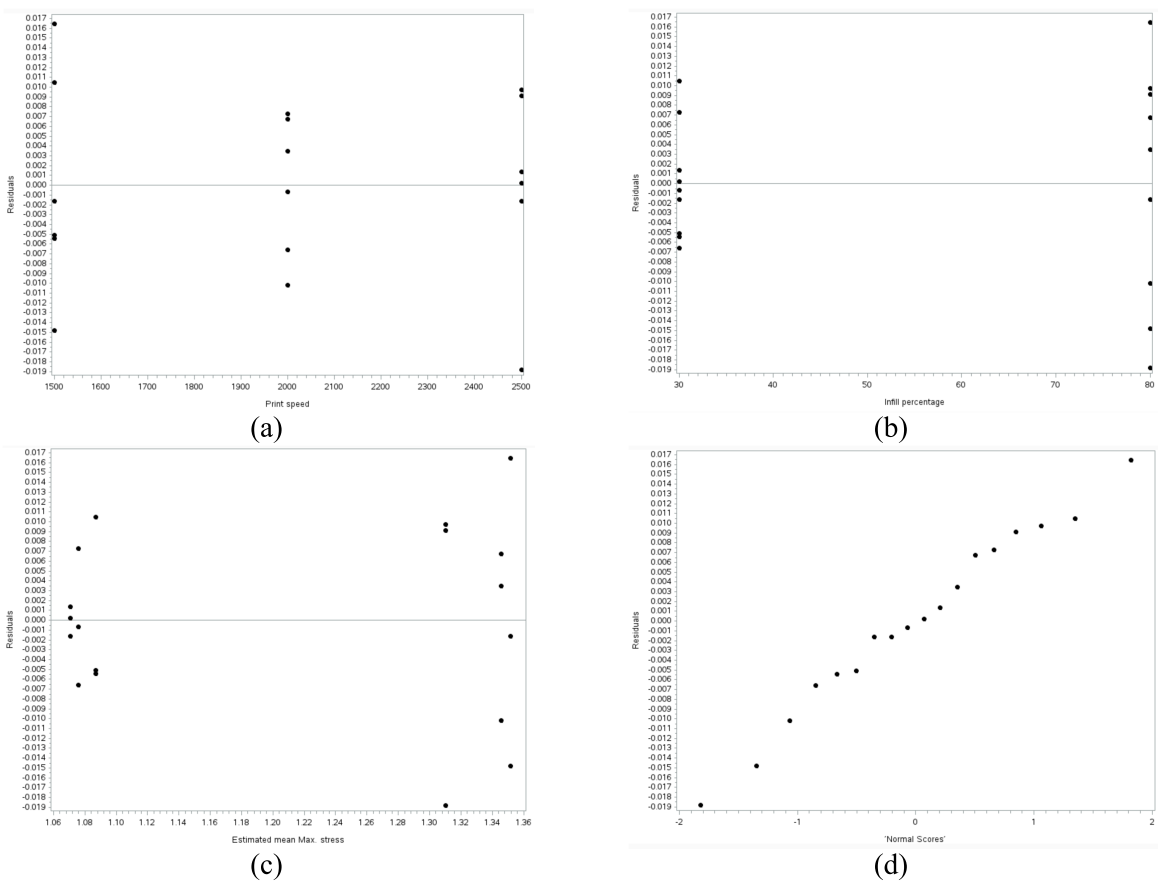
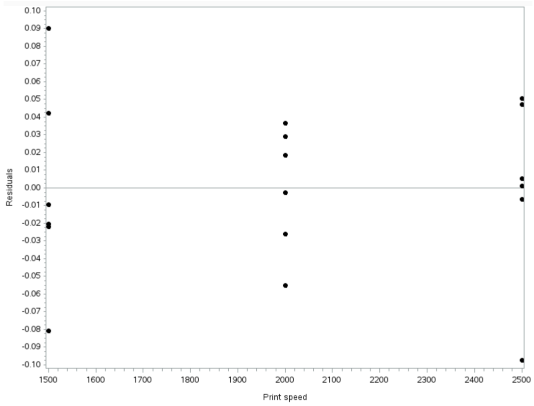


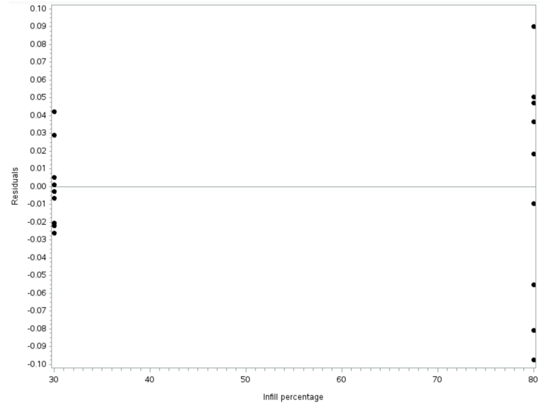
Figure 9: Residual plots after applying log transformation on y (a) Residual vs. Print speed (b) Residual vs. Infill Percentage (c) Residual vs. $\hat{y}$ (d) Normal probability plot

3.2. Sqrt. Transformation

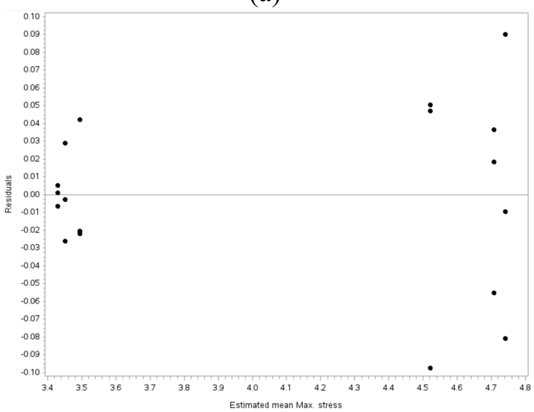
Figures 10 (a)-(c) show the residual plots and it can be seen that applying sqrt. transform has not completely fixed the problem with non-constant variance as the funnel shape in residual versus $\hat{y}$ plot is still seen and there is a difference in the spread of residuals at various factor levels in residual versus factors plots. NPP plot in Figure 10 (d) shows that normality is un affected by the transformation and is not satisfied.



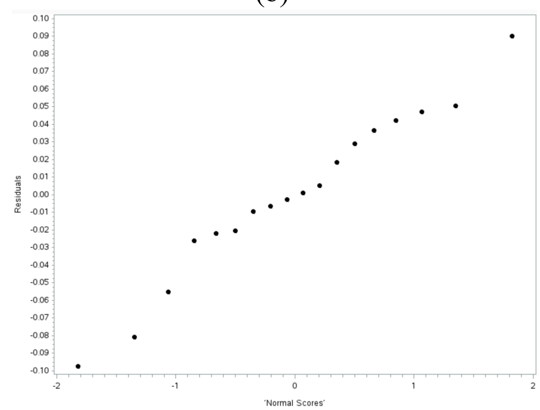
(a)



(b)



(c)

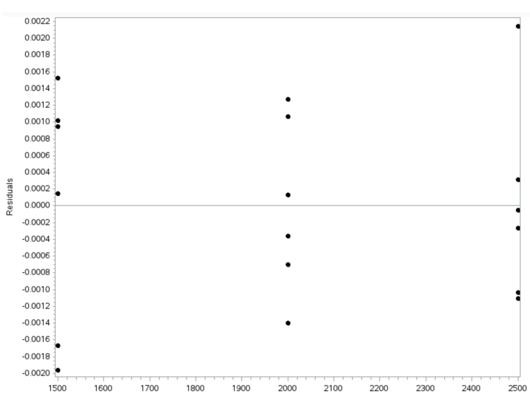


(d)

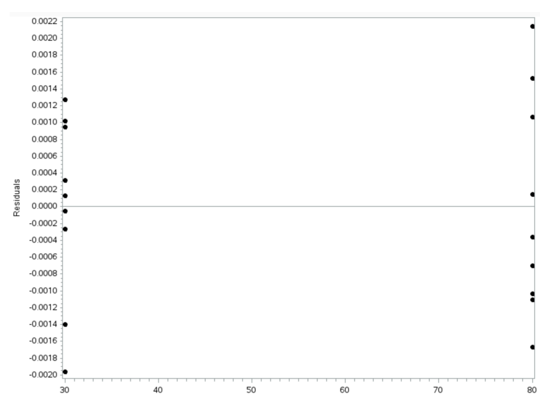
Figure 10: Residual plots after applying sqrt. transformation on y (a) Residual vs. Print speed (b) Residual vs. Infill Percentage (c) Residual vs. $\hat{y}$ (d) Normal probability plot

3.3. Inverse Transformation

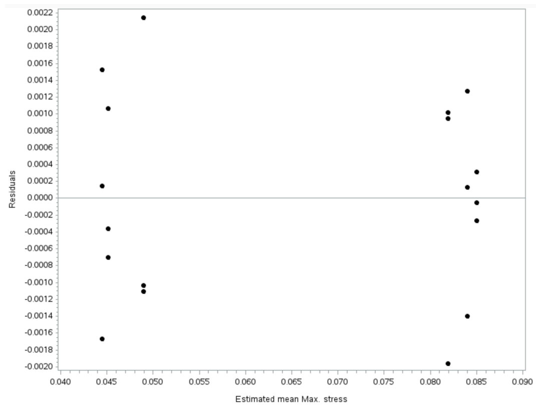
In Figures 11 (a)-(c) it can be seen that applying inverse transform has fixed the problem with non-constant variance because funnel shape is not seen in the residual versus $\hat{y}$ plot and the residual are randomly scattered with similar spread in the residual plots. The NPP plot in Figure 11 (d) shows that normality is satisfied. Hence, the inverse transformation of y is chosen for further analysis.



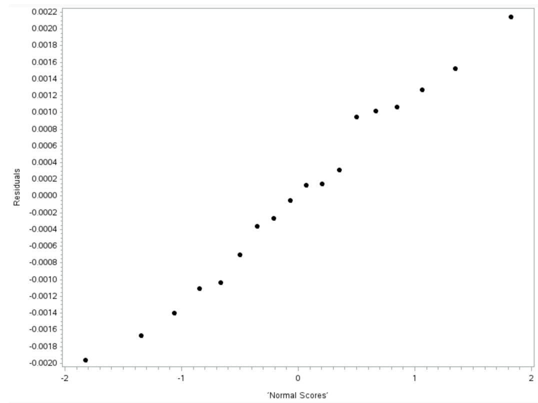
(a)



(b)



(c)



(d)

Figure 11: Residual plots after applying inverse transformation on y (a) Residual vs. Print speed (b) Residual vs. Infill Percentage (c) Residual vs. $\hat{y}$ (d) Normal probability plot

III. Analysis of Variance

As identified in our preliminary analysis, inverse transformation on y is necessary to satisfy the constant variance and normality assumptions thereby making the full-interaction model form valid for further analysis.

1. Interaction Plots

The interaction plot shown in Figure 12 will helps to asses if the main effects and interaction effects of the factors print speed and infill are important or not on the maximum stress. The slope of lines where infill is 30 and 80 are not perfectly zero, hence we anticipate print speed effects might be slightly important. Since lines with infill being 30 and 80 are at a distance, we anticipate infill effects might be important and will need to be tested. Furthermore, since these two lines are not perfectly parallel but almost parallel, the interaction effects may not be important and will need to be tested.

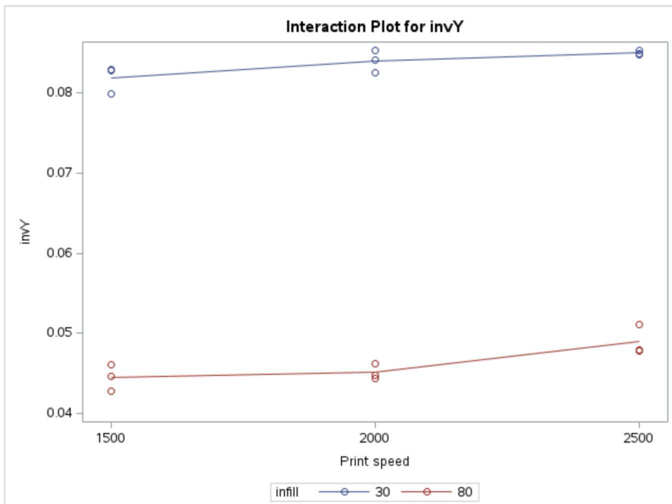


Figure 12 Interaction plots

2. Analysis of Variance

We have designated "print speed" as factor A and "infill percentage" as factor B. The figure below shows the output from SAS where a full interaction model was run. It can be seen that the Type I SS and Type III SS are same because the design is orthogonal and there is no multicollinearity.

The GLM Procedure

Dependent Variable: invY

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	0.00635475	0.00127095	652.70	<.0001
Error	12	0.00002337	0.00000195		
Corrected Total	17	0.00637812			

R-Square	Coeff Var	Root MSE	invY Mean
0.996336	2.149653	0.001395	0.064914

Source	DF	Type I SS	Mean Square	F Value	Pr > F
speed	2	0.00004390	0.00002195	11.27	0.0018
infill	1	0.00630499	0.00630499	3237.94	<.0001
speed*infill	2	0.00000586	0.00000293	1.51	0.2610

Source	DF	Type III SS	Mean Square	F Value	Pr > F
speed	2	0.00004390	0.00002195	11.27	0.0018
infill	1	0.00630499	0.00630499	3237.94	<.0001
speed*infill	2	0.00000586	0.00000293	1.51	0.2610

Figure 13: SAS output for a full-interaction model after applying inverse transformation on y

Based on Type III SS, it can be observed that the contribution of Infill percentage is higher than the contribution of Speed indicating higher importance of Infill percentage. The decomposition of the SS is shown below

$$SSTot = SSA + SSB + SSAB + SSE$$

$$SSTr = SSA + SSB + SSAB$$

Where:

$$SSA = 0.0000439 \qquad SSTr = 0.00635475$$
$$SSB = 0.00630499 \qquad SSE = 0.00002337$$
$$SSAB = 0.00000586 \qquad SSTot = 0.00637812$$

To know the correct model for future work, we can now conduct hypothesis tests to know if interaction is significant.

2.1. Hypothesis Test for AB (Print Speed & Infill Percentage) Interaction

H_0^{AB} : AB interaction is negligible (Additive Model)

H_1^{AB} : AB interaction is not negligible (Full Interaction Model)

From the SAS output for the full interaction model, we see that for print speed and infill percentage interaction, p-value = 0.261. Considering an $\alpha = 0.05$,

p-value > α => FTR H_0^{AB}.

Therefore, the print speed and infill percentage interactions are insignificant. We now have to test main effects.

2.2. Hypothesis Test for Main A (Print Speed)

H_o^A : Main effect for A is negligible → $H_o^A : \alpha_i = 0 \; \forall \; i = 1,2,3$

H_1^A : Main effect for A not negligible → $H_o^A: Not \; all \; \alpha_i = 0 \; \forall \; i = 1,2,3$

From the SAS output for the full interaction model, we see that for print speed, p-value = 0.0018. Considering an α = 0.05,

p-value < α => Reject H_o^A. Therefore, it indicates that the main effects for print speed are significant and we need to conduct multiple comparisons on factor level effects of print speed.

2.3. Hypothesis Test for Main B (Infill Percentage)

H_o^B : Main effect for B is negligible → $H_o^B : \beta_j = 0 \; \forall \; j = 1,2$

H_1^B : Main effect for B not negligible → $H_o^B: Not \; all \; \beta_j = 0 \; \forall \; j = 1,2$

From the SAS output for the full interaction model, we see that for infill percentage, p-value < 0.0001. Considering an α = 0.05,

p-value < α => Reject H_o^B. Therefore, it indicates that the main effects for infill percentage are significant and we need to conduct multiple comparisons on factor level effects of infill percentage.

Since the main effects for print speed and infill percentage are both significant and the interaction effects are insignificant, the additive model is the correct model for future work.

3. Estimating Effects

To estimate the effects, we need to first estimate $\hat{\mu}_{..}$ using $\bar{y}_{...}$. From Figure showing SAS output for the full interaction model, we can see that

$$\hat{\mu}_{..} = \bar{y}_{...} = 0.064914$$

3.1. Main A Effects (Print Speed α_i)

Figure 14 shows the mean max. stress for various levels of print speed which lets us estimate $\hat{\mu}_{i.}$ using $\bar{y}_{i..}$.

speed	invY LSMEAN
1500	0.063209
2000	0.064551
2500	0.066982

Figure 14: Estimates of mean max. stress at various print speeds

$$\hat{\mu}_{1.} = \bar{y}_{1..} = 0.063209$$
$$\hat{\mu}_{2.} = \bar{y}_{2..} = 0.064551$$
$$\hat{\mu}_{3.} = \bar{y}_{3..} = 0.066982$$

Using $\hat{\alpha}_i = \hat{\mu}_{i.} - \hat{\mu}_{..}$, we can estimate the main print speed effects

$$\hat{\alpha}_1 = \hat{\mu}_{1.} - \hat{\mu}_{..} = 0.063209 - 0.064914 = -0.001705$$

$$\hat{\alpha}_2 = \hat{\mu}_{2.} - \hat{\mu}_{..} = 0.064551 - 0.064914 = -0.000363$$

$$\hat{\alpha}_3 = \hat{\mu}_{3.} - \hat{\mu}_{..} = 0.066982 - 0.064914 = 0.002068$$

3.2. Main B Effects (Infill Percentage β_j)

Figure 15 shows the mean max. stress for various levels of infill percentage which lets us estimate $\hat{\mu}_{.j}$ using $\bar{y}_{.j.}$.

infill	invY LSMEAN
30	0.08362986
80	0.04619847

Figure 15: Estimates of mean max. stress at two infill percentages

$$\hat{\mu}_{.1} = \bar{y}_{.1.} = 0.08363$$
$$\hat{\mu}_{.2} = \bar{y}_{.2.} = 0.046198$$

Using $\hat{\beta}_j = \hat{\mu}_{.j} - \hat{\mu}_{..}$, we can estimate the main infill percentage effects

$$\hat{\beta}_1 = \hat{\mu}_{.1} - \hat{\mu}_{..} = 0.08363 - 0.064914 = 0.018716$$

$$\hat{\beta}_2 = \hat{\mu}_{.2} - \hat{\mu}_{..} = 0.046198 - 0.064914 = -0.018716$$

3.3. Interaction Effects ($\alpha\beta_{ij}$)

Figure 16 shows the mean max. stress for various levels of print speed and infill percentage combination which lets us estimate $\hat{\mu}_{ij}$ using $\bar{y}_{ij.}$.

speed	infill	invY LSMEAN	LSMEAN Number
1500	30	0.08190161	1
1500	80	0.04451607	2
2000	30	0.08397739	3
2000	80	0.04512561	4
2500	30	0.08501059	5
2500	80	0.04895373	6

Figure 16 Estimates of mean max. stress at various treatment combinations

$$\hat{\mu}_{11} = \bar{y}_{11.} = 0.0819$$
$$\hat{\mu}_{12} = \bar{y}_{12.} = 0.044516$$
$$\hat{\mu}_{21} = \bar{y}_{21.} = 0.083977$$
$$\hat{\mu}_{22} = \bar{y}_{22.} = 0.045126$$
$$\hat{\mu}_{31} = \bar{y}_{31.} = 0.085$$
$$\hat{\mu}_{32} = \bar{y}_{32.} = 0.04895$$

Using $\widehat{\alpha\beta}_{ij} = \hat{\mu}_{ij} - \hat{\mu}_{i.} - \hat{\mu}_{.j} + \hat{\mu}_{..}$, we can estimate the interaction effects

$$\widehat{\alpha\beta}_{11} = \hat{\mu}_{11} - \hat{\mu}_{1.} - \hat{\mu}_{.1} + \hat{\mu}_{..} = 0.0819 - 0.063209 - 0.08363 + 0.064914 = -0.000025$$

$$\widehat{\alpha\beta}_{12} = \hat{\mu}_{12} - \hat{\mu}_{1.} - \hat{\mu}_{.2} + \hat{\mu}_{..} = 0.044516 - 0.063209 - 0.046198 + 0.064914 = 0.000023$$

$$\widehat{\alpha\beta}_{21} = \hat{\mu}_{21} - \hat{\mu}_{2.} - \hat{\mu}_{.1} + \hat{\mu}_{..} = 0.083977 - 0.064551 - 0.08363 + 0.064914 = 0.00071$$

$$\widehat{\alpha\beta}_{22} = \hat{\mu}_{22} - \hat{\mu}_{2.} - \hat{\mu}_{.2} + \hat{\mu}_{..} = 0.045126 - 0.064551 - 0.046198 + 0.064914 = -0.000709$$

$$\widehat{\alpha\beta}_{31} = \hat{\mu}_{31} - \hat{\mu}_{3.} - \hat{\mu}_{.1} + \hat{\mu}_{..} = 0.085 - 0.066982 - 0.08363 + 0.064914 = -0.000698$$

$$\widehat{\alpha\beta}_{32} = \hat{\mu}_{32} - \hat{\mu}_{3.} - \hat{\mu}_{.2} + \hat{\mu}_{..} = 0.04895 - 0.066982 - 0.046198 + 0.064914 = 0.000684$$

IV. Analysis of Effects

Since interaction effects were insignificant, we only conduct pairwise comparisons for print speed and infill percentage.

1. Pairwise Comparisons - Factor A (Print Speed with 3 levels)

As there are 3 levels for print speed, there will be 3 pairwise comparisons which are:

$$D_1: \mu_{1.} - \mu_{2.} \qquad \widehat{D}_1: \hat{\mu}_{1.} - \hat{\mu}_{2.} = 0.063209 - 0.064551 = -0.001342$$
$$D_2: \mu_{2.} - \mu_{3.} \qquad \widehat{D}_2: \hat{\mu}_{2.} - \hat{\mu}_{3.} = 0.064551 - 0.066982 = -0.002431$$
$$D_3: \mu_{1.} - \mu_{3.} \qquad \widehat{D}_3: \hat{\mu}_{1.} - \hat{\mu}_{3.} = 0.063209 - 0.066982 = -0.003773$$

The contrasts shown above have the following coefficients:

$$C_1 = (1, -1, 0)$$

$$C_2 = (0, 1, -1)$$

$$C_3 = (1, 0, -1)$$

In our experiment, infill percentage has two levels and print speed has three levels and each treatment combination was replicated 3 times. Hence, $a = 3$, $b = 2$ and $r = 3$.

From the ANOVA table, we have MSE $= 0.00000195$.

$$se(\widehat{D}_1) = se(\widehat{D}_2) = se(\widehat{D}_3) = \sqrt{MSE * \sum_{i=1}^{a} \frac{C_i^2}{br}} = \sqrt{0.00000195 * \frac{1^2 + 1^2}{2 * 3}} = 0.000806226$$

To conduct pairwise comparisons, we need to first evaluate which method gives the narrowest CI.

$$df_{Error} = 12$$

$$\alpha = 0.01$$

$$W_S = \sqrt{(a-1) * F_{a-1, df_{Error}, \alpha}} = \sqrt{(3-1) * F_{3-1,12,0.01}} = \sqrt{2 * 6.93} = 3.723$$

$$W_T = \frac{1}{\sqrt{2}} * q_{a, df_{Error}, \alpha} = \frac{1}{\sqrt{2}} * q_{3,12,0.01} = \frac{1}{\sqrt{2}} * 5.05 = 3.57$$

As the narrowest CI is given by Tukey method, below are the simultaneous CI

$$\widehat{D}_1 \pm W_T * se(\widehat{D}_1) = -0.001342 \pm 3.57 * 0.000806226 = (-0.0042202, 0.0015362)$$

$$\widehat{D}_2 \pm W_T * se(\widehat{D}_2) = -0.002431 \pm 3.57 * 0.000806226 = (-0.0053092, 0.00044723)$$

$$\widehat{D}_3 \pm W_T * se(\widehat{D}_3) = -0.003773 \pm 3.57 * 0.000806226 = (-0.0066512, -0.00089477)$$

We can now conduct pairwise comparisons by using the following hypothesis test:

$H_0: D_i = 0 \; \forall \; i = 1,2,3$

$H_1: D_i \neq 0 \; \forall \; i = 1,2,3$

For D_1 and D_2, 0 is in CI and hence we fail to reject H_0.

For D_3, 0 is not in the CI and hence we Reject H_0.

Therefore,

- Mean max. stress when factor level of print speed is at level 1 (1500) is same as the mean max. stress when factor level of print speed is at level 2 (2000).
- Mean max. stress when factor level of print speed is at level 2 (2000) is same as the mean max. stress when factor level of print speed is at level 3 (2500).
- Mean max. stress when factor level of print speed is at level 1 (1500) is different from the mean max. stress when factor level of print speed is at level 3 (2500).

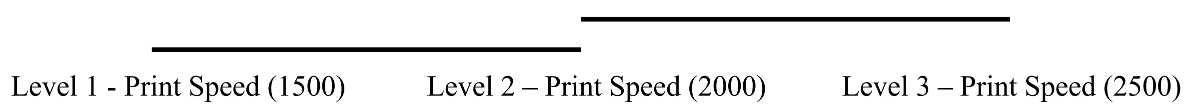


Figure 17: Line plot for factor levels of print speed

2. Pairwise Comparisons - Factor B (Infill Percentage with 2 levels)

As there are 2 levels for infill percentage, there will be 1 pairwise comparison which is:

$$D_4: \mu_{.1} - \mu_{.2} \qquad \widehat{D}_4: \hat{\mu}_{.1} - \hat{\mu}_{.2} = 0.08363 - 0.046198 = 0.037432$$

The contrast shown above have the following coefficients:

$$K_4 = (1, -1)$$

In our experiment, infill percentage has two levels and print speed has three levels and each treatment combination was replicated 3 times. Hence, $a = 3$, $b = 2$ and $r = 3$.

From the ANOVA table, we have MSE $= 0.00000195$.

$$se(\widehat{D}_4) = \sqrt{MSE * \sum_{j=1}^{b} \frac{k_j^2}{ar}} = \sqrt{0.00000195 * \frac{1^2 + 1^2}{3 * 3}} = 0.0006583$$

To conduct pairwise comparison, we need to first evaluate which method gives the narrowest CI.

$$df_{Error} = 12$$

$$\alpha = 0.01$$

$$W_S = \sqrt{(b-1) * F_{b-1, df_{Error}, \alpha}} = \sqrt{(2-1) * F_{2-1,12,0.01}} = \sqrt{1 * 9.33} = 3.0545$$

$$W_T = \frac{1}{\sqrt{2}} * q_{b, df_{Error}, \alpha} = \frac{1}{\sqrt{2}} * q_{2,12,0.01} = \frac{1}{\sqrt{2}} * 4.32 = 3.0547$$

As the narrowest CI is given by Scheffe method, below is the CI

$$\widehat{D}_4 \pm W_S * se(\widehat{D}_4) = 0.037432 \pm 3.0545 * 0.0006583 = (0.03542, 0.039443)$$

We can now conduct pairwise comparison by using the following hypothesis test:

H_0: $D_4 = 0$

H_1: $D_4 \neq 0$

0 is not in the CI and hence we Reject H_0.

Therefore, mean max. stress when infill percentage is at level 1 (30) is different from the mean max. stress when infill percentage is at level 2 (80).

Figure 18: Line plot for factor levels of infill percentage

3. Multiple Comparisons on Pre-selected Contrasts

Below are the 3 pre-selected contrasts which will be compared simultaneously:

1. Main effect Print speed contrast: Print speed < 2500 vs. Print speed = 2500

$$L_4 = \frac{\mu_{1.} + \mu_{2.}}{2} - \mu_{3.}$$

2. Main effect Print speed contrast: Print speed > 1500 vs. Print speed = 1500

$$L_5 = \mu_{1.} - \frac{\mu_{2.} + \mu_{3.}}{2}$$

3. Print speed < 2500 vs. Print speed = 2500 for Infill percentage = 30 vs. Infill percentage = 80

$$L_6 = \left(\frac{\mu_{11} + \mu_{21}}{2} - \mu_{31}\right) - \left(\frac{\mu_{12} + \mu_{22}}{2} - \mu_{32}\right)$$

$$C_4 = \left(\frac{1}{2}, \frac{1}{2}, -1\right)$$

$$C_5 = \left(1, -\frac{1}{2}, -\frac{1}{2}\right)$$

$$D = \left(\frac{1}{2}, \frac{1}{2}, -1, -\frac{1}{2}, -\frac{1}{2}, 1\right)$$

$$\hat{L}_4 = \frac{\hat{\mu}_{1.} + \hat{\mu}_{2.}}{2} - \hat{\mu}_{3.} = \frac{0.063209 + 0.064551}{2} - 0.066982 = -0.003102$$

$$\hat{L}_5 = \hat{\mu}_{1.} - \frac{\hat{\mu}_{2.} + \hat{\mu}_{3.}}{2} = 0.063209 - \frac{0.064551 + 0.066982}{2} = -0.0025575$$

$$\hat{L}_6 = \left(\frac{\hat{\mu}_{11} + \hat{\mu}_{21}}{2} - \hat{\mu}_{31}\right) - \left(\frac{\hat{\mu}_{12} + \hat{\mu}_{22}}{2} - \hat{\mu}_{32}\right)$$

$$\hat{L}_6 = \left(\frac{-0.000025 + 0.00071}{2} - (-0.000698)\right) - \left(\frac{0.000023 - 0.000709}{2} - 0.000684\right)$$

$$\hat{L}_6 = 0.0010405 - (-0.001027) = 0.0020675$$

From the ANOVA table, we have MSE $= 0.00000195$.

$$se(\hat{L}_4) = se(\hat{L}_5) = \sqrt{MSE * \sum_{i=1}^{a} \frac{C_i^2}{br}} = \sqrt{0.00000195 * \frac{\frac{1}{2}^2 + \frac{1}{2}^2 + 1^2}{2 * 3}} = 0.00069821$$

$$se(\hat{L}_6) = \sqrt{MSE * \sum_{i=1}^{a} \sum_{j=1}^{b} \frac{D_{ij}^2}{r}} = \sqrt{0.00000195 * \frac{\frac{1}{2}^2 + \frac{1}{2}^2 + 1^2 + \frac{1}{2}^2 + \frac{1}{2}^2 + 1^2}{3}} = 0.00139642$$

$$n = 18, a = 3, b = 2$$

$$v = a * b = 6$$

$$\alpha = 0.01$$

$$m = 3$$

$$W_B = t_{n-v, \frac{\alpha}{2m}} = t_{18-6, \frac{0.01}{2*3}} = t_{12, 0.00167} = 3.6478$$

Below are the simultaneous Bonferroni CIs

$$\hat{L}_4 \pm W_B * se(\hat{L}_4) = -0.003102 \pm 3.6478 * 0.00069821 = (-0.005649, -0.00055507)$$

$$\hat{L}_5 \pm W_B * se(\hat{L}_5) = -0.0025575 \pm 3.6478 * 0.00069821 = (-0.0051044, -0.00001057)$$

$$\hat{L}_6 \pm W_B * se(\hat{L}_6) = 0.0020675 \pm 3.6478 * 0.00139642 = (-0.00302636, 0.00716136)$$

Based on these CI, we can conduct the following hypothesis test

H_0: $L_i = 0 \; \forall \; i = 4,5,6$

H_1: $L_i \neq 0 \; \forall \; i = 4,5,6$

For L_4 and L_5, 0 is not in the CI and hence we Reject H_0.

For L_6, 0 is in the CI and hence we fail to reject H_0.

Therefore, we can conclude that,

- Mean max. stress when print speed is less than 2500 is different from the mean max. stress when print speed is 2500
- Mean max. stress when print speed is greater than 1500 is different from the mean max. stress when print speed is 1500
- The difference in the mean max. stress when print speed is less than 2500 vs. print speed is equal to 2500 is the same for both level of infill percentage, 30 and 80

V. Final Discussion

In this full factorial experiment, the maximum stress sustained by ASTM D638 Type 5 dogbones made of poly lactic acid (PLA) printed on a 3D printer under 6 different treatment settings were studied. Two factors were involved in the study, which were print speed at 3 levels and infill percentage at 2 levels. There were 3 replications per treatment combination and hence a total of 18 samples were 3D printed.

After we ran proc GLM procedure on the data, we found that the constant variance and normality assumptions were violated. Therefore, we had to apply variance stabilizing transformations on y (max. stress) to resolve this issue. We tried log, sqrt. And inverse transform and found that inverse transform resulted in the constant variance of residuals and the normality also appeared to be satisfied. After this, we found the interactions between the print speed and infill percentage to be insignificant and hence we then tried to determine if the individual factors are significant. We found that both print speed and infill percentage are significant.

As the interactions are not significant, we conducted pairwise comparisons on the factor levels of print speed and infill percentage separately. For print speed, we found that the mean max. stress at 1500 mm/min is same as that at 2000mm/min. Similarly, the mean max. stress at 2000 mm/min is same as that at 2500 mm/min. However, the mean max. stress at 1500 mm/min is different from that at 2500 mm/min. For infill percentage, we found that the mean max. stress is different for 30% and 80%.

We conducted multiple comparisons for 3 pre-selected contrasts using Bonferroni method where we found that the mean max. stress at a print speed of 1500 mm/min is different from that at print speeds greater than 1500 mm/min. Similarly, the mean max. stress at print speed of 2500 mm/min is different from that at print speeds less than 2500 mm/min. We then found that the difference in the mean max. stress when the print speed is less than 2500 vs. the print speed of 2500 is the same for infill percentage of 30 and 80.

In the future experiments, we would like to add a third factor to our study which is the infill pattern. Infill pattern represents how the infill of a part is laid out during 3D printing process and based on the pattern; a part can have a different tensile strength. In the current study, the dogbones were printed at an angle of 45 degree. This value can be set to 0, 30, 45 or 60 degrees.

Overall, the initial understanding that the print speed having no effect on the tensile strength of fused filament fabricated PLA part did not hold true while the infill percentage did clearly have an effect on the tensile strength of the part. The concluded study recommends additive model for any future investigation through these two factors: print speed and infill percentage.