IE 5309 – Stochastic Processes Project

Stock Price Analysis Using Markov Chains

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I. Introduction

Background

Many companies in the world which are involved in economic activities like manufacturing, healthcare, consulting, finance, banking to name a few, need capital to invest in their business and pursue growth opportunities. To accomplish this, the companies cannot always cover the capital requirement only with loans from banks. Hence, these companies try to raise funds and investments from individuals who want to purchase the shares/stocks of their company. This process of offering a stake in the company through shares/stock is called IPO (Initial Public Offering). Once the shares are offered through an IPO and purchased by the individuals, these shares can be traded at the stock exchanges like NASDAQ and NYSE.

Google, Amazon, Apple, Facebook, Tesla, Ford and John Deere are some examples of publicly traded companies. When the shares of a company are being traded at the stock exchange, the price of the stock changes based on the performance of the company, investor sentiment, economic and political situation in the country. Hence, the share price can increase or decrease over time. If the share was purchased by an individual when the price is high and is sold when the price is low, the individual will lose money. On the other hand, if the share is sold when the price is more than what it was purchased at, the individual makes a profit. Therefore, whether to sell or invest or do nothing is a decision that a shareholder should always make.

Events like new government policy, new technology in products of a company influence the stock price and investors are always observing these events keenly. Apart from this, historical data of the stock price is also observed closely by investors in deciding whether or not to invest or sell a stock. Hence, it is important to analyze and use the past data to one's advantage.

Problem Statement

As the price of a stock changes randomly based on many factors, it can be considered to be stochastic. For this project, we are using the stock price of Apple to conduct an analysis on the return offered by the stock and understand how the historical data can be exploited to our advantage to make profit by taking data driven decisions. The data was retrieved from "Google Finance". Markov chains is a promising tool in analyzing a company's stock and by studying the historical data and defining states of the stock price, we can look at the probabilities of transition of states which indirectly tells us the odds of a company's stock price going up or down.

Motivation

Many individuals have made money by investing in stocks. However, they have relied on data driven decisions to do so. This is what motivates us to apply the methods like Markov Chains to a problem like stock price analysis and improve our understanding of the application of these

methods. IE 5309 will help us in understanding and applying the concepts in stochastic processes in this project.

II. Methodology

Stock prices at the end of the end of 2035 days is shown in Figure 1. After the pandemic there has been a steep increase in the price of the stock and this is was because of an increase in the demand of Apple's products as everyone was working remotely. Apart from this, Apple also introduced the latest M1 chip which is faster than the intel chip used previously be Apple. This demonstrates why it is important to keep an eye on the economic situation, company's product launch and current affairs which can help predict stock price change. However, the historic stock price variations do help in making investment decisions and hence, for this project, we have used daily closing stock price and observed the daily return of the stock as shown in Figure 2.

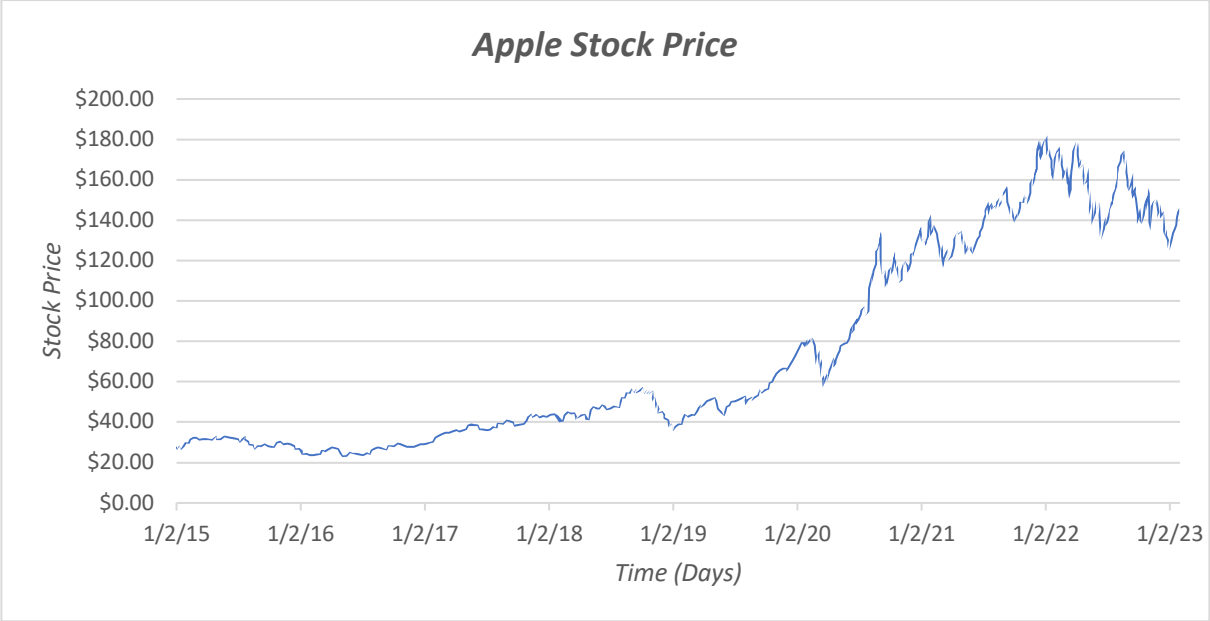

Figure 1 Apple's Daily Closing Stock Price

The daily return of Apple's stock is not perfectly normally distributed as seen in Figure 2 and it has fat tails indicating volatility in the stock price. A high-level methodology used in this project is as follows:

- Obtain stock price data from Google Finance
- Calculate the daily return of stock
- Rank the values and divide them into four percentile ranges (Four States)
- Calculate the P-matrix and associated long-run probabilities and mean recurrence times
- Simulate the stock price variations using python
- Interpret the results

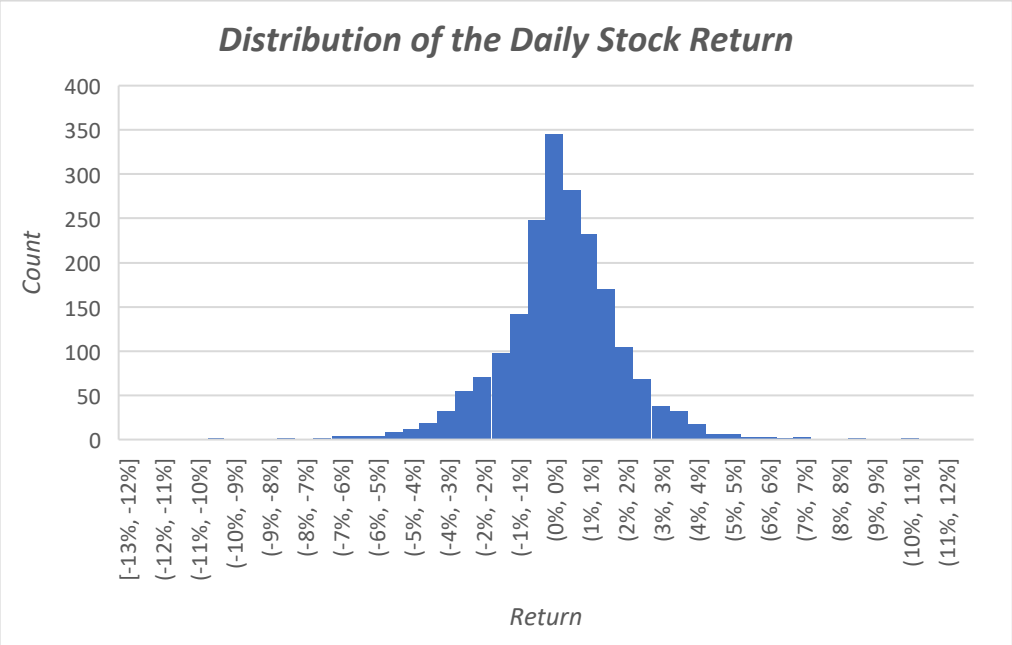

Figure 2 Histogram of the daily stock return

The states and their respective descriptions are shown below. We divided the return into four states by dividing the distribution shown above into four regions with top 25th percentile of the return values classified as being in state '4' and the bottom 25th percentile classified as being in state '1'. Classifying the states in this manner allows us to find the transition probabilities which can be used to simulate the stock price using Monte-Carlo simulation to evaluate the mean stock price after a certain time.

Table 1 Possible states in which the stock price change percent

State	***Description***	***Percentile Range of "Return" on the Stock Price***	***Corresponding Actual "Return" Range***
1	Bear	0 – 25 %	-12.87 % to -0.76 %
2	Stagnant	26 % - 50 %	-0.77 % to 0.08 %
3	Low Return	51 % - 75 %	0.09 % to 1.03 %
4	Bull	76 % - 100 %	1.04 % - 11.97 %

Now, the 16 possible instances of daily state transitions were counted and a one-step state transition matrix (P-matrix) was created. This matrix shows the probability of transition from one state to another. Summation of the transition probabilities from a fixed state to every other state is 1 i.e.,

$$\sum_{j=1}^{4} P_{ij} = 1 \quad \forall\, i = 1,2,3,4$$

P =

From/to	1	2	3	4
1	0.297	0.169	0.22	0.313
2	0.234	0.263	0.285	0.218
3	0.207	0.308	0.266	0.219
4	0.259	0.261	0.228	0.251

$$P_{ij} > 0 \;\; \forall \, i, j = 1,2,3,4$$

This Markov chain is 'Ergodic' and irreducible because all the transition probabilities are greater than zero. We also notice that the probability of moving from state Bear to state Bull is slightly higher than other transition probabilities. We now find the long-run probabilities and then try to simulate the stock price using the P-matrix shown above in python and look at the performance of the stock. We consider a 50-day period of ownership of the stock and observe the closing stock price every single day and evaluate the mean stock price.

III. Results and Discussion

As discussed in the previous section, we now calculate long-run or steady-state probabilities which will tell us the probability of being in a given state in the long run after many state transitions.

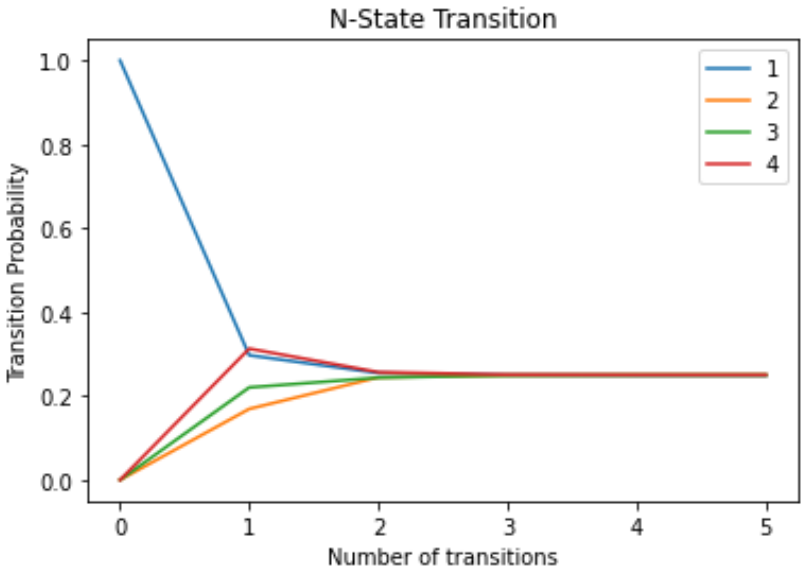

Figure 3 Long-run probability

As seen in Figure 3, if the stock return is initially in state 1, then the probabilities converge after 2 transitions indicating a steady state is achieved. It is equally likely that in the long run, the return of the stock will be in any of the four states 25% of the time.

$$\pi = (0.2493, 0.2504, 0.25, 0.2503)$$

The corresponding mean recurrence time, i.e., the average number of transitions required to reach the same state from which the transition began are shown below:

$$\mu_{jj} = \frac{1}{\pi_j}$$
$$\mu_{11} = 4.0112$$
$$\mu_{22} = 3.994$$
$$\mu_{33} = 4$$
$$\mu_{44} = 3.995$$

For instance, it will take four state transitions on average to have a Bull run again if the current return is in state Bull. Now, we attempt to simulate Apple's stock price by assuming the starting price of the stock to be $166 and the initial state to be 1.

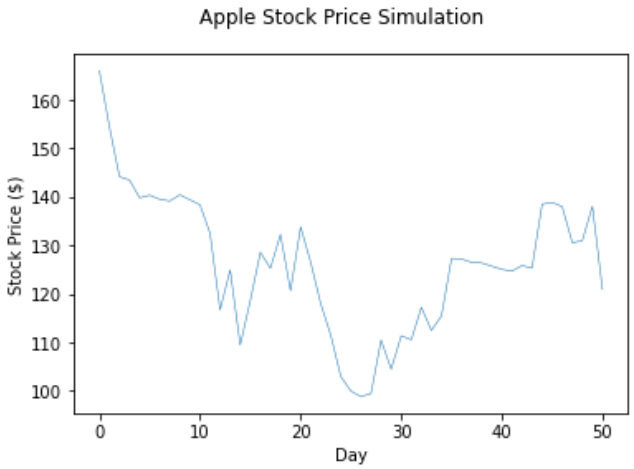

Figure 4 A single 50-day simulation of Apple stock price

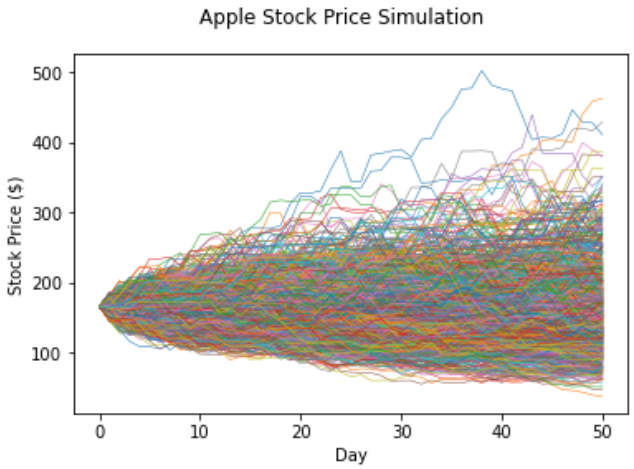

Figure 5 Thousand 50-day simulations of Apple's stock price

A typical 50-day simulation run of the future stock price is shown in Figure 4 and in this instance the closing value of the stock after 50 days of owning the stock was $120 and the investor loses money if he sells it on day 50. However, if he purchased the stock on Day 25, and sells it on Day 50, he will make a profit.

Now, if the simulation is run for 1000 times as shown in Figure 5, there are many possible trends and the investor can now get a sense of how the price might vary in the future. Figure 6 shows a histogram of the closing price on day 50 for all the 1000 simulations shown in Figure 5. The mean price of the stock is going to be $161.94 but the figure below shows the range of possible stock prices between $37.25 and

$462.95. The 5th percentile of the closing price on day 50 is $79.9 and the 95th percentile is $275.7.

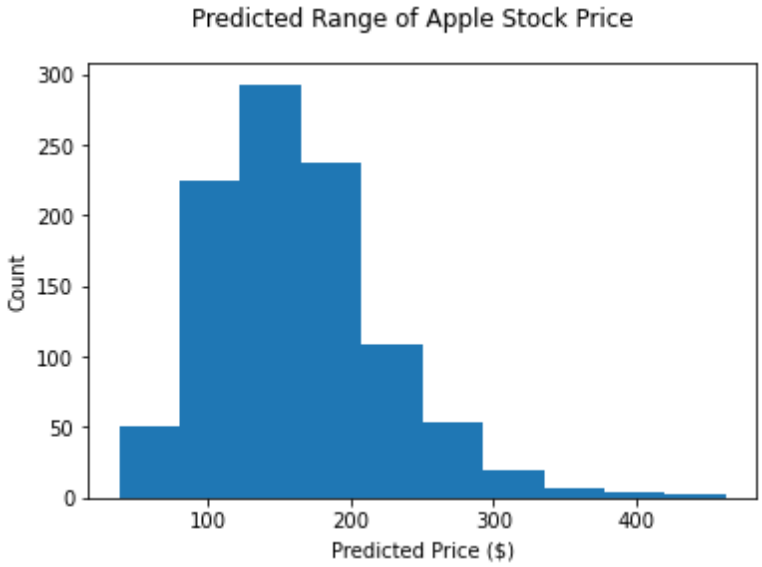

Figure 6 Histogram of the predicted price of Apple's stock at the end of day 50

IV. Conclusion

This class project demonstrates the application of Markov Chains to analyze a company's stock price and model it to simulate the future stock price. This was executed using Apple's stock price by calculating the return on the stock which was then ranked and divided into four states. By calculating the daily (one-step) transition probabilities of these four states, we were able to calculate the long-run probability i.e., the probability that in the long run the stock will give a return in the range specified by the states described in Table 1. We also found that on average, it would take a stock return value (state), 4 transitions to return to the same state/ return value.

Thousand 50-day simulations were executed with the initial price of the stock being $166 and the initial state being 1 and it revealed that though, on average, the stock price will be at $ 161.94 and 90% of the times, the value of the stock will lie between $ 79.9 and $ 275.7.

Hence, a similar simulation can be run for a shorter duration of 10 days or 5 days as well to see if there is a better time to buy or sell a stock and make a profit from it. It is also equally important to look out for the news on the company strategy, global event (example: pandemic), new product launch, general economic situation and change in government regulations before making investment decisions.