Mathematical Expectation

$\mu_X = E(X) = \sum_x x f(x)$, $E(X) = \int_{-\infty}^{+\infty} x f(x) dx$

$E(g(x)) = \sum_x g(x) f(x)$, $E(g(x)) = \int_{-\infty}^{+\infty} g(x) f(x) dx$

$\sigma_X^2 = V(X) = E(X^2) - (E(X))^2$

$\sigma_{XY} = Cov(X,Y) = E(XY) - E(X) * E(Y)$

If X & Y are Independent then $E(XY) = E(X) * E(Y) \Rightarrow$
$Cov(X,Y) = 0$.

Properties of Expectation.

1. $E(aX+b) = aE(X) + b$.
2. $E(X \pm Y) = E(X) \pm E(Y)$.
3. $E(XY) = E(X) * E(Y)$, if X & Y are Indep.

Properties of $V(X) = \sigma_X^2$.

(i) $V(C) = 0$ (ii) $V(CX) = c^2 V(X)$

(iii) $V(X \pm Y) = V(X) + V(Y) \pm 2\, Cov(X,Y)$

$= \sigma_X^2 + \sigma_Y^2 \pm 2\sigma_{XY}$

Note

$V(aX+bY) = \sigma_{aX+bY}^2 = a^2\sigma_X^2 + b^2\sigma_Y^2 + 2ab\,\sigma_{XY}$

If X & Y are independent Random Variables, then

$$\sigma_{aX+bY}^2 = a^2\sigma_X^2 + b^2\sigma_Y^2.$$

Wal
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4.43. Given $f(x) = \frac{1}{4} e^{-x/4}$, $x>0$, $Y = 3X-2$

$\mu_Y = ?$, $\sigma_Y^2 = ?$

Sol:

$$\mu_Y = E(Y) = E(3X-2) = 3E(X) - 2$$

$$\because E(X) = \frac{1}{4}\int_0^\infty x\, e^{-x/4} dx = \frac{1}{4}\Gamma 2 (4)^2 = 4$$

$$\because \int_0^\infty x^{\alpha-1} e^{-x/\beta} dx = \Gamma\alpha\, \beta^\alpha$$

$$\because \Gamma n = (n-1)!$$

$$\Rightarrow \mu_Y = 3(4) - 2 = 10 \text{ Ans}$$

$$\sigma_Y^2 = V(Y) = V(3X-2) = (3)^2 V(X) = 9\sigma_X^2 = 9(16) = 114 \text{ Ans}$$

$$\because \sigma_X^2 = E(X^2) - (E(X))^2$$

$$E(X^2) = \frac{1}{4}\int_0^\infty x^2 e^{-x/4} dx = \frac{1}{4}\Gamma 3 (4)^3 = 2\times 4^2 = 32$$

$$\sigma_X^2 = 32 - (16) = 16$$

4.55

x	-3	6	9
$f(x)$	$1/6$	$1/2$	$1/3$

Find $E(X)$ & $E(X^2)$ & $E(2X+1)^2 = ?$

$$E(X) = \sum_x x f(x) = -3\times\frac{1}{6} + 6\times\frac{1}{2} + 9\times\frac{1}{3} = 5.5$$

$$E(X^2) = \sum_x x^2 f(x) = (-3)^2\times\frac{1}{6} + (6)^2\times\frac{1}{2} + (9)^2\times\frac{1}{3} = 46.5$$

$$E(2X+1)^2 = E(4X^2 + 4X + 1) = 4E(X^2) + 4E(X) + 1$$

$$= 4(46.5) + 4(5.5) + 1 = \qquad \text{Ans}$$

4.58 :- Suppose X & Y are Independent R.V

$f(x,y)$	x: 2	4	$h(y)$
y 1	0.10	0.15	0.25
3	0.20	0.30	0.50
5	0.10	0.15	0.25
$g(x)$	0.40	0.60	1.00

Find
$E(2X-3Y) = ?$
$E(XY) = ?$

Sol. $E(2X-3Y) = 2E(X) - 3E(Y) = 2(\quad) - 3(\quad) =$

$E(X) = 2\times 0.40 + 4(0.6) =$

$E(Y) = 1\times 0.25 + 3\times(0.5) + 5(0.25) =$

$E(XY) = \sum_x \sum_y xy f(x,y)$

$= 2\times 1\times 0.10 + 2\times 3\times 0.20 + 2\times 5\times 0.10 + 4\times 1\times 0.15 + 4\times 3\times 0.30 + 4\times 5\times 0.15 =$

4.64: If X & Y are independent R.V's & $\sigma_x^2 = 5$, $\sigma_y^2 = 3$,
$Z = -2X + 4Y - 3$ $\quad V(Z) = ?$

Sol: $V(Z) = (-2)^2 V(X) + (4)^2 V(Y) = 4\times 5 + 16\times 3 = 68$ Ans

4.65: if $\sigma_{xy} = 1$ then $V(Z) = ?$

$V(Z) = (-2)^2 V(X) + (4)^2 V(Y) + 2(-2)(4)\, Cov(X,Y)$

$= 4\times 5 + 16\times 3 - 16\times 1 = 52$ Ans

4.70: Suppose that X & Y are independent R.V's with
$g(x) = 8/x^3$, $x > 2$, $\quad h(y) = 2y$, $0 < y < 1$

Let $Z = XY$, then $E(Z) = ?$

④

$$E(Z) = E(XY) = \iint_{y\,x} xy\, f(x,y)\, dx\, dy$$

$\because \quad f(x,y) = g(x) \times h(y) = \frac{8}{x^3} \times 2y \quad , \quad 2 < x < \infty$
$, \quad 0 < y < 1$

$$E(Z) = \int_0^1 \int_2^\infty xy \times \frac{16y}{x^3}\, dx\, dy$$

$$= 16 \int_0^1 \int_2^\infty \frac{y^2}{x^2}\, dx\, dy = 16 \int_0^1 \left| \frac{x^{-2+1}}{-2+1} \right|_2^\infty y^2 dy$$

$$E(Z) = +16 \int_0^1 \left| \frac{-1}{+x} \right|_2^\infty y^2 dy = \frac{16}{2} \int_0^1 y^2 dy = 8 \times \left| \frac{y^3}{3} \right|_0^1 = 8/3 \text{ Ans}$$

4.69/4.72

$X = 1, 2, 3, 4, 5, 6 \qquad Y = 1, 2, 3, 4, 5, 6$

$f(x) : 1/6, 1/6, 1/6, 1/6, 1/6, 1/6 \qquad f(y) = 1/6, 1/6, 1/6, 1/6, 1/6, 1/6$

$$E(X) = \frac{1}{6}(1+2+3+4+5+6) = \frac{21}{6} = 3.5 = E(Y)$$

(a) $E(X+Y) = E(X) + E(Y) = 3.5 + 3.5 = 7$ Ans

3.5

6|21

18

3|

(b) $E(X-Y) = E(X) - E(Y) = 3.5 - 3.5 = 0$ Ans

(c) $E(XY) = E(X) * E(Y) = 3.5 \times 3.5 = 12.25$ Ans

$V(2X - Y) = 4V(X) + V(Y)$

$V(X + 3Y - 5) = V(X) + 9V(Y) \qquad \because X \,\&\, Y$ are independent

$\cancel{V(X)}$ $E(X^2) = \left[(1)^2 + (2)^2 + (3)^2 + (4)^2 + (5)^2 + (6)^2 \right] \times \frac{1}{6}$

$$= \frac{1}{6}(1 + 4 + 9 + 16 + 25 + 36) = \frac{91}{6}$$

⑤

$$V(X) = \frac{91}{6} - (3.5)^2 = 2.92 = V(Y)$$

$$V(2X - Y) = 4(2.92) + 2.92 = 14.583$$

$$V(X + 3Y - 5) = 2.92 + 9(2.92) = 29.2$$

<u>4.71</u>:-

$$f(x,y) = \frac{2}{7}(x+2y), \quad 0<x<1, \quad 1<y<2$$

$$E\left(\frac{X}{Y^3} + X^2Y\right) = ?$$

Sol.

$$E\left(\frac{X}{Y^3} + X^2Y\right) = E\left(\frac{X}{Y^3}\right) + E(X^2Y)$$

$$E\left(\frac{X}{Y^3}\right) = \int_1^2\int_0^1 \frac{x}{y^3}\,\frac{2}{7}(x+2y)\,dx\,dy = \frac{2}{7}\int_1^2\int_0^1\left(\frac{x^2}{y^3} + \frac{2x}{y^2}\right)dx\,dy$$

$$= \frac{2}{7}\left[\int_1^2 \frac{1}{y^3}\left|\frac{x^3}{3}\right|_0^1 dy + 2\int_1^2 \frac{1}{y^2}\left|\frac{x^2}{2}\right|_0^1 dy\right]$$

$$= \frac{2}{7}\left[\frac{1}{3}\left|\frac{y^{-3+1}}{-3+1}\right|_1^2 + \left|\frac{y^{-2+1}}{-2+1}\right|_1^2\right]$$

$$= \frac{2}{7}\left[-\frac{1}{6}\left(\frac{1}{y^2}\right|_1^2 - \left(\frac{1}{y}\right|_1^2\right]$$

$$= \frac{2}{7}\left[-\frac{1}{6}\left(\frac{1}{4}-1\right) - \left(\frac{1}{2}-1\right)\right]$$

$$E(X/Y^3) = \frac{2}{7}\left(\frac{1}{8} + \frac{1}{2}\right) = 5/28$$

$$E(X^2Y) = \int_1^2\int_0^1 x^2y\,(\,)\,dx\,dy$$

$$= \int_1^2\int_0^1 (x^3y + 2x^2y^2)\,dx\,dy.$$

$x^2y \times (x+2y)$

$(x^3y + 2x^2y^2)$

$\frac{1}{y^3} \times \frac{2}{4} \times \frac{1}{6}$

$\left(\frac{1}{8} + \frac{4}{8}\right)$