

2. Determine whether each of these functions is $O(x^2)$.

a) $f(x) = 17x + 11$

$$O(x), 17x + 11 = C(x), k = 1, C = 28, O(x) \text{ therefore also } O(x^2)$$

b) $f(x) = x^2 + 1000$

$$O(x^2), x^2 + 1000 = C(x^2) \quad k = 10, C = 11$$

c) $f(x) = x \log x$

$$O(x), x \log x = C(x), k = 1, C = 2$$

d) $f(x) = x^4/2$

$$O(x^4) \text{ therefore not } O(x^2)$$

e) $f(x) = 2^x$

$$O(2^x) \text{ therefore not } O(x^2)$$

f) $f(x) = \lfloor x \rfloor \cdot \lceil x \rceil$

$$\text{floor}(x) * \text{ceil}(x) \leq x * (x+1) = x^2 + x = O(x^2), x^2 + x = C(x^2), k = 1, C = 2$$

4. Use the definition of “ $f(x)$ is $O(g(x))$ ” to show that $2^x + 17$ is $O(3^x)$.

$$2^x + 17 < 2^x + 2^x \text{ for } x > 5$$

In summary, we found that $2^x + 17 < 2 * 3^x$ for all $x > 5$.
Hence, $2^x + 17$ is $O(3^x)$.

8. Find the least integer n such that $f(x)$ is $O(x^n)$ for each of these functions.

a) $f(x) = 2x^2 + x^3 \log x \rightarrow O(x^4)$

b) $f(x) = 3x^5 + (\log x)^4 \rightarrow O(x^5)$

c) $f(x) = (x^4 + x^2 + 1)/(x^4 + 1) \rightarrow O(1)$

d) $f(x) = (x^3 + 5 \log x)/(x^4 + 1) \rightarrow n=-1$