

FIRST-ORDER LOGIC

RECAP

Our KB is a conjunction (\land) of

all rules, i.e.,

KB: $R_1 \wedge R_2 \wedge R_3 \wedge R_4 \wedge R_5$

 $R_3: B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$

 $R_4: \neg B_{1,1}$

 $R_5 : B_{2.1}$

 $(\alpha \wedge \beta) \equiv (\beta \wedge \alpha)$ commutativity of \wedge $(\alpha \vee \beta) \equiv (\beta \vee \alpha)$ commutativity of \vee $((\alpha \wedge \beta) \wedge \gamma) \equiv (\alpha \wedge (\beta \wedge \gamma))$ associativity of \wedge $((\alpha \vee \beta) \vee \gamma) \equiv (\alpha \vee (\beta \vee \gamma))$ associativity of \vee $\neg(\neg \alpha) \equiv \alpha$ double-negation elimination $(\alpha \Rightarrow \beta) \equiv (\neg \beta \Rightarrow \neg \alpha)$ contraposition $(\alpha \Rightarrow \beta) \equiv (\neg \alpha \lor \beta)$ implication elimination $(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha))$ biconditional elimination $\neg(\alpha \land \beta) \equiv (\neg \alpha \lor \neg \beta)$ De Morgan $\neg(\alpha \lor \beta) \equiv (\neg \alpha \land \neg \beta)$ De Morgan

	1.0		
1,3	2,3	3,3	4,3
1,2	2,2 P?	3,2	4,2
OK 1,1 V	2,1 A B	3,1 P?	4,1
OK	OK		

FIRST-ORDER LOGIC

- Propositional Logic is Useful --- but has Limited Expressive Power
 - E.g., cannot say "Pits cause breezes in adjacent squares."
 - except by writing one sentence for each square
 - Needs to refer to objects in the world,
 - Needs to express general rules
- First Order Predicate Calculus (FOPC), or First Order Logic (FOL).
 - → FOPC has greatly expanded expressive power, though still limited.
- New Ontology
 - → The world consists of OBJECTS (for propositional logic, the world was facts).
 - → OBJECTS have PROPERTIES and engage in RELATIONS and FUNCTIONS.
- New Syntax
 - Constants, Predicates, Functions, Properties, Quantifiers.

FIRST ORDER LOGIC

Objects (described by nouns in English, e.g., cat, John, house, wumpus, pit etc)
Relations (described by verbs in English e.g., has color, is bigger than, etc)
Facts describe relationship between objects and evaluate to True or False.

ONTOLOGICAL COMMITMENT

The primary difference between propositional and first-order logic lies in the **ontological commitment** made by each language—that is, what it assumes about the nature of *reality*.

Propositional logic assumes that there are facts that either hold or do not hold in the world.

First-order logic assumes more; namely, that the world consists of objects with certain relations among them that do or do not hold

PROPOSITIONAL LOGIC VS FIRST ORDER LOGIC

Language	Ontological Commitment (What exists in the world)	Epistemological Commitment (What an agent believes about facts)
Propositional logic First-order logic Temporal logic Probability theory Fuzzy logic	facts facts, objects, relations facts, objects, relations, times facts facts with degree of truth $\in [0,1]$	true/false/unknown true/false/unknown true/false/unknown degree of belief $\in [0, 1]$ known interval value

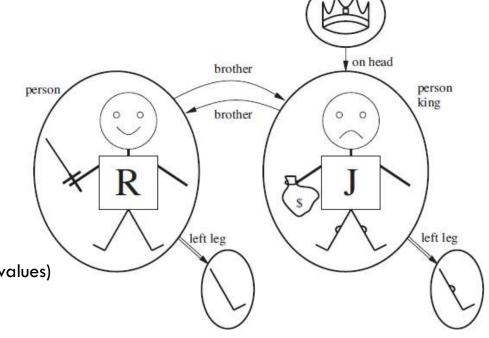
SYNTAX

Three kinds of symbols,

Constant: objects

Predicate: relations

Functions: functions (can return objects instead of binary values)



crown

Predicate and Functions have arity (number of arguments of a function)

Symbols have an interpretation

Complex Sentences: \neg King(Richard) \Rightarrow King(John) (If Richard isn't the king then john must be.

SYNTAX OF FOL: BASIC SYNTAX ELEMENTS ARE SYMBOLS

- Constant Symbols (correspond to English nouns)
 - Stand for objects in the world.
 - ◆ E.g., KingJohn, 2, UCI, ...
- Predicate Symbols (correspond to English verbs)
 - Stand for relations (maps a tuple of objects to a truth-value)
 - ★ E.g., Brother(Richard, John), greater_than(3,2), ...
 - P(x, y) is usually read as "x is P of y."
 - ◆ E.g., Mother(Ann, Sue) is usually "Ann is Mother of Sue."
- Function Symbols (correspond to English nouns)
 - Stand for functions (maps a tuple of objects to an object)
 - E.g., Sqrt(3), LeftLegOf(John), ...
- Model (world) = set of domain objects, relations, functions
- Interpretation maps symbols onto the model (world)
 - Very many interpretations are possible for each KB and world!
 - Job of the KB is to rule out models inconsistent with our knowledge.

SYNTAX OF FOL: ATOMIC SENTENCES

- Atomic Sentences state facts (logical truth values).
 - An atomic sentence is a Predicate symbol, optionally followed by a parenthesized list of any argument terms
 - ➤ E.g., Married(Father(Richard), Mother(John))
 - → An atomic sentence asserts that some relationship (some predicate) holds among the objects that are its arguments.
- An Atomic Sentence is true in a given model if the relation referred to by the predicate symbol holds among the objects (terms) referred to by the arguments.

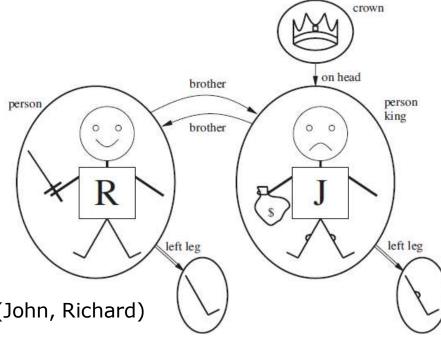
SYNTAX OF FOL: CONNECTIVES & COMPLEX SENTENCES

- Complex Sentences are formed in the same way, and are formed using the same logical connectives, as we already know from propositional logic
- The Logical Connectives:
 - ⇔ biconditional
 - ⇒ implication
 - ∧ and
 - v or
 - − ¬ negation
- **Semantics** for these logical connectives are the same as we already know from propositional logic.

SYNTAX

```
Sentence \rightarrow AtomicSentence \mid ComplexSentence
          AtomicSentence \rightarrow Predicate \mid Predicate(Term,...) \mid Term = Term
         ComplexSentence \rightarrow (Sentence) \mid [Sentence]
                                      \neg Sentence
                                      Sentence \land Sentence
                                      Sentence \lor Sentence
                                      Sentence \Rightarrow Sentence
                                      Sentence \Leftrightarrow Sentence
                                      Quantifier Variable, . . . Sentence
                        Term \rightarrow Function(Term,...)
                                       Constant
                                       Variable
                  Quantifier \rightarrow \forall \mid \exists
                   Constant \rightarrow A \mid X_1 \mid John \mid \cdots
                    Variable \rightarrow a \mid x \mid s \mid \cdots
                   Predicate \rightarrow True \mid False \mid After \mid Loves \mid Raining \mid \cdots
                   Function \rightarrow Mother | LeftLeg | \cdots
OPERATOR PRECEDENCE : \neg, =, \land, \lor, \Rightarrow, \Leftrightarrow
```

MODELS IN FOL

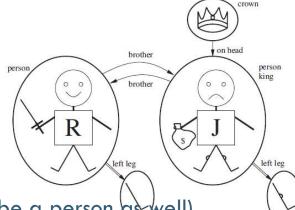


Examples

- Brother(Richard, John) ∧ Brother(John, Richard)
- King(Richard) v King(John)
- King(John) => ¬ King(Richard)

One possible model

UNIVERSAL QUANTIFICATION \(\neq \)



 \forall x King(x) \Rightarrow Person(x) (For every x such that x is a king, x will be a person as well)

 \Rightarrow says every king is a person, but every person is not a king.

 $M=\{x \to Richard, x \to John, x \to Richard's left leg, x \to John's left leg, x \to the crown\}$

Out of these possible five models, there's only one for which (King(x) is True which is $x \to J$ ohn and for this value of x, Person(x) will also be True)

We are not claiming for other 4 values that each one of those 4 cannot be a person!

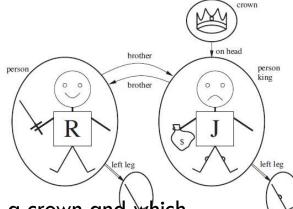
UNIVERSAL QUANTIFICATION

- Universal quantification is equivalent to:
 - Conjunction of all sentences obtained by substitution of an object for the quantified variable.
- All Cats are Mammals.
 - \forall x Cat(x) ⇒ Mammal(x)

Conjunction of all sentences obtained by substitution of an object for the quantified variable:

```
Cat(Spot) \Rightarrow Mammal(Spot) \land Cat(Rebecca) \Rightarrow Mammal(Rebecca) \land Cat(LAX) \Rightarrow Mammal(LAX) \land Cat(Shayama) \Rightarrow Mammal(Shayama) \land Cat(France) \Rightarrow Mammal(France) \land Cat(Felix) \Rightarrow Mammal(Felix) \land
...
```

EXISTENTIAL QUANTIFICATION =



 \exists x Crown(x) \land onHead(x,John) (There exists atleast one x that is a crown and which is placed on John's head.

Intuitively, the sentence $\exists x P$ says that P is true for at least one object x.

 $M=\{x \to Richard, x \to John, x \to Richard's left leg, x \to John's left leg, x \to the crown\}$. Therefore one of the following statements MUST BE correct.

- Richard the Lionheart is a crown ∧ Richard the Lionheart is on John's head;
- King John is a crown ∧ King John is on John's head;
- Richard's left leg is a crown ∧ Richard's left leg is on John's head;
- John's left leg is a crown ∧ John's left leg is on John's head;
- The crown is a crown ∧ the crown is on John's head

\exists , COMPARING \Rightarrow WITH \land

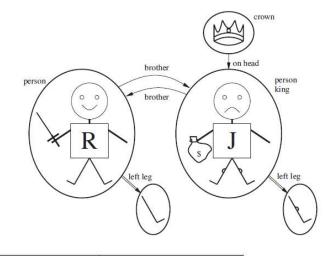
$\exists x \text{ Crown}(x) \land \text{ onHead}(x, \text{John})$

• There exists atleast one x that is a crown and which is placed on John's head.

$\exists x \text{ Crown}(x) \Rightarrow \text{onHead}(x, John)$

• If x exists such that it's a crown, then its placed on John's head.

But what for that x, which isn't a crown? i.e., for which Crown(x) is False, it can still imply onHead(x,John) is True.



P	Q	$P \Rightarrow Q$	
false	false	true	
false	true	true	
true	false	false	_
true	true	true	

EXISTENTIAL QUANTIFICATION

- Existential quantification is equivalent to:
 - → Disjunction of all sentences obtained by substitution of an object for the quantified variable.
- Spot has a sister who is a cat.
 - $\exists x \; Sister(x, Spot) \land Cat(x)$

Disjunction of all sentences obtained by substitution of an object for the quantified variable:

```
Sister(Spot, Spot) \land Cat(Spot) \lor Sister(Rebecca, Spot) \land Cat(Rebecca) \lor Sister(LAX, Spot) \land Cat(LAX) \lor Sister(Shayama, Spot) \land Cat(Shayama) \lor Sister(France, Spot) \land Cat(France) \lor Sister(Felix, Spot) \land Cat(Felix) \lor
```

...

NESTED QUANTIFIERS

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\forall x \forall y \text{ Brother } (x, y) \Rightarrow \text{Sibling}(x, y)
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All brothers are siblings

 \forall x,y Sibling(x, y) \Leftrightarrow Sibling(y, x) (siblinghood is a symmetric relationship)

• Consecutive quantifiers of the same type can be written as one quantifier with several variables.

ORDER OF QUANTIFICATION

Loves(x, y) means x loves y, (not necessarily the other way around), then,

```
• \forall x \exists y Loves(x, y) ..... Everybody loves somebody / For every x, there exists a y \forall x (\exists y Loves(x, y)) ..... that x loves
```

\blacksquare \exists y \forall x Loves(x, y)	There is someone who is loved by everyone / There	∃ y (∀ x
Loves(x, y))	exists a y that is loved by every x	

X	Y	
John	Sara	
Rob	Elizabeth	
Richard	Meryl	
Ned	Cristina	

```
\forall y (\exists x Loves(x, y)) ..... (meaning?) 
\exists x (\forall y Loves(x, y)) ..... (meaning?)
```

CONNECTIONS BETWEEN \forall AND \exists

The two quantifiers are actually intimately connected with each other, through negation.

 $\forall x \neg Likes(x, apples)$ is equivalent to $\neg \exists x Likes(x, apples)$

• everyone dislikes apples is the same as asserting there does not exist someone who likes them

 \forall x Likes(x, IceCream) is equivalent to $\neg \exists$ x \neg Likes(x, IceCream)

• Everyone likes ice cream means that there is no one who does not like ice cream

DEMORGAN'S LAWS

$$\forall x \neg P \equiv \neg \exists x P$$

• For every x, P is false \equiv There doesn't exists an x for which P is True

$$\neg \forall x P \equiv \exists x \neg P$$

• Not for every x is P true \equiv There exists an x for which P is False

$$\forall x P \equiv \neg \exists x \neg P$$

• For every x, P is True \equiv There does not exists an x for which P is False

$$\exists x P \equiv \neg \forall x \neg P$$

• There is an x for which P is True \equiv Not for every x is P False

EQUALITY

We can use the **equality symbol** to signify that two terms refer to the same object.

Father (John)=Henry

To say that Richard has at least two brothers, we would write

■ $\exists x, y \text{ Brother } (x, \text{Richard }) \land \text{ Brother } (y, \text{Richard }) \land \neg(x=y)$

The following sentence is true in the model where Richard has only one brother, i.e., where x and y refer to the same object

 \blacksquare \exists x, y Brother (x,Richard) \land Brother (y,Richard)

In both sentences, Richard can have more than 2 brothers

The notation $x \neq y$ is sometimes used as an abbreviation for $\neg(x=y)$

Brothers are siblings

Brothers are siblings

 $\forall \, x,y \; \, Brother(x,y) \, \Rightarrow \, Sibling(x,y).$

"Sibling" is symmetric

Brothers are siblings

 $\forall \, x,y \; \, Brother(x,y) \, \Rightarrow \, Sibling(x,y).$

"Sibling" is symmetric

 $\forall\, x,y \;\; Sibling(x,y) \; \Leftrightarrow \; Sibling(y,x).$

One's mother is one's female parent

Brothers are siblings

 $\forall \, x,y \;\; Brother(x,y) \; \Rightarrow \; Sibling(x,y).$

"Sibling" is symmetric

 $\forall\, x,y\ Sibling(x,y)\ \Leftrightarrow\ Sibling(y,x).$

One's mother is one's female parent

 $\forall \, x,y \;\; Mother(x,y) \; \Leftrightarrow \; (Female(x) \land Parent(x,y)).$

A first cousin is a child of a parent's sibling

Brothers are siblings

 $\forall \, x,y \; \, Brother(x,y) \, \Rightarrow \, Sibling(x,y).$

"Sibling" is symmetric

 $\forall x, y \ Sibling(x, y) \Leftrightarrow Sibling(y, x).$

One's mother is one's female parent

 $\forall x, y \; Mother(x, y) \Leftrightarrow (Female(x) \land Parent(x, y)).$

A first cousin is a child of a parent's sibling

 $\forall \, x,y \;\; FirstCousin(x,y) \; \Leftrightarrow \; \exists \, p,ps \;\; Parent(p,x) \land Sibling(ps,p) \land Parent(ps,y)$

- "All persons are mortal."
- [Use: Person(x), Mortal (x)]

"All persons are mortal."

[Use: Person(x), Mortal (x)]

- $\forall x \ Person(x) \Rightarrow Mortal(x)$
- Equivalent Forms:
- ∀x ¬Person(x) vMortal(x)
- Common Mistakes:
- ∀x Person(x) ∧ Mortal(x)

- "Fifi has a sister who is a cat."
- [Use: Sister(Fifi, x), Cat(x)]

ullet

- "Fifi has a sister who is a cat."
- [Use: Sister(Fifi, x), Cat(x)]
- $\exists x \; Sister(Fifi, x) \land Cat(x)$
- Common Mistakes:
- $\exists x \; Sister(Fifi, x) \Rightarrow Cat(x)$

• "For every food, there is a person who eats that food."

[Use: Food(x), Person(y), Eats(y, x)]

"For every food, there is a person who eats that food."

```
[Use: Food(x), Person(y), Eats(y, x)]
```

- $\forall x \exists y \ Food(x) \Rightarrow [Person(y) \land Eats(y, x)]$
- Equivalent Forms:
- $\forall x \text{ Food}(x) \Rightarrow \exists y \text{ [Person}(y) \land \text{Eats}(y, x) \text{]}$
- $\forall x \exists y \neg Food(x) \lor [Person(y) \land Eats(y, x)]$
- $\forall x \exists y [\neg Food(x) \lor Person(y)] \land [\neg Food(x) \lor Eats(y, x)]$
- $\forall x \exists y [Food(x) \Rightarrow Person(y)] \land [Food(x) \Rightarrow Eats(y, x)]$
- Common Mistakes:
- $\forall x \exists y [Food(x) \land Person(y)] \Rightarrow Eats(y, x)$
- $\forall x \exists y \ Food(x) \land Person(y) \land Eats(y, x)$

• "Every person eats every food."

[Use: Person (x), Food (y), Eats(x, y)]

"Every person eats every food."

```
[Use: Person (x), Food (y), Eats(x, y)]
```

- $\forall x \ \forall y \ [\ Person(x) \land Food(y) \] \Rightarrow Eats(x, y)$
- Equivalent Forms:
- ∀x ∀y ¬Person(x) ∨¬Food(y) ∨Eats(x, y)
- $\forall x \ \forall y \ \mathsf{Person}(x) \Rightarrow [\ \mathsf{Food}(y) \Rightarrow \mathsf{Eats}(x, y) \]$
- $\forall x \ \forall y \ \mathsf{Person}(x) \Rightarrow [\neg \mathsf{Food}(y) \lor \mathsf{Eats}(x, y)]$
- $\forall x \ \forall y \ \neg Person(x) \ \lor [Food(y) \Rightarrow Eats(x, y)]$
- Common Mistakes:
- $\forall x \ \forall y \ \mathsf{Person}(x) \Rightarrow [\mathsf{Food}(y) \land \mathsf{Eats}(x, y)]$
- ∀x ∀y Person(x) ∧ Food(y) ∧ Eats(x, y)

• "All greedy kings are evil."

[Use: King(x), Greedy(x), Evil(x)]

• "All greedy kings are evil."

```
[Use: King(x), Greedy(x), Evil(x)]
```

- $\forall x [Greedy(x) \land King(x)] \Rightarrow Evil(x)$
- Equivalent Forms:
- ∀x ¬Greedy(x) ∨¬King(x) ∨Evil(x)
- $\forall x \text{ Greedy}(x) \Rightarrow [\text{ King}(x) \Rightarrow \text{Evil}(x)]$
- Common Mistakes:
- $\forall x \text{ Greedy}(x) \land \text{King}(x) \land \text{Evil}(x)$

• "Everyone has a favorite food."

[Use: Person(x), Food(y), Favorite(y, x)]

"Everyone has a favorite food."

```
[Use: Person(x), Food(y), Favorite(y, x)]
```

Equivalent Forms:

- $\forall x \exists y \ Person(x) \Rightarrow [\ Food(y) \land Favorite(y, x)]$
- $\forall x \ Person(x) \Rightarrow \exists y \ [\ Food(y) \land \ Favorite(y, x) \]$
- ∀x ∃y ¬Person(x) √[Food(y) ∧ Favorite(y, x)]
- $\forall x \exists y [\neg Person(x) \forall Food(y)] \land [\neg Person(x)]$

• $\forall x \exists y [Person(x) \Rightarrow Food(y)] \land [Person(x) \Rightarrow Favorite(y, x)]$

Common Mistakes:

- $\forall x \exists y [Person(x) \land Food(y)] \Rightarrow Favorite(y, x)$
- $\forall x \exists y \ Person(x) \land Food(y) \land Favorite(y, x)$

• "There is someone at UCI who is smart."

[Use: Person(x), At(x, UCI), Smart(x)]

"There is someone at UCI who is smart."

[Use: Person(x), At(x, UCI), Smart(x)]

- $\exists x \ Person(x) \land At(x, UCI) \land Smart(x)$
- Common Mistakes:
- $\exists x [Person(x) \land At(x, UCI)] \Rightarrow Smart(x)$

"Everyone at UCI is smart."

[Use: Person(x), At(x, UCI), Smart(x)]

"Everyone at UCI is smart."

[Use: Person(x), At(x, UCI), Smart(x)]

- $\forall x [Person(x) \land At(x, UCI)] \Rightarrow Smart(x)$
- Equivalent Forms:
- $\forall x \neg [Person(x) \land At(x, UCI)] \lor Smart(x)$
- ∀x ¬Person(x) ∨¬At(x, UCI) ∨Smart(x)
- Common Mistakes:
- $\forall x \ Person(x) \land At(x, UCI) \land Smart(x)$
- $\forall x \ Person(x) \Rightarrow [At(x, UCI) \land Smart(x)]$

•

• "Every person eats some food."

[Use: Person (x), Food (y), Eats(x, y)]

"Every person eats some food."

```
[Use: Person (x), Food (y), Eats(x, y)]
```

Equivalent Forms:

• $\forall x \ \mathsf{Person}(x) \Rightarrow \exists y \ [\ \mathsf{Food}(y) \land \mathsf{Eats}(x,y) \]$

 $\forall x \exists y \ \mathsf{Person}(x) \Rightarrow [\ \mathsf{Food}(y) \land \mathsf{Eats}(x, y) \]$

- $\forall x \exists y \neg Person(x) \lor [Food(y) \land Eats(x, y)]$
- ∀x ∃y [¬Person(x) VFood(y)] ∧ [¬Person(x) VEats(x, y)]

Common Mistakes:

- $\forall x \exists y [Person(x) \land Food(y)] \Rightarrow Eats(x,y)$
- $\forall x \exists y \ Person(x) \land Food(y) \land Eats(x, y)$

•

"Some person eats some food."

[Use: Person (x), Food (y), Eats(x, y)]

"Some person eats some food."

[Use: Person (x), Food (y), Eats(x, y)]

- $\exists x \exists y \ Person(x) \land Food(y) \land Eats(x, y)$
- Common Mistakes:
- $\exists x \exists y [Person(x) \land Food(y)] \Rightarrow Eats(x, y)$