NAME: Kulsoom Khurshid

Reg # SP20-BCS-044

Question 1)

//input: int x, int n, array of integers.

//output: $a_nx^n + a_{n-1}x^{n-1} + \ldots\ldots + a_1x + a_0$

```
BruteForcePolynomialEvalution(P[0 …… n], x)

p <- p[0];

power <- 1;

for i <- to n do

        power <- power * x

        p <- p + p [i] * power

return p;
```

Complexity:

$$\sum_{i=0}^{n}(2)$$

n(2) = 2n = O(n)

Question 2)

//input: Array[0 …… n-1] (having 0's and 1's).

//output: Array[0 …… n-1] (sorted as 000… 1111…)

```
SortBalls(A[0 …… n-1])

for i <- 2n-1 to 1 do{

        flag <- false

        for j <- 0 to i – 1{

                if (A[i] > A[j+1]){

                        swap A[j + 1] and A[j]

                        flag <- true

                }}

        if (! flag)
```

return

}

Complexity:

The first iteration runs the condition for n times, for second iteration the condition runs for n times so $T(n) = n * n = n^2$

$O(n^2)$

Question 3)

```
//input: string s.

//output: int num

Algo1(s)

n <- s.length

num <- 0;

for i <- 1 to n do

        for j <- i + 1 to n do

                if a[i] = 'A' and a[j] = 'B'

                        num++

return num
```

Complexity:

$\sum_{i=1}^{n}(1) \sum_{j=i+1}^{n}(1)$

$O(n^2)$

Algo2(s)

```
n <- s.length

if A[n] = 'B'

        B[n] <- 1

else

        B[n] <- 0

For i <- n – 1 to 1 do

        If A[i] = 'B'

                B[i] <- B[i +1] + 1
```

```
        else
                B[i] <- B[i+1]
num <- 0
for i <- 1 to n – 1 do
        if A[i] = ‘A’ then
                num <- num + B[i+1]
return num
```

<u>Complexity:</u>

O(n)

Question 4)

```
//input: Boolean adjacency matrix A[0 … n-1, 0 … n-1], where n > 3
//output: ring = 1, star = 2, mesh = 3
M0 <- 0
for i <- 1 to n – 1 do
        M0 <- M0 + A[0,i]
        If M0 = 2
                return 1
        else if M0 = 1
                return 2
M1 <- 0
 for j <- 0 to n – 1 do
        M1 <- M1 + A[1,j]
        If M0 = M1
                return 3
        else
                return 2
```

<u>Complexity:</u>

$\sum_{i=0}^{n-1}(2) \sum_{j=0}^{n-1}(1)$

$n^2 = O(n^2)$

Question 5)

//input: array of coordinates of points, n (criteria of closeness)

//output: array without any points

b <- array of size a.length

for i <- 0 to a.length – 1

    //distance of a[i] from origin and insert in empty array b

//sort the new array b using merge sort

for i <- 0 to b.length – 1

    if distance(b[i],b[i+1]) $\leq$ n

        remove b[i] from array b

Complexity:

$$\sum_{i=0}^{a.len-1}(1) + n\log n + \sum_{j=0}^{a.len-1}(1)$$

n + n log n + n

O (n log n)

Question 6)

//input: array a, int num

//output: (i,j) or none

n <- a.length

//sort the array

for i <- 0 to n – 1

    j <- find (num – a[i])

    if j $\geq$ 0 and $\leq$ n

        return (i,j)

return none

Complexity:

$$\text{n}\log n + \sum_{i=0}^{n}(\log n)$$

n log n + n log n = 2 logn

O (n log n)

Question 7)

```
//input: array a, array b, int num
//output: (i,j) or none
n <- a.length
//sort array b
for i <- 0 to n – 1
        j <- find (num – a[i])
        if j ≥ b.length and 0 ≤ j
                return (i,j)
return none
```

<u>Complexity:</u>

$$\mathrm{n}\log n + \sum_{i=0}^{n} (\log n)$$

n log n + n log n = 2 logn

O (n log n)

Question 8)

```
T(s,e,array){
        If s == e
                Return array[s]
}
```

merge(TS(S,Le/3), array), TS(le/3) + 1, 2(le/3),array), TS(2 le/3) + e, n,array)

TS(n) {C if s == e, 3TS(n/3) + n

<u>Complexity:</u>

TS(n) = 3TS(n/3) + n

TS(n/3) = 3 TS(n/9) + n/3

TS(n/9) = 3TS(n/27)+ n/9

$T(n) = 3(3\ TS(n/9)+n/3) + 4$

$T(n) = 9(3\ TS(n/27) + 3n/9) + 24$

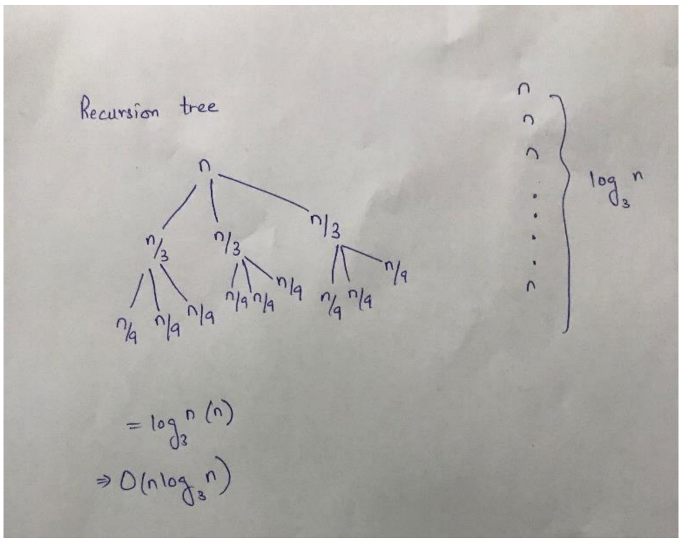
$T(n) = 3^i\ (TS(\ n / 3^i) + in$

$n/3^i = 1 \Rightarrow n = 3^i \Rightarrow \log_3 n = i$

$3^{\log_3 n} + n \log_3 n$

$O(n\log_3 n)$

Recursion Tree:



Master Theorem:

$a = 3,\ b = 3,\ d = 1$

$a = b^d$

$O(n \log_3 n)$

4. Binary merge sort is better because it compute less and has a time complexity $\log_3 n$ which is better. While in terms of space complexity it is same.

Question 9)

TS(l, r, val, arr)

If r>= 1

mid1 = 1 +(r-1)/3

mid2 = r + (r –l) /3

if val == arr[mid1]

return mid1

else if val == arr[mid2]

return mid2

if val < arr[mid1]

return TS(l, mid1-l, val, arr)

else if val > arr[mid2]

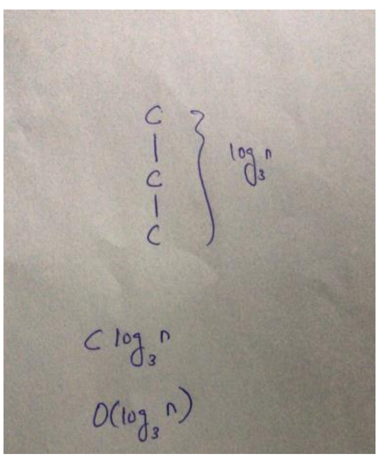
return TS(mid2+l, r, val, arr)

else

TS(mid1 +l, mid2 – l , val, arr)

TS(n) = {c if val == mid1 or val == mid2   TS(n/3) +c

I. <u>Back substitution</u>
   TS(n) = TS(n / 3) + c
   TS(n) = TS (n/9) + 2c
   TS(n) = TS($n/3^i$) + ic
   $n/3^i = 1 \Rightarrow i = \log_3 nc$
   O($\log_3 n$)
II. <u>Recursion tree:</u>



III. <u>Master theorem:</u>
   a = 1, b = 3, d = 0
   $a = b^d$
   $n^d \log_3 n$
   O($\log_3 n$)

Question 10)

I. <u>Brute force:</u>

```
ans <-  ans
for i <- 2 to n
        ans <- ans * a
return ans
```

<u>Complexity:</u>

$$\sum_{i=2}^{n} 1$$

n => O (n)

II. <u>Decrease by 1</u>:

```
f(a,n){
        if n = 1

                return a

        return a *f(a,n-1);

}
```

<u>Complexity:</u>

f(n) = f(n-1) + c

f(n) = f(n-2) + 2c

f(n) = f(n – n + 1) + (n – 1)c

f(n) = 1 + n – c

O(n)

III. <u>Decrease by factor:</u>

```
f(a,n){
        if n = 1
                return a
return f(a, (n/2))* f(a, (n/2))
}
```

<u>Complexity:</u>

T(n) = { c if n= 1 2T(n/2) + c

a = 2 , b = 2, n = 0

$a = b^d$

$n^{\log_2 2}$ => n

O(n)

IV. <u>Divide and conquer:</u>

f(a,n){

if n = 1

return a

return f(a, (n/2))* f(a, (n/2))

}

Complexity:

T(n) = { c   if n= 1   2T(n/2) + c

a = 2 , b = 2, n = 0

$a = b^d$

$n^{\log_2 2}$ => n

O(n)

Question 11)

a) Unsorted
max <- a[0]
small <- a[10]
for i <- 1 to n – 1
if max < a[i]
max = a[i]
else if small > a[i]
small = a[i]
return max – small

Complexity:

$$\sum_{i=1}^{n-1}(1)$$

=> O(n)

b) Sorted:
max <- a[a.len]
small <- a[0]
return max – small

Complexity:

C + C = 2 C => O(1)

c) An array concatenation of 2
max1 <- arr[last item of first sorted list]

max2 <- arr[last item of second sorted list]
min1 <- arr[first element of first list]
min2 <- arr[first item of second sorted list]
max <- max(max1, max2)
small <- min(min1, min2)
return max – small

Complexity:

C + C + C + C + C + C + C= 7 C => O(1)

d) Sorted linked list

max <- last element of list

min <- first element of list

return max – min

Complexity:

n + C = 7 C => O(n)

e) BST

max <- root

min <- smallest item

return max – min

Complexity:

C + log n= O(log n)

f) Comparison

It has same space complexity. While the sorted arrays has least operations to perform hence least time complexity.

Question 12)

a) $^nC_r = \frac{n!}{(n-r)!r!}$

b) $^nC_k = \binom{n-1}{k} + \binom{n-1}{k-1}$

c) dynamic programming is the best designing technique for this task because many subproblems are being repeated.

d) Brute force

For $\frac{n!}{(n-r)!r!}$
nfic <- 1
rfic <- 1
diffic <- 1
ans <-1

```
for i <- 1 to max(n,r)
      ans <- ans * i
      if i == n
            nfic <- ans
      if (n-r) == i
            diffic <- ans
      if r == n
            rfic <- ans
return nfic /(diffic * rfic)
```

<u>Complexity:</u>

$$\sum_{i=0}^{\max(n,r)} (1) + C$$

O(max(n,r))

<u>For</u> $\binom{n-1}{k} + \binom{n-1}{k-1}$

```
kfic <- 1
nlfic <- 1
klfic <- 1
dif1 <- 1
dif2 <- 1
ans <-1
for i <- 1 to max(n-1,k)
      ans <- ans * i
      if i == n-1
            nlfic <- ans
      if k-1 == i
            klfic <- ans
      if k == i
            kfic <- ans
      if (n-1) – (k-1) == i
            dif2 <- ans
      if (n-1)-k == i
            dif1 <- ans
return nlfic /(dif1 * kfic) + nlfic / difl + klfic
```

<u>Complexity:</u>

$$\sum_{i=0}^{\max(n-1,k)} (1) + C$$

O(max(n-1,k))

e) **Decease by one:**

For $\frac{n!}{(n-r)!r!}$

Let x = (n-r)

```
F(n,r,x){
    If 2n ≤ 1 and r ≤ 1
            return 1
    if x == 1
            return (n/ r) * f(n-1,r,x)

    if r == 1

            return n * f (n-1, 1, 1)

    return (n/(x*r))* f(n-1, r-1, x-1)

}
```

Complexity:

F(x) = { C if n ≤ 1

= { f(n-1) + c

f(n-1) = f(n-2) + 2C = f(n-3) + 3C

f(n-n) mC => nC

O(n)

f) Divide and Conquer:

For $\binom{n-1}{k} + \binom{n-1}{k-1}$

```
f(n,k){
    if (k > h)
            return 0
    if k = 0 || k == 1
            return 1
    return f(n-1, k-1)+ f(n-1, k)
}
```

Complexity:

F(x) = { C if n < k or k =1 or k = 0

= { T(n-1,k) + T(n-1,k-1)

O(2^n)

g) In this case, brute force approach is considered the best as it hae time complexity n and less space complexity.

Question 13)

The algorithm needs to list all possible walks of the robot (where the robot can take steps of 1 or 2 or 3 meter only), thus let us call the algorithm listwalk3.

//input: positive integer n and some string s (that is set to the null string).

- When n = 1, then there is l way for the robot to walk 1 meter: one step of 1 meter.
- When n = 2, then there are 2 ways for the robot to walk 2 meters: two steps of 1 meter each or one step of 2 meter.
- When n = 3, then there is 4 ways for the robot to walk 3 meters: three steps of 1 meter each, one step of 2 meter followed by one step of 1 meter, one step of 1 meter followed by one step of 2 meter, or one step of 3 meter.
- When n > 3, then the robot first takes a step of 1, 2 or 3 meter and there are then sn-1, Sn-2, and sn-3 Ways to walk the remaining meters.

//Input: n, s

//Output: A listing of all possible walks of the robot.

```
listwalk3(n,s) {

        if (n == 1) then

                println(s+"take one step of length 1")

                return

        if (n == 2) then

                println(s+"take two steps of length 1")

                println(s+"take one step of length 2")

                return

        if(n==3) then

                println( s+"take three steps of length 1")

                println(s+"take one step of length 2"-"take one step of length 1")

                println(s+"take one step of length 1"+"take one step of length 2")

                println(s+"take one step of length 3")

                return

        t=s+"take one step of length 1"

        listwalk3(n-1,t)

        u=s+"take one step of length 2"
```

```
        listwalk3(n -2,u)

        v=s+"take one step of length 3"

        listwalk3(n-3,v)

}
```