ASSIGNMENT 2

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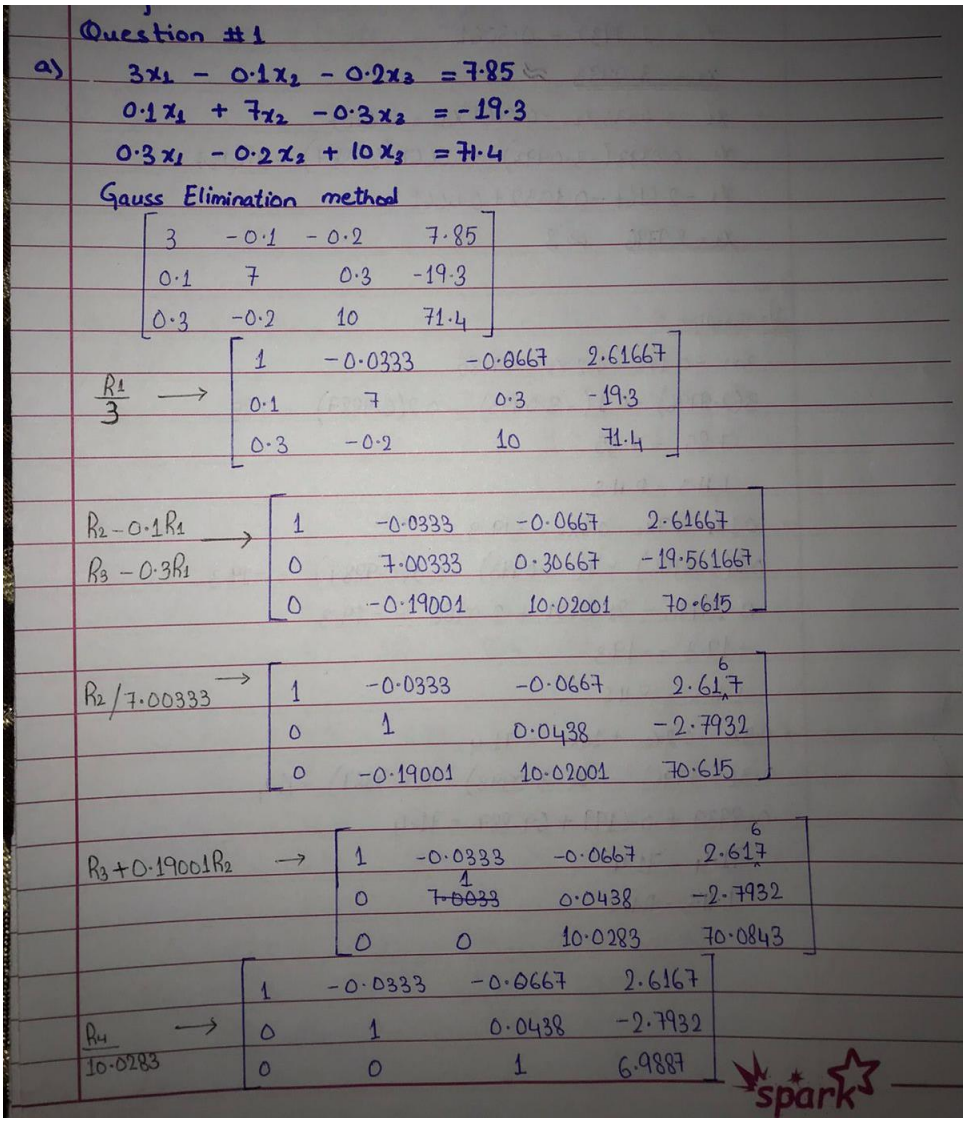
Reg #: Sp20-BCS-044

Course: Numerical Computing

Question 1)

Part a)



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→ $x_3 = 6.9887 \approx 7$

→ $x_2 + 0.0438\, x_3 = -2.7932$

$x_2 + 0.0438(6.9887) = -2.7932$

$x_2 = -2.7932 - 0.3061$

$x_2 = -3.0993 \approx -3$

→ $x_1 - 0.0333\, x_2 - 0.0667\, x_3 = 2.6167$

$x_1 - 0.0333(-3.0993) - 0.0667(6.9887) = 2.6167$

$x_1 = 2.6167 - 0.1032 + 0.4661$

$x_1 = 2.9796 \approx 3$

Verification :-

→ $3x_1 - 0.1x_2 - 0.2x_3 = 7.85$

$3(2.9796) - 0.1(-3.0993) - 0.2(6.9887) = 7.85$

$7.85 = 7.85$

L.H.S = R.H.S

→ $0.1x_1 + 7x_2 - 0.3x_3 = -19.3$

$0.1(2.9796) + 7(-3.0993) - 0.3(6.9887) = -19.3$

$0.29796 - 21.6951 + 2.0966 = -19.3$

$-19.3 = -19.3$

L.H.S = R.H.S

→ $0.3x_1 - 0.2x_2 + 10x_3 = 71.4$

$0.3(2.9796) - 0.2(-3.0993) + 10(6.9887) = 71.4$

$0.8939 + 0.6199 + 69.887 = 71.4$

$71.4 = 71.4$

L.H.S = R.H.S

Part b)

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b) $4x_1 - 2x_2 - 3x_3 + 6x_4 = 12$

$-6x_1 + 7x_2 + 6.5x_3 - 6x_4 = -6.5$

$x_1 + 7.5x_2 + 6.25x_3 + 5.5x_4 = 16$

$-12x_1 + 22x_2 + 15.5x_3 - x_4 = 17$

$$\begin{bmatrix} 4 & -2 & -3 & 6 & 12 \\ -6 & 7 & 6.5 & -6 & -6.5 \\ 1 & 7.5 & 6.25 & 5.5 & 16 \\ -12 & 22 & 15.5 & -1 & 17 \end{bmatrix}$$

Rearrange R_1 & R_3 (swap) $\longrightarrow$

$$\begin{bmatrix} 1 & 7.5 & 6.25 & 5.5 & 16 \\ -6 & 7 & 6.5 & -6 & -6.5 \\ 4 & -2 & -3 & 6 & 12 \\ -12 & 22 & 15.5 & -1 & 17 \end{bmatrix}$$

$R_2 + 6R_1$

$R_3 - 4R_1 \longrightarrow$

$R_4 + 12R_1$

$$\begin{bmatrix} 1 & 7.5 & 6.25 & 5.5 & 16 \\ 0 & 52 & 44 & 27 & 89.5 \\ 0 & -32 & -28 & -16 & -52 \\ 0 & 112 & 90.5 & 65 & 209 \end{bmatrix}$$

$\frac{R_2}{52} \longrightarrow$

$$\begin{bmatrix} 1 & 7.5 & 6.25 & 5.5 & 16 \\ 0 & 1 & 0.8462 & 0.5192 & 1.7212 \\ 0 & -32 & -28 & -16 & -52 \\ 0 & 112 & 90.5 & 65 & 209 \end{bmatrix}$$

$R_3 + 32R_2 \longrightarrow$

$R_4 - 112R_2$

$$\begin{bmatrix} 1 & 7.5 & 6.25 & 5.5 & 16 \\ 0 & 1 & 0.8462 & 0.5192 & 1.7212 \\ 0 & 0 & -0.9216 & 0.6144 & 3.0784 \\ 0 & 0 & -4.2744 & 6.8496 & 16.2256 \end{bmatrix}$$

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$\frac{R_3}{-0.9216} \longrightarrow$

1	7.5	6.25	5.5	16
0	1	0.8462	0.5192	1.7212
0	0	1	−0.6667	−3.3403
0	0	−4.2744	6.8496	16.2256

$R_4 + 4.2744 R_3 \longrightarrow$

1	7.5	6.25	5.5	16
0	1	0.8462	0.5192	1.7212
0	0	1	−0.6667	−3.3403
0	0	0	3.9999	1.9478

$\frac{R_4}{3.9999} \longrightarrow$

1	7.5	6.25	5.5	16
0	1	0.8462	0.5192	1.7212
0	0	1	−0.6667	−3.3403
0	0	0	1	0.4870

$\rightarrow x_4 = 0.4870$

$\rightarrow x_3 = -3.3403 + 0.3247$

$x_3 = -3.0156$

$\rightarrow x_2 = 1.7212 + 2.5518 - 0.2529$

$x_2 = 4.0201$

$\rightarrow x_1 = 16 - 30.1508 + 18.8475 - 2.6785$

$x_1 = 2.0182$

Verification:

$\rightarrow 4(2.0182) - 2(4.0201) - 3(-3.0156) + 6(0.4870) = 12$

$12.0 = 12$

$\rightarrow -6(2.0182) + 7(4.0201) + 6.5(-3.0156) - 6(0.4870) = -6.5$

$-6.5 = -6.5$

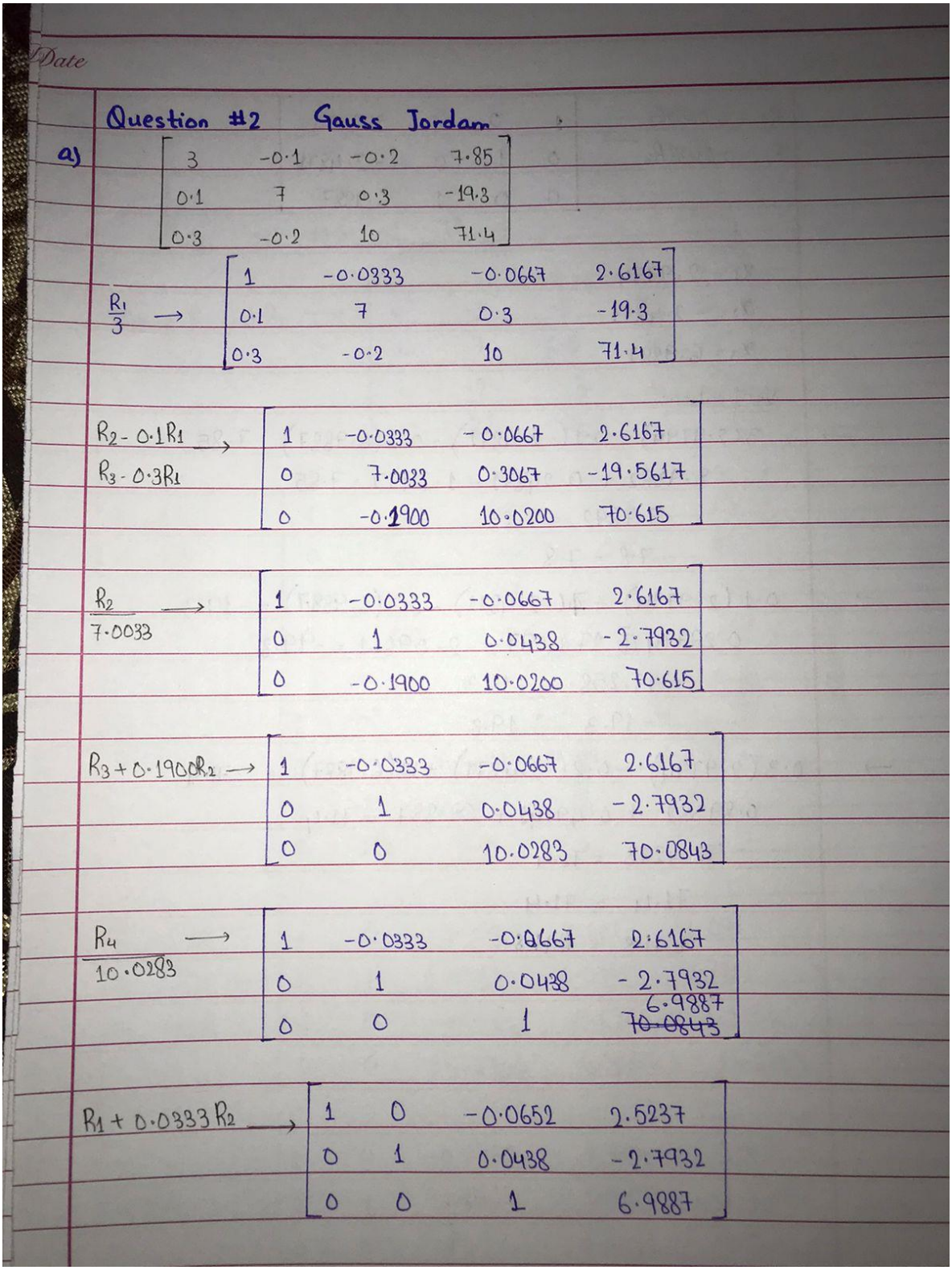
$\rightarrow 2.0182 + 7.5(4.0201) + 6.25(-3.0156) + 5.5(0.4870) = 16 \Rightarrow 16 = 16$

$\rightarrow -12(2.0182) + 22(4.0201) + 15.5(-3.0156) - 0.4870 = 17$

$17 = 17$

Question 2)

Part a)



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Question #2 Gauss Jordan

a)

$$\begin{bmatrix} 3 & -0.1 & -0.2 & 7.85 \\ 0.1 & 7 & 0.3 & -19.3 \\ 0.3 & -0.2 & 10 & 71.4 \end{bmatrix}$$

$\frac{R_1}{3} \longrightarrow$

$$\begin{bmatrix} 1 & -0.0333 & -0.0667 & 2.6167 \\ 0.1 & 7 & 0.3 & -19.3 \\ 0.3 & -0.2 & 10 & 71.4 \end{bmatrix}$$

$R_2 - 0.1R_1$, $R_3 - 0.3R_1 \longrightarrow$

$$\begin{bmatrix} 1 & -0.0333 & -0.0667 & 2.6167 \\ 0 & 7.0033 & 0.3067 & -19.5617 \\ 0 & -0.1900 & 10.0200 & 70.615 \end{bmatrix}$$

$\frac{R_2}{7.0033} \longrightarrow$

$$\begin{bmatrix} 1 & -0.0333 & -0.0667 & 2.6167 \\ 0 & 1 & 0.0438 & -2.7932 \\ 0 & -0.1900 & 10.0200 & 70.615 \end{bmatrix}$$

$R_3 + 0.1900R_2 \longrightarrow$

$$\begin{bmatrix} 1 & -0.0333 & -0.0667 & 2.6167 \\ 0 & 1 & 0.0438 & -2.7932 \\ 0 & 0 & 10.0283 & 70.0843 \end{bmatrix}$$

$\frac{R_4}{10.0283} \longrightarrow$

$$\begin{bmatrix} 1 & -0.0333 & -0.0667 & 2.6167 \\ 0 & 1 & 0.0438 & -2.7932 \\ 0 & 0 & 1 & 6.9887 \\ & & & \cancel{70.0843} \end{bmatrix}$$

$R_1 + 0.0333R_2 \longrightarrow$

$$\begin{bmatrix} 1 & 0 & -0.0652 & 2.5237 \\ 0 & 1 & 0.0438 & -2.7932 \\ 0 & 0 & 1 & 6.9887 \end{bmatrix}$$

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$R_1 + 0.0652R_3$	1	0	0	2.9794
$R_2 - 0.0438R_3$ →	0	1	0	−2.4871
	0	0	1	6.9887

$x_1 = 2.9794$

$x_2 = -2.4871$

$x_3 = 6.9887$

Verification:

→ $3(2.9794) - 0.1(-2.4871) - 0.2(6.9887) = 7.85$

$8.9382 + 0.24871 - 1.3977 = 7.85$

$7.7892 = 7.85$

$7.8 = 7.8$

→ $0.1(2.9794) + 7(-2.4871) - 0.3(6.9887) = -19.3$

$0.29794 \ominus 17.4097 - 2.09661 = -19.3$

$-19.258 = -19.3$

$-19.3 = -19.3$

→ $0.3(2.9794) - 0.2(-2.4871) + 10(6.9887) = 71.4$

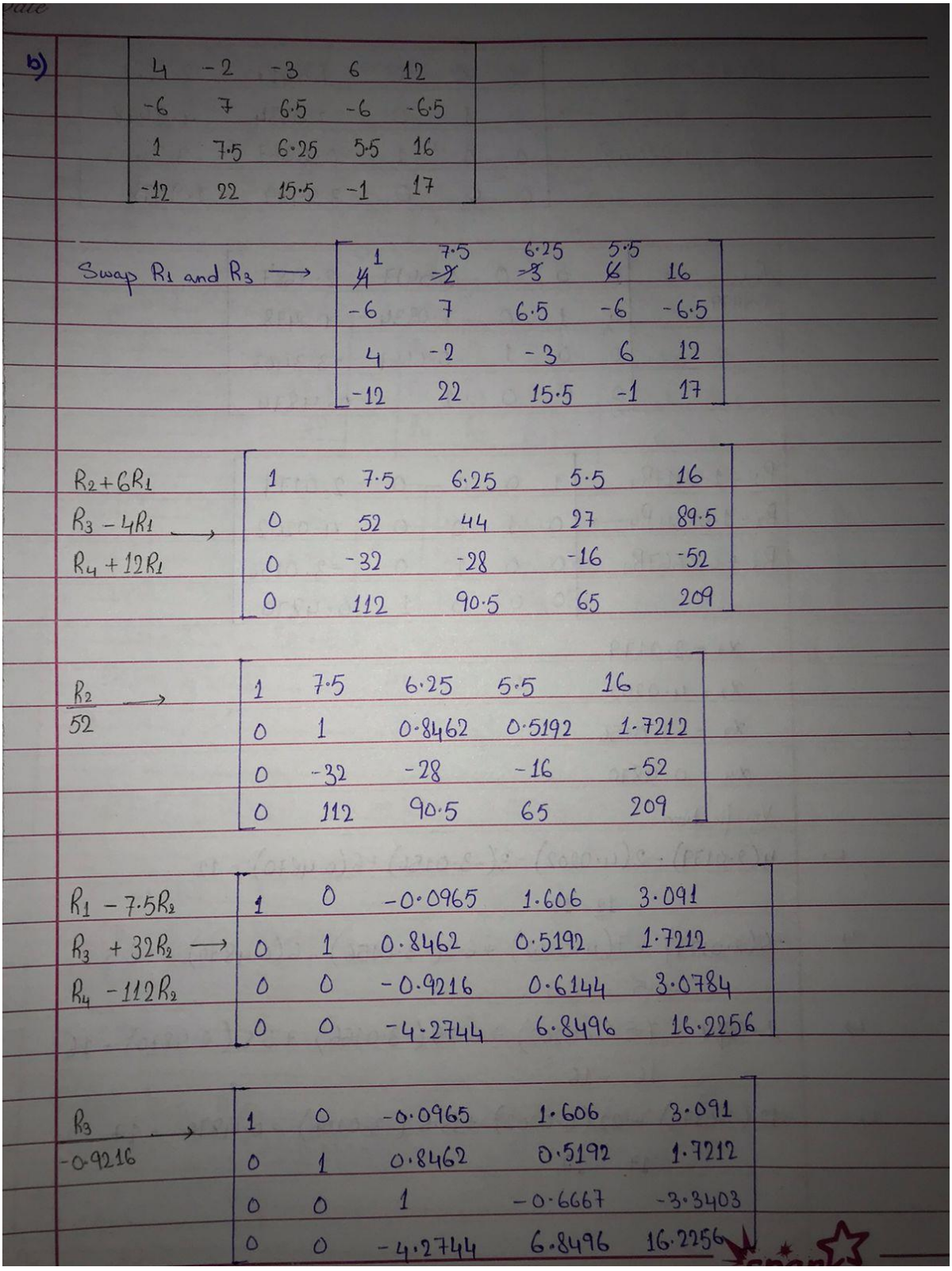
$0.89382 + 0.49742 + 69.887 = 71.4$

$71.4278 = 71.4$

$71.4 = 71.4$

L.H.S = R.H.S

Part b)



b)

4	-2	-3	6	12
-6	7	6.5	-6	-6.5
1	7.5	6.25	5.5	16
-12	22	15.5	-1	17

Swap R_1 and R_3 →

1	7.5	6.25	5.5	16
-6	7	6.5	-6	-6.5
4	-2	-3	6	12
-12	22	15.5	-1	17

$R_2 + 6R_1$
$R_3 - 4R_1$ →
$R_4 + 12R_1$

1	7.5	6.25	5.5	16
0	52	44	27	89.5
0	-32	-28	-16	-52
0	112	90.5	65	209

$\frac{R_2}{52}$ →

1	7.5	6.25	5.5	16
0	1	0.8462	0.5192	1.7212
0	-32	-28	-16	-52
0	112	90.5	65	209

$R_1 - 7.5R_2$
$R_3 + 32R_2$ →
$R_4 - 112R_2$

1	0	-0.0965	1.606	3.091
0	1	0.8462	0.5192	1.7212
0	0	-0.9216	0.6144	3.0784
0	0	-4.2744	6.8496	16.2256

$\frac{R_3}{-0.9216}$ →

1	0	-0.0965	1.606	3.091
0	1	0.8462	0.5192	1.7212
0	0	1	-0.6667	-3.3403
0	0	-4.2744	6.8496	16.2256

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$R_1 + 0.0965R_3$	1	0	0	1.5417	2.7687
$R_2 - 0.8462R_3 \longrightarrow$	0	1	0	1.0834	4.5478
$R_4 + 4.2744R_3$	0	0	1	-0.6667	-3.3403
	0	0	0	3.9999	1.9478

$R_4 \longrightarrow$ 3.9999					
	1	0	0	1.5417	2.7687
	0	1	0	1.0834	4.5478
	0	0	1	-0.6667	-3.3403
	0	0	0	1	0.4870

$R_1 - 1.5417R_4$	1	0	0	0	2.0179
$R_2 - 1.0834R_4 \longrightarrow$	0	1	0	0	4.0202
$R_3 + 0.6667R_4$	0	0	1	0	-3.0156
	0	0	0	1	0.4870

$x_1 = 2.0179$

$x_2 = 4.0202$

$x_3 = -3.0156$

$x_4 = 0.4870$

Verification:

↳ $4(2.0179) - 2(4.0202) - 3(-3.0156) + 6(0.4870) = 12$

$12 = 12$

↳ $-6(2.0179) + 7(4.0202) + 6.5(-3.0156) - 6(0.4870) = -6.5$

$-6.5 = -6.5$

↳ $2.0179 + 7.5(4.0202) + 6.25(-3.0156) + 5.5(0.4870) = 16$

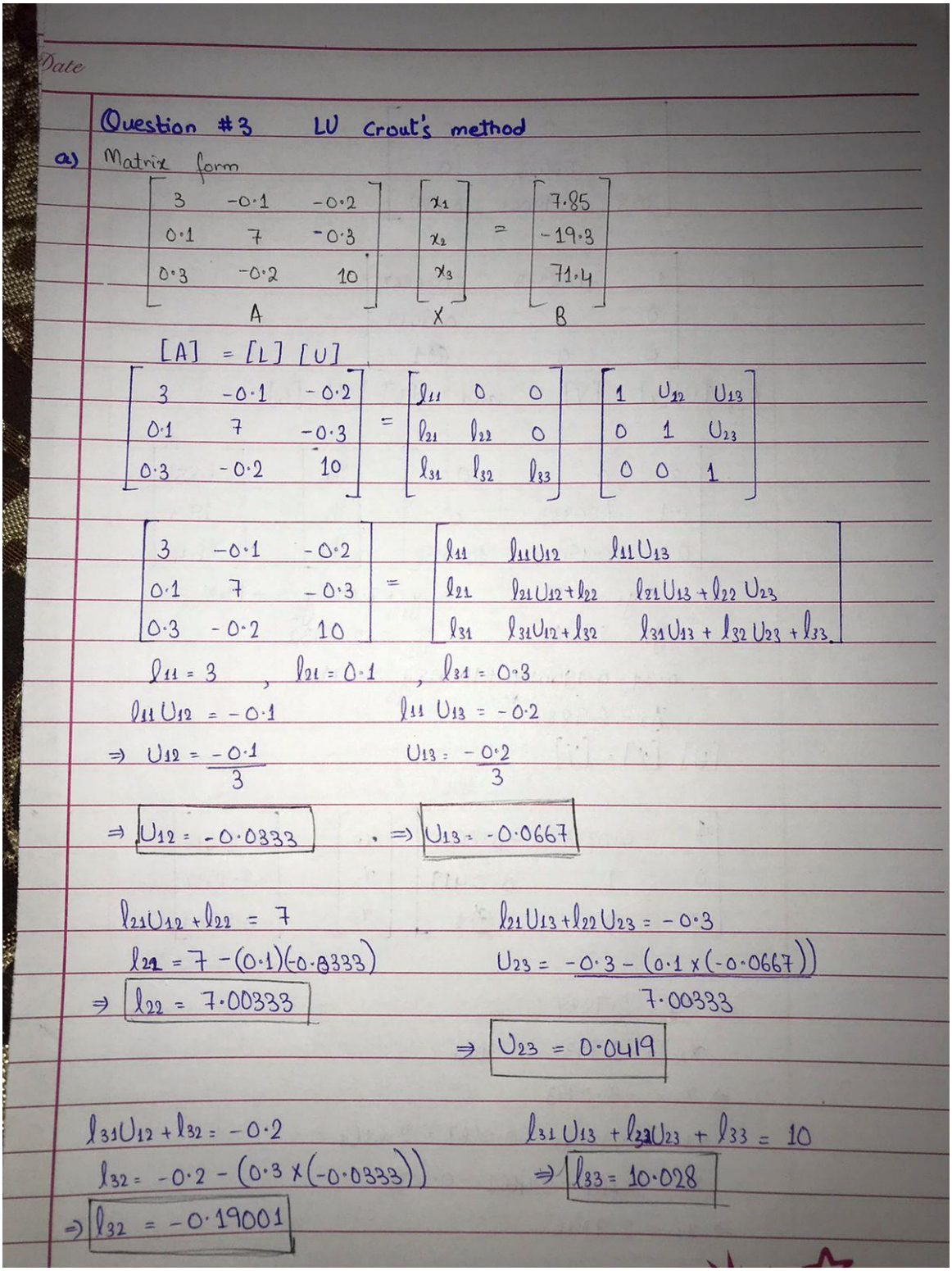
$16 = 16$

↳ $-12(2.0179) + 22(4.0202) + 15.5(-3.0156) - 0.4870 = 17$

$17 = 17$

Question 3)

Part a)



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Question #3 LU Crout's method

a) Matrix form

$$\begin{bmatrix} 3 & -0.1 & -0.2 \\ 0.1 & 7 & -0.3 \\ 0.3 & -0.2 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7.85 \\ -19.3 \\ 71.4 \end{bmatrix}$$

A X B

$[A] = [L][U]$

$$\begin{bmatrix} 3 & -0.1 & -0.2 \\ 0.1 & 7 & -0.3 \\ 0.3 & -0.2 & 10 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & U_{12} & U_{13} \\ 0 & 1 & U_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -0.1 & -0.2 \\ 0.1 & 7 & -0.3 \\ 0.3 & -0.2 & 10 \end{bmatrix} = \begin{bmatrix} l_{11} & l_{11}U_{12} & l_{11}U_{13} \\ l_{21} & l_{21}U_{12}+l_{22} & l_{21}U_{13}+l_{22}U_{23} \\ l_{31} & l_{31}U_{12}+l_{32} & l_{31}U_{13}+l_{32}U_{23}+l_{33} \end{bmatrix}$$

$l_{11} = 3$, $l_{21} = 0.1$, $l_{31} = 0.3$

$l_{11}U_{12} = -0.1$ $\quad$ $l_{11}U_{13} = -0.2$

$\Rightarrow U_{12} = \frac{-0.1}{3}$ $\quad$ $U_{13} = \frac{-0.2}{3}$

$\Rightarrow \boxed{U_{12} = -0.0333}$ $\quad$ $\Rightarrow \boxed{U_{13} = -0.0667}$

$l_{21}U_{12} + l_{22} = 7$ $\quad$ $l_{21}U_{13} + l_{22}U_{23} = -0.3$

$l_{22} = 7 - (0.1)(-0.0333)$ $\quad$ $U_{23} = \frac{-0.3 - (0.1 \times (-0.0667))}{7.00333}$

$\Rightarrow \boxed{l_{22} = 7.00333}$ $\quad$ $\Rightarrow \boxed{U_{23} = 0.0419}$

$l_{31}U_{12} + l_{32} = -0.2$ $\quad$ $l_{31}U_{13} + l_{32}U_{23} + l_{33} = 10$

$l_{32} = -0.2 - (0.3 \times (-0.0333))$ $\quad$ $\Rightarrow \boxed{l_{33} = 10.028}$

$\Rightarrow \boxed{l_{32} = -0.19001}$

$$L = \begin{bmatrix} 3 & 0 & 0 \\ 0.1 & 7.0033 & 0 \\ 0.3 & -0.19001 & 10.028 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & -0.0333 & -0.0667 \\ 0 & 1 & 0.0419 \\ 0 & 0 & 1 \end{bmatrix}$$

As, $[U][x] = [Y]$ and $[L][Y] = [b]$

$$\begin{bmatrix} 3 & 0 & 0 \\ 0.1 & 7.00333 & 0 \\ 0.3 & -0.19001 & 10.028 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 7.85 \\ -19.3 \\ 71.4 \end{bmatrix}$$

$3y_1 = 7.85$ $\quad 0.1y_1 + 7.0033y_2 = -19.3$

$y_1 = 2.6167$ $\quad y_2 = -2.7932$

$0.3y_1 - 0.19001y_2 + 10.028y_3 = 71.4$

$y_3 = 6.9889$

$[U][x] = [Y]$

$$\begin{bmatrix} 1 & -0.0333 & -0.0667 \\ 0 & 1 & 0.0419 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2.6167 \\ -2.7932 \\ 6.9889 \end{bmatrix}$$

$\Rightarrow x_3 = 6.9889$

$x_2 = -2.7932 - 0.3061$

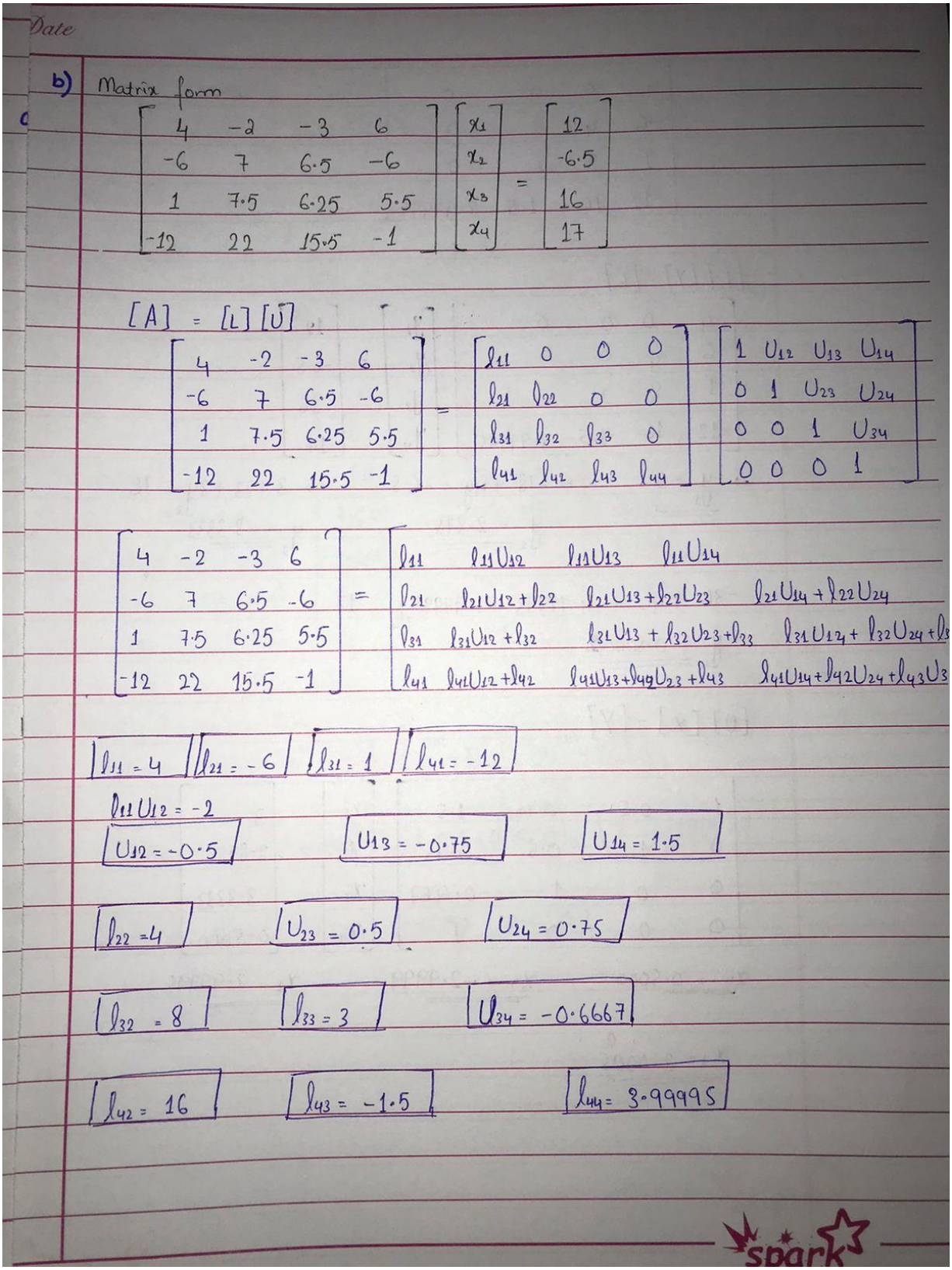
$\Rightarrow x_2 = -3.0993$

$x_1 - 0.0333x_2 - 0.0667 = 2.6167$

$x_1 = 2.6167 + 0.4661 - 0.1032$

$\Rightarrow x_1 = 2.9796$

Part b)



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$$L = \begin{bmatrix} 4 & 0 & 0 & 0 \\ -6 & 4 & 0 & 0 \\ 1 & 8 & 3 & 0 \\ -12 & 16 & -1.5 & 3.99995 \end{bmatrix}$$

$[L][Y] = [b]$

$$\begin{bmatrix} 4 & 0 & 0 & 0 \\ -6 & 4 & 0 & 0 \\ 1 & 8 & 3 & 0 \\ -12 & 16 & -1.5 & 3.99995 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 12 \\ -6.5 \\ 16 \\ 17 \end{bmatrix}$$

$\Rightarrow y_1 = 3$ $\quad \Rightarrow -18 + 4y_2 = -6.5$ $\quad \Rightarrow 3 + 23 + 3y_3 = 16$

$y_2 = 2.875$ $\quad y_3 = -3.3333$

$-36 + 46 + 4.99995 + 3.99995 y_4 = 17$

$y_4 = 0.50002$

$[U][x] = [Y]$

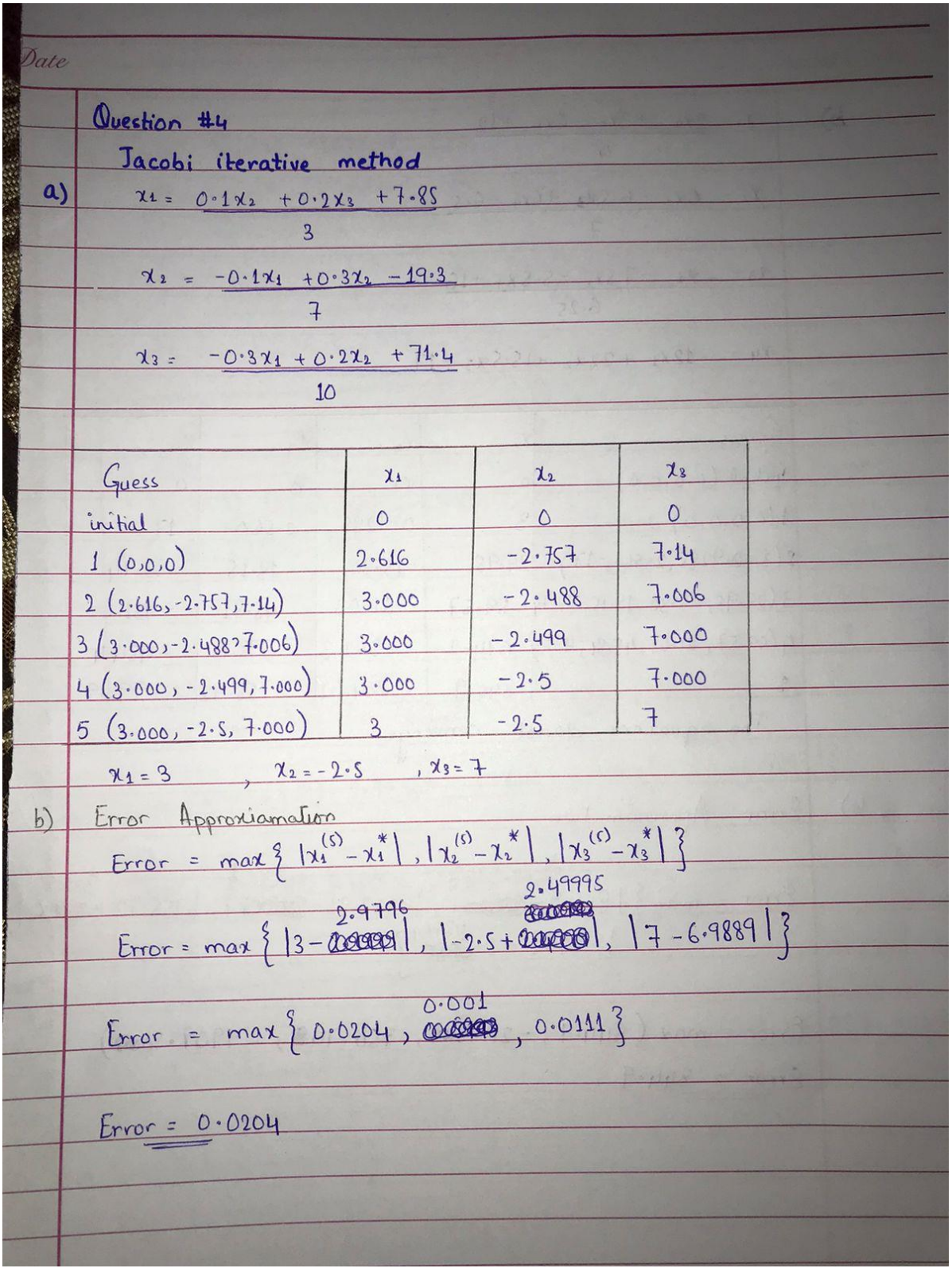
$$\begin{bmatrix} 1 & -0.5 & -0.75 & 1.5 \\ 0 & 1 & 0.5 & 0.75 \\ 0 & 0 & 1 & -0.6667 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 2.875 \\ -3.3333 \\ 0.5000 \end{bmatrix}$$

$x_4 = 0.5000$ $\quad x_3 = -2.9999$ $\quad x_2 = 3.99995$

$x_1 = 2.0005$

Question 4)

Part a) Jacobi Iterative Method and error Approximation



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Question #4

Jacobi iterative method

a)

$$x_1 = \frac{0.1x_2 + 0.2x_3 + 7.85}{3}$$

$$x_2 = \frac{-0.1x_1 + 0.3x_2 - 19.3}{7}$$

$$x_3 = \frac{-0.3x_1 + 0.2x_2 + 71.4}{10}$$

Guess	x_1	x_2	x_3
initial	0	0	0
1 (0,0,0)	2.616	-2.757	7.14
2 (2.616, -2.757, 7.14)	3.000	-2.488	7.006
3 (3.000, -2.488, 7.006)	3.000	-2.499	7.000
4 (3.000, -2.499, 7.000)	3.000	-2.5	7.000
5 (3.000, -2.5, 7.000)	3	-2.5	7

$x_1 = 3$, $x_2 = -2.5$, $x_3 = 7$

b) Error Approximation

$$\text{Error} = \max\left\{ |x_1^{(s)} - x_1^*| , |x_2^{(s)} - x_2^*| , |x_3^{(s)} - x_3^*| \right\}$$

$$\text{Error} = \max\left\{ |3 - 2.9796| , |-2.5 + 2.49995| , |7 - 6.9889| \right\}$$

$$\text{Error} = \max\left\{ 0.0204 , 0.001 , 0.0111 \right\}$$

$$\text{Error} = 0.0204$$

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b)

$$x_1 = \frac{2x_2 + 3x_3 - 6x_4 + 12}{4}$$

$$x_2 = \frac{6x_1 - 6.5x_3 + 6x_4 - 6.5}{7}$$

$$x_3 = \frac{-x_1 - 7.5x_2 - 5.5x_4 + 16}{6.25}$$

$$x_4 = -12x_1 + 22x_2 + 15.5x_3 - 17$$

Guess	x_1	x_2	x_3	x_4
Initial (0,0,0,0)	0	0	0	0
1(0,0,0,0)	3	+0.9285	2.560	-17
2(3,-0.9285,2.560,-17)	29.95	-15.30	18.15	-33.74
3(29.95,-15.30,18.15,33.74	59.57	-21.03	45.81	-431.6
4(59.57,-21.03,48.81,431.6	674.2	-362.3	398.0	-4844
5	846.9	-207.81	755.72	-9909

The equations donot converge.

b) Error Approximation

$$\text{Error} = \max\{|846.9 - 2.0182|, |-207.81 - 4.0201|, |755.72 + 3.0156|, |-9909 - 0.4870|\}$$

$$\text{Error} = \max(844.9, -211.8, 758.74, -9909.5)$$

$$\text{Error} = 844.9$$

Part b) Gauss Seidal Method and error Approximation

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ii **Gauss Seidal Method**

a) $x_1 = \frac{0.1x_2 + 0.2x_3 + 7.85}{3}$

$x_2 = \frac{-0.1x_1 + 0.3x_3 - 19.3}{7}$

$x_3 = \frac{-0.3x_1 + 0.2x_2 + 71.4}{10}$

Guess	x_1	x_2	x_3
initial	0	0	0
1 $(0, 0, 0)$, $(2.6167, 0, 0)$, $(2.6167, -2.7945, 0)$	2.6167	-2.7945	7.0056
2 $(2.6167, -2.7945, 7.0056)$,	2.9906	-2.4996	7.0003
3	3.0000	-2.49999	7.0000
4	3.0000	-2.5	7.000
5	3	-2.5	7

$x_1 = 3$ $x_2 = -2.5$ $x_3 = 7$

b) Error = $\max\{|3 - 2.9796|, |-2.5 + 3.0993|, |7 - 6.9887|\}$

Error = $\max\{0.0204, 0.6197, 0.0113\}$

Error = 0.6197

b) $x_1 = \frac{2x_2 + 3x_3 - 6x_4 + 12}{4}$ $x_4 = -12x_1 + 22x_2 + 15.5x_3 - 17$

$x_2 = \frac{6x_1 - 6.5x_3 + 6x_4 - 6.5}{7}$

$x_3 = \frac{-x_1 - 7.5x_2 - 5.5x_4 + 16}{6.25}$

Guess	x_1	x_2	x_3	x_4
1	3	1.642	0.1096	-15.17
2	26.65	0.809	1.074	-126.3
3	197.66	59.23	11.00	-915.36
4	1413.9	416.1	82.53	-6550
5	10097	2962	596.6	-46769

The equations do not converge

b) Error Approximation

$$\text{Error} = \max\{|10097 - 2.0182|, |2962 - 4.0201|, |596.6 + 3.0156|, |-46769 - 0.4870|\}$$

$$\text{Error} = \max\{10094.98, 2957.98, 599.62, -46769.5\}$$

$$\text{Error} = 10094.98$$