



COMSATS University, Islamabad

Department of Computer Science

Assignment 3

CSC475: Numerical Computing

CLO-4

Question # 1:

Use least-squares regression to fit a straight line to

x	1	3	5	7	10	12	13	16	18	20
y	4	5	6	5	8	7	6	9	12	11

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Assignment #3
Numerical Computing

Question #1
Use least-squares regression to fit a straight line

Predicted Values
 $\hat{y} = a + bx = 3.38875 + (0.3725)x$

x	y	x ²	xy
1	4	1	4
3	5	9	15
5	6	25	30
7	5	49	35
10	8	100	80
12	7	144	84
13	6	169	78
16	9	256	144
18	12	324	216
20	11	400	220
$\Sigma x = 105$	$\Sigma y = 73$	$\Sigma x^2 = 1477$	$\Sigma xy = 906$

x	\hat{y}
1	3.76125
3	4.50625
5	5.25125
7	5.99625
10	7.11375
12	7.85875
13	8.23125
16	9.34875
18	10.09375
20	10.83875

$$\bar{y} = \frac{\Sigma y}{n} = \frac{73}{10} = 7.3$$
$$\bar{x} = \frac{\Sigma x}{n} = \frac{105}{10} = 10.5$$
$$b = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\Sigma x^2 - \frac{(\Sigma x)^2}{n}} = \frac{906 - \frac{(105)(73)}{10}}{1477 - \frac{(105)^2}{10}} = \frac{906 - 766.5}{1477 - 1102.5} = \frac{139.5}{374.5} = 0.3725$$
$$a = \bar{y} - b\bar{x} = 7.3 - (0.3725)(10.5) = 3.38875$$
$$y = a + bx$$
$$y = 3.38875 + (0.3725)x$$

Ans

Question # 2:

Fit a second order polynomial to data:

x	0	1	2	3	4	5
y	2.1	7.7	13.6	27.2	40.9	61.1

Question # 2
Fit a second order polynomial to data

x	y	x^2	x^3	x^4	xy	x^2y
0	2.1	0	0	0	0	0
1	7.7	1	1	1	7.7	7.7
2	13.6	4	8	16	27.2	54.4
3	27.2	9	27	81	81.6	244.8
4	40.9	16	64	256	163.6	654.4
5	61.1	25	125	625	305.5	1527.5
$\sum x = 15$	$\sum y = 152.6$	$\sum x^2 = 55$	$\sum x^3 = 225$	$\sum x^4 = 979$	$\sum xy = 585.6$	$\sum x^2y = 2488.8$

$$\begin{bmatrix} n & \sum x_i & \sum x_i^2 \\ \sum x_i & \sum x_i^2 & \sum x_i^3 \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum xy_i \\ \sum x_i^2 y_i \end{bmatrix}$$

$$\begin{bmatrix} 6 & 15 & 55 \\ 15 & 55 & 225 \\ 55 & 225 & 979 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 152.6 \\ 585.6 \\ 2488.8 \end{bmatrix}$$

$$|D| = 6 \begin{bmatrix} 55 & 225 \\ 225 & 979 \end{bmatrix} - 15 \begin{bmatrix} 15 & 225 \\ 55 & 979 \end{bmatrix} + 55 \begin{bmatrix} 15 & 55 \\ 55 & 225 \end{bmatrix}$$

$$|D| = 6(53845 - 50625) - 15(14685 - 12375) + 55(3375 - 3025)$$

$$|D| = 6(3220) - 15(2310) + 55(350)$$

$$|D| = 19320 - 34650 + 19250$$

$$|D| = 3920$$

For $|D_a|$

$$\begin{bmatrix} 152.6 & 15 & 55 \\ 585.6 & 55 & 225 \\ 2488.8 & 225 & 979 \end{bmatrix}$$

$$|D_a| = 152.6 \begin{bmatrix} 55 & 225 \\ 225 & 979 \end{bmatrix} - 585.6 \begin{bmatrix} 15 & 225 \\ 2488.8 & 979 \end{bmatrix} + 55 \begin{bmatrix} 585.6 & 55 \\ 2488.8 & 225 \end{bmatrix}$$

$$|D_a| = 152.6 (53845 - 50625) - 15 (573302.4 - 559980) + 55 (131760 - 136884)$$

$$|D_a| = 152.6 (3220) - 15 (13322.4) + 55 (-5124)$$

$$|D_a| = 491372 - 199836 - 281820$$

$$|D_a| = 9716$$

For $|D_b|$

$$|D_b| = \begin{bmatrix} 6 & 152.6 & 55 \\ 15 & 585.6 & 225 \\ 55 & 2488.8 & 979 \end{bmatrix}$$

$$|D_b| = 6 \begin{bmatrix} 585.6 & 225 \\ 2488.8 & 979 \end{bmatrix} - 152.6 \begin{bmatrix} 15 & 225 \\ 55 & 979 \end{bmatrix} + 55 \begin{bmatrix} 15 & 585.6 \\ 55 & 2488.8 \end{bmatrix}$$

$$|D_b| = 6 (573302.4 - 559980) - 152.6 (14685 - 12375) + 55 (37332 - 32208)$$

$$|D_b| = 6 (13322.4) - 152.6 (2310) + 55 (5124)$$

$$|D_b| = 79934.4 - 352506 + 281820$$

$$|D_b| = 9248.4$$

For $|D_c|$

$$|D_c| = \begin{bmatrix} 6 & 15 & 152.6 \\ 15 & 55 & 585.6 \\ 55 & 225 & 2488.8 \end{bmatrix}$$

$$|D_c| = 6 \begin{bmatrix} 55 & 585.6 \\ 225 & 2488.8 \end{bmatrix} - 15 \begin{bmatrix} 15 & 585.6 \\ 55 & 2488.8 \end{bmatrix} + 152.6 \begin{bmatrix} 15 & 55 \\ 55 & 225 \end{bmatrix}$$

$$|D_c| = 6 (136884 - 131760) - 15 (37332 - 32208) + 152.6 (3375 - 3025)$$

$$|D_c| = 6 (5124) - 15 (5124) + 152.6 (350)$$

$$|D_c| = 30744 - 76860 + 53410$$

$$|D_c| = 7294$$

$$a = \frac{|D_a|}{|D|} = \frac{9716}{3920} = 2.4786$$

$$b = \frac{|D_b|}{|D|} = \frac{9248.4}{3920} = 2.3593$$

$$c = \frac{|D_c|}{|D|} = \frac{7294}{3920} = 1.8607$$

Equation

$$\hat{y} = a + bx + cx^2$$

$$\hat{y} = 2.4786 + 2.3593x + 1.8607x^2$$

Question # 3:

Use multiple linear regression to fit

x	0	1	1	2	2	3	3	4	4
y	0	1	2	1	2	1	2	1	2
z	15	18	12.8	25.7	20.6	35	29.8	45.5	40.3

Question # 3

Use multiple line regression.

x_1	x_2	y	x_1^2	x_2^2	$x_1 x_2$	$x_1 y$	$x_2 y$
0	0	15	0	0	0	0	0
1	1	18	1	1	1	18	18
1	2	12.8	1	4	2	12.8	25.6
2	1	25.7	4	1	2	51.4	25.7
2	2	20.6	4	4	4	41.2	41.2
3	1	35	9	1	3	105	35
3	2	29.8	9	4	6	89.4	59.6
4	1	45.5	16	1	4	182	45.5
4	2	40.3	16	4	8	161.2	80.6
$\sum x_1 = 20$	$\sum x_2 = 12$	$\sum y = 242.7$	$\sum x_1^2 = 60$	$\sum x_2^2 = 20$	$\sum x_1 x_2 = 30$	$\sum x_1 y = 661$	$\sum x_2 y = 331.2$

$$\begin{bmatrix} n & \sum x_1 & \sum x_2 \\ \sum x_1 & \sum x_1^2 & \sum x_1 x_2 \\ \sum x_2 & \sum x_1 x_2 & \sum x_2^2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \sum y \\ \sum x_1 y \\ \sum x_2 y \end{bmatrix}$$

$$\begin{bmatrix} 9 & 20 & 12 \\ 20 & 60 & 30 \\ 12 & 30 & 20 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 242.7 \\ 661 \\ 331.2 \end{bmatrix}$$

$$\begin{aligned}
 |D| &= 9 \begin{bmatrix} 60 & 30 \\ 30 & 20 \end{bmatrix} - 20 \begin{bmatrix} 20 & 30 \\ 12 & 20 \end{bmatrix} + 12 \begin{bmatrix} 20 & 60 \\ 12 & 30 \end{bmatrix} \\
 &= 9(1200 - 900) - 20(400 - 360) + 12(600 - 720) \\
 &= 9(300) - 20(40) + 12(-120) \\
 &= 2700 - 800 - 1440 \\
 &= 460
 \end{aligned}$$

$$\begin{bmatrix} 242.7 & 20 & 12 \\ 661 & 60 & 30 \\ 331.2 & 30 & 20 \end{bmatrix}$$

$$\begin{aligned}
 |D_a| &= 242.7 \begin{bmatrix} 60 & 30 \\ 30 & 20 \end{bmatrix} - 20 \begin{bmatrix} 661 & 30 \\ 331.2 & 20 \end{bmatrix} + 12 \begin{bmatrix} 661 & 60 \\ 331.2 & 30 \end{bmatrix} \\
 |D_a| &= 242.7(1200 - 900) - 20(13220 - 9936) + 12(19830 - 19872) \\
 |D_a| &= 242.7(300) - 20(3284) + 12(-42) \\
 |D_a| &= 72810 - 65680 - 504 \\
 |D_a| &= 6626
 \end{aligned}$$

$$\begin{bmatrix} 9 & 242.7 & 12 \\ 20 & 661 & 30 \\ 12 & 331.2 & 20 \end{bmatrix}$$

$$\begin{aligned}
 |D_b| &= 9 \begin{bmatrix} 661 & 30 \\ 331.2 & 20 \end{bmatrix} - 242.7 \begin{bmatrix} 20 & 30 \\ 12 & 20 \end{bmatrix} + 12 \begin{bmatrix} 20 & 661 \\ 12 & 331.2 \end{bmatrix} \\
 |D_b| &= 9(13220 - 9936) - 242.7(400 - 360) + 12(6624 - 7932) \\
 |D_b| &= 9(3284) - 242.7(40) + 12(-1308) \\
 |D_b| &= 29556 - 9708 - 15696 \\
 |D_b| &= 4152
 \end{aligned}$$

$$\begin{bmatrix} 9 & 20 & 242.7 \\ 20 & 60 & 661 \\ 12 & 30 & 331.2 \end{bmatrix}$$

$$|D_c| = 9 \begin{bmatrix} 60 & 661 \\ 30 & 331.2 \end{bmatrix} - 20 \begin{bmatrix} 20 & 661 \\ 12 & 331.2 \end{bmatrix} + 242.7 \begin{bmatrix} 20 & 60 \\ 12 & 30 \end{bmatrix}$$

$$|D_c| = 9(39660 - 9936) - 20(6624 - 7932) + 242.7(600 - 720)$$

$$|D_c| = 9(29724) - 20(-1308) + 242.7(-120)$$

$$|D_c| = 267516 + 26160 - 29124$$

$$|D_c| = 264552$$

$$|D_c| = 9(19872 - 19830) - 20(6624 - 7932) + 242.7(600 - 720)$$

$$|D_c| = 9(42) - 20(-1308) + 242.7(-120)$$

$$|D_c| = 378 + 26160 - 29124$$

$$|D_c| = -2586$$

$$a = \frac{|D_a|}{|D|} = \frac{6626}{460} = 14.4043$$

$$b = \frac{|D_b|}{|D|} = \frac{4152}{460} = 9.0261$$

$$c = \frac{|D_c|}{|D|} = \frac{-2586}{460} = -5.6217$$

Equation

$$\hat{z}(y) = a + bx + cy$$

$$\hat{z} = 14.4043 + 9.0261x - 5.6217y$$