

## ASSIGNMENT 2

Maximum Likelihood equation

$$\ln p(t|x, w, \beta) = + \frac{N}{2} \ln \beta - \frac{\beta}{2} \sum_{n=1}^N (y(x_n, w) - t_n)^2 - \frac{N}{2} \ln 2\pi \dots \textcircled{1}$$

Differentiating  $\textcircled{1}$  w.r.t to  $w$ , we get :-

$$-\frac{\beta}{2} \times \sum_{n=1}^N (y(x_n, w) - t_n) \times \frac{d}{dw} y(x_n, w) = 0$$

$$\Rightarrow \sum_{n=1}^N (y(x_n, w) - t_n) = 0 \quad \text{or} \quad \frac{d}{dw} y(x_n, w) = 0$$

$$\text{Let } \boxed{y = w_1 x + w_0} \therefore w_1 x + w_0 - y = 0 \dots \textcircled{2}$$

Given dataset :-  $(1, 1.2), (2, 1.9), (3, 3.2)$

$$\Rightarrow \sum_{n=1}^3 (w_1 x + w_0 - t_n) = 0$$

$$\Rightarrow (w_1 \times 1 + w_0 - 1.2) + (w_1 \times 2 + w_0 - 1.9) + (w_1 \times 3 + w_0 - 3.2) = 0$$

$$\Rightarrow 6w_1 + 3w_0 - (1.2 + 1.9 + 3.2) = 0$$

$$6w_1 + 3w_0 = 6.3$$

$$\boxed{2w_1 + w_0 = 2.1} \dots \textcircled{3}$$

$$\rightarrow \boxed{w_0 = 2.1 - 2w_1} \dots \textcircled{5}$$

Differentiating  $\textcircled{1}$  w.r.t to  $\beta$ , we get.

$$\frac{N}{\beta} = \frac{1}{2} \sum_{n=1}^N (y(x_n, w) - t_n)^2$$

$$\Rightarrow \boxed{\frac{1}{\beta} = \frac{1}{N} \sum_{n=1}^N (y(x_n, w) - t_n)^2} \dots \textcircled{4}$$

Given :-  $\beta = 1, N = 3$

$$\Rightarrow 3 = \sum_{n=1}^3 (w_1 x + w_0 - t_n)^2$$

$$\Rightarrow 3 = \sum_{n=1}^3 (w_1 x + 2.1 - 2w_1 - t_n)^2$$

$$\Rightarrow 3 = ((w_1 + 2 \cdot 1 - 2w_1 - 1.2)^2 + (2w_1 + 2 \cdot 1 - 2w_1 - 1.9)^2 + (3w_1 + 2 \cdot 1 - 2w_1 - 3.2)^2)$$

$$\Rightarrow 3 = ((0.9 - w_1)^2 + (0.2)^2 + (w_1 - 1.1)^2)$$

$$\Rightarrow 3 = 0.81 + \cancel{w_1^2} - 1.8w_1 + 0.04 + w_1^2 - 2.2w_1 + 1.21 - 2.2w_1$$

$$\Rightarrow 3 = 2w_1^2 - 4w_1 + 2.06$$

$$\Rightarrow 2w_1^2 - 4w_1 - 0.94 = 0$$

$$w_1^2 - 2w_1 - 0.47 = 0$$

$$w_1 = \frac{2 \pm \sqrt{4 + 4 \times 0.47}}{2}$$

$$w_1 = \frac{2 \pm \sqrt{5.88}}{2}$$

$$w_1 = \frac{2 \pm 2.42}{2}$$

$$\therefore w_1 = 2.21 \text{ or } -0.21$$

For  $w_1 = 2.21$

$$y = 2.21x - 2.32$$

for,  $x_1 = 1, y_1 = -0.11$

$x_2 = 2, y_2 = 2.1$

$x_3 = 3, y_3 = 4.31$

Calculating difference

$$(-0.11 - 1.2)^2 + (2.1 - 1.9)^2 + (4.31 - 3.2)^2$$

$$\Rightarrow 1.7161 + 0.04 + 1.2321$$

$$\Rightarrow \boxed{2.9882}$$

For  $w_1 = -0.21$

$$y = -0.21x + 2.63$$

for,  $x_1 = 1, y_1 = 2.42$

$x_2 = 2, y_2 = 2.21$

$x_3 = 3, y_3 = 2.00$

Calculating difference

$$(2.42 - 1.2)^2 + (2.21 - 1.9)^2 + (2.00 - 3.2)^2$$

$$\Rightarrow 1.48 + 0.096 + 1.44$$

$$\Rightarrow \boxed{3.025}$$

So, our required equation is:  $y = 2.21x - 2.32$